IJE TRANSACTIONS C: Aspects Vol. 25, No. 3, (September 2012) 223-230

International Journal of Engineering

Journal Homepage: www.ije.ir

TECHNICAL NOTE

Working Vacation Queue with Second Optional Service and Unreliable Server

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PAPER INFO

Paper history:
Received 2 January 2010
Accepted in revised form 17 May 2012

Keywords:
Working Vacation
Second Optional Service
Matrix Geometric Technique
Server Breakdown
Repair
Queue Size

ABSTRACT

An M/M/1 queueing system with second optional service and unreliable server is studied. We consider that the server works at different rate rather than being idle during the vacation period. The customers arrive to the system according to Poisson process with state dependent rates depending upon the server's status. All customers demand the first essential service whereas only some of them demand the second optional service. A customer either may leave the system after the first essential service with probability (1-r) or at the completion of the first essential service go for second optional service with probability r ($0 \le r \le 1$). The server may breaks down according to Poisson process during the busy and working vacation duration. Both service times in vacation and in service period are exponentially distributed. The matrix geometric technique is used for the analysis of the concerned queueing system. The sensitive analysis is also performed to examine the variation of the system performance characteristics with various input parameters.

doi: *10.5829/idosi.ije.2012.25.03c.06*

1. INTRODUCTION

Queueing model with vacations provide performance prediction of many congestion situations encountered in computer, communication, manufacturing, production systems, etc.. Several researchers have contributed significantly in this direction [1]. Lee et al. [2] studied $M^x/G/1$ queueing system with N-policy and single vacation. GI/M/c queue with two classes of vacation mechanism was considered by Chao et al. [6, 3]; in the first case all servers take vacation simultaneously whereas in second class of vacation, each server takes its own vacation. Bacot et al. [4] generalized a single server bulk input batch service queues with multiple vacations. The classical single server vacation model was generalized by Seri and Finn [5] by considering working vacation. Simple explicit formulae for the mean, variance of the number of customers in the system were provided. Arumuganathan et al. [6] considered $M^x/G/1$ queueing system with multiple vacations, setup times and closedown times under N-policy. An M/G/1 queue was studied by Wu et al. [7] by considering the multiple vacations and exhaustive service discipline; the server was assumed to work at different service rates rather than completely stopping the service during vacation. A queueing system with c servers and a threshold type vacation has been considered by Tian et al. [8]. Ke [9] studied the operating characteristics of an $M^x/G/1$ queueing system under a variant vacation policy. Lin and Ke developed a cost model to determine the optimal values of the number of the servers and working vacation rate for M/M/R queue with vacation.

In many real cases, the server may experience breakdowns, so that a more realistic queueing model is that which incorporates the assumption of unreliable server. Cao et al. [10] investigated an M/G/1 queueing model with repairable server. Reliability analysis of M/G/1 queueing system with server breakdowns and vacations has been examined by Li et al. [11]. Wang et al. [12] analyzed the retrial queue with server breakdowns and repairs. Ke [13] studied the N-policy M/G/1 queue with server vacations, startup and breakdowns. Almasi et al. [14] examined a single server retrial queue with finite number of homogeneous sources of calls and a single removable server. Stability conditions are provided by Sherman et al. [15] for an M/M/1 retrial queue with infinite capacity orbit. $M^x/G/1$

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system under vacation policies with startup/closedown times has been examined by Ke [16]. Wang et al. [17] considered a single unreliable server in an $M^x/M/1$ queueing system with multiple vacations. M/G/1 queueing system with a single removable and unreliable server has been considered by Wang et al. [18], by assuming that the server needs a startup time before providing the service. Yang and Alfa [19] studied a multi server queueing system with identical unreliable server with phase type distributed service time.

Most studies on queueing models have considered the main service. However, in many real service systems some customers require the main as well as subsidiary services provided by the server. A pioneer work on such queueing situation was done by Madan [20], who first introduced the concept of second optional service while studying the time dependent as well as steady behavior of an M/G/1 queueing system with no waiting capacity, using supplementary variable technique. An M/G/1 queueing system with second optional service has also studied by Madan [20]. Medhi [21] proposed an M/G/1 queueing model with second optional channel and developed the explicit expressions for the mean queue length and mean waiting time. Supplementary variable technique was used to develop the time dependent probability generating function in terms of their Laplace transform for M/G/1 queue by Al- Jararha and Madan [22]. $M^x/G/1$ queueing system with two phases of heterogeneous service under N-policy was examined by Choudhury and Paul [23]. Wang [24] considered an M/G/1 queueing system with second optional service and server breakdowns based on supplementary variable technique. $M^x/G/1$ queueing system was studied with additional second phase of optional service and unreliable server by Choudhury et al. [25].

Many authors have paid attention on matrix geometric approach which is used to solve the more complex queueing problems having phase arrivals/services [26]. A matrix geometric approximation for tandem queues has been examined by Gomez [27] with blocking and repeated attempts. Matrix geometric solution of the nested QBD chains was given by Choi et al. [28]. The queue length distribution was computed by Gray et al. [29] using matrix geometric method for a queueing model with multiple types of server breakdowns. Ke and Wang [30] used matrix geometric theory in studying the machine repair problem with R servers who take vacation of random length. A single server working vacation queueing model with multiple types of server breakdown via matrix geometric approach was described by M. Jain and A. Jain [31].

In this paper, we develop a working vacation queueing model with second optional service and unreliable server. The server may breaks down during the vacation as well as while working. The paper is organized as follows. In section 2, we outline the underlying assumptions and notations to develop mathematical model under study. The governing steady state equations are constructed by taking appropriate transition rates. In section 3, matrix geometric solution of the system is given. In section 4, various performance characteristics of the system are formulated explicitly in terms of steady sate probabilities. In section 5, we perform comparative study of the system characteristics for various input parameters. In the last section, conclusions and future scopes of the work are provided.

2. MATHEMATICAL MODEL

We consider a single server vacation model with second optional service and unreliable server. The following assumptions are made to describe the model;

- A single server queue begins with a working vacation, when the system is empty. During the vacation the customers arrive in Poisson fashion to the system with rate λ_V.
- The customers are served at mean rate of μ_V during the working vacation and the server is prone to breakdown with mean rate α_V. When the server breaks down during vacation, it is sent for repair with rate β_V.
- When the server is not on working vacations, the customers arrive to the system according to Poisson process with rate λ_B.
- Two types of services are provided to the customers. The first essential service is needed to all arriving customers with mean rate μ_1. As soon as the first service of a customer is completed, then with probability r, he may opt for the second service or else with probability (1-r), he may opt to leave the system. The second service times are assumed to be exponentially distributed with mean service rate μ_2.
- Assume that the life time of the server is exponentially distributed with mean $1/\alpha_1$ in first essential service. In second optional service, the server may fails according to exponential distribution with rate α_2.
- After breakdown, the server immediately sent for repair. The repair time distributions while server fails during essential and optional service phases are exponentially distributed with mean rate β_1 and β_2, respectively.
- The server immediately starts to serve the customers after it is fixed.

For mathematical formulation purpose, we define the following steady state probabilities:

P(0,n,V) Probability that there are n \ customers in the system when the server is on working vacation

P(1,n,V) Probability that there are n customers in the system when the server is in broken down state during working vacation

P(1,n,B) Probability that there are n customers in the system when the server is rendering first essential service

P(2,n,B) Probability that there are n customers in the system when the server is rendering second optional service

P(3,n,B) Probability that there are n customers in the system and the server is in broken down state while rendering first essential service

P(4,n,B) Probability that there are n customers in the system and the server is in broken down state while rendering second optional service

The steady state equations governing the model are constructed as follows:

$$(\lambda_V + \lambda_B)P(0,0,V) = \mu_V P(0,1,V) + (1-r)\mu_1 P(1,1,B) + \mu_2 P(2,1,B) \quad (1)$$

$$(\lambda_V + \alpha_V + \mu_V)P(0,n,V) = \mu_V P(0,n+1,V) + \lambda_V P(0,n-1,V) + \beta_V P(1,1,V),\ n>0 \quad (2)$$

$$(\lambda_V + \beta_V)P(1,1,V) = \alpha_V P(0,1,V) \quad (3)$$

$$(\lambda_V + \beta_V)P(1,n,V) = \alpha_V P(0,n,V) + \lambda_V P(1,n-1,V),\ n>1 \quad (4)$$

$$(\alpha_1 + \lambda_B + (1-r)\mu_1 + r\mu_1)P(1,1,B) = \lambda_B P(0,0,V) + \beta_1 P(3,1,B) + \mu_2 P(2,2,B) + (1-r)\mu_1 P(1,2,B) \quad (5)$$

$$(\alpha_1 + \lambda_B + (1-r)\mu_1 + r\mu_1)P(1,n,B) = \lambda_B P(1,n-1,B) + \beta_1 P(3,n,B) + \mu_2 P(2,n+1,B) + (1-r)\mu_1 P(1,n+1,B),\ n>1 \quad (6)$$

$$(\lambda_B + \alpha_2 + \mu_2)P(2,1,B) = r\mu_1 P(1,1,B) + \beta_2 P(4,1,B) \quad (7)$$

$$(\lambda_B + \alpha_2 + \mu_2)P(2,n,B) = r\mu_1 P(1,n,B) + \beta_2 P(4,n,B) + \lambda_B P(2,n-1,B),\ n>1 \quad (8)$$

$$(\lambda_B + \beta_1)P(3,1,B) = \alpha_1 P(1,1,B) \quad (9)$$

$$(\lambda_B + \beta_1)P(3,n,B) = \alpha_1 P(1,n,B) + \lambda_B P(3,n-1,B),\ n>1 \quad (10)$$

$$(\lambda_B + \beta_2)P(4,1,B) = \alpha_2 P(2,1,B) \quad (11)$$

$$(\lambda_B + \beta_2)P(4,n,B) = \alpha_2 P(2,n,B) + \lambda_B P(4,n-1,B),\ n>1 \quad (12)$$

3. MATRIX GEOMETRIC SOLUTION

The theory of matrix geometric approach was developed [26] to solve the stationary state probabilities for the vector state Markov Process with repetitive structure. Consider the generator matrix Q as shown below:

$$Q = \begin{bmatrix} B_{00} & B_{01} & & & & & & \\ B_{10} & A_1 & A_0 & & & & & \\ & A_2 & A_1 & A_0 & & & & \\ & & & --- & & & & \\ & & & & --- & & & \\ & & & & A_2 & A_1 & A_0 & \\ & & & & & A_2 & A_1 & A_0 \\ & & & & & & & --- \end{bmatrix}$$

The matrix can be decomposed in to sub matrices B_{00}, B_{01}, B_{10}, A_0, A_1, A_2 as follows:

$$B_{00} = [-(\lambda_V + \lambda_B)];$$

$$B_{01} = [\lambda_V \quad 0 \quad \lambda_B \quad 0 \quad 0 \quad 0]$$

$$B_{10} = \begin{bmatrix} \mu_V \\ 0 \\ (1-r)\mu_1 \\ \mu_2 \\ 0 \\ 0 \end{bmatrix}$$

$$A_0 = \begin{bmatrix} \lambda_V & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_V & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_B & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_B & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_B & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda_B \end{bmatrix}$$

$$A_1 = \begin{bmatrix} -(\lambda_V + \alpha_V + \mu_V) & \alpha_V & 0 & 0 & 0 & 0 \\ \beta_V & -(\lambda_V + \beta_V) & 0 & 0 & 0 & 0 \\ 0 & 0 & -(\lambda_B + \alpha_1 + \mu_1) & r\mu_1 & \alpha_1 & 0 \\ 0 & 0 & 0 & -(\lambda_B + \alpha_2 + \mu_2) & 0 & \alpha_2 \\ 0 & 0 & \beta_1 & 0 & -(\lambda_B + \beta_1) & 0 \\ 0 & 0 & 0 & \beta_2 & 0 & -(\lambda_B + \beta_2) \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \mu_V & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & (1-r)\mu_1 & 0 & 0 & 0 \\ 0 & 0 & \mu_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Let X be the vector of the steady state probabilities with Q as coefficient matrix, such that XQ=0, and the normalizing condition is Xe=1, where e is the column vector of appropriate dimension with all elements equal to 1. Let us partition X as $X = [X_0, X_1, X_2, ...]$ where:

$$X_0 = [X_{0,0,V}]$$

$$X_i = [X_{0,i,V}, X_{1,i,V}, X_{1,i,B}, X_{2,i,B}, X_{3,i,D}, X_{4,i,D}],\ i \geq 1$$

We examine the existence of a solution of the form,

$$X_i = X_{i-1}R \quad \text{or} \quad X_i = X_1 R^{i-1}, i \geq 1 \quad (13)$$

Since, X_i depends only on the state transition between level (i-1) and level i, the balance equations are given by:

$$X_0 B_{00} + X_1 B_{10} = 0 \quad (14)$$

$$X_0 B_{01} + X_1 A_1 + X_2 A_2 = 0 \quad (15)$$

$$X_{i-1}A_0 + X_i A_1 + X_{i+1}A_2 = 0,\ i > 1 \tag{16}$$

In Equation (13), R is a square matrix and is the unique minimal non negative solution to the non linear matrix equation

$$A_0 + RA_1 + R^2 A_2 = 0 \tag{17}$$

The matrix R can be computed by successive substitution in the recurrence relation R(0)=0

$$R(n+1) = -A_0 A_1^{-1} - R^2(n) A_2 A_1^{-1},\ n \geq 0 \tag{18}$$

Finally, we are interested to calculate the vector $X=[X_0, X_1, X_2, \ldots\ldots]$ for this purpose. The balance equations for the boundary states given by Equations (14) and (15) can be written in matrix form as:

$$X_0(A_1 + RA_2) = 0 \tag{19}$$

$$X_0(I-R)^{-1}e = 1 \tag{20}$$

where, e is the column matrix of suitable dimension having all elements 1; this gives a unique solution for $X=[X_0, X_1, X_2, \ldots\ldots]$.

4. PERFORMANCE MEASURES

The validity of the model and the system performance characteristics can be analyzed by computing the system performance characteristics, in terms of the steady state probabilities explicitly. Some of the system performance indices are as follows:

- Probability that the server is on working vacation is obtained as:

$$P(W_v) = \sum_{i=0}^{\infty} P(0,i,V) \tag{21}$$

- Probability that the server is in broken down state while fails during working vacation is:

$$P(D_v) = \sum_{i=1}^{\infty} P(1,i,V) \tag{22}$$

- Probability that the server is busy with first essential service is:

$$P(B_1) = \sum_{i=1}^{\infty} P(1,i,B) \tag{23}$$

- Probability that the server is busy with second optional service is:

$$P(B_2) = \sum_{i=1}^{\infty} P(2,i,B) \tag{24}$$

- Probability that the server is in broken down state while failed during first essential service is:

$$P(D_1) = \sum_{i=1}^{\infty} P(3,i,D) \tag{25}$$

- Probability that the server is in broken down state while failed during second optional service is:

$$P(D_2) = \sum_{i=1}^{\infty} P(4,i,D) \tag{26}$$

- Average number of customers in the system is:

$$E[N] = \sum_{i=0}^{\infty} i\left[P(0,i,V) + P(1,i,V)\right] + \sum_{i=1}^{\infty} i\left[P(1,i,B) + P(2,i,B) + P(3,i,D) + P(4,i,D)\right] \tag{27}$$

- Throughput is given by:

$$T(P) = \mu_V \sum_{i=0}^{\infty}[P(0,i,V) + P(1,i,V)] + \sum_{i=1}^{\infty}[\mu_1 P(1,i,B) + \mu_2 P(2,i,B) \tag{28}$$

5. SENSITIVITY ANALYSIS

In order to determine the performance of the working vacation queueing system with second optional service and unreliable server, we perform a computational experiment by a program developed in MATLAB using matrix geometric technique discussed in section 3. The Tables 1-5 and Figures 1-6 depict the variation of the system performance characteristics with respect to the various input parameters. We assume the following basic input data for different tables:

TABLE 1:	$\lambda_B=0.3$, $\mu_1=6$, $\mu_2=4$, $\mu_V=2$, $\alpha_1=0.05$, $\alpha_2=0.03$, $\alpha_V=0.02$, $\beta_1=3.5$, $\beta_2=2.5$, $\beta_V=1.5$, r=0.2
TABLE 2:	$\lambda_V=0.1$, $\mu_1=6$, $\mu_2=4$, $\mu_V=2$, $\alpha_1=0.05$, $\alpha_2=0.03$, $\alpha_V=0.02$, $\beta_1=3.5$, $\beta_2=2.5$, $\beta_V=1.5$, r=0.2
TABLE 3:	$\lambda_V=0.1$, $\lambda_B=0.3$, $\alpha_1=0.05$, $\alpha_2=0.03$, $\alpha_V=0.02$, $\beta_1=3.5$, $\beta_2=2.5$, $\beta_V=1.5$, r=0.2
TABLE 4:	$\lambda_V=0.1$, $\lambda_B=0.3$, $\mu_1=6$, $\mu_2=4$, $\mu_V=2$, $\beta_1=3.5$, $\beta_2=2.5$, $\beta_V=1.5$
TABLE 5:	$\lambda_V=0.1$, $\lambda_B=0.3$, $\alpha_1=0.05$, $\alpha_2=0.03$, $\alpha_V=0.02$, r=0.2, $\mu_1=6$, $\mu_2=4$

Tables 1 and 2 show the variation in long run probabilities of different states of the server by varying arrival rates. It is clear from tables that P_{WV} and P_{DV} show decreasing trend with increased value of arrival rates whereas P_{B1}, P_{B2}, P_{D1}, P_{D2} have increased values for higher arrival rates. Table 3 summarizes the effect of increased service rates on the long run probabilities. It is evident from table that the increased service rate results in decreased P_{B1}, P_{B2}, P_{D1}, P_{D2}, keeping the working vacation service rate μ_V as constant. Table 4 illustrates that the probabilities of the server being busy and under repair state, follow increasing trend for increased server breakdown rates α_1, α_2 whereas P_{WV} and P_{DV} have very dormant decrement with the increase in failure rates. Table 5 presents the comparison between probabilities of server status for different repair rates β_1 and β_2 by keeping β_V as constant. We observe that the vacation state probabilities of the server increase with the increase in the value of repair rate while the probabilities of the server being busy and under repair show decreasing pattern.

In Figures 1-6, we plot the variation of the queue length with respect to different parameters such as failure rate, repair rate, service rate, etc.. Figures 1-6 depict that the average queue length E[N] follow an increasing trend for higher values of arrival rate λ.

In Figures 1 and 2, E[N] depicts a clearly increasing pattern with the increase in server breakdown rate. This trend indicates that the system performance tremendously affected by the server breakdown and will lead to increased congestion. From Figures 3 and 4, it is noticed that initially there is a gradual decrement in the average queue length but later on a sharp decrement can be seen in E[N] on increasing the repair rate of the server. Therefore, by keeping higher repair rate, the performance of the system can be improved. Figures 4-6 depict how service rates affect the average queue length.

TABLE 1. Long run probabilities of the server status by varying λ_v

λ_v	P_{wv}	P_{Dv}	P_{B1}	P_{B2}	P_{D1}	P_{D2}
0.1	0.4071	0.3029	0.1040	0.1685	0.1147	0.0785
0.2	0.4009	0.3015	0.1301	0.1725	0.1194	0.0975
0.3	0.3498	0.2222	0.1506	0.2256	0.1393	0.1084
0.4	0.3050	0.2107	0.1810	0.2520	0.1577	0.1137
0.5	0.3005	0.2072	0.1978	0.2612	0.1654	0.1262
0.6	0.2777	0.2047	0.2437	0.2734	0.1676	0.1373
0.7	0.2119	0.1453	0.2556	0.3661	0.1666	0.1404
0.8	0.1439	0.1317	0.3271	0.5934	0.1635	0.1469
0.9	0.0994	0.0932	0.3619	0.5812	0.1585	0.1557

TABLE 2. Long run probabilities of the server status by varying λ_B

λ_B	P_{wv}	P_{Dv}	P_{B1}	P_{B2}	P_{D1}	P_{D2}
0.1	0.5975	0.4833	0.0016	0.0519	0.0172	0.0233
0.2	0.4820	0.4847	0.0034	0.0819	0.0212	0.0476
0.3	0.4660	0.4799	0.0055	0.1191	0.0264	0.0743
0.4	0.3493	0.4749	0.0080	0.1641	0.0322	0.1054
0.5	0.3322	0.3697	0.0111	0.2179	0.0381	0.1434
0.6	0.3147	0.3645	0.0150	0.2819	0.0437	0.1917
0.7	0.2968	0.2591	0.0202	0.3582	0.0486	0.2546
0.8	0.1780	0.1536	0.0269	0.4500	0.0524	0.3385
0.9	0.1600	0.0480	0.0358	0.5630	0.0551	0.3520

TABLE 3. Long run probabilities of the server status by varying μ_1 and μ_2

μ_v	μ_1	P_{wv}	P_{Dv}	P_{B1}	P_{B2}	P_{D1}	P_{D2}
	2	0.0054	0.0017	0.3369	0.3169	0.2718	0.1720
	4	0.2329	0.1104	0.2912	0.2778	0.1841	0.0882
	6	0.2607	0.2199	0.1678	0.2542	0.1477	0.0795
	8	0.2890	0.2634	0.1082	0.2091	0.1191	0.0651
3	μ_2	P_{wv}	P_{Dv}	P_{B1}	P_{B2}	P_{D1}	P_{D2}
	1	0.0082	0.0024	0.3301	0.3028	0.2601	0.1102
	3	0.2408	0.1212	0.2721	0.2493	0.1792	0.0828
	5	0.2841	0.2652	0.1390	0.2391	0.1408	0.0729
	7	0.2988	0.2981	0.0998	0.1830	0.1118	0.0539

TABLE 4. Long run probabilities of the server status by varying α_1 and α_2

α_v	α_1	P_{wv}	P_{Dv}	P_{B1}	P_{B2}	P_{D1}	P_{D2}
	0.01	0.0078	0.0025	0.5328	0.3194	0.1691	0.0174
	0.03	0.0064	0.0020	0.6210	0.3209	0.1729	0.0205
	0.05	0.0052	0.0016	0.6350	0.3231	0.1832	0.0247
	0.07	0.0043	0.0011	0.6520	0.3301	0.1929	0.0391
0.02	α_2	P_{wv}	P_{Dv}	P_{B1}	P_{B2}	P_{D1}	P_{D2}
	0.02	0.0059	0.0019	0.5197	0.3273	0.1337	0.0431
	0.04	0.0056	0.0014	0.5431	0.3309	0.1568	0.0910
	0.06	0.0052	0.0011	0.5578	0.3746	0.1691	0.0943
	0.08	0.0049	0.0009	0.5629	0.3809	0.1731	0.0971

TABLE 5. Long run probabilities of the server status by varying β_1 and β_2

β_v	β_1	P_{wv}	P_{Dv}	P_{B1}	P_{B2}	P_{D1}	P_{D2}
	3	0.0951	0.0216	0.5374	0.3294	0.1148	0.1481
	5	0.0953	0.0217	0.4438	0.3019	0.1031	0.1423
	7	0.0954	0.0217	0.3368	0.3000	0.1009	0.1409
	9	0.0959	0.0319	0.2291	0.2902	0.0962	0.1399
2.5	β_2	P_{wv}	P_{Dv}	P_{B1}	P_{B2}	P_{D1}	P_{D2}
	2	0.0944	0.0314	0.5242	0.3616	0.1443	0.1988
	4	0.0958	0.0316	0.5297	0.3220	0.1201	0.1701
	6	0.0961	0.0319	0.4038	0.3201	0.1179	0.1516
	8	0.0979	0.0324	0.3981	0.2201	0.2124	0.0998

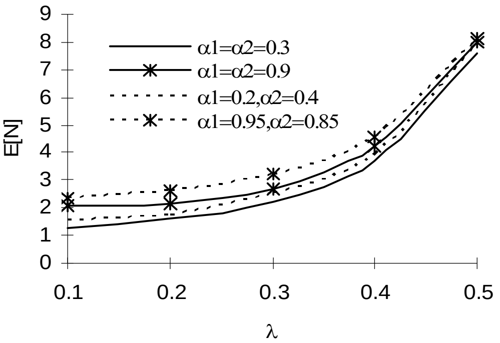

Figure 1. Average queue length vs λ

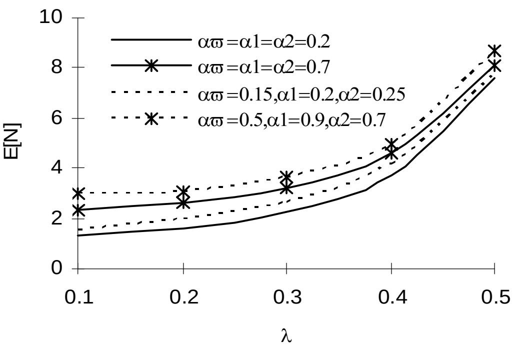

Figure 2. Average queue length vs λ

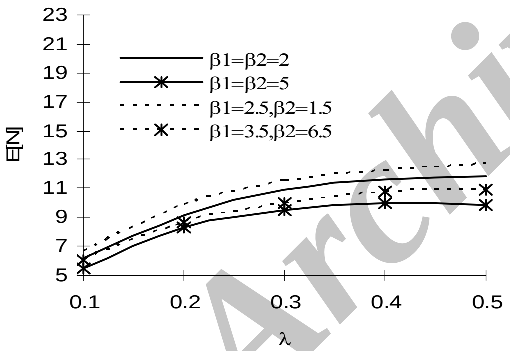

Figure 3. Average queue length vs λ

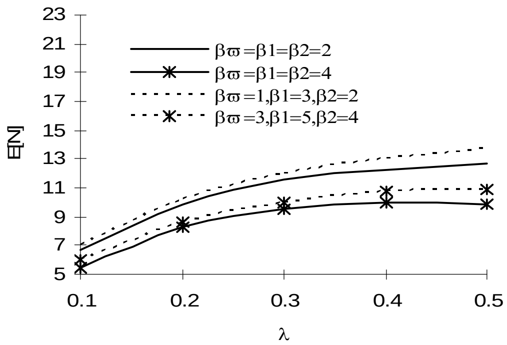

Figure 4. Average queue length vs λ

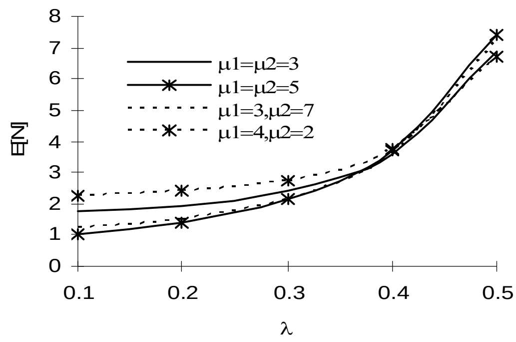

Figure 5. Average queue length vs λ

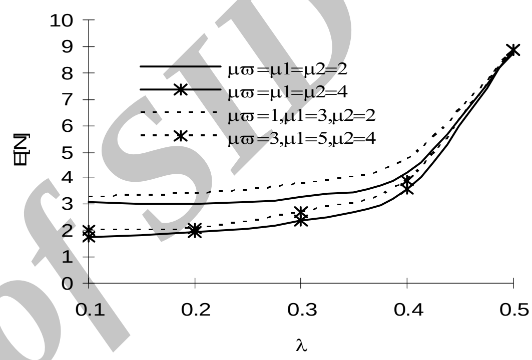

Figure 6. Average queue length vs λ

6. CONCLUDING REMARKS

In many stochastic systems, there may occur a situation in which the first service is essential to all arrivals whereas second service is needed by only some of them. Similarly breakdown is a remarkable and unavoidable phenomenon in the service facility of a queueing system, because system performance deteriorates seriously by the server breakdown and limitations of the repair capacity. In this paper, we have studied the effect of second optional service and unreliable server on the performance measures for working vacation queueing model. The queue with working vacation may be applicable in modeling of many practical situations related to computers, communications, and productions systems, etc., wherein the server works at different service rates rather than completely stopping the service during a vacation.

The inclusions of realistic factors such as unreliable server, optional service, working vacation, etc. make our model more versatile from application point of view. Using matrix geometric solution we have obtained some important performance measures, which may be useful for the system designers and practitioners involved in many industrial organizations operating in congestion scenario.

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Working Vacation Queue with Second Optional Service and Unreliable Server

TECHNICAL NOTE

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PAPER INFO

Paper history:
Received 2 January 2010
Accepted in revised form 17 May 2012

Keywords:
Working Vacation
Second Optional Service
Matrix Geometric Technique
Server Breakdown
Repair
Queue Size

چکیده

یک سیستم رده‌بندی M/M/1 به همراه سرویس پشتیبانی اختیاری و سرور نامعتبر مورد مطالعه قرار گرفته است. ما متوجه شدیم که سرور به جای بی‌کار بودن در زمان تعطیلات، با سرعت‌های متفاوتی کار می‌کند. کاربران طبق فرایند پویسن (Poisson) و با توجه به سرعت (اینترنت) و وضعیت سرور به سیستم دسترسی دارند. همه کاربران خواستار سرویس لازم اولیه هستند در حالی‌که فقط بعضی از انها از سرویس پشتیبانی اختیاری استفاده می‌کنند. ممکن است یک کاربر بعد از (استفاده از) سرویس لازم اولیه با احتمال $(1-r)$ سیستم را ترک کند یا این‌که با احتمال $(0<r<1)$ r در تکمیل (کار) سرویس لازم اولیه، از سرویس پشتیبان اختیاری استفاده کند. طبق فرایند پویسن، طی دوره‌های کاری پرازدحام و تعطیلات ممکن است سرور از کار بیافتد (احتمال خرابی سرور وجود دارد). دفعات سرویس در تعطیلات و هم مدت زمان سرویس به‌صورت (توابع) نمایی توزیع شده است. از روش ماتریس ژئومتریک برای انالیز وابستگی سیستم رده‌بندی استفاده شده است. همچنین انالیز دقیقی برای بررسی تغییرات خصوصیات اجرایی سیستم با پارامترهای ورودی متفاوت انجام شده است.

doi: *10.5829/idosi.ije.2012.25.03c.06*