

On the Self Similar Energy Gain Scaling in Isobaric Approach of ICF Targets

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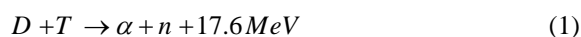
Abstract

In this paper we start with Meyer-Ter-Vehn isobaric fusion model and try to reconstruct all equations by introducing a dimensionless variable $\psi_i = r_i/R_m$. Then we investigate the proper sets of spark confinement parameter and temperature $\{H_s, T_s\}$ which satisfy ignition conditions of spark ignition in deuterium-tritium (DT) equimolar mixture in terms of isentrope parameter, α , implosion velocity, U_{im} . Finally, limiting fuel energy gain curve (G_f^*) as function of ignition energy (E_{ign}) for different implosion velocities is calculated. By this simple modification of our original formulation, we obtain fuel energy scaling as $E_f \propto \alpha^3 U_{im}^{-10}$ for desirable range of ignition parameters. Since in our formulation $H_s T_s$ product has appeared, it enabled us to insert a scaling relation for ignition condition in fuel energy, E_f which this in turn decreases implosion velocity exponent in our scaling relation to 7.0.

Keywords: Isobaric model; Central spark ignition; Self-similar; Energy gain; Energy gain scaling

I. Introduction

The Vast efforts have taken place to achieve the generation of energy from controlled nuclear fusion both in magnetic confinement fusion (MCF) and inertial confinement fusion (ICF) [1]. ICF is an approach to fusion that relies on the inertia of the fuel mass to provide confinement. It is based on micro explosion of thermonuclear targets:



A major challenge has always been to come up with target design that would be appropriate for energy production [2] *i.e.* major scientific requirement for using ICF is to reach high energy gain G_f :

$$G_f = \frac{E_{TN}}{E_f} \quad (2)$$

where E_{TN} is thermonuclear energy released from deuterium-tritium (DT) fusion reaction that carried with its products and E_f is fraction of incident beam which is converted to fuel thermal energy. The central spark ignition idea, which is currently being considered as a scheme for reaching high energy gain, is required to drive fuel to lower specific entropy than its inner part by a suitable set of shocks and refraction during shell implosion. In this concept, ignition occurs at fuel center and then propagates a burning wave to the surrounded dense and cold fuel layer.

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II. Admissible Criterion for Thermonuclear Burn of a Bare DT Microsphere

In inertial confinement fusion, a spherical shell of cryogenic deuterium and tritium (DT) filled with DT gas is accelerated by direct laser irradiation (direct drive) or x rays produced by a high-Z enclosure (indirect drive). When an intense laser beam is uniformly impinged on a spherical fuel pellet with intensity of the order of 10^{14} - 10^{15} Wcm⁻², the laser energy is absorbed on the surface to generate high temperature plasma of 2-3 keV, and eventually an extremely high pressure of a few hundred mega-bars is generated. The mechanism of the acceleration is the same as rocket propulsion [3]. When the accelerated fuel collides at the center, compression and heating occur. If the dynamics are sufficiently spherically symmetric, the central area is heated up to 5-10 keV (is called the 'central spark'), and a fusion reaction starts. In fusion reactor scale implosions, a fuel pellet with an initial radius of about 3 mm should be compressed to a radius of about 100 μ m. For this purpose, highly precise uniformity is required for irradiation intensity distribution over the fuel pellet as well as high quality sphericity and uniformity of the fuel pellet. The implosion velocity for achieving the fusion ignition temperature of 5-10 keV at the centre of fuel is required to be $(3-4) \times 10^7$ cm s⁻¹. Therefore, the pulse length of the laser should be 10-20 ns, and in this time-scale the directed mega-joule energy has to be delivered to a fuel pellet of radius about 3 mm [4].

According to isobaric model of fusion targets in inertial confinement fusion, it is assumed that at the final stage of hydrodynamic implosion, target configuration is composed of two distinct regions called as *spark* and *cold fuel* region [5]. The spark region is an ideal plasma of hydrogen isotopes with density, temperature and radius of ρ_s , T_s and R_s , respectively. On the other hand, the cold fuel region is a degenerate electron gas of parameters ρ_c , T_c and R_m , respectively. It is also assumed that pressure is constant across the whole target at stagnation time (maximum compression). A spark with given values of temperature, T_s , and its confinement parameters, $H_s = \rho_s R_s$, can be produced by hydrodynamic implosion process which then ignites and propagates burning waves in the cold region. There exist processes here which cool and dissipate spark region. Most important of them are bremsstrahlung energy losses of hydrogen isotope plasma and electron thermal conductivity [6].

As a capsule implodes Alpha-particle deposition from thermonuclear burn of DT try to heat the central hot spot region. On the other hand, electron conduction

from the hot spot to cold surrounding fuel as well as radiative losses acts to cool the hot spot. If conduction and radiative losses from the hot spot is too large, the ignition process never occurs. To achieve ignition by the time the implosion process has stopped, hot spot must have a ρr equal to about 0.3-0.4 g/cm² and must achieve a central temperature of about 5-10 KeV. Under this condition, α -particle energy deposition can overcome loss processes and a self sustaining burn wave will be generated. At first, we consider that the dominant radiation loss in fuel target, P_{br} is bremsstrahlung emission which escapes from the fuel and its power density is given by:

$$P_{br} = 5.36 \times 10^{-37} n_e (n_D + n_T) T_e^{1/2} \quad W / m^3 \quad (3)$$

where T_e is electron temperature and n_e , n_D and n_T are electron, deuterium and tritium densities, respectively. Assuming equal number densities ($n_e = n_i = n_s$) and temperatures ($T_e = T_i = T_s$) of electron and ions in hydrogen plasma we have:

$$P_{br} = 5.36 \times 10^{-37} n_s^2 T_s^{1/2} \quad W / m^3 \quad (4)$$

On the other hand, the conduction loss power density, P_{ec} , is obtained by assuming power law dependence for heat conduction coefficient and by applying Spitzer conductivity for deuterium-tritium plasma:

$$P_{ec} = 1.74 \kappa_{ec} \frac{KT_s}{R_s^2} \quad W / m^3 \quad (5)$$

here, $K = 1.6 \times 10^{-16}$ J/KeV is Boltzmann's constant and κ_{ec} is electron heat conduction coefficient.

Now, we introduce a self-similar parameter, ψ , which is defined as ratio of a specific layer to fuel radius at stagnation time. By this definition we would be able to consider the evolution of class of pellet designs with definite values of spark/fuel ratio [7]:

$$\psi_i = \frac{r_i}{R_m} \quad (6)$$

Using this definition, we will try to find appropriate sets of spark confinement parameters, H_s , and corresponding temperature, T_s , dependence on some important fusion parameters like isentrope, α implosion velocity, U_{im} , and spark radius, R_s . In Eq. 6, R_m is outer fuel radius at the time of maximum compression and is a function of isentrope parameter α , implosion velocity U_{im} , spark temperature T_s and also fractional spark radius ψ_s and which can be calculated from energy and mass balance equation and is given by:

$$R_m =$$

$$\left[\left(\frac{Y}{J} \right)^3 \frac{H_s^2 T_s^{-3}}{U_{im}^{10} \psi_s^2 (1 - \psi_s^3)^5} (3JT_s - \psi_s^3 U_{im}^2)^5 \right]^{1/2} \quad (7)$$

where $J = 77.2 \text{ MJ/g.KeV}$ and $Y = 2.18 \times 10^{-3} \text{ Pa.m}^5 \text{.Kg}^{-5/3}$ are constants.

It is obvious that necessary condition for ignition is that the amounts of energy in gain processes overcome loss processes. In our case, this means that the thermonuclear energy release exceeds cooling energy losses process across spark region. Thermonuclear heating rate per unit volume, power density, is obtained from burn rate and the fractional alpha particle deposition and is given by:

$$P_{TN} = \langle \sigma v \rangle_{DT} n_D n_T f_\alpha Q_\alpha \quad W / m^3 \quad (8)$$

where $\langle \sigma v \rangle_{DT}$ is Maxwellian averaged fusion rate of a DT mixture, f_α is α -particles deposition factor i.e. fractions of alpha particles that remain in spark region and deposit their energies there and Q_α is alpha particles birth energy at spark center which is equal 3.5 MeV. The DT fuel in the hot spot region undergoes a thermonuclear spark whenever the net energy release during the confinement period becomes comparable to the initial energy content of DT plasma. In a good approximation, an effective confinement time scale which allows ignition of cold fuel surrounded hot region is given by:

$$\Delta t_c \cong \frac{R_m}{U_{im}} \quad (9)$$

So ignition condition in this frame can be written as:

$$(P_{TN} - P_{ec} - P_{br}) \Delta t_c \cong \frac{3}{2} \sum_i n_i K T_s \quad (10)$$

where sum is over ion and electron densities. With substituting for corresponding expressions its final form is:

$$\begin{aligned} & 7.11 \times 10^{18} \frac{H_s f_\alpha \langle \sigma v \rangle_{DT}}{T_s} \\ & - 26.96 \frac{H_s}{\sqrt{T_s}} - 2.97 \times 10^{-3} \frac{T_s^{5/2}}{H_s} \\ & \cong U_{im} \psi_s \end{aligned} \quad (11)$$

As could be seen, the right hand side of this equation is only a function of products of implosion velocity and spark fractional radius. To obtain a good Physical picture of spark parameter in HT plane we apply a numerical solution of this expression for three right

hand values of 0, 4 and 8 which gives ignition boundary curves for admissible values of spark parameters which satisfy our ignition condition and is shown in Figure 1. In the limit of prolonged confinement period, $U_{im} \psi_s \ll 10^7 \text{ cm/s}$, ignition condition is defined by equilibrium of heating and cooling processes, $P_\alpha = P_{br} + P_{ec}$, then it is independent of implosion velocity and spark fractional radius. In contrast, for short confinement period, $U_{im} \psi_s \gg 10^7 \text{ cm/s}$, ignition boundary shows a sensitive behavior to implosion velocity and spark fractional radius changes. For its larger value ignition boundary curve lies further with respect to prolonged confinement situation and $H_s T_s$ product has a minimum value.

We perform a series of numerical calculations in order to find $H_s T_s$ product dependence on implosion velocity and spark fractional radius for spark temperature $7 \leq T_s (\text{KeV}) \leq 12$, fractional spark radius $0.2 \leq \psi_s \leq 0.9$ and implosion velocity $2 \times 10^7 \leq U_{im} (\text{cm/s}) \leq 6 \times 10^7$. We obtain a linear proportionality for ignition criterion:

$$H_s T_s \propto U_{im} \psi_s \quad (12)$$

The latter relation not only gives ignition threshold for the product $H_s T_s$ but also its minimum value along the ignition boundary which lies between 7 to 12 KeV. It should also be noticed that left branch of these curves which approximately coincide on each other will also give us an important result of self-ignition temperature of about 4.4 KeV for deuterium-tritium plasma at ignition configuration. Higher values of RHS of Eq. (11) correspond to short confinement time and high implosion velocity and its lower values for a prolonged confinement of fusion plasma and small implosion velocity. However if the fuel pellet is optically thick for an opacity length of l_α , it has several thickness units from the center to the edge and most of radiated energy is trapped inside it and radiation loss is reduced.

III. Energy Gain

In this section, we present simple analytical calculations based on Meyer-Ter-Vehn isobaric model of ICF targets by reconstructing all equations in terms of our self-similar variable, ψ_s , and then discuss on their results. We start fuel stagnation configuration according to ideal assumptions of isobaric model [5]. According to this model, it is supposed that compressed fuel has a spherical shape of radius R_m (fuel external radius at maximum compression) and step-like density profile,

$$\rho(\psi) = \begin{cases} \rho_s, & 0 \leq \psi \leq \psi_s \\ \rho_c, & \psi_s < \psi \leq 1 \end{cases} \quad (13)$$

where $\rho_s < \rho_c$ and subscripts denote hot spark and cold main fuel regions, respectively and the pressure, P , is constant in entire fuel volume. Assuming ideal gas equation of state for spark region, we have:

$$P_s = J \rho_s T_s \quad (14)$$

On the other hand, adopting a constant value of isentrope parameter throughout cold fuel region:

$$\alpha = \frac{P_c}{P_{\text{deg}}} = \frac{\text{(pressure of main fuel)}}{\text{(Fermi pressure at the density of cold main fuel)}} \quad (15)$$

So, our final expression for pressure balance in compressed fuel is obtained:

$$P_s = J \rho_s T_s = Y \alpha \rho_c^{5/3} \quad (16)$$

Using Eq. (16), we can easily evaluate spark and fuel region densities dependence. So, we have constructed our ignition configuration with five independent parameters H_s (g/cm^2), ψ_s , α , T_s (KeV), and U_{im} (in units of 10^7 cm/sec). Using equation of mass:

$$M_f = \frac{4\pi}{3} R_m^2 [H_s \psi_s^2 + \rho_c R_m (1 - \psi_s^3)] \quad (17)$$

the following expression for spark and cold region energies can be obtained:

$$E_s = H_s T_s R_m^2 \frac{4\pi \psi_s^2 K}{m_{DT}} \quad (18)$$

$$E_c = 5.18 \times 10^{16} H_s T_s R_m^2 \frac{(1 - \psi_s^3)}{\psi_s} \quad (19)$$

Now, we can use energy balance equation to obtain an explicit expression for compressed fuel radius in terms of ignition parameters:

$$E_f = \frac{3}{2} P_s V = \frac{1}{2} M_f U_{\text{im}}^2 \quad (20)$$

By substitution of Eq. (7) in Eq. (20) we will obtain a relation for fuel energy, E_f as function of ψ_s , α , U_{im} , T_s and H_s :

$$E_f = 2\pi \times 10^{-13} (JY)^3$$

$$\times \frac{\alpha^3 (H_s T_s)^3}{U_{\text{im}}^{10} \psi_s^3 (1 - \psi_s^3)^5} \left(3 - \frac{\psi_s^3 U_{\text{im}}^2}{J T_s} \right)^5 \quad (21)$$

Above equation shows a weak dependence on implosion velocity and it does not give us exact power law dependence for energy scaling. Since we look for a fuel energy scaling relation of the form $E_f \propto \alpha^{3.0} U_{\text{im}}^B$ where B is constant, we perform a series of numerical calculation to obtain a power law dependence on implosion velocity for fuel energy by assuming $U_{\text{im}} \leq 5$, $5 \leq T_s (\text{KeV}) \leq 12$, $0.4 \leq H_s (\text{g/cm}^2) \leq 1.0$ and $0.2 \leq \psi_s \leq 0.9$ for our isobaric configuration

$$E_f \propto \psi_s^{-2.35} \alpha^{3.0} (H_s T_s)^{3.0} U_{\text{im}}^{-10.0} \quad (22)$$

By this modification, our results reproduced in a good manner the energy scaling relation of the Basko self-similar fusion model [8] and Meyer-Ter-Vehn model [9].

Thermonuclear energy gain with respect to energy invested in the fuel is calculated as:

$$G_f = \frac{E_{TN}}{E_{DT}} = \varepsilon_{DT} f_b \frac{M_f}{E_f} \quad (23)$$

where ε_{DT} is the specific DT energy, 3.34×10^{11} MJ/g and f_b is fuel burn fraction:

$$f_b = \frac{H_F}{H_F + 7.0} = \frac{H_s + \rho_c R_m (1 - \psi_s)}{H_s + \rho_c R_m (1 - \psi_s) + H_B} \quad (24)$$

where H_F is fuel areal density.

The optimum value of fractional spark radius for $U_{\text{im}} \leq 5$ is $\psi_{s,\text{opt}} \cong 0.56-0.58$. It tells us about the most economical regime of ignition where maximum fuel mass can be ignited by a given ignition energy [8]. In our reconstruction of the original isobaric model formula in terms of self similar variable, ψ , a product of $H_s T_s$ in Eq. (21) has appeared which this matter help us to invoke to our previous result in Sec. II at ignition time and insert it in our fuel energy relation, which it cause to suppress our previous energy scaling exponent of implosion velocity to 7.0:

$$E_f \propto \psi_s^{0.14} \alpha^{3.0} (H_s T_s)^{3.0} U_{\text{im}}^{-7.0} \quad (25)$$

The results of our numerical calculation is obtained for $U_{\text{im}} \leq 5$ cm/s, $5 \leq T_s (\text{KeV}) \leq 12$ and $0.2 \leq \psi_s \leq 0.9$. Eq. (22) also gives us a more realistic scaling relation which coincides in better approximation with Basko scaling formula [8].

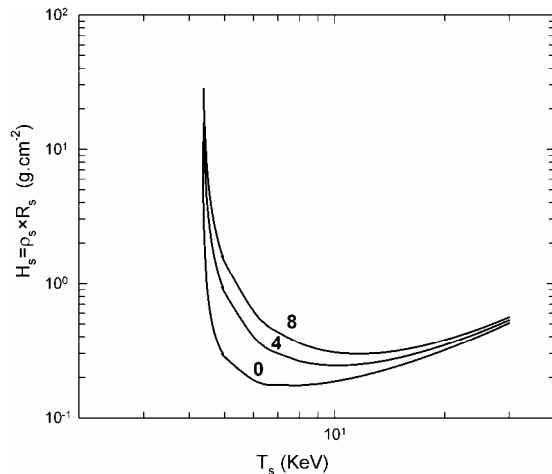


Figure 1. Ignition boundary for three set of RHS values 0, 4 and 8.

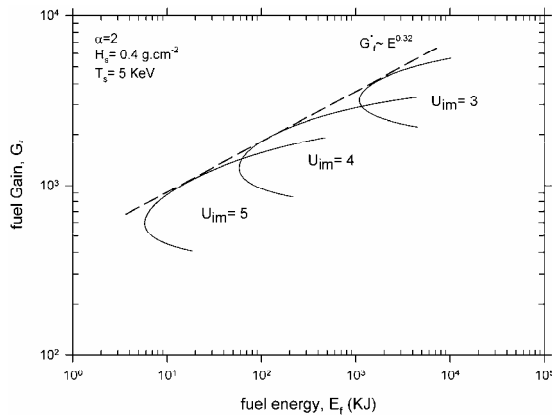


Figure 2. Fuel gain curves (full lines) and limiting fuel gain curve (dash line) as function of compressed fuel energy.

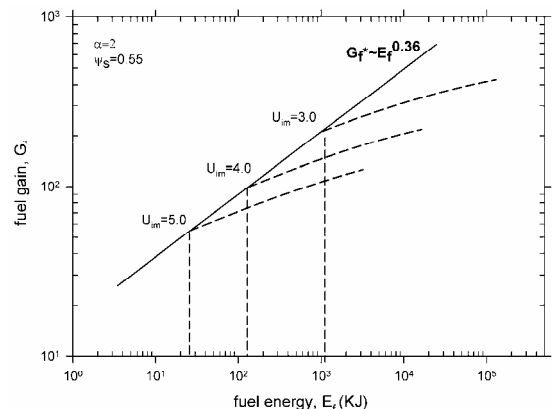


Figure 3. Fuel gain curves (dash lines) and limiting fuel gain curve (full line) as function of compressed fuel energy for case that we insert $H_s T_s$ in our relations.

IV. Limiting Energy Gain

The question about required amount of energy for a proper ignition of fusion capsule is one of the aims in ICF research and has a lot of importance in optimization of target design and drivers (lasers or accelerators), since we want to reach to the highest value of energy gain with a given driver energy, $G_f(E_{dr})$. A gain curve that is obtained for a given value of implosion velocity represents a class of capsule design with different sizes. Any such curve has an ignition threshold, $E_{dr,min}$, since for a given implosion velocity, U_{im} , arbitrary fuel masses for ignition is not allowed. The envelope of gain curves for different implosion velocity is the limiting (fuel) gain curve, $(G_f^*) G_T^*$. If we plot fuel gain as function of ignition energy for fixed implosion velocity and limited values of spark fractional radius $0.1 < \psi_s < 0.9$, we will obtain a series of gain curves. The envelope of these gain curves has been shown with a dash line G_f^* and is proportional to $E_f^{0.32}$ which was calculated numerically and is shown in Figure 2 [10]. This result is in good agreement with one that obtained by Basko by self-similar solution of hydrodynamics equations for a bare DT sphere in its simple approximation without any additional constraints like hydrodynamics instability and drive asymmetry. When we take into account direct dependence of our relation to $H_s T_s$ and insert it in scaling relation, we could obtain a more relaxed scaling expression for limiting energy gain line by applying a series of numerical calculation for $0.1 < \psi_s < 0.9$ which is shown in Figure 3 [11]. In this diagram we fixed spark radius near to its optimum condition. These results lead to limiting fuel gain $E_f^{0.36}$ which is the same as Basko scaling [8].

V. Conclusion

In this paper, it is shown that for ICF targets considered with plasma cooling processes, a limited sets of $\{H_s, T_s\}$ values are obtained that satisfy ignition conditions. Ignition boundary curve gives us universal admissible values of spark parameters. Then we perform a series of numerical calculation to obtain $H_s T_s$ dependence on stagnation configuration. It yields direct proportionality dependence to implosion velocity and spark fractional radius.

Further, we present a simple formulation based on Meyer-Ter-Vehn [5] isobaric model and change it in terms of self-similar variable ψ_s , which opens a new window for this standard model results and enable us to compare its results with more sophisticated one. So, we obtained a five variable relation for fuel energy and then simplify it to evaluate $E_f \propto \alpha^{3.0} U_{im}^{-10.0}$. We reproduce

energy scaling Basko [8] and Meyer-Ter-Vehn [9]. Then due to the appearance of $H_s T_s$ product in our fuel energy relation we are able to find a better scaling relation by inserting $H_s T_s$ were derived in Sec.II in our relations which it in turn suppresses implosion velocity exponent in energy scaling. In this way, we obtain final expression for energy scaling $E_f \propto U_{im}^{-7.0}$.

After that, a series of fuel energy gain curves as function of fuel energy for different implosion velocity were plotted and the envelope of the gain curves was calculated numerically without applying $H_s T_s$ scaling. The relation $G^*_f \propto E_f^{0.32}$ was obtained which is in good accordance with the relation obtained by Basko [8]. In continue we look for a limiting gain curve slope by inserting $H_s T_s$ scaling obtained in Sec. II. This causes to suppress implosion velocity exponent in energy scaling to 7.0. The nature of both calculations are the same but a small difference in details appears due to some more realistic assumptions about pressure, density and temperature profile of compressed fuel at ignition time.

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