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# The Differential Transform Method for Solving the Model Describing Biological Species Living Together

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Abstract. F. Shakeri and M. Dehghan presented the variational iteration method for solving the model describing biological species living together. Here we suggest the differential transform (DT) method for finding the numerical solution of this problem.
To this end, we give some preliminary results of the DT and by proving some theorems, we show that the DT method can be easily applied to mentioned problem. Finally several test problems are solved and compared with variational iteration method.

**Keywords:** Biological species living together, Differential transform method, Volterra integro-differential equations, Variational iteration method.

**2000 Mathematics subject classification:** 65R20

## 1. Introduction

The DT method is a numerical method for solving differential, integral and integro-differential equations. The concept of DT was first introduced by Zhou [15] in 1986 for solving linear and nonlinear initial value problems in electric analysis (see also [5]).
Up to now, the differential transform method has been developed for solving various types of differential and integral equations. In [2, 3], an extension of the DT method has been presented for solving system of differential equations

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and differential-algebraic equations. In [4, 5], this method has been applied for partial differential equations and in [1, 11], for one dimensional Voltrra integral and integro-differential equations. In [12], the generalized form of DT method has been applied to differential equations of fractional order and in [7], to multi order fractional differential equations. Also in [14], the DT method has been developed for solving the two dimensional Volterra integral equations.
The subject of presented paper is to apply the DT method for solving the system of nonlinear Volterra integro-differential equations which obtain in modeling the problem of biological species living together. This system has the following form

$$\begin{cases} \frac{dn_1}{dt} = n_1(t)\left[k_1 - \gamma_1 n_2(t) - \int_{t-T_0}^{t} f_1(t-\tau)n_2(\tau)d\tau\right] + g_1(t) \\ \qquad\qquad , \ k_1, \gamma_1 > 0, \ \ 0 \le t \le l \\ \frac{dn_2}{dt} = n_2(t)\left[-k_2 + \gamma_2 n_1(t) + \int_{t-T_0}^{t} f_2(t-\tau)n_1(\tau)d\tau\right] + g_2(t) \\ \qquad\qquad , \ k_2, \gamma_2 > 0, \ \ 0 \le t \le l \end{cases} \tag{1.1}$$

with the supplementary conditions

$$n_1(0) = \alpha_1, \qquad n_2(0) = \alpha_2, \tag{1.2}$$

where $f_1$, $f_2$, $g_1$ and $g_2$ are given functions while $n_1$ and $n_2$ are unknown functions, and $T_0 \epsilon R$. This system is obtained from mathematical modeling of the problem of biological species living together (for more information see [9]).
The rest of this paper organized as follows. In Section 2, we introduce the DT and give some preliminary results of this method. In Section 3, we prove some theorems for developing the DT for (1.1), then we describe the method. In Section 4, we give some numerical examples to present a clear overview of discussion. In Section 5, a conclusion of this paper is given.

## 2. Preliminary results of the differential transform

The basic definition of DT and corresponding fundamental theorems can be found in [1-5], [11] and [15], however for convenience of the reader, in this section we present a review of the DT. We define differential transform of the function $f(x)$ (see [11]) in $x_0 = \alpha$ as

$$F_\alpha(n) = \frac{1}{n!}\left[\frac{d^n f(x)}{dx^n}\right]_{x=\alpha} \tag{2.1}$$

then its inverse transform is defined as

$$f(x) = \sum_{n=0}^{\infty} F_\alpha(n)(x-\alpha)^n. \tag{2.2}$$

The relations (2.1) and (2.2) imply that

$$f(x)=\sum_{n=0}^{\infty}\frac{1}{n!}\left[\frac{d^n f(x)}{dx^n}\right]_{x=\alpha}(x-\alpha)^n \tag{2.3}$$

which is the Taylor series of function $f(x)$.
In the following theorem, we summarize some fundamental properties of the differential transform (see [11]).

**Theorem 2.1.** *If $F_0(n)$, $U_0(n)$ and $V_0(n)$ are the differential transforms of functions $f(x)$, $u(x)$ and $v(x)$ in $x_0=0$ respectively, then*
**a.** *If $f(x)=u(x)\pm v(x)$ then*

$$F_0(n)=U_0(n)\pm V_0(n).$$

**b.** *If $f(x)=au(x)$ then*

$$F_0(n)=aU_0(n).$$

**c.** *If $f(x)=u(x)v(x)$ then*

$$F_0(n)=\sum_{k=0}^{n}U_0(k)V_0(n-k).$$

**d.** *If $f(x)=x^k$ then*

$$F_0(n)=\delta_{n,k}.$$

**e.** *If $f(x)=sin(ax+b)$ then*

$$F_0(n)=\frac{a^n}{n!}sin(\frac{n\pi}{2}+b).$$

**f.** *If $f(x)=cos(ax+b)$ then*

$$F_0(n)=\frac{a^n}{n!}cos(\frac{n\pi}{2}+b).$$

**g.** *If $f(x)=e^{ax}$ then*

$$F_0(n)=\frac{a^n}{n!}.\qquad\square$$

We also recall the following theorem from [4] to apply the DT method for the differential parts of (1.1).

**Theorem 2.2.** *If $F_0(n)$, $U_0(n)$ and $V_0(n)$ are the differential transforms of functions $f(x)$, $u(x)$ and $v(x)$ in $x_0=0$ respectively, then*
**a.** *If $f(x)=\frac{d^r u(x)}{dx^r}$, $\quad r=1,2,\cdots$ then*

$$F_0(n)=(n+1)(n+2)\cdots(n+r)U_0(n+r)$$

**b.** *If $f(x)=\frac{du(x)}{dx}\frac{dv(x)}{dx}$ then*

$$F_0(n)=\sum_{k=0}^{n}(k+1)(n-k+1)U_0(k+1)V_0(n-k+1)\qquad\square$$

## 3. Main results

In this section, we prove some theorems for extension of the DT to the system (1.1).

**Theorem 3.1.** *If $F_\alpha(m)$ is the differential transform of function $f(x)$ in $x_0 = \alpha$, then the differential transform of $f(x)$ in $x_0 = 0$ is*

$$F_0(m) = \sum_{k=m}^{\infty} F_\alpha(k) \binom{k}{m} (-\alpha)^{k-m} \tag{3.1}$$

*Proof.* Since

$$f(x) = \sum_{m=0}^{\infty} F_\alpha(m)(x-\alpha)^m.$$

Therefore

$$f(x) = \sum_{m=0}^{\infty} F_\alpha(m) \left[ \sum_{k=0}^{m} \binom{m}{k} x^k (-\alpha)^{m-k} \right]$$

$$= \sum_{m=0}^{\infty} \left[ \sum_{k=m}^{\infty} F_\alpha(k) \binom{k}{m} (-\alpha)^{k-m} \right] x^m$$

and (3.1) is obtained. □

**Theorem 3.2.** *If $F_0(m)$ is the differential transform of function $f(x)$ in $x_0 = 0$, then the differential transform of $f(x)$ in $x_0 = \alpha$ is*

$$F_\alpha(m) = \sum_{k=m}^{\infty} F_0(k) \binom{k}{m} \alpha^{k-m} \tag{3.2}$$

*Proof.* We have

$$f(x) = \sum_{m=0}^{\infty} F_0(m) x^m = \sum_{m=0}^{\infty} F_0(m)((x-\alpha)+\alpha)^m$$

and similar to the previous theorem, the result can be obtained. □

**Theorem 3.3.** *If $h(t) = \int_0^t f(t-\tau)n(\tau)d\tau$ then for the differential transform of $h(t)$ in $x_0 = 0$, we have*

$$H_0(0) = 0$$

$$H_0(k) = \sum_{l=0}^{k-1} \frac{l!(k-l-1)!}{k!} F_0(l) N_0(k-l-1), \quad k = 1, 2, \ldots \tag{3.3}$$

*where $F_0$ and $N_0$ are the differential transforms of functions $f(x)$ and $n(x)$ in $x_0 = 0$, respectively.*

*Proof.* We have

$$h(0) = 0 \quad \Rightarrow \quad H_0(0) = 0$$

and

$$h^{(k)}(t) = \int_0^t f^{(k)}(t-\tau)n(\tau)d\tau + \sum_{l=0}^{k-1} f^{(l)}(0)n^{(k-l-1)}(t), \quad k = 1, 2, \ldots$$

therefore

$$H_0(k) = \frac{1}{k!}h^{(k)}(0) = \frac{1}{k!}\sum_{l=0}^{k-1} f^{(l)}(0)n^{(k-l-1)}(0)$$

$$= \frac{1}{k!}\sum_{l=0}^{k-1}\Big[\, l!F_0(l)\Big]\Big[\,(k-l-1)!N_0(k-l-1)\Big]$$

$$= \sum_{l=0}^{k-1}\frac{l!(k-l-1)!}{k!}F_0(l)N_0(k-l-1), \quad k = 1, 2, \ldots$$

so the proof is completed. □

**Theorem 3.4.** *If $h(t) = \int_0^{t-T_0} f(t-\tau)n(\tau)d\tau$ then the differential transform of $h(t)$ in $x_0 = 0$, is of the form*

$$H_0(0) = \sum_{m=1}^{\infty}\sum_{r=0}^{m-1}\sum_{l=r}^{\infty}\frac{l!(m-r-1)!}{m!(l-r)!}F_0(l)N_0(m-r-1)T_0^{l-r}(-T_0)^m \qquad (3.4)$$

*and*

$$H_0(k) = \sum_{m=k}^{\infty}\sum_{r=0}^{m-1}\sum_{l=r}^{\infty}\frac{l!(m-r-1)!}{k!(m-k)!(l-r)!}F_0(l)N_0(m-r-1)T_0^{l-r}(-T_0)^{m-k} \qquad (3.5)$$

*for $k = 1, 2, \ldots$ .*

*Proof.* By definition of $h(t)$ we have

$$h(T_0) = 0 \quad \Rightarrow \quad H_{T_0}(0) = 0 \qquad (3.6)$$

and

$$h^{(m)}(t) = \int_0^{t-T_0} f^{(m)}(t-\tau)n(\tau)d\tau + \sum_{r=0}^{m-1} f^{(r)}(T_0)n^{(m-r-1)}(t-T_0), \;\; m = 1, 2, \ldots$$

therefore

$$H_{T_0}(m) = \frac{1}{m!}h^{(m)}(T_0) = \frac{1}{m!}\sum_{r=0}^{m-1} f^{(r)}(T_0)n^{(m-r-1)}(0)$$

$$= \frac{1}{m!}\sum_{r=0}^{m-1}\Big[\, r!F_{T_0}(r)\Big]\Big[\,(m-r-1)!N_0(m-r-1)\Big], \quad m = 1, 2, \ldots$$

and by substituting $F_{T_0}(r)$ of (3.2)

$$H_{T_0}(m) = \frac{1}{m!}\sum_{r=0}^{m-1} r!(m-r-1)!\left[\sum_{l=r}^{\infty} F_0(l)\binom{l}{r}T_0^{l-r}\right]N_0(m-r-1) \quad (3.7)$$

for $m = 1, 2, \dots$ .
Therefore by theorem 3.1 we have

$$H_0(k) = \sum_{m=k}^{\infty} H_{T_0}(m)\binom{m}{k}(-T_0)^{m-k}$$

$$= \sum_{m=k}^{\infty}\sum_{r=0}^{m-1}\sum_{l=r}^{\infty}\frac{r!(m-r-1)!}{m!}\binom{m}{k}\binom{l}{r}F_0(l)N_0(m-r-1)T_0^{l-r}(-T_0)^{m-k}$$

$$= \sum_{m=k}^{\infty}\sum_{r=0}^{m-1}\sum_{l=r}^{\infty}\frac{l!(m-r-1)!}{k!(m-k)!(l-r)!}F_0(l)N_0(m-r-1)T_0^{l-r}(-T_0)^{m-k}, \ k = 1, 2, \dots.$$

Also note that for $k = 0$ by substituting from (3.6) into (3.1) we can write

$$H_0(0) = \sum_{m=1}^{\infty} H_{T_0}(m)(-T_0)^m$$

and by substituting from (3.7), the relation (3.4) is obtained. □

Now we can obtain the differential transform of the system (1.1) in $x_0 = 0$. First note that in the remaining part of this paper we assume that all differential transforms are in $x_0 = 0$, hence for the sake of simplicity, we denote all differential transforms without the zero index.
For simplicity, we also set

$$h_1(t) = \int_0^t f_1(t-\tau)n_2(\tau)d\tau, \quad h_2(t) = \int_0^{t-T_0} f_1(t-\tau)n_2(\tau)d\tau$$

and

$$h_3(t) = \int_0^t f_2(t-\tau)n_1(\tau)d\tau, \quad h_4(t) = \int_0^{t-T_0} f_2(t-\tau)n_1(\tau)d\tau$$

so

$$\int_{t-T_0}^{t} f_1(t-\tau)n_2(\tau)d\tau = h_1(t) - h_2(t)$$

$$\int_{t-T_0}^{t} f_2(t-\tau)n_1(\tau)d\tau = h_3(t) - h_4(t)$$

therefore the system (1.1) can be written as

$$\begin{cases} \frac{dn_1}{dt} = k_1 n_1(t) - \gamma_1 n_1(t)n_2(t) - n_1(t)h_1(t) + n_1(t)h_2(t) + g_1(t) \\ \qquad , \ k_1, \gamma_1 > 0, \quad 0 \le t \le l \\ \\ \frac{dn_2}{dt} = -k_2 n_2(t) + \gamma_2 n_1(t)n_2(t) + n_2(t)h_3(t) - n_2(t)h_4(t) + g_2(t) \\ \qquad , \ k_2, \gamma_2 > 0, \quad 0 \le t \le l \end{cases}$$

and by theorems 2.1 and 2.2 the differential transform of it is

$$\begin{cases} (n+1)N_1(n+1) = k_1N_1(n) - \gamma_1 \sum_{k=0}^{n} N_1(k)N_2(n-k) \\ \qquad - \sum_{k=0}^{n} H_1(k)N_1(n-k) + \sum_{k=0}^{n} H_2(k)N_1(n-k) + G_1(n) \\ \\ (n+1)N_2(n+1) = -k_2N_2(n) + \gamma_2 \sum_{k=0}^{n} N_1(k)N_2(n-k) \\ \qquad + \sum_{k=0}^{n} H_3(k)N_2(n-k) - \sum_{k=0}^{n} H_4(k)N_2(n-k) + G_2(n) \end{cases}$$

where $N_1, N_2, H_1, H_2, H_3, H_4, G_1$ and $G_2$ denote the differential transforms of functions $n_1, n_2, h_1, h_2, h_3, h_4, g_1$ and $g_2$ in $x_0 = 0$, respectively.
By substituting $H_1(k)$ and $H_3(k)$ from theorem 3.3 and $H_2(k)$ and $H_4(k)$ from theorem 3.4 we obtain

$$\begin{cases} (n+1)N_1(n+1) - k_1N_1(n) + \gamma_1 \sum_{k=0}^{n} N_1(k)N_2(n-k) + \sum_{k=0}^{n} \sum_{l=0}^{k-1} \\ \left(\frac{l!(k-l-1)!}{k!} F_1(l)N_2(k-l-1)N_1(n-k)\right) - \sum_{k=0}^{n} \sum_{m=k}^{\infty} \sum_{r=0}^{m-1} \sum_{l=r}^{\infty} \\ \left(\frac{l!(m-r-1)!}{k!(m-k)!(l-r)!} F_1(l)N_1(n-k)N_2(m-r-1)T_0^{l-r}(-T_0)^{m-k}\right) - G_1(n) = 0 \\ \\ (n+1)N_2(n+1) + k_2N_2(n) - \gamma_2 \sum_{k=0}^{n} N_1(k)N_2(n-k) - \sum_{k=0}^{n} \sum_{l=0}^{k-1} \\ \left(\frac{l!(k-l-1)!}{k!} F_2(l)N_1(k-l-1)N_2(n-k)\right) + \sum_{k=0}^{n} \sum_{m=k}^{\infty} \sum_{r=0}^{m-1} \sum_{l=r}^{\infty} \\ \left(\frac{l!(m-r-1)!}{k!(m-k)!(l-r)!} F_2(l)N_1(m-r-1)N_2(n-k)T_0^{l-r}(-T_0)^{m-k}\right) - G_2(n) = 0 \end{cases} \tag{3.8}$$

for $n = 0, 1, ..., N-1$.
We also have from initial conditions

$$N_1(0) = \alpha_1, \quad N_2(0) = \alpha_2.$$

If we set $N$ instead of $\infty$, a nonlinear algebraic system of equations is obtained and by solving this system, the unknowns $N_1(1), N_1(2), ..., N_1(N), N_2(1), N_2(2), ..., N_2(N)$ are obtained.
Finally we use the truncated form

$$n_i(t) = \sum_{n=0}^{N} N_i(n)t^n, \qquad i = 1, 2 \tag{3.9}$$

to get approximate solution of (1.1) and (1.2).

## 4. Numerical Examples

In this section, we give some examples of [13] to clarify accuracy of the presented method. The results also are compared with variational iteration method of [13].
All computations were done by programming in Maple software.

**Example 4.1.** Consider the system of integro-differential equations (1.1) and (1.2) with

$$f_1(t) = 1, \quad f_2(t) = t - 1$$

$$k_1 = 1, \quad k_2 = 2$$

$$\gamma_1 = \frac{1}{3}, \quad \gamma_2 = 1$$

$$T_0 = \frac{1}{2}$$

$$\alpha_1 = 1, \quad \alpha_2 = 0$$

$$g_1(t) = -\frac{5}{2}t^3 + \frac{49}{12}t^2 + \frac{17}{12}t - \frac{23}{6}$$

and

$$g_2(t) = \frac{15}{8}t^3 - \frac{1}{4}t^2 + \frac{3}{8}t - 1,$$

with the exact solution as $n_1(t) = -3t + 1$ and $n_2(t) = t^2 - t$.

By solving the system (3.8) with this data for $N = 3$, we obtain approximate solution as

$$n_1(t) = 1 - 3t + 0.166963 \times 10^{-19}t^2 - 0.656782 \times 10^{-20}t^3$$

$$n_2(t) = -t + t^2 - 0.571352 \times 10^{-21}t^3$$

which is indeed the exact solution of the problem.
For comparing, we give the results obtained in [13] by variational iteration method in Table 1. This table shows the absolute errors(A.E.) for $n_1(t)$ and

$n_2(t)$ in some points.

Table 1: Numerical results of [13] for example 4.1.

| $t$ | $A.E.(n_1)$ | $A.E.(n_2)$ |
|---|---|---|
| 0.1 | $0.315188e-3$ | $0.334119e-4$ |
| 0.2 | $0.427289e-3$ | $0.854529e-4$ |
| 0.3 | $0.472313e-3$ | $0.133352e-3$ |
| 0.4 | $0.485540e-3$ | $0.179896e-3$ |
| 0.5 | $0.474363e-3$ | $0.222780e-3$ |
| 0.6 | $0.445981e-3$ | $0.237116e-3$ |
| 0.7 | $0.436823e-3$ | $0.162689e-3$ |
| 0.8 | $0.535814e-3$ | $0.107083e-3$ |
| 0.9 | $0.910002e-3$ | $0.731024e-3$ |
| 1.0 | $0.182947e-2$ | $0.190776e-2$ |

Note that the solution obtained by the differential transform method (DTM) at all of the above points is exact (errors are equal to zero). □

**Example 4.2.** As second example, consider the system (1.1) and (1.2) with

$$f_1(t) = 2t - 3, \quad f_2(t) = t$$

$$k_1 = 2, \quad k_2 = 2$$

$$\gamma_1 = 1, \quad \gamma_2 = 1$$

$$T_0 = \frac{1}{3}$$

$$\alpha_1 = 0, \quad \alpha_2 = 0$$

$$g_1(t) = t^2\left(2 - 3te^{-t} - \frac{7}{2}e^{-t} + \frac{13}{6}te^{\frac{1}{3}-t} + \frac{22}{8}e^{\frac{1}{3}-t}\right) - 2t$$

and

$$g_2(t) = \frac{1}{648}e^{-t}\left(342t^3 - 8t^2 + 325t + 324\right)$$

with the exact solution $n_1(t) = -t^2$ and $n_2(t) = \frac{1}{2}te^{-t}$. Table 2 shows the absolute errors(A.E.) for $n_1(t)$ and $n_2(t)$ by the DTM and variational iterative method (VIM) from [13].

Table 2: Numerical results of example 4.2.

| $t$ | $A.E.n_1(VIM)$ | $A.E.n_1(DTM)$ | $A.E.n_2(VIM)$ | $A.E.n_2(DTM)$ |
|---|---|---|---|---|
| 0.1 | $0.450227e-9$ | $0.394175e-9$ | $0.980983e-7$ | $0.162746e-11$ |
| 0.2 | $0.407215e-8$ | $0.370379e-8$ | $0.693367e-7$ | $0.638333e-10$ |
| 0.3 | $0.472344e-7$ | $0.159002e-7$ | $0.269708e-6$ | $0.319748e-9$ |
| 0.4 | $0.364798e-6$ | $0.494973e-7$ | $0.355407e-6$ | $0.105576e-8$ |
| 0.5 | $0.203596e-5$ | $0.127306e-6$ | $0.249470e-5$ | $0.287591e-8$ |
| 0.6 | $0.880599e-5$ | $0.286453e-6$ | $0.108724e-4$ | $0.699789e-8$ |
| 0.7 | $0.312110e-4$ | $0.582449e-6$ | $0.385228e-4$ | $0.159106e-7$ |
| 0.8 | $0.944467e-4$ | $0.109259e-5$ | $0.114881e-3$ | $0.350059e-7$ |
| 0.9 | $0.251619e-3$ | $0.191171e-5$ | $0.300927e-3$ | $0.765393e-7$ |
| 1.0 | $0.604002e-3$ | $0.317562e-5$ | $0.711284e-3$ | $0.168549e-6$ |

The results show the high accuracy of DTM. □

**Example 4.3.** We consider the third case of system (1.1) and (1.2) with

$$f_1(t) = 1, \quad f_2(t) = e^{-t}$$

$$k_1 = \frac{1}{3}, \quad k_2 = \frac{1}{2}$$

$$\gamma_1 = 2, \quad \gamma_2 = 1$$

$$T_0 = \frac{3}{10}$$

$$\alpha_1 = 0, \quad \alpha_2 = 0$$

$$g_1(t) = \frac{1}{4}cost - \frac{1}{4}sint\left(\frac{1}{3} + \frac{1}{2}sint - \frac{1}{4}cost + \frac{1}{4}cos(t - \frac{3}{10})\right)$$

and

$$g_2(t) = -\frac{1}{4}cost + \frac{1}{4}sint\left(-\frac{1}{2} + \frac{3}{8}sint - \frac{1}{8}cost + \frac{1}{8}e^{-\frac{3}{10}}\left(cos(t - \frac{3}{10}) - sin(t - \frac{3}{10})\right)\right).$$

The exact solution of this problem is $n_1(t) = \frac{1}{4}sint$ and $n_2(t) = -\frac{1}{4}sint$. Table 3 shows the absolute errors in points

$$x = (0.1)i, \quad i = 1, 2, \cdots 10.$$

for DTM and VIM.

Table 3: Numerical results of example 4.3.

| $t$ | $A.E.n_1(VIM)$ | $A.E.n_1(DTM)$ | $A.E.n_2(VIM)$ | $A.E.n_2(DTM)$ |
|---|---|---|---|---|
| 0.1 | $0.522144e-9$ | $0.105188e-9$ | $0.463282e-9$ | $0.712377e-10$ |
| 0.2 | $0.623816e-8$ | $0.216504e-8$ | $0.288070e-8$ | $0.149437e-8$ |
| 0.3 | $0.152983e-6$ | $0.911417e-8$ | $0.752128e-7$ | $0.610308e-8$ |
| 0.4 | $0.124549e-5$ | $0.251960e-7$ | $0.609076e-6$ | $0.158714e-7$ |
| 0.5 | $0.612075e-5$ | $0.582985e-7$ | $0.297314e-5$ | $0.331595e-7$ |
| 0.6 | $0.220311e-4$ | $0.124355e-6$ | $0.106280e-4$ | $0.613616e-7$ |
| 0.7 | $0.639936e-4$ | $0.254473e-6$ | $0.306475e-4$ | $0.106237e-6$ |
| 0.8 | $0.158733e-3$ | $0.505411e-6$ | $0.754263e-4$ | $0.178142e-6$ |
| 0.9 | $0.348563e-3$ | $0.973945e-6$ | $0.164226e-3$ | $0.295292e-6$ |
| 1.0 | $0.694383e-3$ | $0.181554e-5$ | $0.324140e-3$ | $0.488066e-6$ |

Comparing the results show high accuracy of the DTM in this example too.□

**Example 4.4.** Finally consider the system (1.1) and (1.2) with

$f_1(t) = t, \quad f_2(t) = t{+}1$

$k_1 = 1, \quad k_2 = 1$

$\gamma_1 = \frac{1}{2}, \quad \gamma_2 = 3$

$T_0 = \frac{1}{4}$

$\alpha_1 = 0, \quad \alpha_2 = -1$

$$g_1(t) = 2t{-}1{-}(t^2{-}t)\left(1 + \frac{11}{18}e^{-3t} - \frac{1}{36}e^{\frac{3}{4}-3t}\right)$$

and

$$g_2(t) = \frac{1}{3072}e^{-3t}\left(10080t^2 - 10304t + 6275\right)$$

and exact solution $n_1(t) = t^2 - t$ and $n_2(t) = -e^{-3t}$.

Table 4 shows the results.

Table 4: Numerical results of example 4.4.

| $t$ | $A.E.n_1(VIM)$ | $A.E.n_1(DTM)$ | $A.E.n_2(VIM)$ | $A.E.n_2(DTM)$ |
|---|---|---|---|---|
| 0.1 | $0.359861e-6$ | $0.200346e-7$ | $0.109370e-4$ | $0.923480e-8$ |
| 0.2 | $0.266169e-6$ | $0.279956e-6$ | $0.155271e-4$ | $0.424967e-8$ |
| 0.3 | $0.466523e-6$ | $0.101643e-5$ | $0.822708e-5$ | $0.258151e-7$ |
| 0.4 | $0.164191e-4$ | $0.240353e-5$ | $0.926457e-4$ | $0.889430e-7$ |
| 0.5 | $0.693119e-4$ | $0.456668e-5$ | $0.395232e-3$ | $0.174112e-6$ |
| 0.6 | $0.173602e-3$ | $0.754251e-5$ | $0.937210e-3$ | $0.242934e-6$ |
| 0.7 | $0.323762e-3$ | $0.110685e-4$ | $0.162796e-2$ | $0.187246e-6$ |
| 0.8 | $0.492632e-3$ | $0.139459e-4$ | $0.226796e-2$ | $0.331710e-6$ |
| 0.9 | $0.641424e-3$ | $0.125358e-4$ | $0.263143e-2$ | $0.238148e-5$ |
| 1.0 | $0.737216e-3$ | $0.222504e-4$ | $0.257459e-2$ | $0.913622e-5$ |

The above results show the high accuracy of the DTM with respect to VIM. □

## 5. Conclusion

Differential transform method has been successfully applied for solving a nonlinear system of Volterra integro-differential equations which describe biological species living together. As examples show the presented method has a high accuracy and a simple structure. Therefore this method is recommended for solving similar problems in applied science and engineering. For example the Schrodinger equation [8], the Fisher-like equation [6] and the Burgers' equation [10] can be solved by DTM.

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