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A State of the Art Review of the Finite Element Modelling of Rolling Tyres

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ABSTRACT

This paper is devoted to review of the publications on the finite element modelling of rolling tyres performed during the past two decades. Starting with the description of the scope and motivation behind this work, a brief history of the finite element modelling of tyres is given. Then, a comprehensive review on the published works on modelling techniques of the simulation of the rolling tyres is presented. The applied aspects of the subject including the energy loss (rolling resistance) and temperature prediction, interaction between tyre and road, noise, failure, and stability are considered too. It is then followed by introducing the most applied computer codes available in the numerical and computational market. A summary on the published works with particular focus to compare different developed methods and especially the current situation of the applied aspect of the finite element modelling of rolling tyres are presented and discussed. Finally, a conclusion with some remarks is drawn.

Key Words:

tyre;
review;
rolling;
finite element;
simulation.

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INTRODUCTION

Tyre as one of the most important components of vehicles requires to fulfil a fundamental set of functions as follows [1]:

- Provide load-carrying capacity
- Provide cushioning and dampening
- Transmit driving and braking torque
- Provide cornering force
- Provide dimensional stability
- Resist abrasion
- Generate steering response
- Have low rolling resistance
- Provide minimum noise and minimum vibration
- Durability throughout the expected life span

Figure 1 shows various parts of a typical modern radial tyres used in passenger cars. The mechanical properties of a tyre describe the tyre's characteristics in response to the application of load, torque, and steering input, resulting in the generation of external forces and deflection. Such mechanical properties are interrelated, and thus a design decision affecting one factor will influence the other factors, either positively or negatively [1].

Knowledge of how the tyre operates can give engineers insight into design considerations. In order to achieve these tasks many investigators have tried during the past three decades to develop robust mathematical models and simulation schemes for the description of kinematics and dynamics of the rolling of pneumatic tyres on rigid and non-rigid surfaces. Most of these models are based on the use of finite element (FE) technique. However, modelling of a tyre in rolling state by finite element method poses certain difficulties that prevents researcher to adopt a standard and unique procedure in achieving an accurate and reliable solution for a specific problem.

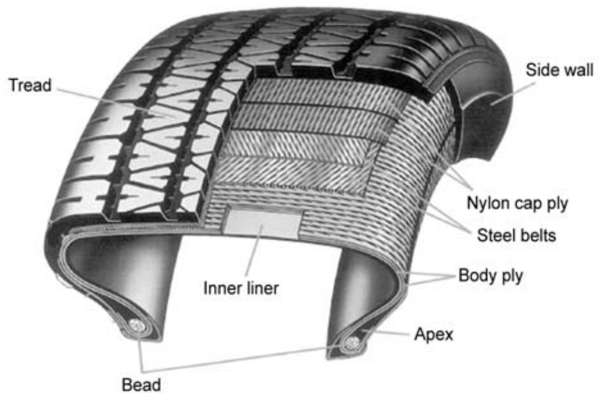

Figure 1. A cross-sectional view of a radial tyre with its components.

Therefore, in this paper, we have reviewed the published works devoted to finite element modelling of the rolling tyres. Our main objective is to give an overall perspective of this intricate engineering field that may help researchers and/or engineers on selecting correct steps towards obtaining a precise and applicable solution for their problems. Furthermore, in order to provide a better viewpoint of the applied works accomplished in this area, we also present the results of the computer simulation of a radial tyre under steady rolling condition carried out by a powerful finite element code. This will tie a close connection between what the reader finds and the practical results which can lead to application of the available algorithms and codes in a modern tyre design cycle.

Scope and Motivation

A literature survey clearly reveals that there exist a large number of published works in journals and conference proceedings in the field of structural analysis of pneumatic tyres by computer simulation. In addition, there are also many unpublished papers in this field generated by R&D and technical departments of tyre companies. This obviously shows that the computer simulation of this complicated structure is still of interest to researchers and engineers involved in tyre design and optimization. The main objective is to develop more realistic models under different rolling conditions. There are different methods developed so far to tackle the rolling tyre problem. This is mainly due to the complex structure of the tyres and also non-linear nature of the deformation as well as complicated loads applied on tyre in service.

Investigations have shown that there are generally two approaches available. In the first one, the kinematics and finite element modelling have been studied. For example, type of framework such as Lagrangian or ALE (Arbitrary Lagrangin Eulerian), material models (linear versus non-linear elastic, viscoelastic), type of time domain (transient versus steady-state) and type of analysis (isothermal versus non-isothermal or thermomechanical) constitutes part of the researchers' point of view on this subject. The second approach, however, is focused on the applied aspects of this type of simulation, i.e., temperature prediction, calculation of the rolling resistance, and interaction between tyre and soft road (soil, snow, etc.). It is therefore the purpose of this review to present a comprehensive state of the art on the finite element modelling of rolling tyre. It is tried here to consider all aspects of this complicated and interesting engineering subject based on up-to-date published papers and monographs.

In the following sections, we first give a brief description on the finite element analysis of pneumatic tyres, and then, the simulation of the rolling tyres which is our main spotlight. The main challenge here is to cover the different techniques presented by researchers to find the best solution for a rotating body in contact with road. Some practical results of the finite element modelling of the rolling of a passenger car radial tyre constitute this section as well. To continue, we turn our attention to the applied aspects and thus review those papers which deal with application of the rolling tyre models. The fourth section is totally devoted to the current commercial finite element codes with potential analysis of a tyre rolling problem. In the fifth section, a concise summary is given on the work in conjunction with the current state of the finite element modelling of rolling tyres. Finally, to complete this work, we conclude our review in the sixth section and cited references are given at the end.

History of the Finite Element Modelling of Pneumatic Tyres

The first application of the finite element method in tyre industries dates back to early 1970s. With the emergence of the journal of *Tire Science and Technology* since 1973, research papers in this field have begun to be published. Zorowski [2] has presented the idea of using finite element method for the study of the tyre dynamic behaviour. Later, Ridha [3] developed a linear finite element model for determining tyre deformation due to shrinkage. The analysis was applied to relate the mould shape to the final shape of the tyre. Despite the apparent symmetrical shape of tyres that may facilitate the use of two-dimensional models, three-dimensional analyses are generally required to achieve accurate and complete results. Consequently, the development of the finite element formulation in this field has been based on the creation of both 2D and 3D models. Two-dimensional models are easy to create but their applicability is limited to those cases in which the applied loads remain in the tyre meridional plane such as shrinkage upon mould opening, applying inflation pressure and rim mounting (e.g., Kennedy et al. [4], DeEskinazi et al. [5], Patel et al. [6] and more recently Ghoreishy [7,8]).

On the other hand, the process of developing three-dimensional models along with the interpretation of the results is time-consuming and a relatively enduring task. However, all tyre loading cases can be virtually simulated using these models and thus much applied results are obtained especially when dealing with rolling problems. An early work in this field was performed by DeEskinazi et al. [9] in which they developed a finite element procedure for the displacement and stress analysis of the homogeneous and isotropic inner tube mounted on a rim and in contact with a flat surface. The geometrical non-linear effect was taken into account by including the quadratic terms in the strain-displacement equations. They showed that the computer results are in reasonable agreement with the experimental data.

Modelling the tyre under static vertical load (footprint analysis) constituted the next stage of development. Having obtained the results of the analysis under the load of vehicle weight, researchers tried to develop more realistic formulations for tyre structural analysis. Many of these works particularly those published in the last decade are dealt with rolling problems which is reviewed in the following sections. It should be, however, noted that the development of finite element analyses under static loads are still being performed for specific purposes (e.g., Ghoreishy [10] and Yanjin et al. [11]).

SURVEY ON FE MODELLING OF ROLLING TYRES

In this section we focus solely on the development of the various techniques for the simulation of the rolling tyres. An early attempt in this field belongs to

Padovan et al. [12]. In order to solve the rolling tyre problem, a model based on moving total Lagrangian coordinates was created. It was a so-called travelling Hughes type contact strategy. Employing the modified contact scheme in conjunction with a travelling finite element strategy, an overall solution methodology was developed to handle transient and steady viscoelastic rolling contact. To verify the scheme, the results of both experimental and analytical benchmarking were presented. The experimental benchmarking included the handling of rolling tyres up to their upper bound behaviour, namely the standing wave response.

Watanabe [13] presented a finite element model for camber thrust analysis in a bias-ply motorcycle tyre. By inclining and pressing this finite element model onto the road plane, the forces prevailing in the contact patch of an inclined standing tyre were calculated and the asymmetry-dependent mechanism of camber thrust generation was analyzed. Then, by rotating the inclined standing tyre at a constant vertical deflection, the roll-distance-dependent mechanism of camber thrust generation was also analyzed.

Padovan et al. [14] developed a number of finite element modelling schemes enabling the simulation of rolling tyres. In particular, the moving Lagrangian type strategy was extended to 3D type isoparametric elements. Because of its generality, the formulation could handle kinematic and material non-linearity, as well as viscoelastic behaviour. To benchmark the procedure, a full tyre simulation was treated.

Kennedy et al. [15] also analyzed a radial automobile tyre undergoing steady-state rotation by a finite element method. A special formulation was used which allowed the finite element equations to be solved as a quasi-static problem using static analysis solution procedures, rather than as a dynamic problem requiring solution in the time domain. This was accomplished through a transformation of variable that changes time derivatives, present through inertia, to spatial derivatives. The solution time for analysis was thereby considerably shortened. The tyre was first modelled as a two-dimensional ring on an elastic foundation. Then, its full three-dimensional geometry was created. Rotational speeds, at which resonance occurs, have been computed so that the dynamics can be easily studied and the response may be verified. The models were subjected to point load excitation or ground contact.

Nakajima et al. [16] developed a generalized finite element methodology to handle travelling load problems involving large deformation fields in structure composed of viscoelastic media. The main thrust of their work was to develop an overall finite element methodology and associated solution algorithms to handle the transient aspects of moving problems involving contact impact type loading fields.

Based on methodologies and algorithms formulation, several numerical experiments were considered, including the rolling/sliding impact of tyres with road obstructions. Oden et al. [17] developed mathematical models of finite deformation of a rolling viscoelastic cylinder in contact with a rough foundation in preparation for a general model of rolling tyres. Variational principles and finite element models were derived. Numerical results were obtained for a variety of cases, including that of a pure elastic rubber cylinder, a viscoelastic cylinder, the development of standing waves, and frictional effects.

Faria et al. [18] presented a steady state formulation of the rolling contact problem with friction that allows the analysis of free rolling, cornering, acceleration, and braking. The formulation was applied to the finite element analysis of tyres. A layered shell finite element with shear deformation that allows for large deflection and rotation was developed. In each layer, orthotropic Hookean materials or Mooney-Rivlin type materials with fibre reinforcements can be used and the incompressibility constraint is enforced with Lagrange multipliers. The contact constraint was enforced with a penalty and the friction term, instead of the usual Coulomb friction law was regularized by a differentiable form that makes it more suitable for numerical analysis. The work was consisted as a numerical example of the developed model.

Kazempour et al. [19,20] employed an instantly centered moving Lagrangian observer (ICMLO) to develop a finite element scheme and associated solution algorithms for the modelling of rotating components, each with its own distinct rotational history. The methodology can handle the steady and transient response of ground based automotive type vehicular systems, including the modelling of roadway-multiple tyre-suspension-vehicular structural interactions during: (i) steady rolling, (ii) obstacle envelopment, and (iii) motion on generalized trajectories.

In particular, the simulation accounts for such aspects as: (i) contact impact, (ii) rolling friction, and (iii) potential lift-off, free motion, and re-contact during obstacle envelopment. To illustrate the scheme, a full vehicular simulation was presented. Faria et al. [21] extended their previous models [18] for the simulation of large deformation steady state behaviour of tyre structures. Their new developments include the extension of the material modelling capabilities to consider viscoelastic materials and a generalization of the formulation of the rolling contact problem to include special non-linear constraints. These constraints include normal contact load, applied torque, and constant pressure-volume.

Several new test problems and examples of analysis were carried out for a P195/70R14 radial tyre. Padovan et al. [22] tried to modify the existing formulations to develop more direct cost-effective solution of the steady rolling contact critical speeds of aircraft, truck, and automotive tyres. Their work was concentrated on the modification of the numerical algorithms used for this type of problems.

Mousseau et al. [23] implemented the ring on elastic foundation model (REF) in a non-linear, finite element program with elements that accommodate large deformation and contact. The main aim was to investigate the suitability of the model to reproduce hub forces during large deformation and quasi-static contact. Two types of tyre contact problems were considered. The first problem was a tyre deforming against a flat plate and the second was a tyre rolling over a 50 mm step. It was shown that the best match with experimental data was accomplished with a model that entirely consisted of beam elements and included a softening foundation. Also, it was shown that the hub force was strongly dependent on the radial stiffness of the foundation and the inflation pressure.

Goldstein [24] used the ABAQUS code to simulate the slow (quasi-static) rolling of a radial truck tyre subjected to ground plane tractions. Three conditions were considered, namely, (1) straight free rolling, (2) cornering, and (3) braking. Lateral and longitudinal slip was calculated by analyzing the motion of a moveable road surface relative to the wheel plane. Footprint moments were calculated for the cornering and braking condition. In addition, cornering stiffness, braking stiffness, and aligning stiffness were calculated and compared to measured results.

Since the mid 1990s researchers have begun to use the commercial finite element programmes in the modelling of rolling tyres. Kao et al. [25] used the LS-DYNA3D, an explicit FE program, to simulate a simple tyre test, demonstrating that it is possible to predict the tyre dynamic responses from the tyre design data. Geometry, material properties of various components and the fibre reinforcement, layout, etc. of a commercial tyre were used to create the tyre FE model. Tyre carcass composite properties were calculated from a strain energy function derived for the fibre-reinforced rubber. The Mooney constitutive law was adopted for the elastic properties of the rubbers. The tyre model was coupled with a rigid wheel model and inflated to a specified inflation pressure. The tyre-wheel model was then loaded against a rotating rigid cylinder with an attached semi-circular cleat. The calculated tyre centre reaction forces showed good correlation with laboratory measurements.

Kamoulakos et al. [26] used the capability of PAM-SHOCK, for studying the transient dynamic responses of a rolling tyre impacting a road imperfection (bump). An industrial test case, that was, a tyre rolling on a spinning drum that contains a cleat, was simulated using PAM-SHOCK. The simulation included pressurization of the tyre, loading the tyre against the drum and progressive rotating of the drum until the tyre model reached a steady state rotation speed corresponding to 30 mph spindle velocity, and finally, impacting of the tyre with the cleat. The excess vertical and horizontal spindle force histories extracted from this simulation were studied and compared against experimental measurements and the similarities between them were identified.

The simulation was further extended for six more repeated impacts corresponding to 21 tyre revolutions to demonstrate the reliability of the programme in providing a stable solution to the tyre impact problem. Koishi et al. [27] have also studied the feasibility of tyre cornering simulation using the explicit finite element code, PAM-SHOCK. To demonstrate the efficiency of the proposed simulation, computed cornering forces for a 175SR14 tyre were compared with experimental results from an MTS Flat-Trac tyre test system. The computed cornering forces agreed well with experimental results. In addition, they have conducted a series of parametric studies by using the proposed simulation.

In a work by Negrus [28], the tyre radial vibrations and the standing waves were studied through finite element modelling and experimental measurements. 3D and 2D finite element tyre models were used in the modal analysis and in the study of tyre circumference deformed shapes at critical velocity of rolling. The experimental research was achieved on a175/70R13 passenger car tyre. Modal analysis was made on the tyre mounted on a common steel rim. The results showed the models employed in the study predicted well the tyre vibration behaviour.

Zhang et al. [29] developed a complete finite element tyre model for the purpose of vehicle dynamics analyses and full vehicle finite element model real time proving ground simulations. The tyre model was validated through simulations of some of the very important global, static, and dynamic mechanical properties such as the tyre radial and lateral stiffness, free-drop test, and low-speed rolling cornering stiffness. The three-dimensional free vibration and harmonic/randomly forced vibrations with ground contact of the tyre model were studied. One of the main purposes of this work was to provide a new approach towards tyre and vehicle NVH studies. All the analyses were nonconventional in the sense that, instead of NASTRAN-type modal analysis used in other works, the explicit non-linear dynamic finite element code LS/DYNA3D was used to conduct all the analyses in the time domain, and the vibration modes were decomposed via fast Fourier transformation.

Zhang et al. [30] studied the effects of tyre properties and their interaction with the ground and the suspension system on vehicle dynamic behaviours using a newly developed finite element analysis method. This analysis method used the explicit non-linear dynamic code LS-DYNA as a solver and contained finite element models for both the vehicle body structure and subsystems like chassis/suspension. The case presented in this paper was curb impact. Different tyre properties such as tyre/wheel assembly mass, tyre stiffness, tyre inflation pressure, tyre size, etc., as well as different curb heights were used with the same vehicle body and suspension system. Simulation results of the impact forces, wheel centre jumps, and vehicle body roll/pitch angles at impact were compared for different parameters of the tyres and the curb. The analyses provided an accurate and practical method for tyre and vehicle dynamics analysis.

Campanac et al. [31] presented the theories of rolling tyre vibrations. In their work the heterogeneity caused by a tread pattern on the tyre belt was introduced and it was shown that vibrations can be described by linear equations with time periodic coefficients. Firstly, the perturbation method was applied for a nearly smooth tyre, and the 'self-excitation' phenomenon, a general feature in time periodic linear systems, was illustrated with the semi-analytic expressions obtained. Then, the generalization to a strong heterogeneity was achieved using the Bloch wave theory. This theoretical background suggests the decomposition of experimental data of noise in time signals for a given phase as compared to the wheel rotation. Finally, an effective method for numerical computations of vibrations was proposed which used the Floquet theory, a consequence of the Bloch theory. Finite element formulation and algorithm were derived for the heterogeneous 'circular ring model'.

Mancosu et al. [32] presented a new 3D mathematical-physical tyre model. The model considered not only the handling behaviour of the tyre but also its comfort characteristics, i.e., the dynamic properties in the lateral and the vertical planes. The model was divided into two parts, the structural model and the contact area model. The structural parameters were identified by comparison with frequency responses of a 3D finite element model of the tyre, whereas the contact parameters were directly calculated with a finite element model of the tread pattern.

The 3D physical model allowed predicting both steady state and transient behaviour of the tyre without the need of any experimental tests on the tyre. The steady state analysis also allowed obtaining the friction circle diagram, i.e., the plot of the lateral force against the longitudinal force for different slip angles and for longitudinal slip, and the Gough plot, i.e., the diagram of the self-aligning torque versus the lateral force. The transient analysis allowed obtaining the dynamic behaviour of the tyre for any maneuver given to the wheel. Among its outputs there were the relaxation length and the dynamic forces and torque transmitted to the suspension of the vehicle. Combining the tyre model with the vehicle model it was possible to perform any kind of maneuver such as overtaking, changing of lane, and steering pad at growing speed with or without braking, or accelerating.

Shiraishi et al. [33] simulated the dynamically rolling tyre by using an explicit finite element method. In their simulation, the complicated pattern shape and internal construction of the tyre were modelled exactly since both these factors are very important for the performance properties of the tyre. The authors described the model used in the simulation and report on the results of several properties under various rolling conditions of the tyre evaluated by this method. The correlation between the simulation and the experiment appeared to be good.

Kabe et al. [34] conducted a comprehensive tyre cornering simulation with implicit and explicit finite element analysis using ABAQUS code under both steady state and transient conditions for a 235/45ZRR17 radial tyre. In the case of implicit FEA (steady state), finite element model of tyre required the fine mesh only in the contact region because of formulation by the moving reference frame technique. On the other hand, for explicit FEA (transient), fine mesh was required in the circumferential direction of tyre.

Predicted cornering forces of passenger car's tyre using implicit/explicit FEM were compared with experimental results obtained from MTS Flat-Test tyre test systems. It was shown that a good correlation exist between predicted and experimental data. In addition, the CPU time for cornering simulation using implicit FEM was shorter than that of explicit FEM.

Bauchau et al. [35] published a paper which was concerned with the modelling of wheels within the framework of finite element-based dynamic analysis of non-linear, flexible multibody systems. The overall approach to the modelling of wheels was broken into four distinct parts: (1) a purely kinematic part describing the configuration of the wheel and contacting plane, (2) a unilateral contact condition giving rise to a contact force, (3) the friction forces associated with rolling and/or sliding, and (4) a model of the deformations in the wheel tyre. The formulation of these various aspects of the problem involves a combination of holonomic and non-holonomic constraints enforced via the Lagrange multiplier technique. The work was developed within the framework of energy-preserving and decaying time integration schemes that provide unconditional stability for non-linear, flexible multibody systems involving wheels. Strategies for dealing with the transitions from rolling to sliding and vice-versa were discussed and found to be more efficient than the use of continuous friction law. Numerical examples are presented that demonstrate the efficiency and accuracy of the proposed approach [35].

Darnell et al. [36] developed a new and special purpose finite element model for simulating tyre spindle force and moment response during side slip. They have considered a vertically loaded tyre deforming laterally on a flat surface and a tyre rolling straight ahead under a prescribed sideslip angle. Experimental data was also presented to verify the force predictions.

Dorsch et al. [37] carried out a series of friction tests over a range of different loading conditions, velocities, and temperatures. They have proposed a phenomenological law for the friction coefficient. It was then incorporated into a finite element programme. Comparison of the experimental data with finite element computations demonstrated the validity of the model. The main applications of the friction model were in simulations of the rolling tyre during braking and/or cornering. The results obtained include global braking and side forces as well as local mechanical processes in the contact patch, for example, the distribution of contact pressure and sliding velocities. From these results, the criteria for important aspects of tyre performance such as wear can be computed.

Zhang et al. [38] employed a non-linear finite element model of a truck tyre to conduct a parametric study on the shear stresses developed in different belt layers. The parametric study incorporated the geometric and anisotropic material properties of the individual layers in the multi-layered system and the orientations of the cords in different layers. These parameters embodied the geometry of the tyre including the aspect ratio, rim radius, and tread depth. The parameters related to structural features and material properties of the individual layers in the belts, such as the cord angle, total number of belt layers under the crown and the number of twisted cords per unit width of an individual layer was also considered. The influence of these parameters on the maximum shear stresses developed in individual belt layers was investigated for a non-rolling radial truck tyre. The analysis was performed using the ANSYS software and the results were used to derive a more desirable set of structural parameters so that the maximum shear stresses in the belt layers of a loaded radial truck tyre can be reduced.

The maximum inter-ply shear stresses computed using the proposed set of parameters was compared with the stresses derived corresponding to the pre-estimated nominal parameters. It was concluded that a tyre designed on the basis of the proposed set of parameters can yield considerably lower maximum shear stresses in the multi-layered system of a radial truck tyre compared to tyres designed by the other conventional methods.

Olatunbosun et al. [39] modelled a rolling of 195/65R15 radial tyre in the time domain against a rotating drum using MSC-NASTRAN code. They have also compared the results with experimental data obtained from the same system.

Chang [40] developed a full non-linear finite element model for a P205/70R14 passenger car radial-ply tyre run on a 1.7 meter-diameter spinning test drum model at a constant speed of 50 km/h in order to investigate the tyre transient response characteristics. The reaction forces of the tyre axle in longitudinal (X axis), lateral (Y axis) and vertical (Z axis) directions were recorded when the tyre rolled over a cleat on the test drum, and then the FFT algorithm was applied to examine the transient response information in the frequency domain. The result showed that this tyre has clear peaks of 45, 40, and 84 Hz transmissibility in the longitudinal, lateral, and vertical directions, respectively. This result was validated by analytical, experimental, and other finite element approaches and showed excellent agreement. The parameters adopted in this tyre model were also compared with experimental work and the extraordinary agreement was also confirmed. The tyre three-dimensional free vibration modes transmissibility was successfully detected virtually.

Duni [41] presented a numerical methodology based on the finite element method used for the transient dynamic simulation of the full vehicle rolling on different kind of obstacles. Some issues related to the tyre finite element model development and its validation, by numerical-experimental comparison, have been discussed. The strategy to combine the static simulations such as the tyre inflating, the vehicle weight application and suspension pre-loading, with transient dynamic analysis of the car rolling over the obstacle has been chosen. The methodology, based on integration of ABAQUS Implicit and Explicit codes has been successfully applied for the dynamic simulation of a car passing over comfort and pothole obstacle.

Liu [42] used the ABAQUS/Explicit to simulate the transient dynamics of a rolling passenger tyre to examine the vibrations induced by the entrance of tread pattern discontinuities into the contact patch. The loaded tyre rolls on a 3.05 m (10 ft) diameter drum at terminal speeds of 4.3 km/h (2.7 mph) and 34.4 km/h (21.4 mph). Three models with augmented discontinuities in the circumferential ribs have been examined: (1) continuous (control), (2) angled lateral grooves, and (3) transverse grooves. They were ordered in increasing severity of rib discontinuity. The induced rolling vibrations were evaluated by identifying the 'perturbed' dynamic responses of (2) and (3) from (1).

The results indicated that any discontinuity or abruptness in tread patterns could induce vibrations which were similar in nature for the two speeds studied. The higher rolling speed would only change the amplitudes, but not the frequencies of the induced oscillations at the free spindle. The perturbed vibrations for both speeds manifest at the spindle as translational (vertical) and rotational (axial) oscillations of 29 and 44 Hz, respectively, for the tyre studied.

Sobhanie [43] also used ABAQUS/Explicit code to simulate the rolling and transient impact of a tyre. The ABAQUS/Explicit modelling results were compared to ABAQUS/Standard results. The comparison included the tyre forces, footprint pressure distribution at a free rolling condition, and resonant frequencies. In addition, the modelling results of a tyre/suspension system traversing an obstacle were presented. The suspension components, except spring and shock were modelled by rigid elements connected together.

Yan [44] also presented a finite element model for the simulation of the steady state rolling of a 9.00R20 truck tyre using an in-house developed computer code. The friction term of the tyre-foundation contact under a steady rolling tyre was treated using the regularized Coulomb's law.

Zheng [45] developed a procedure based on steady state rolling contact finite element analysis (FEM) to predict tyre cross section tread wear profile under specified vehicle driving conditions. The procedure not only considers the tyre construction effects, but also includes the effects of materials, vehicle setup, test course, and driver's driving style. In this algorithm, the vehicle driving conditions were represented by the vehicle acceleration histogram. Vehicle dynamic simulations were done to transform the acceleration histogram into tyre loading condition distributions for each tyre position.

Tyre weight loss rates for different vehicle accelerations were generated based on a steady state rolling contact simulation algorithm. Combining the weight loss rate and the vehicle acceleration histogram, nine typical tyre loading conditions were chosen with different weight factors to represent tyre usage conditions.

It is shown that the tyre tread wear rate profile changes continuously as the tyre is worn. Simulation of a new tyre alone cannot be used to predict the tyre cross-section tread wear profile. For this reason, an incremental tread wear simulation procedure was performed to predict the tyre cross-section tread wear profile. Compared with actual tyre cross-section tread wear profiles, good results were obtained from the simulations.

Chae et al. [46] developed a detailed non-linear finite element model of a radial ply truck tyre using the explicit finite element code, PAM-SHOCK. The tyre model was constructed to its extreme complexity with three-dimensional solid, layered membrane, and beam elements. In addition to the tyre model itself, a rim model was included and rotated with the tyre with proper mass and rotational inertial effects. The in-plane sidewall transitional stiffness and damping constants of the tyre model were determined by rotating the tyre on a cleat-drum. The other constants, such as in-plane rotational stiffness and damping constants, were determined by applying and releasing a tangential force on the rigid tread band of the FE tyre model. The tyre axle, spindle, and reaction force histories at longitudinal and vertical directions were recorded. In addition, the FFT algorithm was applied to examine the transient response in frequency domain. The tyre steering characteristics were also determined.

These parameters were used as input for a simplified rigid ring tyre model. The dynamic responses for the developed tyre model were compared with the dynamics predicted using the rigid ring model. The results showed a successful attempt to capture the transient response of a tyre rolling over a complex road profile.

Hall et al. [47,48] presented the results of the transient macroscopic behaviour of a rolling automobile tyre. The aim of the research has been to develop the modelling methodology for the advanced LS-DYNA finite element simulation of a rolling tyre that can be used to provide internal transient stresses (and strains) to support the development of sensor systems technology. The methodology has been developed for a 195/65R15 tyre, which has provided physical test data to benchmark the performance of the numerical predictions for the free-rolling analysis on a rigid horizontal surface.

Simulation results for a normal load of 3 kN and a speed of 20 km/h were presented to show the characteristics of stresses at the tyre/ground interface and, for the first time, internal stresses and strains at specific locations inside the tyre structure. Such numerical results of the internal behaviour will be valuable to tyre technologists who want to determine the optimal location of their sensor systems, and also to vehicle dynamists who want to establish the dynamic relationships between the contact patch stresses and the global tyre forces.

Nackenhorst [49] has recently presented the idea of Arbitrary Lagrangian/Eulerian (ALE) formulation. Essential differences between the Lagrangian description and the ALE-description of rolling were shown up and discussed in detail. For a comprehensive presentation the finite element discretization is restricted to the case of a deformable wheel rolling on a rigid plane surface. The efficiency of the numerical algorithms developed so far was discussed by the analysis of a simple three-dimensional example.

Olatunbosun et al. [50] presented a three-dimensional finite element tyre model using ABAQUS.

They have focused on some rolling tyre output responses, such as lateral forces and self-aligning moment generated due to steering input during vehicle manoeuvring and vibration responses to road disturbances. The model is also applicable in investigating other tyre design issues such as the heat generated in the carcass of the rolling tyre.

Tan et al. [51] developed an in-house three-dimensional non-linear finite element analysis system with cord-rubber composite material models to simulate and predict mechanical properties of pneumatic tyres under different working conditions. This system has been used in tyre steady rolling analysis, dynamic analysis, rolling resistance and temperature analysis. Some comparisons between tyre finite element analysis and experiment results are also discussed.

The inclusion of the details of the tread pattern in the finite element modelling of the rolling tyres has gained interest more recently. Transient dynamic response of a rolling tyre impacting with a small cleat was analyzed by Cho et al. [52] using an explicit finite element method. A 3D tyre model considering detailed tread blocks was used to accurately simulate the local tyre-cleat impact process. The frictional dynamic contact problem was formulated by making use of total Lagrangian scheme and the penalty method. By imposing mass-proportional damping to only the tyre parts showing the significant lateral deformation, the dynamic viscosity effect was artificially reflected. Time-history and frequency responses of the dynamic forces exerted on the tyre axis were numerically predicted and assessed through the comparison with experimental results. In addition, the effects of the tyre rolling speed and the inflation pressure on the transient dynamic response were parametrically investigated.

Mundl et al. [53] have developed a simulation procedure of a ply steer residual aligning torque (PRAT) using a stationary global rolling finite-element method (FEM) tyre model combined with a detailed local FEM tread pattern model. The simulated results of the PRAT for eight tread pattern variants were compared with measured values of experimental tyres that showed a high correlation. Additionally, simulated local contact stress distributions of the tread blocks on a rolling tyre showed a high degree of similarity compared with measured distributions generated with a specific in-house test drum. The local contact stresses were also mechanically interpreted to obtain a basic understanding of how the PRAT was generated by the tread pattern.

Ghoreishy [54] developed a finite element model for the steady state rolling of a steel-belted radial tyre by the use of ABAQUS code. The model was then employed to study the effect of the belt construction on the tyre mechanical behaviour. The effects of the cord angle and cap ply have also been examined.

Chae et al. [55] have constructed two finite element analysis quarter-vehicle models (QVMs) using developed non-linear 3-and 4-groove tread FEA radial-ply truck tyre models. In addition to the FEA models, a rigid ring QVM was developed to observe the dynamic response of the rigid ring tyre model under the effect of the sprung mass vertical motions. In the rigid ring QVM, the suspension characteristics were similar to that used in the FEA QVMs. Simulations were conducted using explicit FEA simulation software, PAM-SHOCK. The FEA tyre model predictions of contact patch area, static vertical stiffness, first mode of free vertical vibration, and yaw oscillation frequency response were compared with measurements and found to be in good agreement.

After the successful validation tasks, the FEA QVMs was subjected to a durability test on a 74 cm long and 8.6 cm deep water drainage ditch to observe the dynamic tyre responses. In addition, measurements were conducted using a tractor-semitrailer. The vertical acceleration of the front axle that moves vertically together with front tyres was measured and compared with the results from the QVMs. The predicted vertical accelerations from the QVMs exhibit similar results in magnitude and trend to each other.

However, the measured peak values were lower than those observed from the QVMs due to a dynamic coupling effect from roll and pitch motions. Reasonable agreement between predicted and measured vertical acceleration was observed at higher speeds because the dynamic coupling effect was less significant on the front axle of the tractor-semi-trailer at higher speeds.

In order to compare the dynamic tyre responses of the QVMs with measured values, special test equipment similar to the QVM was required to obtain the actual dynamic tyre responses in the same quarter-vehicle environment.

Ghoreishy [56,57] has studied the effect of different belt angles on the steady state rolling behaviour of a steel belted radial tyre with slip angle. To achieve this goal, a finite element model has been developed using ABAQUS computer software.

The simulation started with an axisymmetric model to analyze the tyre under inflation pressure. Then a full 3D model was generated to model the tyre under static vertical load. Having obtained the tyre configuration under contact load, a steady state rolling analysis was conducted using a mixed Lagrangian/Eulerian technique. The final stage of the modelling was the inclusion of the slip angle in the model. Each set of simulations was repeated for three belt angles and the effect of the belt angle variation on the tyre structural variables, including contact pressure and area, lateral force, interlayer shear stress and total strain energy was examined. In addition, the computed value of the number of revolutions per kilometre was compared with experimentally reported data which confirms the accuracy of the present model.

Stanciulescu et al. [58] analyzed the steady state rolling of inflated tyres. They focused on adherent (i.e., no slip) friction formulations, and in addition to analyzing the numerical difficulties associated with such problems, they also studied the interaction of frictional conditions with bifurcation phenomena. Such phenomena are observed in the context of multiple solutions (both stable and unstable) of the discretized system, and are also manifested in the behaviour of the iterative map used to solve the non-linear algebraic system of equations.

Yanjin et al. [59] developed a finite element model for a rolling radial tyre.The rebar model was employed to simulate complex multilayer rubber-cord composites and to directly define the cord directions varying with their positions. A 3D finite element model has been built with MSC-MARC software according to the actual construction of a 195/60R14 radial tyre.

The model considered the geometric non-linearity due to large deformation, material non-linearities of cord-rubber composites, and the non-linear boundary conditions from tyre-rim contact and tyre-road contact. Based on the non-linear finite element model, the influence of the belt cord angle on the radial tyre under different rolling states was numerically studied. It was found that the shear strain of the radial tyre concentrates on the edge of the belted layer, the surface of the shoulder, and the tread groove. In addition, the strain energy density of the belt end decreases with the increase of the belt cord angle.

Ghoreishy et al. [60] have developed a three-dimensional finite element model for the modelling of a 155/65R13 steel-belted tyre under inflation pressure, vertical static (footprint) load, and steady state rolling. The model was created for the ABAQUS/Standard code and used to carry out a series of parametric studies. The main purpose was to examine the effect of some structural and operational parameters including type of body and cap plies cords, cord belt angle, and friction coefficient between tyre and road on the mechanical behaviour of the tyre under both static and rolling conditions. It is shown that the belt angle is the most important constructional variable among the selected parameters that affect the tyre behaviour, significantly. The changes of friction coefficient have had great influence on tyre variables at contact zone, e.g., pressure field and relative shear between the tread elements and road as well. The accuracy and reliability of the results obtained in this simulation were also confirmed by comparing two deformational characteristics of the tyre (static deflection and free rolling perimeter) with those of two commercial tyres.

Liu [61] carried out an explicit transient dynamic finite element analysis using ABAQUS used to model a rolling passenger tyre (195/75R14) subjected to a slip-angle sweep of 0 to -1 degree. The computation tracks the rolling and yawing history of the tyre on a 3 m (10 ft) diameter drum. Various loads and inflation pressures were applied and the computed forces and moments at a slip angle of -1 deg were compared to identify their sensitivities to these parameters. It was found that for the current small slip angle used, the lateral force was quite insensitive to inflation and vertical load. The moment, however, was highly dependent on both. The difference in sensitivities was caused by the strong dependence of moment on footprint size, which was controlled by both load and inflation.

Lopez et al. [62] have developed an approach to model the vibrations of a deformed rolling tyre at low frequencies (below 500 Hz). It was used to calculate the dynamic response of a rolling tyre including the details of its complex build up to relate the tyre design parameters to its vibro-acoustic properties. The natural frequencies and mode shapes of a deformed tyre were calculated using a full non-linear finite element model. Subsequently, this modal base was transformed to determine the response of the rotating tyre in a fixed (Eulerian) reference frame. Furthermore, this approach made it possible to define a receptance matrix for the rotating tyre. Results from relatively simple tyre models showed that the effects of rotation were modelled correctly and were in accordance with results from literature.

Nakajima [63] applied a new tyre design tool to the actual design, which was capable of determining the optimum tyre contour, optimum compound characteristics in each tyre material, optimum tyre construction and optimum pitch sequence. This technology was developed by combining the finite element method or analytical method with an optimization technique, such as mathematical programming or genetic algorithm, and was applied not only to the shape optimization, such as tyre crown shape, but also to the topological optimization, such as composite construction. The new design procedure was verified to improve the manoeuvrability, rolling resistance, durability, tyre noise and other tyre performances through indoor drum tests and field tests, if the appropriate design variable, i.e., the constraints and the objective function were selected.

Cho et al. [64] developed an explicit finite element model for the modelling of the tread pattern wear and compared the model predictions with the indoor drum test results. They have first selected a pattern and determined the parameters of the wear equation which was composed of slip velocity and tangential stress under a single driving condition. Then, this equation was used for two other patterns with the same size (225/45ZR17). Having combined the indoor wear test with outdoor (actual) results, they could show that their finite element model can be used for the modelling of the wear phenomenon in tyres under real conditions. The effect of the four tread patterns on the rolling behaviour of a 155/65R13 radial tyre was examined in a work by Ghoreishy et al. [65].

Ghoreishy et al. [66] have also applied their previously finite element model which was developed for ABAQUS, to a 12-24 truck tyre. They showed that by combining the so-called lift equation and the rebar element, it is possible to simulate the rolling behaviour of the truck tyres with bias construction.

Rao et al. [67] have developed two finite element models by the use of ABAQUS code. Their models consisted of explicit and mixed Lagrangian/Eulerian algorithms. These models were used to simulate the rolling of a tyre. They have compared the force and moment (FM) results with experimental measurements. The numerical performance of these models was also studied.

Finnveden et al. [68] have recently developed a new finite element formulation known as waveguide which is claimed to be used for curved structures such as tyres that have constant properties along one axis. However, the results are only compared with test problems and those reported in literature.

Ziefle et al. [69] have developed a new full implicit algorithm based on a time discontinuous Galerkin method for solution of the rolling contact problem of viscoelastic bodies. The advantage of their model is to integrate the slip velocity along the path-lines of the rolling structures (like tyre). The dissipative effects due to both, inelastic behaviour and friction were considered.

Here, we present the results obtained by the steady state analysis of a 175/70R14 84T steel-belted radial tyre [70]. It is a full three-dimensional finite element model with tread pattern. The ABASQUS/Standard v. 6.6 was used in this work [70].

Figure 2 shows the finite element mesh of the tyre under deformed and rolling state. The total number of elements and nodes are 26497 and 32386, respectively. The rubber components and the reinforcing fibres were modelled by the Mooney-Rivlin hyperelastic and the linear elastic material models, respectively. The mentioned finite element model was used in four consecutive simulations. In the first step, an internal pressure of 0.248 MPa was applied inside the tyre surface to obtain its inflated shape. In the second stage, the contact of the tyre tread with ground

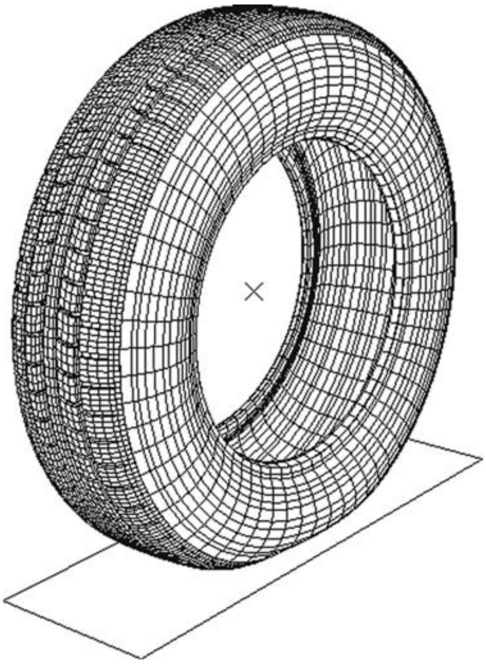

Figure 2. Finite element mesh of the tyre.

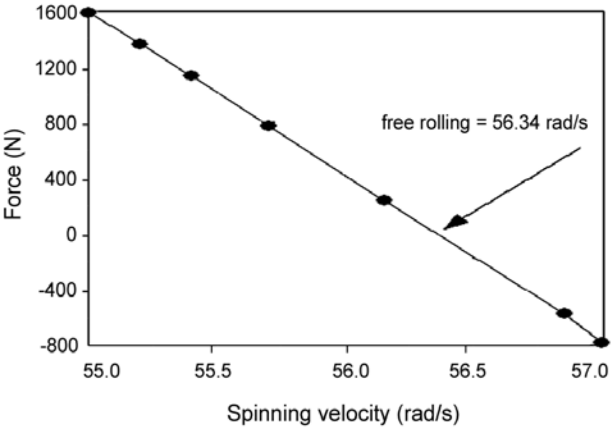

Figure 3. Longitudinal force vs. spinning velocity of the tyre.

(footprint analysis) was simulated. To achieve this goal, a pair of surfaces was taken into account. First, a rigid surface was defined that resembled the ground and considered as the master surface. This was an analytical (non-discretized) surface defined using spatial coordinates. The second surface was, however, defined on the tread surface using elemental faces exposing to ground. A point load (or concentrated load) of 4900 N (500 kg) corresponding to load index of 84 was applied on master (analytical rigid) surface. This was resulted in moving of contact surface towards inflated tyre and the deflected shape under static vertical load was obtained. The friction between contact surface and tyre tread has been ignored (frictionless) in this stage.

The third step restarts from the results of the footprint analysis and a steady state rolling analysis was performed in an ALE (Arbitrary Eulerian/ Lagrangian) framework implemented in the ABAQUS code. This capability uses a reference frame that is attached to the axle of the rotating tyre. In this frame the tyre is not moving, although the material of which the tyre is made is moving through those points that were used to create the finite element mesh. This enabled us to tackle the problem of modelling the rolling tyre using the traditional Lagrangian formulation since the frame of reference in which motion is described is attached to the material. A constant ground velocity of 60 km/h was assumed for the tyre. The inertia effect and friction between tyre tread and ground were also considered in this step. A special formulation of the well-known Coulomb's law with a friction factor of 1.0 was selected to model the frictional effect between tread surface and contact road.

The final stage was devoted to the inclusion of slip angle to model the rolling tyre in cornering condition. The tyre was analyzed using two slip angles of 1.5º and 3º, respectively. In all analyses the tyre rim was assumed to be constructed from very stiff materials so that the tyre was completely attached to the rim. Consequently, a fixed rim was selected as the boundary condition for this model.

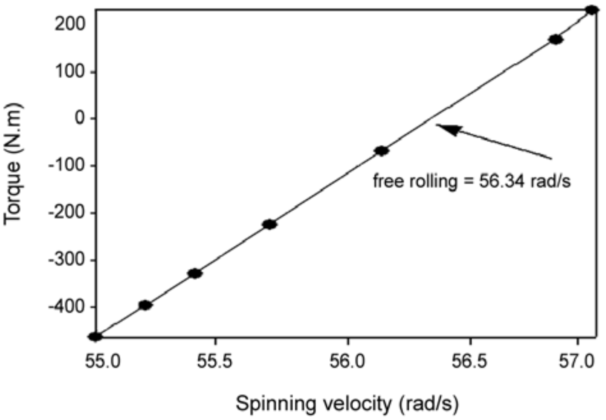

Figure 4. Rolling resistance moment vs. spinning velocity of the tyre.

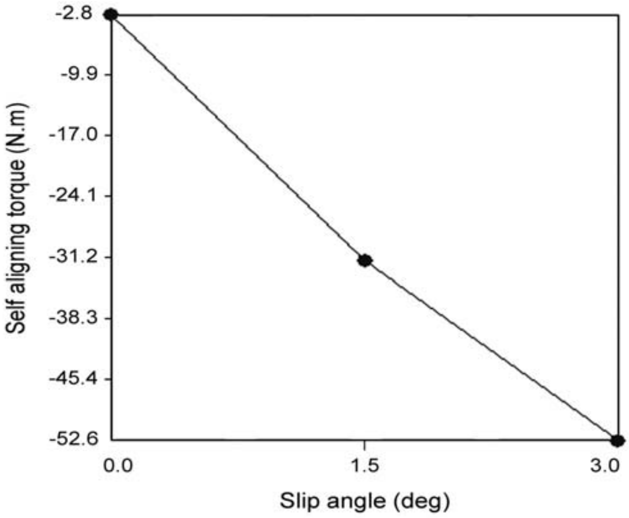

Figure 5. Self aligning torque vs. slip angle at free rolling condition.

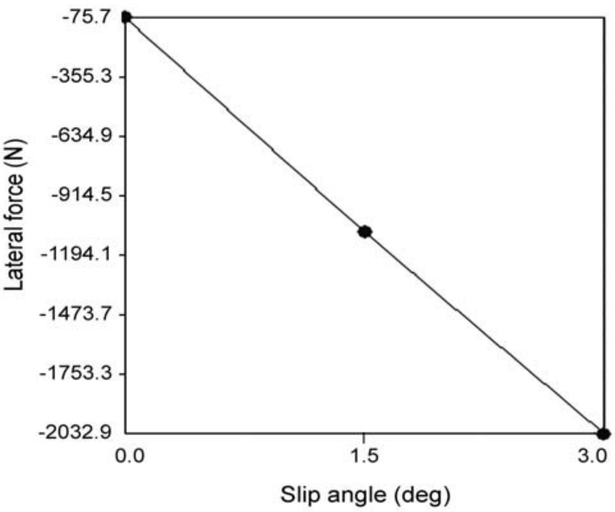

Figure 6. Lateral force vs. slip angle at free rolling condition.

The tyre spinning velocity (rotational speed) was changed within a range from 55 rad/s to 57 rad/s to find the tyre free rolling condition and its associated rolling speed. The SAE terminology has been adopted in this work for the representation of the forces and moments in which the x, y, and z stand for the longitudinal, lateral and vertical directions, respectively.

Figure 3 shows the calculated longitudinal force (F_x) against spinning velocity within the mentioned range (55-57 rad/s) at steady state condition with no slip angle (α = 0). As it can seen the free rolling conditions occur at ω = 56.34 rad/s that corresponds to the point where tractive (longitudinal) force is zero. Figure 4 also gives the similar results by plotting the rolling resistance moment (M_y) against spinning velocity with same condition. In addition to these force and moment, there are two other important components that are related to the lateral behaviour of the tyre especially cornering in situation ($\alpha \neq 0^o$) namely as self aligning torque (M_z) and lateral (transverse) force (F_y). The variations of these factors with slip angles are shown in Figures 5 and 6, respectively. Increasing the slip angle, will increase both

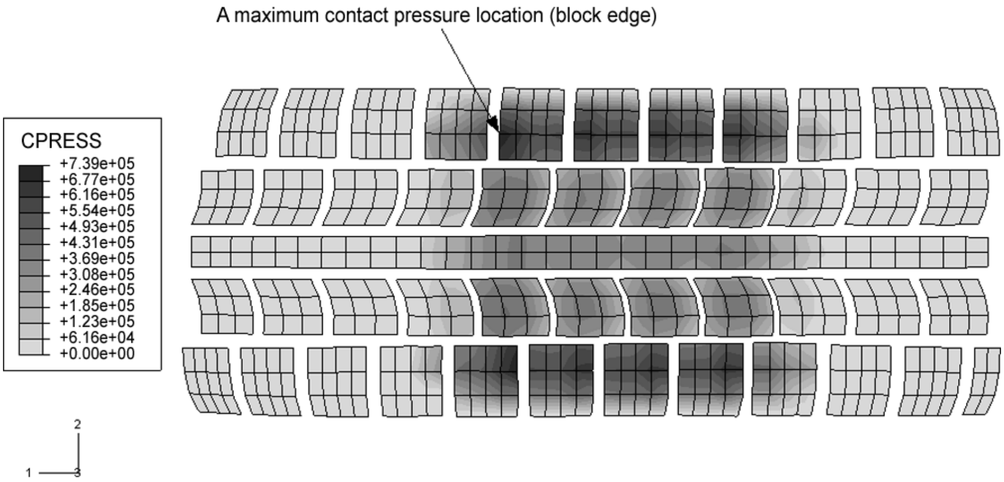

Figure 7. Distribution of the contact pressure (Pa) at free rolling condition with no slip.

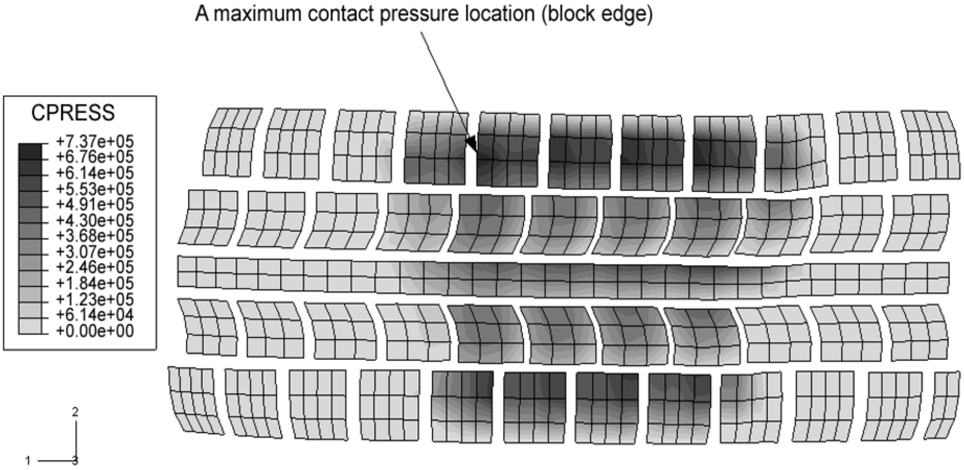

Figure 8. Distribution of the contact pressure (Pa) at free rolling condition with slip angle = 1.5º.

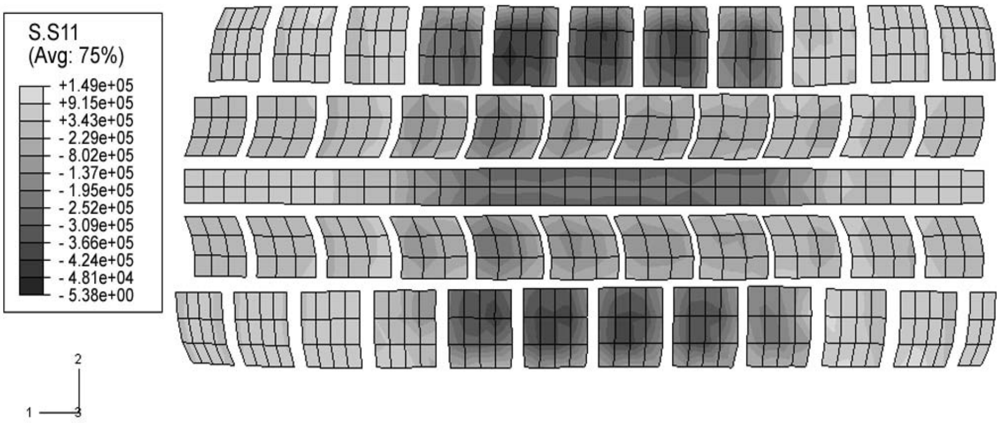

Figure 9. Distribution of the normal stress (σ_{xx}) (Pa) at free rolling condition with no slip.

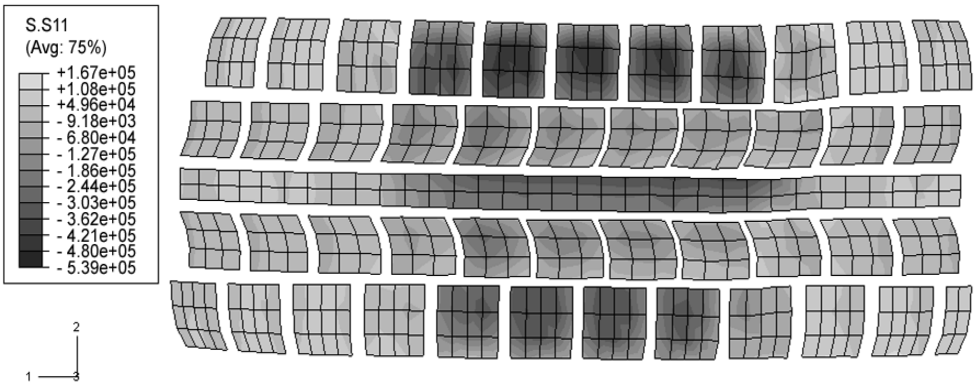

Figure 10. Distribution of the normal stress (σ_{xx}) (Pa) at free rolling condition with slip angle = 1.5º.

(M_z) and (F_y), accordingly. This is as expected since they are trying to restore tyre to realign its direction of travel with the direction of the heading.

Referring to the field results, the distribution of the contact pressure for no slip and slip (α = 1.5º) are shown in Figures 7 and 8, respectively. In both cases, the maximum pressure is found to be at shoulder zones. In addition, at each individual blocks on the tread surface, the maximum pressure is located at the block edge. This is due to the inclusion of the frictional effect and generation of a tangential resistance between rubber and ground. However, when the slip angle (α) is set to 1.5º (Figure 8), because of the increasing of the lateral force, more deformations are obtained specially at the shoulder areas. Figures 9 and 10 show the distributions of the normal stress in x direction (σ_{xx}) at the same conditions (α = 0, 1.5º). It can be seen that, those parts of the tread that are in contact with road undergo compression while the other sections are set to tension state. These situations exist for both no slip and slip conditions.

APPLICATION OF ROLLING TYRE MODELS

In this section we review the published works on the application of the rolling tyre models including energy loss (for the calculation of the rolling resistance) and temperature prediction, interaction between tyre and road (especially soft roads such as soil and snow), noise generation, failure and stability analysis.

Energy Loss (Rolling Resistance) and Temperature Prediction

One of the most important features that a tyre analyst is always seeking is to find a robust technique for the prediction of the energy loss and temperature rise in a rolling tyre. This is a very well-known phenomenon that is due to the viscoelastic nature of the rubber and cords in tyres. To achieve this goal, two more steps in addition to deformation module in a conventional rolling analysis are generally required which are dissipation and thermal modules. In dissipation stage, the energy loss is computed via a hysteretic model and in thermal step, the heat transfer equations are solved in conjunction with the generated heat calculated from dissipation stage to predict temperature distribution. If the main purpose is to predict the rolling resistance of the tyre then thermal module is not necessary otherwise the three mentioned stages should be accomplished repeatedly until a complete thermo-mechanical solution is obtained.

An early work in this field belongs to Sarkar et al. [71]. They have described an approach for the complete thermo-mechanical analysis of a pneumatic tyre to estimate cyclic changes in the stresses and strains and obtain the pseudo steady state temperature profile in a tyre rolling under load. The approach involved a simplified two-dimensional representation of the tyre geometry to reduce computation time. The analysis included three stages: inflation analysis, contact analysis, and temperature analysis.

Weiss et al. [72] have investigated the influence of five belt constructions on high speed endurance, ride comfort, and rolling resistance for a high performance 225/50R16 92V radial passenger car tyre.

Luchini et al. [73] presented a new theory for the description of the hysteretic behaviour of the carbon black filled rubbers called the Directional Incremental Hysteresis (DIH) theory. It was a strain-based model which includes an incremental formulation to deal with non-sinusoidal cycles within tyres. This model used in conjunction within a finite element code to predict rolling resistance of tyres.

McAllen et al. [74,75] developed a complete finite element model to predict the temperature rise in aircraft tyres under free rolling conditions. They first studied the deformation characteristics of the tyre to determine the heat generation due to the inelastic deformation (viscoelasticity). The heat generation was then used as an input to solve the heat transfer problem. A methodology which considers a 2D formulation with the contribution of out-of-plane forces is presented.

This modelling considered the deformation process of the tyre to be a steady-state problem, where all concurrent cycles were also assumed to be the same as the first. The inelastic energy was determined by imposing a phase lag between the strain and the stress fields. The phase lag was assumed to be frequency independent in the range of interest, in keeping with the experimental observations in aircraft tyre materials. It was further assumed that the inelastic energy was completely converted into volumetric heat input for a transient thermal conduction analysis. A conduction heat transfer model was described and results are compared against thermocouple data recorded by experimental measurements.

In a more rigorous work, Park et al. [76] presented a complete thermo-mechanical analysis for a 205/60R15 tyre under steady state rolling condition. They have used the ABAQUS code and implemented the necessary routines to accomplish the energy dissipation calculations. They have also compared their numerical results with both experimentally measured rolling resistance and temperate rise performed on a test drum.

Song et al. [77] and Ebbot et al. [78] also performed similar simulation to predict the temperature distribution in steady state rolling tyres. Shida et al. [79] presented a practical rolling resistance simulation method for tyres using a static finite element method. The method is claimed to fulfil three requirements of easy input data preparation, shorter computation time, and adequate accuracy. The method was based on a static deflection analysis together with the loss factors of the materials that were determined separately. The latter were used to estimate the energy dissipation of a rolling tyre. It was shown that the rolling resistance simulation of a passenger radial tyre using this approach accurately captures the trends of an actual tyre.

Futamura and Goldstein [80] proposed a simplified method to predict the temperature rise of rolling tyres. First, the sensitivity of the tyre elastic response of the tyre to changes in material stiffness was characterized using the "deformation index". Then, using a commercial finite element programme and an appropriate user subroutine, heat generation was expressed as a function of the local temperature using a simple algebraic expression involving the temperature dependent material properties and the deformation indices. Temperatures were then computed using the finite element programme with the coupling information contained in the user subroutine. The result was a simplified method for a fully coupled thermo-mechanical analysis of a tyre for steady-state and transient thermal analysis. The accuracy and the simplicity of the method are demonstrated using a small "tyre-like" model. The simplified method was compared to the fully coupled iterative method for a steady-state thermal solution.

Lin et al. [81] have developed a numerical procedure for investigating the temperature distribution in a smooth tread bias light truck tyre, operated under different speeds, inflation pressures, and loading conditions. Prior to simulation, two separate sets of testing, namely dynamic mechanical testing and material testing, have been conducted in relation to the evaluation of hysteresis (H) and total strain energy (U_{sed}), respectively. Hysteresis loss energy was given as ($H \times U_{sed}$) and considered to relate directly to heat generation rate. Temperature rise was assumed to be due to the energy dissipation from periodic deformation. This dissipation of energy may be equated to be the primary heat generation source. Hysteresis energy loss was used as a bridge to link the strain energy density to the heat source in rolling tyres; temperature distribution of rolling tyres may be obtained by the steady-state thermal analysis. The above procedure has been shown to facilitate the simulation of the temperature distribution in the rolling tyre. They have used ANSYS/LS-Dyna and ANSYSMechanical for dynamic and thermal analyses, respectively.

Narasimha et al. [82,83] have used a three-stage sequential model using finite element (FE) technique to determine rolling resistance and temperature prediction for tyres with smooth and circumferential groove tread profiles. A finite element algorithm was developed with Petrov-Galerkin Eulerian technique in cylindrical coordinates and was used to determine three-dimensional tyre-operating temperatures. A sensitivity analysis has been made with various operating conditions and tyre design attributes that will aid tyre designers to realize improved designs with reduced rolling resistance and acceptable temperatures. They have also extended their work to provide a technique that is more appropriate for non-axisymmetric tyres with tread patterns. However, the developed algorithm has been implemented for the prediction of three-dimensional operating temperatures for axisymmetric tyres through a simple decoupled procedure.

Tyre/Road Interaction

Most of the works accomplished in this area are devoted to the interaction between tyres and soft roads, particularly soft soils and snow. Pi [84,85] has developed a dynamic tyre/soil contact surface interaction model for aircraft ground operations. The formulation used a finite-element kernel function approach. It was based on the concept of the quasi-steady motion of a tyre rolling at a constant speed on a linear viscoelastic layer (soil).

In the soil model, the Young's modulus, Poisson's ratio, and shear modulus were treated as three independent parameters, and the inertia and viscous damping effects were included. Numerical examples were given to correlate the experimental results from a high-flotation test program. The predicted drag ratio versus the speed results compared well with the test data trend for soils of various strengths. The analyses indicated that the high drag force and severe rutting occur when the wheel forward speed is near the shear wave velocity of the soil.

Tielking et al. [86] studied the effect of the contact pressure distribution calculated by the finite element method on the loads applied to road pavement. It was found that non-uniform pavement contact pressure, as produced by a real tyre, causes significantly higher pavement strains than those calculated with the conventional assumptions of uniform contact pressure.

Mousseau et al. [87] in their paper described how changes in the tyre and suspension impact the dynamic spindle force response of a tyre rolling over an obstacle in the roadway. The investigation was conducted with a simple vehicle dynamics simulation coupled to a comprehensive 2D dynamic non-linear finite element model of the tyre. The tyre model used an inextensible circular membrane to approximate the shape and an empirical non-linear term to account for forces that result from the elastic deformation of the sidewall.

Both the tyre structure and contact between the tyre and roadway were modelled with the finite element method. This presented approach allowed the tyre model to reproduce the tyre response accurately during large deformations in a computationally efficient manner. They have also made a comparison between measured and simulated tyre contact forces for the case of a slowly rolling tyre. The effect of vehicle speed, suspension stiffness, suspension damping, tyre mass, tyre damping, and tyre inflation pressure on the spindle forces was also presented.

Shoop et al. [88] developed a three-dimensional model of a wheel moving through snow using commercial finite element software ABAQUS. Because of the large deformation of the snow relative to the tyre, a rigid wheel was used to simplify computations. The snow was modelled as both an elastic-plastic material and as a crushable foam material. Models of uniaxial compression and plate sinkage tests in snow were used to explore the snow material model and match measured and observed snow deformation to model results. These constitutive models were then applied to the three-dimensional tyre-snow model.

A new ALE adaptive meshing formulation was also evaluated for improvements in handling the large deformations encountered in tyre-snow interactions. The predicted snow deformation was compared to sinkage, displacement, and changes in snow densities. The modelled reaction forces on the wheel were compared with tyre forces measured using an instrumented vehicle.

Seta et al. [89] have developed a three-dimensional prediction model in which the interaction between snow and a rolling tyre with tread pattern is considered. An explicit finite element method (FEM) and a finite volume method (FVM) are used to model tyre and snow, respectively. Snow deformation was calculated by the Eulerian formulation to solve the complex interaction between snow and tyre tread pattern. Coupling between a tyre and snow was automatically computed by the coupling element. Numerical modelling of snow was essential to the tyre performance prediction on snow.

In this study, snow is assumed to be homogeneous and considered to be an elasto-plastic material. The Mohr-Coulomb yield model, in which the yield stress was a single function of pressure adopted. This function was investigated by tyre traction tests under a wide range of tyre contact pressures using several tyres with different inflation pressures and patterns. The predicted results using the Mohr-Coulomb yield model were compared with those using the Capped Drucker-Pragger and the Cam-Clay yield models. Snow traction of a tyre featuring different tread patterns was simulated by this technology. Results were shown to be in good qualitative agreement with experimental data.

Cho et al. [90] developed a finite element model for the braking distance estimation of tyre controlled by anti-lock brake system (ABS). The frictional heat dissipation at disc pad was derived analytically and the tyre frictional energy loss was computed by the 3D dynamic rolling analysis of patterned tyre.

Shoop et al. [91] has applied a three-dimensional dynamic finite element model of a wheel rolling over soil to simulate local vehicle traffic on a secondary unpaved road. These simulations were used to study the effects of vehicle speed, load, suspension system, wheel torque, and wheel slip on rutting and washboard formation. Modelling results were also compared to field measurements and observations.

Kuwajima et al. [92] developed a two-dimensional explicit finite element model and used it for modelling of abrasion of the tread area in contact with abrasive materials. They have studied the effect of the contact length, partial slip, and friction factor on the interfacial phenomenon between rubber and abrasive material. They have showed that the rolling/sliding friction at low slip ratio was affected by local frictional behaviour at micro slip regions at asperity contacts.

Shoop et al. [93] constructed a three-dimensional finite element model to simulate a tyre rolling over snow. The snow was modelled as an inelastic material using critical-state constitutive modelling and plasticity theory. The snow material model was generated from experiments on the mechanical deformation of snow and was validated using a plate sinkage test. The model results were also compared to vehicle mobility predictions made. These comparisons showed the agreement between the finite element models and field measurements of motion resistance forces and snow deformation under the tyre.

Moving of the tyres on wet roads is a very important aspect of the study of the interaction between tyre and road. This phenomenon is known as the “hydroplaning”. There are also a few published works on the simulation of the hydroplaning in tyres.

Seta et al. [94] established a numerical procedure for hydroplaning. They considered three important factors; fluid/structure interaction, tyre rolling, and practical tread pattern. The tyre was analyzed by the finite element method with Lagrangian formulation, and the fluid was analyzed by the finite volume method with Eulerian formulation. Since the tyre and the fluid can be modelled separately and their coupling was computed automatically, the fluid/structure interaction of the complex geometry, such as tyre with tread pattern can be analyzed. Since their focus was on the simulation of dynamic hydroplaning with thick water films, they ignored the effect of fluid viscosity. They verified the predictability of the hydroplaning simulation in the different parameters such as water flow, velocity dependence of hydroplaning, and effect of tread pattern on hydroplaning. These parameters could be predicted qualitatively. They have also developed the procedure of the global-local analysis to apply the hydroplaning simulation to a practical tyre tread pattern design and found that the sloped block tip is effective in improving the hydroplaning performance.

Another example in this area is the work carried out by Cho et al. [95]. They have investigated the hydroplaning characteristics of patterned tyre on wet road by using finite volume and finite element methods. A detailed 3D patterned tyre model was constructed by an in-house modelling programme and the rainwater flow was considered as an incompressible and inviscid fluid. Meanwhile, the fluid-structure interaction between the highly complicated tyre tread and the rainwater flow was effectively treated by the general coupling method. Through the numerical experiments, the rainwater flow drained through tyre grooves, hydrodynamic pressure and contact force were investigated and compared with those of the three-grooved tyre model.

Noise

Optimization of the tyre design especially tread pattern for noise reduction is another important aspect of the rolling tyre simulation. This area of research has received particular attentions in recent years due to more restricted environmental regulations. Larsson et al. [96] stated that the interaction between tyre and road constitutes the dominant noise source for road vehicles at speeds above 50 km/h. They have developed a finite element model for individual blocks in order to investigate their first eigen-frequencies and mode shapes.

This information was used to build an equivalent model consisting of a simple mass and springs. The equivalent model has the advantage of being handier when coupling to a model of the tyre structure. The impedance coupling method was used. The results of the driving point mobility in the radial and tangential directions to the surface of the block were compared with measurements on tyres. The results showed good agreement for the radial direction, while for the tangential direction, the agreement was poor. This was mainly due to the fact that the model for the tyre structure does not include in-plane motion.

The results also showed that, for the frequency range up to 3 kHz, the influence of the blocks depends strongly on their geometry. The geometry of the tread blocks determines the contact geometry as a kind of macro roughness. It also determines the eigen-frequencies which for typical tread blocks are expected to be situated, at least, in the above range (up to 3 kHz).

Wullens et al. [97] developed a three-dimensional contact model for the tyre/road interaction for the estimation of tyre/road noise. The contact model was formulated in the time domain for non-linearity reasons. The dynamic radial contact forces the local deformation due to roughness indenting and the normal forced vibrations of the tyre structure were calculated.

Biermann et al. [98] presented a numerical model based on a simulation process that may be split into several analysis steps including computation of the non-linear stationary rolling process, analysis of the tyre dynamics caused by the road roughness, and computation of the sound radiation. Their work was concerned with the latter part of the analysis procedure. For the sound radiation analysis, the vibrations on the tyre surface were extracted from a preceding structural analysis and used as boundary conditions in the acoustic model.

The acoustic simulation process was based on the finite/infinite element approach, where an improved variant of the so-called Astley-Leis elements was used to model the sound radiation. By evaluating the sound pressure field, it was possible to compare the acoustic performance of specific tyre/road systems, and the influence of certain parameters, e.g., the road texture or the impedance on the noise radiation could be studied. The current work focused on the validation of the computational model. Hence, characteristic results from the numerical simulations were compared with corresponding measurement data obtained from different test setups, including standing as well as rolling tyres.

In addition, more recently Brinkmeier et al. [99,100] have carried out a series of simulations on the prediction of the sound radiating from rolling tyres. They have performed detailed finite element calculations for the dynamics of tyre/road systems with emphasis on rolling noise prediction. The analysis was split into sequential steps, namely, the non-linear analysis of the stationary rolling problem within an arbitrary Lagrangian/Eulerian framework and a subsequent analysis of the transient dynamic response due to the excitation caused by road surface roughness. A modal superposition approach was employed using complex eigen value analysis. Finally, the sound radiation analysis of the rolling tyre/road system was performed.

In a recent work, Brinkmeier et al. [101] have developed a finite element model to study the high frequency response of tyre. Their emphasis was placed upon the efficient numerical treatment of the complex-valued eigen problems for large scale gyroscopic systems.

Failure and Stability

The last topic that will be reviewed in this section is the failure analysis and also stability which is mostly focused on the prediction of the standing waves in rolling tyres. Chang et al. [102] have developed a full non-linear finite-element P185/70R14 passenger car radial-ply tyre model and run on a 1.7 metre-diameter spinning test drum model up to 300 km/h. To investigate the tyre transient response characteristics, the virtual tyre/drum finite-element model was constructed and tested using the non-linear finite element analysis software, PAM-SHOCK. It was found that standing wave formation resulted in the increase of vehicle's receptions, internal energy, and contact energy assumptions.

Han et al. [103] developed a three-dimensional finite element local model to calculate the energy release rate at the belt edge region. The local model uses a three-dimensional fracture analysis based on a steady-state rolling assumption, in conjunction with a global-local technique in ABAQUS. Within the local model, a J-integral variation study was performed in the crack region. This consisted of a prediction of the crack propagation direction and a mesh density analysis of the crack model. Furthermore, the study was used to determine the crack growth rate analysis. It is assumed that a flaw exists inside the tyre, in the local model, due to a mechanical inhomogeneity introduced during the manufacturing of tyres. This work also considered how different driving conditions, such as free-rolling, braking, and traction can contribute to the detrimental effects of belt separation in tyre failure.

Ok et al. [104] modelled the belt edge separation. The finite element method and fracture mechanics concepts were combined to study the initiation and growth of a rectangular crack at the belt edge. Implicit steady state formulation of rolling and the global-local modelling technique were used in the analysis. J-integral was calculated as part of the fatigue crack growth in the circumferential and meridional directions. The reliability and accuracy of J-integral calculations for crack models with sharp corners was studied by comparing the results with smoother crack models as well as more robust methods of calculating energy release rate.

Jeong et al. [105] developed a finite element model for the simulation of the non-uniformity in tyres. They have studied that how imperfections such as variations in stiffness or geometry can affect the tyre uniformity. It was shown that the radial force variation strongly depends on the geometrical variations of the tyre.

Kaliske et al. [106] developed a fully three-dimensional finite element model based on fracture mechanical approach which was served as a basis for tyre durability simulations. The approach which was called material force approach was employed as an elegant alternative characterization of the energy release rate or the J-integral to describe discrete cracks.

FINITE ELEMENT CODES

It is quite obvious that the finite element calculations required for a complete analysis of a tyre under rolling state are highly complicated and thus the use of commercial codes is generally considered to be a critical task. Consequently, the selection of proper finite element software in reaching to a reliable, accurate, and applicable result is crucial and needs careful attention.

On the other hand, a pneumatic tyre is not only a very complex structure but the loads and working conditions are also too complicated which dictates some in-house developed codes might be necessary in conjunction with available packages to carry out a thorough and complete simulation. In recent years, software vendors have tried to add special features to their codes to take the most complicated aspects of a tyre simulation into account. We intend in this section to give a brief review of some of the most famous and applied finite element programmes used by giant tyre companies and research institutes and consider their specifications from tyre analysis point of view. It should be noted that these data are generally available in vendors' websites which we do not cite their address as they can be reached easily in the web by any available search engine.

ABAQUS

This finite element programme is now recognized among the leading vendors as the best solution for tyre analysis. Within a unified and integrated environment known as ABAQUS/CAE, it is possible to virtually analyse any complicated structures such as tyres under different loading conditions. The two main processing modules namely as ABAQUS/Standard and ABAQUS/Explicit help tyre analysts to simulate the rolling behaviour of rolling structures in both steady and transient states. However, some of its features such as generating of three-dimensional models from their corresponding two-dimensional schemes cannot be achieved by working inside its graphical environment (ABAQUS/CAE) which is a main drawback of this code.

This programme like the other powerful codes has the capability of developing and linking of user-defined subroutines. Simulation of the steady state rolling and the use of rebar elements that allows the proper modelling of reinforcement as well as advanced contact modelling capabilities are the best features of this code for tyre analysis.

ANSYS and LS-DYNA

ANSYS is one of the oldest companies involved in finite element modelling in different fields of physics and mechanics. Recently some of the other vendors joined it so that this company now offers the other well-known codes as well. This code also has some good features such as rebar elements for the modelling of reinforcement in tyres. However, the number of publications which used this programme is much lower than those used ABAQUS as the main solver. There are several modules in this package in which LS-DYNA that is on of its explicit codes has been extensively used by researchers for the modelling of rolling tyres in transient states.

MSC Software

MSC Software is another leading company that its codes are widely used for simulation of tyres. This is a computer package that consists of several finite element codes, each designed to cover a wide range of applications. Among them, MSC-NASTRAN and -MARC are the most important modules extensively used for the simulation of the tyres. In addition, DYTRAN is an explicit finite element analysis solution that can be used for the modelling of transient behaviour of rolling tyres. ADAMS is another code in this package that can be employed for the modelling of the dynamic behaviour of non-static structures such as tyres and cars.

Other Codes

There are other finite element codes such as COSMOS/M, ALGOR, ADINA, NISA which are also used for the simulation of tyres but their applications in tyre industries have been limited.

SUMMARY

Despite the substantial progresses achieved in finite element modelling of pneumatic tyres, the full analysis of these complicated structures is still a formidable task. This is mainly due to the wide range of various parameters existing in this area which makes their coverage in one step to be almost impossible. As it can be seen in previous sections, none of the published works up to the date of this review is capable of giving a full analysis of the tyre under different loading conditions. Instead, each work tried to focus on some critical aspects of tyres and investigates those specific features as deep as possible. To give an understanding position of the finite element modelling of tyres based on the methodologies and results presented in the reviewed works, it is tried to summarize and bold the most important items which should be brought into mind when modelling a tyre by this technique.

1- The most challenging point is to adopt valid and real material models for both rubber and reinforcing parts. Except steel, all other materials used in manufacturing of a pneumatic tyre are viscoelastic which means that not only their properties changes with time but also they dissipate energy so that an isothermal analysis cannot inherently describe the real behaviour of tyres under rolling conditions. The development of valid material models for the description of the viscoelastic behaviours as well as numerical methods for utilization and testing in tyre applications are demanding tasks and should be covered by researchers involved in this field.

2- While hyperelastic and linear elastic models are widely used for rubbers and reinforcing cords, their effectiveness is limited to static cases such as rim mounting, inflation (axisymmetric models) and footprint loadings (static and vertical load). In theses cases, the differences between real situations (determined by experimental measurements) and model predictions are trivial and thus, the developed models can be used with high degree of confidence (e.g., [8,10,11]).

3- Geometrical modelling is almost a finished item. The exceptional point is modelling the tread patterns. This is very important since the inclusion of the tread patterns not only significantly increases the computational time and effort but also in many cases the creation of valid elements becomes very difficult specially when the pitching (non-uniform distribution of tread blocks in circumferential direction of the tyre) is taken into account (e.g., [31,32,70]).

4- Referring to rolling conditions, the steady state algorithm implemented in two top level finite element codes, i.e., ABAQUS and MARC which were used by some researchers to provide a simple and inclusive methods for the modelling of tyres (e.g., [35,54,56,57]). However, they are not capable to consider the non-isothermal conditions for the prediction of the temperature rise and energy loss in tyres. A separate module should be developed and linked to current codes for this purpose (e.g., [76,78,81]).

5- Tyres are not always rolling in steady state conditions. In many situations, they show transient behaviour. Therefore, finite element models that are devoted to modelling of tyres in transient conditions are of great importance. Similar to steady state cases, most of the works developed in this area have modelled the tyres under isothermal conditions. The critical point here is to develop methods that reduce the use of computer resources as low as possible. Models developed based on explicit algorithm give the best compromise between time and accuracy (e.g., [34,42,43,50]).

6- Modelling of tyres for especial purposes such as studying the interaction between tyre tread and non-rigid surfaces (like soil and snow), noise generation, failure and endurance phenomena is not a completed work, although some very good works have been accomplished in recent years (e.g., [89,101]). It means that this line of work is opened to future and researchers should continue their efforts at different fields such as development of models for soil, snow, and other soft materials, noise generation models, especially, at higher frequencies, and failure of cord-rubber composite.

CONCLUSION

Pneumatic tyre is a very complex composite structure that comprises of different materials with various physical and mechanical properties and its computer simulations is still a challenging task in tyre and automotive industries. This is mainly due to its distinguishing and unique features such as large deformation and strains, incompressibility and viscoelasticity, material and geometrical non-linearity, and rolling contact. Therefore, as it is shown in this monograph, there are many published works during the past two decades to simulate the rolling of tyres under different conditions. It is tried in this paper to give a complete review of the attempts accomplished in this field focusing on different aspects of the subject. Furthermore, to give a general overview on the topic and direct the reader toward practical and applied use of this information, the most important and applicable computer codes and some practical results of using this technique were also presented.

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