A New Dynamic Random Fuzzy DEA Model to Predict Performance of Decision Making Units

Ali Yaghoubi[a,]*, Maghsoud Amiri[b], Azamdokht Safi Samghabadi[c]

[a] *Assistant Professor, Department of Industrial Engineering, Faculty of Engineering, Payam-e-Noor University, Tehran, Iran*

[b] *Professor, Department of Industrial Management, Allameh Tabataba'i University, Tehran, Iran*

[c] *Assistant Professor, Department of Industrial Engineering, Faculty of Engineering, Payam-e-Noor University, Tehran, Iran*

Received 2 October, 2014; Revised 3 June, 2015; Accepted 26 June, 2015

Abstract

Data envelopment analysis (DEA) is a methodology for measuring the relative efficiency of decision making units (DMUs) which consume the same types of inputs and producing the same types of outputs. Assuming that future planning and predicting the efficiency are very important for DMUs, this paper first presents a new dynamic random fuzzy DEA model (DRF-DEA) with common weights (using multi objective DEA approach) to predict the efficiency of DMUs under mean chance constraints and expected values of the objective functions. In the initial proposed DRF-DEA model, the inputs and outputs are assumed to be characterized by random triangular fuzzy variables with normal distribution, in which data are changing sequentially. Under this assumption, the solution process is very complex. So we then convert the initial proposed DRF-DEA model to its equivalent multi-objective stochastic programming, in which the constraints contain the standard normal distribution functions, and the objective functions are the expected values of functions of normal random variables. In order to improve in computational time, we then convert the equivalent multi-objective stochastic model to one objective stochastic model with using fuzzy multiple objectives programming approach. To solve it, we design a new hybrid algorithm by integrating Monte Carlo (MC) simulation and Genetic Algorithm (GA). Since no benchmark is available in the literature, one practical example will be presented. The computational results show that our hybrid algorithm outperforms the hybrid GA algorithm which was proposed by Qin and Liu (2010) in terms of runtime and solution quality.

Keywords: Stochastic Data envelopment analysis; Dynamic programming; random fuzzy variable; Monte Carlo simulation; Genetic algorithm.

1. Introduction

Data envelopment analysis is an important managerial tool for evaluating and improving the performance of decision making units in systems. Data envelopment analysis (DEA) which was initially proposed by Charnes, Cooper and Rhodes (1978) has been widely applied to evaluate the relative efficiency of a set of DMUs based on multiple criteria. Since the first DEA model (CCR), it has been surveyed by researchers very quickly to various areas. The advantage of this technique is that it does not require the explicit specification of functional relations between the multiple inputs and outputs or either a priori weights. However, when we measure the efficiency of DMUs, the data in traditional DEA models are often limit to crisp data and the efficiency scores of DMUs are very sensitive to data variations and don't allow the stochastic variations in the data, such as data entry errors and measurement errors. With considering stochastic variations in outputs and inputs, a DMU which is efficient relative to other DMUs may convert to be inefficient. Cooper, Huang and Li (1996) is the first one who developed a stochastic DEA model with chance constrained programming reflecting theories of behavior in social psychology. Usually we obtain fuzzy data from DMUs, because various experts may have various ideas, especially one expert may have different ideas during different times. Sengupta (1992) is the first one who considered fuzziness both in objective and constraints and analyzed the fuzzy DEA model. Entani, Maeda and Tanaka (2002) changed fuzzy input and output data in to intervals by using α-level sets, and suggested two different interval DEA models. Since in real world problems, decision makers may encounter a hybrid uncertain environment where randomness and fuzziness coexist in a decision system, they represent the inputs and outputs in these systems by random fuzzy variables to characterize the hybrid uncertainty. Recently, the random fuzzy variable (Kwakernaak, 1978), possibility theory (Z.Q. Liu and Y.K. Liu, 2010), credibility theory and

* Corresponding author E-mail: phd_yaghoubi@yahoo.com

mean chance theory (Liu, 2004) have been proposed to treat fuzzy phenomena existing in real-life problems. Liu (2005) presented a new class of random fuzzy minimum-risk problems (RFMRPs) via the mean chance, and applied the RFMRP to the capacitated location-allocation problem with random fuzzy demands (Y. Liu and B. Liu, 2005). Wang and Watada (2009) discussed the analytical properties of mean chance distribution functions and critical value functions of random fuzzy variables and obtained several useful continuity theorems. Razavi, Amoozad and Zavadskas (2013) developed a new fuzzy data envelopment analysis approach based on parametric programming. The basic idea of the proposed method is to transform the original DEA model to an equivalent linear parametric programming model, applying the notion of α-cuts. Azadi, Jafarian and Farzipoor (2014) proposed a new fuzzy DEA model for evaluation the efficiency and effectiveness of suppliers in sustainable supply chain management context. They developed an integrated DEA enhanced Russell measure (ERM) model in fuzzy context to select the best sustainable suppliers. Bray et al. (2015) proposed a Fuzzy theory-based DEA model to assess efficiency of transportation systems and services considering uncertainty in data, as well as in the evaluation result. They focused on the "delay time" that is an uncertain input data. In the current literature, all researchers focused on evaluating current efficiency of DMUs with using DEA models in fuzzy or stochastic environments, without considering the need of predicting performance for future planning of DMUs. So, this paper attempts to establish a new fuzzy stochastic DEA model to predict the efficiency of DMUs.

The rest of this paper is organized as follows. Section 2 reviews the previous studies on DEA models with common weights in static and dynamic environments. Section 3 reviews some basic concepts of fuzzy theory such as credibility approach and mean chance of fuzzy random event, which are needful in the next section. In order to predict efficiency of DMUs, firstly section 4 presents a new random fuzzy DEA model with common weights in dynamic environment (DRF-DEA), in which the inputs and outputs are assumed to be characterized by random triangular fuzzy variables with normal distribution and data are changing sequentially. Under these assumptions, in Section 5, we then convert the mean chance constraints and expected values of objective functions of DRF-DEA model in to their equivalents representations with applying the established formulas in section3, then we convert the equivalent multi objective stochastic programming to one objective stochastic model with using fuzzy multiple objectives linear programming approach to improve in computational time. In section 6 we integrate MC simulation and GA to design a new hybrid algorithm (MC-GA) for solving the proposed equivalent one objective stochastic programming. Section 7 provides a practical example to illustrate the modeling idea and the effectiveness of our solution method. We also compare the results of our designed algorithm (MC-GA) with the hybrid GA algorithm which was proposed by Qin and Liu (2010) in this section. Finally, Section 8 draws our conclusions.

2. Literature Review

Nowadays, common weights set approach in DEA based on multi-objective programming are an attractive approach for evaluation quality and quantity aspects of performance analysis because there are some weaknesses in classical DEA models such as one-dimensional performance measuring and also, when DEA is employed, it is important to consider the decision makers preference over the adjustment of different outputs and inputs. Golany (1988) is the first one who designed an interactive multi-objective linear programming (MOLP) procedure to select the preferred output targets given the input levels and attempted to present DEA with common weights model to improve the discriminating power of the classical DEA models. DEA models which incorporated the preferences over output–input improvements to attain the corresponding preferred output–input target have been established by Thanassoulis and Dyson (1992).

Many studies have been accomplished about DEA models with common weights set in static environment. Sengupta (1995) is the first one who presents stochastic and dynamic extensions in classic multi-objective DEA models and two different types of inputs (quasi-fixed inputs and variable inputs) incorporated into a framework of dynamic DEA by Nemoto and Goto (2003). This problem has been surveyed by researchers. These studies find that their research has not documented how to measure returns to scale (RTS) in the analytical framework of the dynamic multi-objective DEA. Sueyoshi and Sekitani (2005) presented a new type of multi-objective DEA efficiency measure within a framework of dynamic DEA. They extended the dynamic DEA of Nemoto and Goto (2003) in a manner that the concept of returns to scale (RTS) is incorporated into the dynamic DEA. Chen (2005) presented a non-radial set of DEA preference structure models which incorporate a user's preference over alternative paths to the targets. The output (input) change rates in DEA preference structure models can be greater than, less than or equal to one. He showed that Golany's MOLP based DEA target setting or forecasting approach can be improved and it is able to consider multi-criteria multiple constraints in the DEA context. A new dynamic DEA model with common weights which produced aggregate efficiency of the total planning horizon has been developed by Teimoori (2006). Lozano and Villa (2007) presented two common weights DEA approaches for target setting. One of them is an interactive procedure that allows for a progressive articulation of the preferences of the decision maker. The other is a lexicographic approach that solves a sequence of models which try to improve, in a weighted manner, selected outputs and inputs. Yang, Wong, Xu and Stewart

(2008) presented an equivalence model between MOLP and dynamic DEA models and present how a dynamic DEA problem can be solved interactively without any prior judgments by transforming it into an MOLP formulation. Fukuyama (2012) presented a dynamic network DEA model with common weights to measure productivity change for 269 Japanese Shinkin banks during 2002 to 2009. Omrani (2013) introduced a robust optimization approach to find common weights in DEA with uncertainty in data. Ramezani and Khodabakhshi (2013) proposed model to ranking DMUs with using common weights set in DEA in dynamic environment. The aim of this study is to show the criteria used by Wong. Wang, Lu and Liu (2014) proposed a new multi-objective two stage fuzzy DEA model in dynamic environment for evaluating the performance of US bank holding companies. This model provides a set of common weights for comparing performance and increases the discriminating power. Kawaguchi, Tone and Tsutsui (2014) estimated the efficiency of Japanese hospitals using a dynamic network DEA model with common weights. Tone and Tsutsui (2014) proposed a new slacks-based measure approach in dynamic DEA with network structure and common weights set. It can use for input oriented model and analysis efficiency variations in network. Tavana et al. (2015) presented a common set of weights (CSW) model for ranking the DMUs with the stochastic data and the ideal point concept. The proposed method minimizes the distance between the evaluated DMUs and the ideal DMU. Hatami-Marbini et al. (2015) proposed a DEA model for centrally imposed resource or output reduction. They used a common set of weights method for controlling the weight flexibility and reducing the computational complexities.

The crisp outputs and inputs in traditional DEA models become random fuzzy variables in fuzzy stochastic environment, and modeling with such data is meaningless directly because the meanings of the constraints and the objective are not clear at all. In fact, we have faced such situation in fuzzy and stochastic environment, in which we deal with the fuzzy data and random data with credibility and probability, respectively, to obtain a meaningful model. In fuzzy stochastic programming, the mean chance plays the same role as credibility in fuzzy environments and probability role in stochastic environments (Y. Liu and B. Liu, 2005). So, in order to obtain a meaningful model in fuzzy stochastic environments and predict the efficiencies of DMUs, we employ the expected value to objective functions and the mean chance to constraints with given confidence levels to propose a new random fuzzy DEA model with common weights in dynamic environment. In general, the mean chance functions in the constraints are difficult to compute, so we can convert the mean chance constraints to their equivalent stochastic representations according to the formula for the mean chance function (in section 3). To compute the objective functions, under the assumption that the inputs and outputs are random triangular fuzzy vectors, we present the equivalent stochastic representation of the objective functions. As a consequence, the initial proposed model can be converted to its equivalent stochastic programming one. Since the objective functions are the mathematical expectation of functions of the normal random variables, we cannot solve it via the conventional optimization algorithm. To overcome the difficulty, we combine MC simulation technique and genetic algorithm to solve it.

3. Preliminaries

3.1. Credibility approach

Let ξ be a fuzzy variable with a possibility distribution function μ. The credibility of a fuzzy event {ξ ≥r} for $r \in R$ is defined as (Qin and Liu, 2010):

$$Cr\{\xi \geq r\} = \frac{1}{2}(1 + \sup_{t \geq r} \mu(t) - \sup_{t < r} \mu(t)) \tag{1}$$

Also “Cr” has the following property:

$$\mathrm{Cr}\{\xi \geq r\} + \mathrm{Cr}\{\xi < r\} = 1 \tag{2}$$

And the expected value of random fuzzy variable (ξ) is defined as:

$$E[\xi] = \int_0^{\infty} Cr\{\xi \geq r\} dr - \int_{-\infty}^{0} Cr\{\xi \leq r\} dr \tag{3}$$

Let ξ be an n-dimensional random fuzzy vector, and B a Borel subset of R. The mean chance of a random fuzzy event {ξ ∈B} is defined as:

$$Ch\{\xi \in \beta\} = \int Cr\{\xi \in \beta\} P\{\xi \in \beta\} \tag{4}$$

And the expected value of random fuzzy variable (ξ) is defined as (Qin and Liu, 2010):

$$E[\xi] = \int_0^{\infty} Ch\{\xi \geq r\} dr - \int_{-\infty}^{0} Ch\{\xi \leq r\} dr \tag{5}$$

3.2. Mean chance distributions for random triangular fuzzy variables

This section establishes some useful formulas for the mean chance functions of random triangular fuzzy variables, which will be used in the next section.

Theorem 3.2.1 Let $\xi = (X - a, X, X + b)$ be a continuous random triangular fuzzy variable, in which X is a random variable, and a,b being positive numbers. If $X \sim N(\mu, \sigma^2)$ then we have:

$$Ch\{\xi \ge r\} = \frac{\sigma(b-a)}{2ab\sqrt{2\pi}}\exp(-\frac{(r-\mu)^2}{2\sigma^2}) + \frac{\sigma}{2b\sqrt{2\pi}}\exp(-\frac{(b+\mu-r)^2}{2\sigma^2}) - \frac{\sigma}{2a\sqrt{2\pi}}\exp(-\frac{(r+a-\mu)^2}{2\sigma^2})$$
$$+\frac{\mu-a-r}{2a}\varphi(\frac{r+a-\mu}{\sigma}) - \frac{(\mu-r)(b-a)}{2ab}\varphi(\frac{r-\mu}{\sigma}) + \frac{\mu+b-r}{2b}\varphi(\frac{b+\mu-r}{\sigma}) + \frac{r+b-\mu}{2b} \quad (6)$$

where $\phi(0)$ is the probability distribution of standard normal distribution function (Qin and Liu, 2010).

Theorem 3.2.2 Let $\xi_i = (X_i - a_i, X_i, X_i + b_i)$ be mutually independent triangular fuzzy variables (i=1,2,…,n). If $X_i \sim N(\mu_i, \sigma_i^2)$ with $a_i, b_i \succ 0$, then we have:

$$Ch\left\{\sum_{i=1}^n x_i\xi_i \ge r\right\} = \frac{\sqrt{\sum_{i=1}^n x_i^2\sigma_i^2}\sum_{i=1}^n x_i(b_i-a_i)}{2\sqrt{2\pi}\sum_{i=1}^n x_ia_i\sum_{i=1}^n x_ib_i}\exp(-\frac{(r-\sum_{i=1}^n x_i\mu_i)^2}{2\sum_{i=1}^n x_i^2\sigma_i^2}) + \frac{\sqrt{\sum_{i=1}^n x_i^2\sigma_i^2}}{2\sqrt{2\pi}\sum_{i=1}^n x_ib_i}\exp(-\frac{(\sum_{i=1}^n x_i(b_i+\mu_i)-r)^2}{2\sum_{i=1}^n x_i^2\sigma_i^2}) -$$
$$\frac{\sqrt{\sum_{i=1}^n x_i^2\sigma_i^2}}{2\sqrt{2\pi}\sum_{i=1}^n x_ia_i}\exp(-\frac{(r+\sum_{i=1}^n x_i(a_i-\mu_i))^2}{2\sum_{i=1}^n x_i^2\sigma_i^2}) - \frac{(\sum_{i=1}^n x_i\mu_i - r)\sum_{i=1}^n x_i(b_i-a_i)}{2\sum_{i=1}^n x_ia_i\sum_{i=1}^n x_ib_i}\phi(\frac{r-\sum_{i=1}^n x_i\mu_i}{\sqrt{\sum_{i=1}^n x_i^2\sigma_i^2}}) + \frac{\sum_{i=1}^n x_i(b_i+\mu_i)-r}{2\sum_{i=1}^n x_ib_i}\phi(\frac{\sum_{i=1}^n x_i(b_i+\mu_i)-r}{\sqrt{\sum_{i=1}^n x_i^2\sigma_i^2}})$$
$$+\frac{\sum_{i=1}^n x_i(\mu_i-a_i)-r}{2\sum_{i=1}^n x_ia_i}\phi(\frac{r+\sum_{i=1}^n x_i(a_i-\mu_i)}{\sqrt{\sum_{i=1}^n x_i^2\sigma_i^2}}) + \frac{r+\sum_{i=1}^n x_i(b_i-\mu_i)}{2\sum_{i=1}^n x_ib_i} \quad (7)$$

where x_i, i=1,2,...,n are nonnegative real numbers and at least one of them is nonzero (Qin and Liu, 2010).

Theorem 3.2.3. Let $\xi = (X-a, X, X+b)$ and $\eta = (Y-c, Y, Y+d)$ be two mutually independent random triangular fuzzy variables. If $X \sim N(\mu_1, \sigma_1^2)$, $Y \sim N(\mu_2, \sigma_2^2)$ and a,b,c,d being positive numbers, then we have (Qin and Liu, 2010):

$$Ch\{x_1\xi - x_2\eta \ge r\} = \frac{\sqrt{x_1^2\sigma_1^2 + x_2^2\sigma_2^2}(x_1(b-a)+x_2(c-d))}{2(x_1a+x_2d)(x_1b+x_2c)\sqrt{2\pi}}\exp(-\frac{(r-(x_1\mu_1-x_2\mu_2))^2}{2(x_1^2\sigma_1^2+x_2^2\sigma_2^2)})$$
$$+\frac{\sqrt{x_1^2\sigma_1^2+x_2^2\sigma_2^2}}{2\sqrt{2\pi}(x_1b+x_2c)}\exp(-\frac{(x_1(b+\mu_1)+x_2(c-\mu_2)-r)^2}{2(x_1^2\sigma_1^2+x_2^2\sigma_2^2)}) - \frac{\sqrt{x_1^2\sigma_1^2+x_2^2\sigma_2^2}}{2\sqrt{2\pi}(x_1a+x_2d)}\exp(-\frac{(r+x_1(a-\mu_1)+x_2(d+\mu_2))^2}{2(x_1^2\sigma_1^2+x_2^2\sigma_2^2)}) \quad (8)$$
$$+\frac{x_1(\mu_1-a)-x_2(\mu_2+d)-r}{2(x_1a+x_2d)}\phi(\frac{r+x_1(a-\mu_1)+x_2(d+\mu_2)}{\sqrt{x_1^2\sigma_1^2+x_2^2\sigma_2^2}}) + \frac{((x_1\mu_1-x_2\mu_2)-r)(x_1(b-a)+x_2(c-d))}{2(x_1a+x_2d)(x_1b+x_2c)}\phi(\frac{r-(x_1\mu_1-x_2\mu_2)}{\sqrt{x_1^2\sigma_1^2+x_2^2\sigma_2^2}})$$
$$+\frac{x_1(b+\mu_1)-x_2(\mu_2-c)-r}{2(x_1b+x_2c)}\phi(\frac{x_1(b+\mu_1)+x_2(c-\mu_2)-r}{\sqrt{x_1^2\sigma_1^2+x_2^2\sigma_2^2}}) + \frac{r+x_1(b-\mu_1)+x_2(c+\mu_2)}{2(x_1b+x_2c)}$$

where x_1 and x_2 are nonnegative real numbers and at least one of them is nonzero.

Theorem 3.2.4. Suppose $\xi = (X-a, X, X+b)$ and $\eta = (Y-c, Y, Y+d)$ are two mutually independent triangular fuzzy variables, in which X,Y ∈R and a,b,c,d being positive numbers, then we have (Qin and Liu, 2010):

$$E\left[\frac{\eta}{\xi}\right] = -\frac{c}{2b} - \frac{d}{2a} + \frac{1}{2b}(Y + \frac{c}{b}X)\ln(1+\frac{b}{X}) +$$
$$\frac{1}{2a}(Y + \frac{d}{a}X)\ln\frac{X}{X-a} \quad (9)$$

4. Dynamic random fuzzy DEA model (DRF-DEA) formulation

The traditional DEA model (CCR), which was proposed by Charnes, Cooper and Rhodes (1978) is built as:

$$Max: \quad Z_0 = \frac{\sum_{r=1}^{s} u_r y_{r0}}{\sum_{i=1}^{m} v_i x_{i0}}$$

$$st: \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1 \qquad j=1,2,...,n$$

$$u_r, v_i \ge \varepsilon \qquad i=1,...,m \quad r=1,...,s \tag{10}$$

where x_{ij} and y_{rj} represent the *i*th input and *r*th output of DMU_j, respectively. u_r and v_i are the weights of the rth output and ith input, respectively. Finding a multipliers (u_r,v_i) for i=1,..,m and r=1,..s is the basic opinion of the efficiency measurement of the CCR model, so that the efficiency ratio in objective function can be maximized for DMU0. Generally, CCR (10) has been applied in static environment in which data supposed to be fixed during evaluation period. But in dynamic environment, it is assumed that there are "n" DMUs and their activities are examined in T periods (t =1,2,…, T) in which data are changing sequentially. In the *t*th period, each DMUj (j=1,..,n) uses two different groups of inputs: K^{t-1} (l dimensional vector of quasi-fixed inputs) and X^t(m dimensional vector of input variables) to produce two different groups of outputs: Y^t (r dimensional vector of output variables) and K^t (l dimensional vector of quasi-fixed outputs) (Nemoto and Goto, 2003). As shown in Fig. 1, the horizontal axis denotes the order of periods and the vertical axis indicates the order of DMUs. In this figure, the quasi-fixed outputs vector (K^t) in the *t*th period is used as the quasi-fixed or feedback inputs vector (link data) at the next (t + 1) period. As for variables mentioned above, the dynamic CCR model is built as following:

$$Max: \quad Z_0^t = \frac{\sum_{r=1}^{s} u_r^t y_{ro}^t + \sum_{l=1}^{L} \rho_l^t k_{l0}^t}{\sum_{i=1}^{m} v_i^t x_{i0}^t + \sum_{l=1}^{L} \beta_l^{t-1} k_{l0}^{t-1}}$$

$$st: \tag{11}$$

$$\frac{\sum_{r=1}^{s} u_r^t y_{rj}^t + \sum_{l=1}^{L} \rho_l^t k_{lj}^t}{\sum_{i=1}^{m} v_i^t x_{ij}^t + \sum_{l=1}^{L} \beta_l^{t-1} k_{lj}^{t-1}} \le 1 \qquad j=1,2,...,n$$

$$u_r^t, v_i^t, \rho_l^t, \beta_l^{t-1} \ge \varepsilon$$

$$i=1,..,m \quad r=1,..,s \quad l=1,..,L \quad t=1,..,T$$

Where ρ_l^t and β_l^{t-1} are the weights of *l*th quasi–fix outputs and quasi–fix inputs at period t (t=1,..,T), respectively. The dynamic CCR model (11) is usually applied to evaluate the relative efficiency of DMUs with crisp outputs and inputs. However, in real world problems, the data are often derived by statistic or given by experts according to their experience, so randomness and fuzziness may exist simultaneously in these data. In many cases, we can only obtain the possibility distributions of the inputs and outputs. Thus in this paper, we assume that the inputs and outputs are random triangular fuzzy variables with normal distributions, following as:

$$X_j^t = \begin{pmatrix} (x_{1j}^t - a_{1j}^t, x_{1j}^t, x_{1j}^t + b_{1j}^t) \\ (x_{2j}^t - a_{2j}^t, x_{2j}^t, x_{2j}^t + b_{2j}^t) \\ \vdots \\ (x_{mj}^t - a_{mj}^t, x_{mj}^t, x_{mj}^t + b_{mj}^t) \end{pmatrix} \qquad Y_j^t = \begin{pmatrix} (y_{1j}^t - c_{1j}^t, y_{1j}^t, y_{1j}^t + d_{1j}^t) \\ (y_{2j}^t - c_{2j}^t, y_{2j}^t, y_{2j}^t + d_{2j}^t) \\ \vdots \\ (y_{sj}^t - c_{sj}^t, y_{sj}^t, y_{sj}^t + d_{sj}^t) \end{pmatrix}$$

$$K_j^t = \begin{pmatrix} (k_{1j}^t - e_{1j}^t, k_{1j}^t, k_{1j}^t + f_{1j}^t) \\ (k_{2j}^t - e_{2j}^t, k_{2j}^t, k_{2j}^t + f_{2j}^t) \\ \vdots \\ (k_{lj}^t - e_{lj}^t, k_{lj}^t, k_{lj}^t + f_{lj}^t) \end{pmatrix} \qquad K_j^{t-1} = \begin{pmatrix} (k_{1j}^{t-1} - e_{1j}^{t-1}, k_{1j}^{t-1}, k_{1j}^{t-1} + f_{1j}^{t-1}) \\ (k_{2j}^{t-1} - e_{2j}^{t-1}, k_{2j}^{t-1}, k_{2j}^{t-1} + f_{2j}^{t-1}) \\ \vdots \\ (k_{lj}^{t-1} - e_{lj}^{t-1}, k_{lj}^{t-1}, k_{lj}^{t-1} + f_{lj}^{t-1}) \end{pmatrix} \tag{12}$$

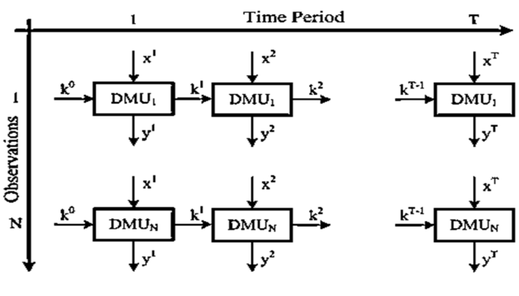

Fig. 1. Execution of the DEA model in dynamic framework

where X_j^t , Y_j^t are the column vectors of random fuzzy inputs and outputs of DMUj (j=1,..,n) in period t, respectively. K_j^t is the quasi–fix random fuzzy outputs column vector of DMUj at period t and the quasi–fix random fuzzy inputs column vector of DMUj in period t+1 (link data). K_j^{t-1} is the quasi–fix random fuzzy inputs column vector of DMUj at period t and the quasi–fix random fuzzy outputs column vector of DMUj in period t-1 (link data). Also suppose $X_{ij}^t \sim N(\mu_{ij}^t, \sigma_{ij}^{2^t})$, $Y_{rj}^t \sim N(\overline{\mu}_{rj}^t, \overline{\sigma}_{rj}^{2^t})$, $K_{lj}^t \sim N(\overline{\overline{\mu}}_{lj}^t, \overline{\overline{\sigma}}_{lj}^{2^t})$ and $K_{lj}^{t-1} \sim N(\overline{\overline{\mu}}_{lj}^{t-1}, \overline{\overline{\sigma}}_{lj}^{2^{t-1}})$ with $a_{ij}^t, b_{ij}^t, c_{rj}^t, d_{rj}^t, e_{lj}^t, f_{lj}^t, e_{lj}^{t-1}, f_{lj}^{t-1}$ being positive numbers for each i (i=1,..,m), r (r=1,..,s) and l (l=1,..,L) which are predicted by decision maker for the next financial period to predict the efficiency of each DMU. In this case, each objective function of model (11) is also a random fuzzy variable, but the meaning of the model (11) is not clear. If we consider the efficiency ratio for all DMUs, we can then establish the multiple objectives programming which uses common weights set to efficiency measurement (Lozano and Villa, 2007). In this paper we will use the multi-objective decision making (MODM) methods to present a new multi-objective DEA model with common weights in dynamic environment in which data are changing sequentially. To build a meaningful model, we utilize the expectation of a random fuzzy variable in each objective function and the mean chance in the constraints to formulate the initial proposed DRF-DEA model with common weights:

$$
\begin{aligned}
&Max: \quad Z_1^t = E\left[\frac{\sum_{r=1}^{s} u_r^t y_{r1}^t + \sum_{l=1}^{L} \rho_l^t k_{l1}^t}{\sum_{i=1}^{m} v_i^t x_{i1}^t + \sum_{l=1}^{L} \beta_l^{t-1} k_{l1}^{t-1}}\right] \\
&\vdots \\
&Max: \quad Z_n^t = E\left[\frac{\sum_{r=1}^{s} u_r^t y_{rn}^t + \sum_{l=1}^{L} \rho_l^t k_{ln}^t}{\sum_{i=1}^{m} v_i^t x_{in}^t + \sum_{l=1}^{L} \beta_l^{t-1} k_{ln}^{t-1}}\right] \qquad (13) \\
&st: \ Ch\left\{(\sum_{i=1}^{m} v_i^t x_{ij}^t + \sum_{l=1}^{L} \beta_l^{t-1} k_{lj}^{t-1}) - (\sum_{r=1}^{s} u_r^t y_{rj}^t + \sum_{l=1}^{L} \rho_l^t k_{lj}^t) \geq 0\right\} \geq 1-\alpha_j^t \qquad j=1,2,...,n \\
&\quad u_r^t, v_i^t, \rho_l^t, \beta_l^{t-1} \geq \varepsilon \qquad i=1,..,m \quad r=1,..,s \quad l=1,..,L \quad t=1,..,T
\end{aligned}
$$

where α_j^t is considered as a risk criterion of failing to satisfy the jth constraint in period t $\left(\alpha_j^t \in [0,1)\right)$. This model contains "n" objective functions and our purpose is to seek a common weights set $(u_r^t, v_i^t, \rho_l^t, \beta_l^{t-1})$ with the maximum value of each objective function in period t (t=1,..,T), while the fuzzy events $\left\{(\sum_{i=1}^{m} v_i^t x_{ij}^t + \sum_{l=1}^{L} \beta_l^{t-1} k_{lj}^{t-1}) - (\sum_{r=1}^{s} u_r^t y_{rj}^t + \sum_{l=1}^{L} \rho_l^t k_{lj}^t) \geq 0\right\}$ are satisfied at least with confidence level $(1-\alpha_j^t)$ for j=1,..,n. In traditional CCR model (10), the value of objective function was used to express the efficiency of DMU0. DMU0 is efficient if and only if the optimal value is equal to 1. In proposed DRF-DEA model (13), we extend the efficiency and adopt α-expected efficient value to express the efficiency of DMUs. The optimal value of common weights set is referred to as the α-expected efficient value.

5. Equivalent Stochastic Programming of DRF-DEA Model

5.1. Equivalent stochastic representation of the mean chance constraints

The constraints of initial proposed DRF-DEA model (13) contain the mean chance of random fuzzy events which are difficult to compute, so we need to compute their values during the solution process. In the following, we discuss their equivalent stochastic representation according to the formulas for the mean chance established by Theorems (3.2.2) and (3.2.3) Suppose X_j^t , Y_j^t, K_j^t and K_j^{t-1} are the fix (quasi–fix) random fuzzy inputs (outputs) column vectors of DMU_j in period t which were defined as (12), so the jth constraint of initial proposed DRF-DEA model can be transformed to the following equivalent stochastic one:

$$
\begin{aligned}
&g_j^t(u_r^t, v_i^t, \rho_l^t, \beta_l^{t-1}) = Ch\left\{(\sum_{i=1}^{m} v_i^t x_{ij}^t + \sum_{l=1}^{L} \beta_l^{t-1} k_{lj}^{t-1}) - (\sum_{r=1}^{s} u_r^t y_{rj}^t + \sum_{l=1}^{L} \rho_l^t k_{lj}^t) \ge 0\right\} \\
&= \frac{\sqrt{\sum_{i=1}^{m} v_i^{2^t} \sigma_{ij}^{2^t} + \sum_{l=1}^{L} \beta_l^{2^{t-1}} \overline{\overline{\sigma}}_{lj}^{2^{t-1}} + \sum_{r=1}^{s} u_r^{2^t} \overline{\sigma}_{rj}^{2^t} + \sum_{l=1}^{L} \rho_l^{2^t} \overline{\overline{\sigma}}_{lj}^{2^t}} \, (\sum_{i=1}^{m} v_i^t \ (b_{ij}^t - a_{ij}^t)^2 + \sum_{l=1}^{L} \beta_l^{t-1} \ (f_{lj}^{t-1} - e_{lj}^{t-1})}{2\sqrt{2\pi}\,(\sum_{i=1}^{m} v_i^t \ a_{ij}^t + \sum_{l=1}^{L} \beta_l^{t-1} \ e_{lj}^{t-1} + \sum_{r=1}^{s} u_r^t \ d_{rj}^t + \sum_{l=1}^{L} \rho_l^t \ f_{lj}^t)(\sum_{i=1}^{m} v_i^t \ b_{ij}^t + \sum_{l=1}^{L} \beta_l^{t-1} \ f_{lj}^{t-1}} \cdots \\
&\cdots \frac{-\sum_{r=1}^{s} u_r^t \ (d_{rj}^t - c_{rj}^t) - \sum_{l=1}^{L} \rho_l^t \ (f_{lj}^t - e_{lj}^t))}{+\sum_{r=1}^{s} u_r^t \ c_{rj}^t + \sum_{l=1}^{L} \rho_l^t \ e_{lj}^t)} * \exp(-\frac{(\sum_{i=1}^{m} v_i^t \ \mu_{ij}^t + \sum_{l=1}^{L} \beta_l^{t-1} \ \overline{\overline{\mu}}_{lj}^{t-1} - \sum_{r=1}^{s} u_r^t \ \overline{\mu}_{rj}^t - \sum_{l=1}^{L} \rho_l^t \ \overline{\overline{\mu}}_{lj}^t)^2}{2(\sum_{i=1}^{m} v_i^{2^t} \sigma_{ij}^{2^t} + \sum_{l=1}^{L} \beta_l^{2^{t-1}} \overline{\overline{\sigma}}_{lj}^{2^{t-1}} + \sum_{r=1}^{s} u_r^{2^t} \overline{\sigma}_{rj}^{2^t} + \sum_{l=1}^{L} \rho_l^{2^t} \overline{\overline{\sigma}}_{lj}^{2^t})}) \\
&+ \frac{\sqrt{\sum_{i=1}^{m} v_i^{2^t} \sigma_{ij}^{2^t} + \sum_{l=1}^{L} \beta_l^{2^{t-1}} \overline{\overline{\sigma}}_{lj}^{2^{t-1}} + \sum_{r=1}^{s} u_r^{2^t} \overline{\sigma}_{rj}^{2^t} + \sum_{l=1}^{L} \rho_l^{2^t} \overline{\overline{\sigma}}_{lj}^{2^t}}}{2\sqrt{2\pi}\,(\sum_{i=1}^{m} v_i^t \ b_{ij}^t + \sum_{l=1}^{L} \beta_l^{t-1} \ f_{lj}^{t-1} + \sum_{r=1}^{s} u_r^t \ c_{rj}^t + \sum_{l=1}^{L} \rho_l^t \ e_{lj}^t)} \\
&* \exp(-\frac{(\sum_{i=1}^{m} v_i^t \ (b_{ij}^t + \mu_{ij}^t) + \sum_{l=1}^{L} \beta_l^{t-1} \ (f_{lj}^{t-1} + \overline{\overline{\mu}}_{lj}^{t-1}) + \sum_{r=1}^{s} u_r^t \ (c_{rj}^t - \overline{\mu}_{rj}^t) + \sum_{l=1}^{L} \rho_l^t \ (e_{lj}^t - \overline{\overline{\mu}}_{lj}^t))^2}{2(\sum_{i=1}^{m} v_i^{2^t} \sigma_{ij}^{2^t} + \sum_{l=1}^{L} \beta_l^{2^{t-1}} \overline{\overline{\sigma}}_{lj}^{2^{t-1}} + \sum_{r=1}^{s} u_r^{2^t} \overline{\sigma}_{rj}^{2^t} + \sum_{l=1}^{L} \rho_l^{2^t} \overline{\overline{\sigma}}_{lj}^{2^t})}) - \\
&\frac{\sqrt{\sum_{i=1}^{m} v_i^{2^t} \sigma_{ij}^{2^t} + \sum_{l=1}^{L} \beta_l^{2^{t-1}} \overline{\overline{\sigma}}_{lj}^{2^{t-1}} + \sum_{r=1}^{s} u_r^{2^t} \overline{\sigma}_{rj}^{2^t} + \sum_{l=1}^{L} \rho_l^{2^t} \overline{\overline{\sigma}}_{lj}^{2^t}}}{2\sqrt{2\pi}\,(\sum_{i=1}^{m} v_i^t \ a_{ij}^t + \sum_{l=1}^{L} \beta_l^{t-1} \ e_{lj}^{t-1} + \sum_{r=1}^{s} u_r^t \ d_{rj}^t + \sum_{l=1}^{L} \rho_l^t \ f_{lj}^t)} \\
&* \exp(-\frac{(\sum_{i=1}^{m} v_i^t \ (a_{ij}^t - \mu_{ij}^t) + \sum_{l=1}^{L} \beta_l^{t-1} \ (e_{lj}^{t-1} - \overline{\overline{\mu}}_{lj}^{t-1}) + \sum_{r=1}^{s} u_r^t \ (d_{rj}^t + \overline{\mu}_{rj}^t) + \sum_{l=1}^{L} \rho_l^t \ (f_{lj}^t + \overline{\overline{\mu}}_{lj}^t))^2 + \sum_{l=1}^{L} \rho_l^t \ (f_{lj}^t + \overline{\overline{\mu}}_{lj}^t))^2}{2(\sum_{i=1}^{m} v_i^{2^t} \sigma_{ij}^{2^t} + \sum_{l=1}^{L} \beta_l^{2^{t-1}} \overline{\overline{\sigma}}_{lj}^{2^{t-1}} + \sum_{r=1}^{s} u_r^{2^t} \overline{\sigma}_{rj}^{2^t} + \sum_{l=1}^{L} \rho_l^{2^t} \overline{\overline{\sigma}}_{lj}^{2^t})}) \\
&+ \frac{\sum_{i=1}^{m} v_i^t \ (\mu_{ij}^t - a_{ij}^t) + \sum_{l=1}^{L} \beta_l^{t-1} \ (\overline{\overline{\mu}}_{lj}^{t-1} - e_{lj}^{t-1}) - \sum_{r=1}^{s} u_r^t \ (\overline{\mu}_{rj}^t + d_{rj}^t) - \sum_{l=1}^{L} \rho_l^t \ (\overline{\overline{\mu}}_{lj}^t + f_{lj}^t)}{2(\sum_{i=1}^{m} v_i^t \ a_{ij}^t + \sum_{l=1}^{L} \beta_l^{t-1} \ e_{lj}^{t-1} + \sum_{r=1}^{s} u_r^t \ d_{rj}^t + \sum_{l=1}^{L} \rho_l^t \ f_{lj}^t)} \\
&* \phi(\frac{\sum_{i=1}^{m} v_i^t \ (a_{ij}^t - \mu_{ij}^t) + \sum_{l=1}^{L} \beta_l^{t-1} \ (e_{lj}^{t-1} - \overline{\overline{\mu}}_{lj}^{t-1}) + \sum_{r=1}^{s} u_r^t \ (d_{rj}^t + \overline{\mu}_{rj}^t) + \sum_{l=1}^{L} \rho_l^t \ (f_{lj}^t + \overline{\overline{\mu}}_{lj}^t)}{\sqrt{\sum_{i=1}^{m} v_i^{2^t} \sigma_{ij}^{2^t} + \sum_{l=1}^{L} \beta_l^{2^{t-1}} \overline{\overline{\sigma}}_{lj}^{2^{t-1}} + \sum_{r=1}^{s} u_r^{2^t} \overline{\sigma}_{rj}^{2^t} + \sum_{l=1}^{L} \rho_l^{2^t} \overline{\overline{\sigma}}_{lj}^{2^t}}}
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{(\sum_{i=1}^{m} v_i^t\ \mu_{ij}^t + \sum_{l=1}^{L} \beta_l^{t-1}\ \bar{\bar{\mu}}_{lj}^{t-1} - \sum_{r=1}^{s} u_r^t\ \bar{\mu}_{rj}^t - \sum_{l=1}^{L} \rho_l^t\ \bar{\bar{\mu}}_{lj}^t)(\sum_{i=1}^{m} v_i^t\ (b_{ij}^t - a_{ij}^t) + \sum_{l=1}^{L} \beta_l^{t-1}\ (f_{lj}^{t-1} - e_{lj}^{t-1}) -}{2(\sum_{i=1}^{m} v_i^t\ a_{ij}^t + \sum_{l=1}^{L} \beta_l^{t-1}\ e_{lj}^{t-1} + \sum_{r=1}^{s} u_r^t\ d_{rj}^t + \sum_{l=1}^{L} \rho_l^t\ f_{lj}^t)(\sum_{i=1}^{m} v_i^t\ b_{ij}^t + \sum_{l=1}^{L} \beta_l^{t-1}\ f_{lj}^{t-1} +}\cdots \\
&\cdots\frac{\sum_{r=1}^{s} u_r^t\ (d_{rj}^t - c_{rj}^t) - \sum_{l=1}^{L} \rho_l^t\ (f_{lj}^t - e_{lj}^t))}{\sum_{r=1}^{s} u_r^t\ c_{rj}^t + \sum_{l=1}^{L} \rho_l^t\ e_{lj}^t)} * \phi\left(\frac{-\sum_{i=1}^{m} v_i^t\ \mu_{ij}^t - \sum_{l=1}^{L} \beta_l^{t-1}\ \bar{\bar{\mu}}_{lj}^{t-1} + \sum_{r=1}^{s} u_r^t\ \bar{\mu}_{rj}^t + \sum_{l=1}^{L} \rho_l^t\ \bar{\bar{\mu}}_{lj}^t}{\sqrt{\sum_{i=1}^{m} v_i^{2^t} \sigma_{ij}^{2^t} + \sum_{l=1}^{L} \beta_l^{2^{t-1}} \bar{\bar{\sigma}}_{lj}^{2^{t-1}} + \sum_{r=1}^{s} u_r^{2^t} \bar{\sigma}_{rj}^{2^t} + \sum_{l=1}^{L} \rho_l^{2^t} \bar{\bar{\sigma}}_{lj}^{2^t}}}\right) \\
&+\frac{\sum_{i=1}^{m} v_i^t\ (b_{ij}^t + \mu_{ij}^t) + \sum_{l=1}^{L} \beta_l^{t-1}\ (f_{lj}^{t-1} + \bar{\bar{\mu}}_{lj}^{t-1}) - \sum_{r=1}^{s} u_r^t\ (\bar{\mu}_{rj}^t - c_{rj}^t) - \sum_{l=1}^{L} \rho_l^t\ (\bar{\bar{\mu}}_{lj}^t - e_{lj}^t)}{2(\sum_{i=1}^{m} v_i^t\ b_{ij}^t + \sum_{l=1}^{L} \beta_l^{t-1}\ f_{lj}^{t-1} + \sum_{r=1}^{s} u_r^t\ c_{rj}^t + \sum_{l=1}^{L} \rho_l^t\ e_{lj}^t)} \\
&* \phi\left(\frac{\sum_{i=1}^{m} v_i^t\ (b_{ij}^t + \mu_{ij}^t) + \sum_{l=1}^{L} \beta_l^{t-1}\ (f_{lj}^{t-1} + \bar{\bar{\mu}}_{lj}^{t-1}) + \sum_{r=1}^{s} u_r^t\ (c_{rj}^t - \bar{\mu}_{rj}^t) + \sum_{l=1}^{L} \rho_l^t\ (e_{lj}^t - \bar{\bar{\mu}}_{lj}^t)}{\sqrt{\sum_{i=1}^{m} v_i^{2^t} \sigma_{ij}^{2^t} + \sum_{l=1}^{L} \beta_l^{2^{t-1}} \bar{\bar{\sigma}}_{lj}^{2^{t-1}} + \sum_{r=1}^{s} u_r^{2^t} \bar{\sigma}_{rj}^{2^t} + \sum_{l=1}^{L} \rho_l^{2^t} \bar{\bar{\sigma}}_{lj}^{2^t}}}\right) \\
&+\frac{\sum_{i=1}^{m} v_i^t\ (b_{ij}^t - \mu_{ij}^t) + \sum_{l=1}^{L} \beta_l^{t-1}\ (f_{lj}^{t-1} - \bar{\bar{\mu}}_{lj}^{t-1}) + \sum_{r=1}^{s} u_r^t\ (\bar{\mu}_{rj}^t + c_{rj}^t) + \sum_{l=1}^{L} \rho_l^t\ (\bar{\bar{\mu}}_{lj}^t + e_{lj}^t)}{2(\sum_{i=1}^{m} v_i^t\ b_{ij}^t + \sum_{l=1}^{L} \beta_l^{t-1}\ f_{lj}^{t-1} + \sum_{r=1}^{s} u_r^t\ c_{rj}^t + \sum_{l=1}^{L} \rho_l^t\ e_{lj}^t)}
\end{aligned}
\tag{14}
$$

5.2. Equivalent stochastic representation of the expectation objective functions

Theorem 5.2.1. Let $X^t = (x^t - a^t, x^t, x^t + b^t)$, $Y^t = (y^t - c^t, y^t, y^t + d^t)$, $K^t = (k^t - e^t, k^t, k^t + f^t)$ and $K^{t-1} = (k^{t-1} - e^{t-1}, k^{t-1}, k^{t-1} + f^{t-1})$ be independent triangular fuzzy variables, where $x^t, y^t, k^t, k^{t-1} \in R$; a^t, b^t are the left-width and right-width of X^t ; c^t, d^t are the left-width and right-width of Y^t ; e^t, f^t are the left-width and right-width of K^t and e^{t-1}, f^{t-1} are the left-width and right-width of K^{t-1}, respectively. Then according to Theorem (3.2.4), we have:

$$
\begin{aligned}
E\left[\frac{Y^t + K^t}{X^t + K^{t-1}}\right] = &-\frac{1}{2}\left(\frac{c^t + e^t}{b^t + f^{t-1}}\right) - \frac{1}{2(b^t + f^{t-1})}\left[y^t + k^t - c^t - e^t + (x^t + k^{t-1} + b^t + f^{t-1}\left(\frac{c^t + e^t}{b^t + f^{t-1}}\right)\right] * Ln\left(\frac{x^t + k^{t-1}}{x^t + k^{t-1} + b^t + f^{t-1}}\right) \\
&-\frac{1}{2}\left(\frac{d^t + f^t}{a^t + e^{t-1}}\right) + \frac{1}{2(a^t + e^{t-1})}\left[y^t + k^t + d^t + f^t + x^t + k^{t-1} - a^t - e^{t-1}\left(\frac{d^t + f^t}{a^t + e^{t-1}}\right)\right] * Ln\left(\frac{x^t + k^{t-1}}{x^t + k^{t-1} - a^t - e^{t-1}}\right)
\end{aligned}
\tag{15}
$$

***Proof*.** Let X_α^t, Y_α^t, K_α^t and K_α^{t-1} are the α-cuts of X^t, Y^t, K^t and K^{t-1}, respectively. Then, for each $\alpha \in (0,1]$, we have:

$$
\begin{aligned}
X_a^t &= \left[x^t - (1-\alpha)a^t, x^t + (1-\alpha)b^t\right] & Y_a^t &= \left[y^t - (1-\alpha)c^t, y^t + (1-\alpha)d^t\right] \\
K_a^t &= \left[k^t - (1-\alpha)e^t, k^t + (1-\alpha)f^t\right] & K_a^{t-1} &= \left[k^{t-1} - (1-\alpha)e^{t-1}, k^{t-1} + (1-\alpha)f^{t-1}\right]
\end{aligned}
\tag{16}
$$

By the continuity of the possibility distributions of X^t, Y^t, K^t and K^{t-1}, and the interval arithmetic, we have:

$$
\left(\frac{Y^t + K^t}{X^t + K^{t-1}}\right)_\alpha = \frac{Y_\alpha^t + K_\alpha^t}{X_\alpha^t + K_\alpha^{t-1}} = \begin{cases} \left[\overbrace{\frac{y^t - (1-\alpha)c^t + k^t - (1-\alpha)e^t}{X^t + (1-\alpha)b^t + k^{t-1} + (1-\alpha)e^{t-1}}}^{\mathrm{I}}, \overbrace{\frac{y^t + (1-\alpha)d^t + k^t + (1-\alpha)f^t}{X^t - (1-\alpha)a^t + k^{t-1} - (1-\alpha)e^{t-1}}}^{\mathrm{II}}\right] & \text{if} \quad \mathrm{I} \le \mathrm{II} \\ \left[\frac{y^t + (1-\alpha)d^t + k^t + (1-\alpha)f^t}{X^t - (1-\alpha)a^t + k^{t-1} - (1-\alpha)e^{t-1}}, \frac{y^t - (1-\alpha)c^t + k^t - (1-\alpha)e^t}{X^t + (1-\alpha)b^t + k^{t-1} + (1-\alpha)e^{t-1}}\right] & \text{if} \quad \mathrm{I} \succ \mathrm{II} \end{cases}
\tag{17}
$$

Where $\left(\frac{Y^t+K^t}{X^t+K^{t-1}}\right)_\alpha$ is the α-cut of the fuzzy variable $\left(\frac{Y^t+K^t}{X^t+K^{t-1}}\right)$ at period t. So we have:

$$E\left[\frac{Y^t+K^t}{X^t+K^{t-1}}\right]=\frac{1}{2}\int_0^1[\frac{y^t-(1-\alpha)c^t+k^t-(1-\alpha)e^t}{x^t+(1-\alpha)b^t+k^{t-1}+(1-\alpha)f^{t-1}}+\frac{y^t+(1-\alpha)d^t+k^t+(1-\alpha)f^t}{x^t-(1-\alpha)a^t+k^{t-1}-(1-\alpha)e^{t-1}}]\quad d\alpha$$

$$=-\frac{1}{2}(\frac{c^t+e^t}{b^t+f^{t-1}})-\frac{1}{2(b^t+f^{t-1})}[y^t+k^t-c^t-e^t+x^t+k^{t-1}+b^t+f^{t-1}(\frac{c^t+e^t}{b^t+f^{t-1}})]*Ln(\frac{x^t+k^{t-1}}{x^t+k^{t-1}+b^t+f^{t-1}})$$
$$-\frac{1}{2}(\frac{d^t+f^t}{a^t+e^{t-1}})+\frac{1}{2(a^t+e^{t-1})}[y^t+k^t+d^t+f^t+x^t+k^{t-1}-a^t-e^{t-1}(\frac{d^t+f^t}{a^t+e^{t-1}})]*Ln(\frac{x^t+k^{t-1}}{x^t+k^{t-1}-a^t-e^{t-1}}) \tag{18}$$

The proof of the theorem is complete.

Suppose X_j^t , Y_j^t, K_j^t and K_j^{t-1} are random triangular fuzzy vectors of DMUj in period t as (12), then we have:

$$\mu_j^t=\sum_{i=1}^m v_i^t\mu_{ij}^t \qquad \overline{\mu}_j^t=\sum_{r=1}^s u_r^t\overline{\mu}_{rj}^t \qquad \overline{\overline{\mu}}_j^t=\sum_{l=1}^L \rho_l^t\overline{\overline{\mu}}_{lj}^t \qquad \overline{\overline{\mu}}_j^{t-1}=\sum_{l=1}^L \beta_l^{t-1}\overline{\overline{\mu}}_{lj}^{t-1}$$
$$\sigma_j^{2^t}=\sum_{i=1}^m v_i^t\delta_{ij}^{2^t} \qquad \overline{\sigma}_j^{2^t}=\sum_{r=1}^s u_r^t\overline{\delta}_{rj}^{2^t} \qquad \overline{\overline{\sigma}}_j^{2^t}=\sum_{l=1}^L \rho_l^t\overline{\overline{\delta}}_{lj}^{2^t} \qquad \overline{\overline{\sigma}}_j^{2^{t-1}}=\sum_{l=1}^L \beta_l^{t-1}\overline{\overline{\delta}}_{lj}^{2^{t-1}}$$
$$a_j^t=\sum_{i=1}^m v_i^t a_{ij}^t \qquad b_j^t=\sum_{i=1}^m v_i^t b_{ij}^t \qquad c_j^t=\sum_{r=1}^s u_r^t c_{rj}^t \qquad d_j^t=\sum_{r=1}^s u_r^t d_{rj}^t$$
$$e_j^t=\sum_{l=1}^L \rho_l^t e_{lj}^t \qquad f_j^t=\sum_{l=1}^L \rho_l^t f_{lj}^t \qquad e_j^{t-1}=\sum_{l=1}^L B_l^{t-1}e_{lj}^{t-1} \qquad f_j^{t-1}=\sum_{l=1}^L \beta_l^{t-1}f_{lj}^{t-1}$$

By Theorem (5.2.1), the *j*th objective function of initial DRF-DEA model (13) has the following equivalent stochastic representation:

$$Z_j^t=E(\frac{\sum_{r=1}^s u_r^t y_{rj}^t+\sum_{l=1}^L \rho_l^t k_{lj}^t}{\sum_{i=1}^m v_i^t x_{ij}^t+\sum_{l=1}^L \beta_l^{t-1}k_{lj}^{t-1}})=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\{-\frac{1}{2}(\frac{c_j^t+e_j^t}{b_j^t+f_j^{t-1}})-\frac{1}{2}(\frac{d_j^t+f_j^t}{a_j^t+e_j^{t-1}})+\frac{1}{2(b_j^t+f_j^{t-1})}[y_j^t+k_j^t+c_j^t+e_j^t+x_j^t(\frac{c_j^t+e_j^t}{b_j^t+f_j^{t-1}})$$
$$+k_j^{t-1}(\frac{c_j^t+e_j^t}{b_j^t+f_j^{t-1}})+b_j^t(\frac{c_j^t+e_j^t}{b_j^t+f_j^{t-1}})+f_j^{t-1}(\frac{c_j^t+e_j^t}{b_j^t+f_j^{t-1}})]*Ln(\frac{x_j^t+k_j^{t-1}}{x_j^t+k_j^{t-1}+b_j^t+f_j^{t-1}})+\frac{1}{2(a_j^t+e_j^{t-1})}[y_j^t+d_j^t+k_j^t+f_j^t+x_j^t(\frac{d_j^t+f_j^t}{a_j^t+e_j^{t-1}})$$
$$+k_j^{t-1}(\frac{d_j^t+f_j^t}{a_j^t+e_j^{t-1}})-a_j^t(\frac{d_j^t+f_j^t}{a_j^t+e_j^{t-1}})-e_j^{t-1}(\frac{d_j^t+f_j^t}{a_j^t+e_j^{t-1}})]*Ln(\frac{x_j^t+k_j^{t-1}}{x_j^t+k_j^{t-1}-a_j^t-e_j^{t-1}})\}*\frac{1}{4\pi^2\sigma_j^t\overline{\sigma}_j^t\overline{\overline{\sigma}}_j^t\overline{\overline{\sigma}}_j^{t-1}}\exp(-\frac{(x_j^t-\mu_j^t)^2}{2\sigma_j^{2^t}}-\frac{(y_j^t-\overline{\mu}_j^t)^2}{2\overline{\sigma}_j^{2^t}}$$
$$-\frac{(k_j^t-\overline{\overline{\mu}}_j^t)^2}{2\overline{\overline{\sigma}}_j^{2^t}}-\frac{(k_j^{t-1}-\overline{\overline{\mu}}_j^{t-1})^2}{2\overline{\overline{\sigma}}_j^{2^{t-1}}})dk_j^{t-1}dk_j^t dy_j^t dx_j^t \quad =\frac{1}{\sigma_j^t\sqrt{2\pi}}\int_{-\infty}^{+\infty}f(x_j^t)\quad dX_j^t-\frac{1}{2}(\frac{c_j^t+e_j^t}{b_j^t+f_j^{t-1}})-\frac{1}{2}(\frac{d_j^t+f_j^t}{a_j^t+e_j^{t-1}}) \tag{19}$$

where:

$$f(x_j^t)=\{\frac{1}{2(b_j^t+f_j^{t-1})}[\overline{\mu}_j^t+\overline{\overline{\mu}}_j^t+c_j^t+e_j^t+x_j^t(\frac{c_j^t+e_j^t}{b_j^t+f_j^{t-1}})+\overline{\overline{\mu}}_j^{t-1}(\frac{c_j^t+e_j^t}{b_j^t+f_j^{t-1}})+b_j^t(\frac{c_j^t+e_j^t}{b_j^t+f_j^{t-1}})+f_j^{t-1}(\frac{c_j^t+e_j^t}{b_j^t+f_j^{t-1}})]$$
$$*Ln(\frac{x_j^t+\overline{\overline{\mu}}_j^{t-1}}{x_j^t+\overline{\overline{\mu}}_j^{t-1}+b_j^t+f_j^{t-1}})+\frac{1}{2(a_j^t+e_j^{t-1})}[\overline{\mu}_j^t+d_j^t+\overline{\overline{\mu}}_j^t+f_j^t+x_j^t(\frac{d_j^t+f_j^t}{a_j^t+e_j^{t-1}})+\overline{\overline{\mu}}_j^{t-1}(\frac{d_j^t+f_j^t}{a_j^t+e_j^{t-1}})$$
$$-a_j^t(\frac{d_j^t+f_j^t}{a_j^t+e_j^{t-1}})-e_j^{t-1}(\frac{d_j^t+f_j^t}{a_j^t+e_j^{t-1}})]*Ln(\frac{x_j^t+\overline{\overline{\mu}}_j^{t-1}}{x_j^t+\overline{\overline{\mu}}_j^{t-1}-a_j^t-e_j^{t-1}})\}*\exp(-\frac{(x_j^t-\mu_j^t)^2}{2\sigma_j^{2^t}})$$

As a result, in the case when the outputs and inputs are mutually independent random triangular fuzzy variables, we have transformed the initial proposed DRF-DEA model (13) to its equivalent multi-objective stochastic programming model (20) according to the formulas established in section 5 as follows:

$$
\begin{aligned}
&Max: \quad Z_1^t \\
&\vdots \\
&Max: \quad Z_n^t \\
&st: \ g_j^t(u_r^t, v_i^t, \rho_l^t, \beta_l^{t-1}) \geq 1-\alpha_j^t \ ; \quad j=1,2,\dots,n \\
&\quad u_r^t, v_i^t, \rho_l^t, \beta_l^{t-1} \geq \varepsilon \\
&\quad i=1,..,m \quad r=1,..,s \quad l=1,..,L \quad t=1,..,T
\end{aligned} \tag{20}
$$

where $g_j^t(u_r^t, v_i^t, \rho_l^t, \beta_l^{t-1})$ and Z_j^t are determined by (14) and (19), respectively.

5.3. Reformation of the proposed equivalent multi-objective stochastic programming model

The proposed equivalent multi-objective stochastic model (20) has "n" objective functions in each period and is established by individually maximizing the efficiency of each DMU which consumes much computational time. It can be converted to equivalent one objective stochastic model by using fuzzy multiple objectives linear programming approach which was proposed by Zimmerman (1991). This approach utilizes the membership function to convert the equivalent multiple objectives programming to one objective model. If $\mu(Z_j^t)=W$ then Z_j^t is a convex combination of Z_j^{tL} and Z_j^{tR}, $\mu(Z_j^t)=WZ_j^{tR}+(1-W)Z_j^{tl}$ with $0 \leq W \leq 1$, in which Z_j^{tL} and Z_j^{tR} denotes the value of the objective

e function Z_j^t in period t such that the degree of membership function is 0 and 1 respectively. Suppose $\underset{j,t}{Min} \quad \mu(Z_j^t)=W.Z_j^{tR}+(1-W).Z_j^{tL}$, therefore the model (20) is converted to following model:

$$
\begin{aligned}
&Max: \ \{W.Z_j^{tR}+(1-W).Z_j^{tL}\} \\
&st: g_j^t(u_r^t, v_i^t, \rho_l^t, \beta_l^{t-1}) \geq 1-\alpha_j^t \quad ; \ j=1,\dots,n \\
&W.Z_j^{tR}+(1-W).Z_j^{tL} \quad Z_j^t \geq \ ; \ j=1,\dots,n \\
&0 \leq W \leq 1 \\
&u_r^t, v_i^t, \rho_l^t, \beta_l^{t-1} \geq \varepsilon \\
&\quad i=1,..,m \quad r=1,..,s \quad l=1,..,L \quad t=1,..,T
\end{aligned} \tag{21}
$$

Since $Z_j^{tL}=0$ and $Z_j^{tR}=1$ for all objective functions, so model (21) can be rewritten as following:

$$
\begin{aligned}
&Max: \ W \\
&st: \quad g_j^t(u_r^t, v_i^t, \rho_l^t, \beta_l^{t-1}) \geq 1-\alpha_j^t \ ; \quad j=1,\dots,n \\
&\qquad Z_j^t \geq W \ ; \quad j=1,\dots,n \\
&\qquad 0 \prec W \leq 1 \\
&u_r^t, v_i^t, \rho_l^t, \beta_l^{t-1} \geq \varepsilon \\
&\quad i=1,..,m \quad r=1,..,s \quad l=1,..,L \quad t=1,..,T
\end{aligned} \tag{22}
$$

Model (22) is the equivalent one objective stochastic model or the final equivalent representation of proposed DRF-DEA model, in which the efficiency scores for all DMUs are predicted with once run of this model in each period and thereupon the computational time is saved. Since the equivalent stochastic constraints (19) in this model are still in form of the integral, we cannot solve it via the conventional optimization algorithms. In order to overcome the difficulty, in the next section we design a new hybrid algorithm to solve final proposed DRF-DEA model by incorporating MC simulation and GA, in which MC simulation will employ to compute the integrals involved in the constraints and GA will use to find the optimal of problem.

6. Solution Methodology

In the computer science field of artificial intelligence, genetic algorithm (GA) is a search metaheuristic that mimics the process of natural selection, it was proposed by Holland and further developed by Goldberg (1989) and others. A simple GA consists of two main operators: crossover and mutation. Crossover is the partial swapping between two parent strings to generate two offspring strings, while Mutation is used to expose unexpected changes in the values of genes. The main characteristic of the GA is that it can explore the solution space. The procedure of our hybrid algorithm for solving the final proposed DRF-DEA model (22) is summarized as follows.

6. 1. Solution representation

Suppose there are pop-size chromosomes in the population, representing the solutions of the final proposed DRF-DEA model (22), in which the decision variables include $u_r^t, v_i^t, \rho_l^t, \beta_l^{t-1} \in (0,1)$ for each i (i=1,..,m), r (r=1,..,s) and l (l=1,..,L) and $R^t=[u_1^t,\dots,u_s^t, v_1^t,\dots,v_m^t, \rho_1^t,\dots,\rho_l^t, \beta_1^{t-1},\dots,\beta_l^{t-1}] \in (0,1)$ is characterized as a chromosome to show a decision vector in period t (t=1,..,T).

6. 2. Initialization process

Generate randomly $u_r^t, v_i^t, \rho_l^t, \beta_l^{t-1}$ from the interval (0,1). Compute $g_j^t(u_r^t, v_i^t, \rho_l^t, \beta_l^{t-1})$ via formula (14) and Z_j^t via formula (19) for j=1,2,..,n at period t, where the integrals in Z_j^t are approximated by MC simulation which will discribe in section 6.5. If R^t satisfies the constraints of model (22), then it is feasible and take it as an initial chromosome. Repeat this process until pop-size initial feasible chromosomes $R_1^t, R_2^t, ...R_{pop-size}^t$ are produced.

6. 3. Recombination process

Firstly by crossover operator, renew the chromosomes $R_p^t, p = 1,2,...,pop-size$. For crossover operation, repeat the following process to determine the parents for choromosomes from p = 1 to pop-size: generate a random real number r from the unit interval [0, 1]. If $r \prec P_c$, the chromosome R_p^t will be selected as a parent choromosome, where the parameter P_c is the probability of crossover. Then group the selected parent choromosomes $R_1^{t,1}, R_2^{t,1}, R_3^{t,1}, ...$ to the pairs $(R_1^{t,1}, R_2^{t,1}), (R_3^{t,1}, R_4^{t,1}), ...$. The crossover process on each pair $(R_p^{t,1}, R_{p+1}^{t,1})$ is showed as follows. Generate a random number λ from the interval (0,1), then the crossover operator on $R_p^{t,1}$ and $R_{p+1}^{t,1}$ will generate two offspring $R_p^{t,2}$ and $R_{p+1}^{t,2}$ as following:

$$\begin{aligned} R_p^{t,2} &= \lambda R_p^{t,1} + (1-\lambda) R_{p+1}^{t,1} \\ R_{p+1}^{t,2} &= (1-\lambda) R_p^{t,1} + \lambda R_{p+1}^{t,1} \end{aligned} \tag{23}$$

If both offsprings satisfy the constraints of model (22), then we replace the parents with them. Otherwise, we keep the feasible one if it exists, and repeat the crossover by reproduce another real number from interval (0,1) until two feasible offsprings are obtained. Finally, we get pop-size chromosomes, including the new generated chromosomes. Secondly, update the chromosomes R_p^t by mutation operator. Repeat the following steps from p = 1 to pop-size: produce a random real number r from the [0,1], if $r \prec P_m$ the chromosome R_p^t will be selected as a parent choromosome, where the parameter P_m is the probability of mutation. For each selected parent $R_p^{t,1}$, the mutation operation is as the following:

$$R_p^{t,2} = R_p^{t,1} + r.M \tag{24}$$

where M is an appropriate large positive number. If $R_p^{t,2}$ is infeasible, then we set M as a random number between 0 and M until it is feasible. We set M = 0, if the above process cannot find a feasible solution in a predetermined number of iterations. Anyway, we replace the parent $R_p^{t,1}$ with its offspring $R_p^{t,2}$. Finally, we get pop-size chromosomes, including the new generated chromosomes.

6. 4. Evolution process

We compute the fitness of each choromosome in each period via formula (19). The chromosomes $R_1^t, R_2^t, ...R_{pop-size}^t$ are assumed to have been rearranged from good to bad according to their ftnesses. Select the chromosomes for a new population, in which the chromosome with higher fitness will have a big chance for selection. After this process pop-size times, we obtain pop-size of chromosomes, denoted also by R_p^t.

6. 5. Monte Carelo (MC) simulation

MC simulation is a method to deal with the stochastic behavior in complex systems (Chuen ,Kuan and Wai, 2012). In order to solve the final proposed DRF-DEA model (22), for any given solution $(u_r^t, v_i^t, \rho_l^t, \beta_l^{t-1})$, we need to check its feasibility. Since the some constraints (19) include integrals $(\int_{-\infty}^{+\infty} f_j(x_j^t) \quad dx_j^t)$ which cannot solve via the conventional optimization algorithm, so we should approximate their values. MC simulation method, firstly changes variable in the function $f_j(x_j^t)$ to convert infinite interval to finite interval as following:

$$\int_{-\infty}^{+\infty} f(x)dx = \int_{-1}^{+1} f\left(\frac{h}{1-h^2}\right) \frac{1+h^2}{(1-h^2)^2} dh \tag{25}$$

Also, the process of MC simulation to approximate integral (25) is described as follows:

Procedure: MC simulation for approximating (25)

begin
n←number-simulation
for i =1 to n do

h_i←Generate a uniform distributed random point in the interval [a,b]=[-1,1]
Determine the average value of the function:

$$\hat{f} = \frac{1}{n} * \sum_{i=1}^{n} f\left(\frac{h_i}{1-h_i^2}\right)\left(\frac{1+h_i^2}{(1-h_i^2)^2}\right)$$

Compute the approximation to the integral:

$$\int_{-\infty}^{+\infty} f(x)dx \approx (b-a)\hat{f}$$

end

By integrating GA and MC simulation, we design a new hybrid algorithm (MC-GA) for solving the equivalent one objective stochastic model (22) which summarized as follows:

- Initialize pop_size chromosomes whose feasibility must be checked by constraints of model (22) and MC simulation.

- Update the chromosomes by crossover and mutation operators in which the feasibility of offsprings must be checked by constraints of model (22) and MC simulation.
- Evaluate the fitness for all the chromosomes by (19) and MC simulation.
- Select the chromosomes by fitness-proportional selection.
- Repeat the second to fourth steps for a given number of generations.
- Select the best chromosome as the optimal solution.

7. Practical Example

In this section, since no benchmark is available in the literature, one practical example as a case study is presented. A general manager of the Iranian petroleum firm was involved in our study and he assisted us in accessing information on the operation of gas stations (DMUs) in order to predict efficiency of them for the next two financial periods. He selected five DMUs along with three inputs and two outputs that all of which are listed in Table 1. The three inputs are: employees salary (unit: 1000 milion Rial per month), operation cost (unit: 1000 milion Rial per month) and net profit (unit: 1000 milion Rial per month). The two outputs are: the total amount of gasoline (unit: kl per month) and net profit. Net profit is quasi-fix input (output) or link data, because it plays an important role and influences to the performance of DMUs in the next period time. Since the outputs and inputs are random triangular fuzzy variables, this study needs to mention how to obtain them. In June 2013, the firm's managers predicted the left-width and right-width of the inputs and outputs for the Autumn (first period) and Winter of 2013 (second period). The inputs and outputs of DMUs in the previous periods follow the normal distribution which are presented in Table 1.

To demonstrate the modeling idea and the effectiveness of the solution methodolgy, the final proposed DRF-DEA model (22) and the proposed MC-GA algorithm are used to predict the efficiency of five gas stations. Table 2 documents the predicted efficiency scores under risk level $\alpha = 0.5$ for the next two financial periods. This practical experiments are performed on a personal computer, using the Microsoft Windows 7 operating system, and hybrid algorithm is written by C++ programming language with the following parameters in the GA: the population size is 30, the probability of crossover is 0.3, the probability of mutation is 0.2 and generation number is 900. From the solution results for each period, we can see the information about each DMU. For example in the first period, DMU2 has the biggest α-expected efficient value 0.982, followed by DMU4, DMU1, DMU3 and DMU5, which implies that DMU2 is the most efficient DMU.

Also, with the proposed hybrid algorithm, the predicted efficiency scores under various risk levels but similar for all DMUs for the next two financial periods are reported in Table 3.

Table 1
The random triangular fuzzy inputs and outputs for five Gas stations (DMUs) about the next two financial periods

periods	DMU_j	Inputs: Employees salary (X_1)	Inputs: Operation cost (X_2)	Inputs: Net profit (K^{t-1})	Outputs: Gasoline (Y_1)	Outputs: Net profit (K^t)
t=1	1	$(x_{11}^1-0.6, x_{11}^1, x_{11}^1+0.6)$ $x_{11}^1 \sim N(3.9,0.02)$	$(x_{21}^1-0.3, x_{21}^1, x_{21}^1+0.3)$ $x_{21}^1 \sim N(2.2,0.01)$	$(k_{11}^0-0.1, k_{11}^0, k_{11}^0+0.1)$ $k_{11}^0 \sim N(7.9,0.01)$	$(y_{11}^1-0.3, y_{11}^1, y_{11}^1+0.3)$ $y_{11}^1 \sim N(4,0.01)$	$(k_{11}^1-0.2, k_{11}^1, k_{11}^1+0.2)$ $k_{11}^1 \sim N(8.2,0.01)$
	2	$(x_{12}^1-0.2, x_{12}^1, x_{12}^1+0.2)$ $x_{12}^1 \sim N(3,0.02)$	$(x_{22}^1-0.2, x_{22}^1, x_{22}^1+0.2)$ $x_{22}^1 \sim N(1.5,0.01)$	$(k_{12}^0-0.1, k_{12}^0, k_{12}^0+0.1)$ $k_{12}^0 \sim N(7.2,0.01)$	$(y_{12}^1-0.2, y_{12}^1, y_{12}^1+0.2)$ $y_{12}^1 \sim N(3.5,0.01)$	$(k_{12}^1-0.1, k_{12}^1, k_{12}^1+0.1)$ $k_{12}^1 \sim N(7.5,0.01)$
	3	$(x_{13}^1-0.6, x_{13}^1, x_{13}^1+0.6)$ $x_{13}^1 \sim N(4.7,0.02)$	$(x_{23}^1-0.5, x_{23}^1, x_{23}^1+0.5)$ $x_{23}^1 \sim N(2.5,0.01)$	$(k_{13}^0-0.1, k_{13}^0, k_{13}^0+0.1)$ $k_{13}^0 \sim N(9.2,0.01)$	$(y_{13}^1-0.1, y_{13}^1, y_{13}^1+0.1)$ $y_{13}^1 \sim N(4.7,0.01)$	$(k_{13}^1-0.3, k_{13}^1, k_{13}^1+0.3)$ $k_{13}^1 \sim N(9.5,0.01)$
	4	$(x_{14}^1-0.8, x_{14}^1, x_{14}^1+0.8)$ $x_{14}^1 \sim N(4.1,0.02)$	$(x_{24}^1-0.5, x_{24}^1, x_{24}^1+0.5)$ $x_{24}^1 \sim N(2.2,0.01)$	$(k_{14}^0-0.1, k_{14}^0, k_{14}^0+0.1)$ $k_{14}^0 \sim N(8.9,0.01)$	$(y_{14}^1-0.5, y_{14}^1, y_{14}^1+0.5)$ $y_{14}^1 \sim N(4.1,0.01)$	$(k_{14}^1-0.1, k_{14}^1, k_{14}^1+0.1)$ $k_{14}^1 \sim N(9.2,0.01)$
	5	$(x_{15}^1-0.4, x_{15}^1, x_{15}^1+0.4)$ $x_{11}^1 \sim N(5.2,0.02)$	$(x_{25}^1-0.2, x_{25}^1, x_{25}^1+0.2)$ $x_{25}^1 \sim N(3.1,0.01)$	$(k_{15}^0-0.1, k_{15}^0, k_{15}^0+0.1)$ $k_{15}^0 \sim N(9.8,0.01)$	$(y_{15}^1-0.4, y_{15}^1, y_{15}^1+0.4)$ $y_{15}^1 \sim N(5.2,0.01)$	$(k_{15}^1-0.2, k_{15}^1, k_{15}^1+0.2)$ $k_{15}^1 \sim N(10.1,0.02)$
t=2	1	$(x_{11}^2-0.6, x_{11}^2, x_{11}^2+0.6)$ $x_{11}^2 \sim N(4.1,0.02)$	$(x_{21}^2-0.3, x_{21}^2, x_{21}^2+0.3)$ $x_{21}^2 \sim N(2.3,0.01)$	$(k_{11}^1-0.2, k_{11}^1, k_{11}^1+0.2)$ $k_{11}^1 \sim N(8.2,0.01)$	$(y_{11}^2-0.3, y_{11}^2, y_{11}^2+0.3)$ $y_{11}^2 \sim N(4.2,0.01)$	$(k_{11}^2-0.2, k_{11}^2, k_{11}^2+0.2)$ $k_{11}^2 \sim N(8.5,0.01)$
	2	$(x_{12}^2-0.2, x_{12}^2, x_{12}^2+0.2)$ $x_{12}^2 \sim N(3.2,02)$	$(x_{22}^2-0.2, x_{22}^2, x_{22}^2+0.2)$ $x_{22}^2 \sim N(1.6,0.01)$	$(k_{12}^1-0.1, k_{12}^1, k_{12}^1+0.1)$ $k_{12}^1 \sim N(7.5,0.01)$	$(y_{12}^2-0.2, y_{12}^2, y_{12}^2+0.2)$ $y_{12}^2 \sim N(3.7,01)$	$(k_{12}^2-0.1, k_{12}^2, k_{12}^2+0.1)$ $k_{12}^2 \sim N(7.7,0.01)$
	3	$(x_{13}^2-0.6, x_{13}^2, x_{13}^2+0.6)$ $x_{13}^2 \sim N(4.9,0.02)$	$(x_{23}^2-0.5, x_{23}^2, x_{23}^2+0.5)$ $x_{23}^2 \sim N(2.6,0.01)$	$(k_{13}^1-0.3, k_{13}^1, k_{13}^1+0.3)$ $k_{13}^1 \sim N(9.5,0.01)$	$(y_{13}^2-0.1, y_{13}^2, y_{13}^2+0.1)$ $y_{13}^2 \sim N(4.6,0.01)$	$(k_{13}^2-0.3, k_{13}^2, k_{13}^2+0.3)$ $k_{13}^2 \sim N(9.6,0.01)$
	4	$(x_{14}^2-0.8, x_{14}^2, x_{14}^2+0.8)$ $x_{14}^2 \sim N(4.3,0.02)$	$(x_{24}^2-0.5, x_{24}^2, x_{24}^2+0.5)$ $x_{24}^2 \sim N(2.3,0.01)$	$(k_{14}^1-0.1, k_{14}^1, k_{14}^1+0.1)$ $k_{14}^1 \sim N(9.2,0.01)$	$(y_{14}^2-0.5, y_{14}^2, y_{14}^2+0.5)$ $y_{14}^2 \sim N(5.3,0.01)$	$(k_{14}^2-0.1, k_{14}^2, k_{14}^2+0.1)$ $k_{14}^2 \sim N(9.5,0.01)$
	5	$(x_{15}^2-0.4, x_{15}^2, x_{15}^2+0.4)$ $x_{15}^2 \sim N(5.3,0.02)$	$(x_{25}^2-0.2, x_{25}^2, x_{25}^2+0.2)$ $x_{25}^2 \sim N(3.2,0.01)$	$(k_{15}^1-0.2, k_{15}^1, k_{15}^1+0.2)$ $k_{15}^1 \sim N(10.1,0.02)$	$(y_{15}^2-0.4, y_{15}^2, y_{15}^2+0.4)$ $y_{15}^2 \sim N(5.8,0.01)$	$(k_{15}^2-0.2, k_{15}^2, k_{15}^2+0.2)$ $k_{15}^2 \sim N(10.5,0.02)$

Table 2
The predicted efficiency scores of DMUs for both periods with $\alpha = 0.5$

period	DMU	Optimal solution $(u_1^t, v_1^t, v_2^t, \rho_1^t, \beta_1^{t-1})$	α-expected efficient value
t=1	1	(0.2851 , 0.0194 , 0.9929 , 0.2521 , 0.0302)	0.927
	2	(0.3839 , 0.1708 , 0.6286 , 0.9724 , 0.0853)	0.975
	3	(0.2312 , 0.7125 , 0.7281 , 0.4173 , 0.5125)	0.884
	4	(0.5554 , 0.3531 , 0.9679 , 0.2232 , 0.8834)	0.954
	5	(0.4138 , 0.0187 , 0.8113 , 0.8266 , 0.0670)	0.837
t=2	1	(0.3151 , 0.0134 , 0.9529 , 0.1521 , 0.1302)	0.915
	2	(0.4209 , 0.1608 , 0.6086 , 0.9724 , 0.1153)	0.945
	3	(0.1981 , 0.8126 , 0.9201 , 0.4173 , 0.4824)	0.846
	4	(0.3994 , 0.4561 , 0.9709 , 0.2232 , 0.6855)	0.885
	5	(0.4935 , 0.0123 , 0.6119 , 0.8266 , 0.2685)	0.934

Table 3
The predicted efficiency scores of DMUs under various risk levels for the next two financial periods

periods	DMU	$\alpha = 0.05$	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.5$	$\alpha = 0.8$
t=1	1	0.823	0.841	0.868	0.927	0.962
	2	0.891	0.921	0.932	0.975	0.994
	3	0.812	0.834	0.854	0.884	0.946
	4	0.827	0.849	0.871	0.954	0.973
	5	0.801	0.825	0.844	0.837	0.900
t=2	1	0.813	0.831	0.858	0.915	0.967
	2	0.888	0.918	0.927	0.945	0.995
	3	0.780	0.812	0.818	0.846	0.911
	4	0.831	0.852	0.874	0.885	0.926
	5	0.885	0.899	0.929	0.934	0.973

Table 4 documents the total amount of actual inputs and outputs for the Autumn and Winter of 2013. This study sampled these results in January 2014 in order to examine whether the predicted efficiency scores are different from actual efficiency scores. So, the results of actual efficiency scores with using conventional DEA (dynamic CCR model (11)) and actual data which are shown in Table 4 for DMUs. Generally there are three types of classification: (a) $\alpha \prec 0.5$ is conservative, (b) $\alpha = 0.5$ is risk-natural and (c) $\alpha \succ 0.5$ is risk-taking in DEA. It is easily thought that the conventional use of DEA belongs to the risk-natural (Nemoto and Goto, 2003). This finding can be easily confirmed by comparing the actual efficiency results with the predicted efficiency results under $\alpha = 0.5$. The two DEA approaches exhibit very similar results on efficiency and ranks scores. For example in second period from table 4, three DMUs (the 1th, 2th, 5th gas stations) are efficient based on actual efficiency scores and have been in the first place, while they are in the first place to third and separated based on the predicted efficiencies. Table 4 indicates that the high Pearson correlation rates have obtained (0.732 and 0.957) for both periods between predicted and actual efficiency scores with $\alpha = 0.5$.

In order to further illustrate the validation of the obtained results, we compare our results against the results of the similar hybrid GA algorithm which was proposed by Qin and Liu (2010). The computational results of the predicted efficiency scores for DMU1 are reported in Table 5, in which parameter "CPU(s)" is the computational time consumed by the two hybrid algorithms to get (near) optimal predicted efficiency score (Z_1^*). It can be from Table 5 that the proposed hybrid GA algorithm solves all instances optimally in an average of less than 61s of CPU time requirement for both periods. As a result, we conclude that our hybrid GA algorithm outperforms the Qin's hybrid GA algorithm in terms of runtime. In addition, the predicted efficiency scores of our hybrid algorithm are closer to the actual efficiencies of computational results of the Qin's hybrid algorithm.

Table 4
Actual inputs and outputs values of DMUs with their actual efficiency scores for both financial periods

Period	DMU_j	Inputs: Employees salaries (X_1)	Inputs: Operation costs (X_2)	Inputs: Net profits (K^{t-1})	Outputs: Gasoline (Y_1)	Outputs: Net profit (K^t)	Actual efficiencies	Predicted efficiencies (with $\alpha = 0.5$)	Real ranks	Predicted ranks
t=1	1	4.01	2.15	7.85	3.95	8.16	0.98	0.927	2	3
	2	3.02	1.51	7.11	3.41	7.41	1	0.975	1	1
	3	4.6	2.34	9.07	4.91	9.52	1	0.884	1	4
	4	4.11	2.11	8.91	4.55	9.24	1	0.954	1	2
	5	5.12	3.03	9.89	5.19	10.2	0.96	0.837	3	5
t=2	1	4.12	2.21	8.16	4.01	8.61	1	0.915	1	3
	2	3.11	1.42	7.41	3.75	7.79	1	0.945	1	1
	3	4.78	2.48	9.52	4.61	9.68	0.97	0.846	3	5
	4	4.15	2.15	9.24	5.3	9.39	0.98	0.885	2	4
	5	5.28	3.22	10.2	5.92	10.62	1	0.934	1	2
Pearson correlation coefficient between real and predicted efficiency scores of DMUs in t=1 :									0.732	
Pearson correlation coefficient between real and predicted efficiency scores of DMUs in t= 2 :									0.957	

Table 5
Comparison between the results of our MC-GA algorithm and Qin's hybrid GA algorithm

Period	α	proposed algorithm Z_1^*	proposed algorithm CPU(s)	Qin's hybrid GA algorithm Z_1^*	Qin's hybrid GA algorithm CPU(s)
t=1	0.05	0.823	60.458	0.784	107.412
	0.1	0.841	61.159	0.816	108.521
	0.2	0.868	60.412	0.847	107.217
	0.5	0.927	60.202	0.881	109.698
	0.8	0.962	60.719	0.901	110.665
Average		0.884	60.59	0.846	108.701
t=2	0.05	0.813	61.408	0.775	108.701
	0.1	0.831	60.798	0.796	107.942
	0.2	0.858	60.719	0.824	109.432
	0.5	0.915	61.062	0.897	110.121
	0.8	0.967	60.911	0.932	110.789
Average		0.877	60.979	0.845	109.397

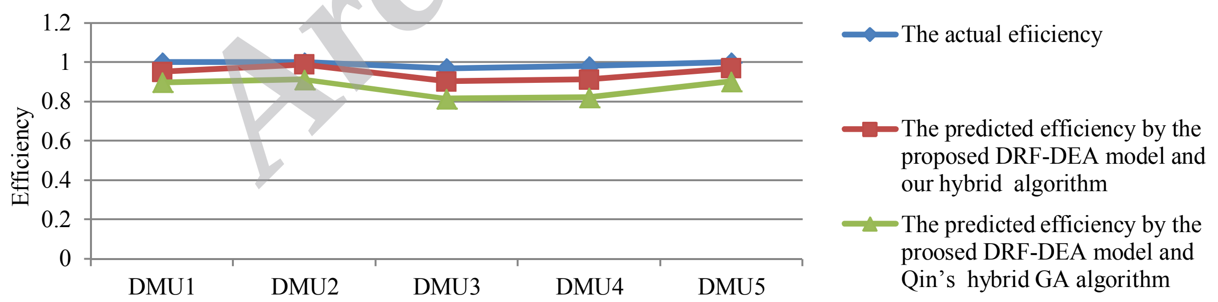

Fig. 2. Comparison between actual efficiencies with predicted efficiency scores

Fig. 2 shows the comparison between actual and predicted efficiency scores under $\alpha = 0.5$. So the proposed hybrid GA algorithm outperforms the Qin's hybrid GA algorithm in term of solution quality. According to paper which was proposed by Qin and Liu (2010), to further test the effectiveness of the proposed hybrid GA algorithm, a careful variations about the probability of mutation P_m and the probability of crossover P_C in GA is made in view of the identification influence on the solution quality for DMU1 with $\alpha = 0.5$ at the first period (t=1). The computational results are collected in Table 6. To compare these results, we give the relative error as follows:

$$\frac{\text{optimal } \alpha\text{-expexted efficient value - actual } \alpha\text{-expexted efficient value}}{\text{optimal } \alpha\text{-expexted efficient value}} \times 100\%$$

Table 6
Comparison solutions for DMU1 under different GA's parameters in first period under $\alpha = 0.5$

P_C	P_m	Gen	α-expected efficient value	Realative error (%)
0.1	0.3	900	0.946	0.83
0.2	0.4	900	0.938	1.67
0.3	0.2	900	0.927	1.36
0.4	0.1	900	0.954	0
0.5	0.4	900	0.940	1.46

where the optimal α-expected efficient value is the maximum one of the five α-expected efficient values in Table 6. Generally, findings from the above tables can be summarized as follows:

Finding 1: Table 3 indicates that the predicted efficiencies of the five DMUs (gas stations) become larger as the risk criterion increase.

Finding 2: Table 4 indicates the high correlation rates have obtained for both periods (0.732 and 0.957) between predicted and real efficiency scores; it can represents the validity of the proposed DRF-DEA model (22).

Finding 3: The comparsion between predicted and real efficienies scores in Table 4 reveals significant improvement in discriminating power.

Finding 4: Table 5 indicates that our hybrid algorithm outperforms the Qin's hybrid GA algorithm in terms of runtime and solution quality.

Finding 5: It can be seen from Table 6 that the relative errors do not exceed 1.67%, which implies that the our MC-GA algorithm is robust for parameters selection and effective for solving the final proposed DRF-DEA model.

8. Conclusions

This paper attempted to present a new random fuzzy DEA model with common weights under mean chance constraints and the expected values of objective functions in dynamic environment; in which data were changing sequentially. The proposed DRF-DEA model incorporates future information on outputs and inputs into its analytical framework to predict efficiency scores of DMUs for the next financial periods. The major results of the paper include the following : (i) A new multi-objective DEA model was built in uncertain and dynamic environments, in which the outputs and inputs are characterized by random traingular fuzzy variables with normal distribution. Under this assumption, the initial proposed DRF-DEA model transformed to its equivalent multi-objecyives stochastic model; in which some constraints contain the standard normal distribution function and others are the mathematical expectation for functions of the normal random variable. (ii) In order to improve in computational time during the solution process, the equivalent multi objecyives stochastic model converted to one objective stochastic model (final repesentation of DRF-DEA model) by fuzzy multiple objectives linear programming approach. (iii) To solve the final DRF-DEA model, this paper designed a new hybrid GA algorithm by incorporating MC simulation and GA, in which the MC simulation was employed to compute the integrals which involved in some of the constraints, while GA was used to find the optimal solution of problem. To document its practicality, the DRF-DEA model was applied to predict efficiencies for five gas stations in a Iranian petroleum company for the next two financial periods. The computational results showed that our hybrid algorithm outperforms the hybrid algorithm which was proposed by Qin and Liu (2010) in terms of runtime and solution quality.

Acknowledgements

The authors wish to thank the Editor and the referees. This work is supported partially by the Tehran Payam-e-Noor University and Islamic Azad University branch of Qazvin in Iran.

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