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## Chaotically Tuples of Unilateral Weighted Backward Shifts Acting On Hilbert Spaces

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### Abstract

We investigate characterize the Chaotically Conditions for the Tuples of unilateral weighted backward shifts on some Hilbert Spaces. The Tuple  $T = (T_1, T_2, T_3, ..., T_n)$  is chaotic if and only if T is Hypercyclic and has a non-trivial periodic point if and only if T has a non-trivial periodic point if and only if the series  $\sum_{m=1}^{\infty} \left(\prod_{k=1}^{m} (e_{k,\lambda})^{-1} e_m\right), \lambda = 1, 2, ..., n$ 

are convergence.

Keywords: Hypercyclicity, periodic point, Chaotically Tuples, Infinity Tuples.

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#### Introduction

Let *B* be an Ordered Banach space and  $T_1, T_2, T_3,...$  are commutative bounded linear mapping on *B*, the infinity Tuple *T* is an infinity components  $T = (T_1, T_2, T_3,...)$ , for every  $x \in B$  defined

 $T(x) = T_1 T_2 T_3 \dots (x) = Sup_n \{ T_1 T_2 T_3 \dots T_n(x) | n \in \mathbb{N}, n = 1, 2, 3, \dots \}$ 

Infinity-Tuple  $T = (T_1, T_2, T_3,...)$  is said to be hypercyclic infinity-tuple if Orb(T, x) is dense in B, that is  $\overline{Orb}(T, x) = B$ .

**Definition 1.1** The Tuple  $T = (T_1, T_2, T_3, ...)$  is called chaotic tuple, if we have tree below conditions together,

- 1. It is topologically transitive, that is, for any given open sets U and V, there exist sequence of positive integers  $\{\delta_j\}_{j=1}^n$  such that  $T_1^{\delta_1}T_2^{\delta_2}...T_n^{\delta_n}(U) \cap (V) \neq \phi$ .
- 2. It has a dense set of periodic points, that is, there is a set P such that for each x in P, we can find  $\{\lambda_j\}_{j=1}^n$  such that  $T_1^{\lambda_1}T_2^{\lambda_2}...T_n^{\lambda_n}(x) = x$ .

It has a certain property called sensitive dependence on initial conditions.

#### Equations

If the Tuple satisfy the bellow theorem, we say that Tuple satisfy The Hypercyclic Criterion.

**Theorem 1.1 [The Hypercyclicity Criterion]** Let *B* be a separable Banach space and  $T = (T_1, T_2, T_3, ...)$  is an infinity tuple of commutative continuous linear mappings on *B*. If there exist two dense subsets *Y* and *Z* in *B* and strictly increasing sequences  $\{m_{k,2}\}_{k=1}^{\infty}$ ,  $\{m_{k,3}\}_{k=1}^{\infty}$ , ... such that:

- 1.  $T_1^{k_{1,j}}T_2^{k_{2,j}}T_3^{k_{3,j}}... \to 0 \text{ on } Y \text{ as } m_{i,j} \to \infty \text{ for } i = 1,2,3,...,$
- 2. There exist function  $\{S_k | S_k : Z \to B\}$  such that for every  $z \in Z$ ,  $S_k z \to 0$  and  $T_1^{k_{1,j}} T_2^{k_{2,j}} T_3^{k_{3,j}} \dots S_k z \to z$

Then  $T = (T_1, T_2, T_3, ..., T_n)$  is a Hypercyclic Tuple.

**Theorem 2.2** Suppose X be an F-sequence space whit the unconditional basis  $\{e_k\}_{k \in N}$  and let  $T_1, T_2, T_3, \dots, T_n$  are unilateral weighted backward shifts with weight



sequence  $\{e_{1,k}\}_{k \in \mathbb{N}}, \{e_{2,k}\}_{k \in \mathbb{N}}, ..., \{e_{n,k}\}_{k \in \mathbb{N}}$  and  $T = (T_1, T_2, T_3, ..., T_n)$  be a tuple of operators

- $T_1, T_2, T_3, \dots T_n$ . Then the following assertions are equivalent:
- 1. T is chaotic,
- 2. T is Hypercyclic and has a non-trivial periodic point,
- 3. T has a non-trivial periodic point,

4. The series 
$$\sum_{m=1}^{\infty} \left( \prod_{k=1}^{m} (e_{k,i})^{-1} e_m \right)$$
 are convergence in X for  $i = 1, 2, ..., n$ .

**Proof.** Proof of the cases  $1 \rightarrow 2$  and  $2 \rightarrow 3$  are trivial, so we just proof  $3 \rightarrow 4$  and  $4 \rightarrow 1$ .

First we proof  $3 \to 4$ , for this, Suppose that T has a non-trivial periodic point, and  $x = \{x_n\}_{x_n \in X}$  be a non-trivial periodic point for T, that is there are positive integers  $\mu_1$ ,  $\mu_2$ , ...,  $\mu_n$  such that  $T_1^{\mu_1}T_2^{\mu_2}...T_n^{\mu_n}(x) = x$ . Comparing the entries at positions,  $k \in N \cup \{0\}$ , of x and  $T_1^{\mu_1}T_2^{\mu_2}...T_n^{\mu_n}(x)$ , we will find that

$$x_{j+kM_1} = \prod_{t=1}^{M_1} (a_{j+kN+t} \cdot x_{j+k+1}), x_{j+kM_2} = \prod_{t=1}^{M_2} (a_{j+kN+t} \cdot x_{j+k+1}), \dots, x_{j+kM_n} = \prod_{t=1}^{M_n} (a_{j+kN+t} \cdot x_{j+k+1})$$

so for  $k \in N \bigcup \{0\}$  and  $\lambda = 1, 2, ..., n$ , we have

$$x_{j+kM_{\lambda}} = \left(\prod_{t=j+1}^{j+kM_{\lambda}} a_{t}\right)^{-1} \cdot x_{j} = c_{\lambda} \cdot \left(\prod_{t=1}^{j+kM_{\lambda}} a_{t}\right)^{-1}, \lambda = 1, 2, ..., n$$

Where  $c_{\lambda} = \prod_{i=1}^{j} (m_{j,\lambda} \cdot x_j), \lambda = 1, 2, ..., n$ . Since  $\{e_{\lambda}\}_{\lambda=1}^{\infty}$  is an unconditional basis for X and  $x \in X$  it

follows that

$$\sum_{k=0}^{\infty} \left( \frac{1}{\prod_{t=1}^{(j+kM_{\lambda})} \dots e_{j+M_{\lambda}}} \right) = \frac{1}{c} \sum_{k=0}^{\infty} \left( x_{j+M_{\lambda}} \dots e_{j+M_{\lambda}} \right), \lambda = 1, 2, \dots, n$$

convergence sequences in X. Without loss of generality we may assume that  $j \ge N$ . Applying the operators  $T, T_2, T_3, ..., T_{k-1}$ , where  $k = Min\{M_1, M_2, ..., M_n\}$ , to this series and note that

$$T_1(e_t) = a_{1,t}e_{t-1}, T_2(e_t) = a_{2,t}e_{t-1}, \dots, T_n(e_t) = a_{n,t}e_{t-1}$$

we deduce that

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$$\sum_{k=0}^{\infty} \left( \frac{1}{\prod_{t=1}^{(j+kM_1-\varepsilon_1)} m_{1,j}} \right) e_{(j+kM_1-\varepsilon_1)}, \sum_{k=0}^{\infty} \left( \frac{1}{\prod_{t=1}^{(j+kM_2-\varepsilon_2)} m_{2,j}} \right) e_{(j+kM_2-\varepsilon_2)}, \dots, \sum_{k=0}^{\infty} \left( \frac{1}{\prod_{t=1}^{(j+kM_n-\varepsilon_n)} m_{n,j}} \right) e_{(j+kM_n-\varepsilon_n)}$$

convergence sequences in X. By adding these series, we see that condition 4 holds. Proof of  $4 \rightarrow 1$ . It follows from theorem (2.1), so under condition 4 the operator T is Hypercyclic. Hence it remains to show that T has a dense set of periodic points. Since  $\{e_{\alpha}\}$  is an unconditional basis, condition 4 with proposition 2.3 implies that for each  $M_1 \in N$  consider the series

$$\varphi_{\lambda}(j,M_{\lambda}) = \sum_{k=0}^{\infty} \frac{1}{\prod_{t=1}^{j+kM_{\lambda}} m_{k,\lambda} \cdot e_{j+kM_{\lambda}}} = \prod_{t=1}^{j} m_{k,\lambda} \cdot \sum_{k=0}^{\infty} \frac{1}{\prod_{t=1}^{j+kM_{\lambda}} m_{k,\lambda} \cdot e_{j+kM_{\lambda}}}, \lambda = 1, 2, \dots, m_{k,\lambda} \cdot e_{j+kM_{\lambda}}$$

All the series converges and define n elements in X. Moreover, if  $Mj \ge 0$  then

$$T_1^{m_{j,1}}T_2^{m_{j,2}}...T_3^{m_{j,n}} = \varphi_1(j, M_1)T_1^{M_1}T_2^{M_2}...T_3^{M_n}.\varphi_1(j, M_1) = \varpi(j, M_1)$$

if  $Nj \ge 0$  then

$$x_{j+kM_1} = \prod_{t=1}^{M_1} (a_{j+kN+t} \cdot x_{j+k+1}), x_{j+kM_2} = \prod_{t=1}^{M_2} (a_{j+kN+t} \cdot x_{j+k+1}), \dots, x_{j+kM_n} = \prod_{t=1}^{M_n} (a_{j+kN+t} \cdot x_{j+k+1})$$

when  $m_{i,j} \ge M$ , i = 1, 2, ..., n. So that each  $\varphi(i, N)$  for i = 1, 2, ..., n is a periodic point for T. We shall show that T has a dense set of periodic points. Since  $\{e_{\lambda}\}_{\lambda=1}^{\infty}$  is a basis, it suffices to show that for every element  $x \in span\{e_{\lambda} : \lambda \in N\}$  there is a periodic point y arbitrarily close to it. For this, let

$$x = \sum_{j=1}^{m} x_j \cdot e_j \text{ and } \varepsilon > 0. \text{ We can assume without lost of generality that}$$
$$\left| x_1 \cdot \prod_{t=1}^{1} a_{1,t} \right| \le 1, \left| x_2 \cdot \prod_{t=1}^{2} a_{2,t} \right| \le 1, \dots, \left| x_n \cdot \prod_{t=1}^{n} a_{n,t} \right| \le 1$$

Since  $\{e_n\}_{n=1}^{\infty}$  is an unconditional basis, then condition 4 implies that there are  $M_j \ge m, j = 1, 2, ..., n$ such that

$$\left\|\sum_{n=M_1+1}^{\infty} \left(\varepsilon_{1,n} \cdot \frac{1}{\prod_{t=1}^{\alpha} a_{t,1} e_t} \cdot e_k\right)\right\| < \frac{\varepsilon}{m_1} \cdot \left\|\sum_{n=M_2+1}^{\infty} \left(\varepsilon_{2,n} \cdot \frac{1}{\prod_{t=1}^{\alpha} a_{t,2} e_t} \cdot e_k\right)\right\| < \frac{\varepsilon}{m_2} \cdot \dots \cdot \left\|\sum_{n=M_n+1}^{\infty} \left(\varepsilon_{k,n} \cdot \frac{1}{\prod_{t=1}^{\alpha} a_{t,n} e_t} \cdot e_k\right)\right\| < \frac{\varepsilon}{m_n} \cdot \dots \cdot \left\|\sum_{n=M_n+1}^{\infty} \left(\varepsilon_{k,n} \cdot \frac{1}{\prod_{t=1}^{\alpha} a_{t,n} e_t} \cdot e_k\right)\right\| < \frac{\varepsilon}{m_n} \cdot \dots \cdot \left\|\sum_{n=M_n+1}^{\infty} \left(\varepsilon_{k,n} \cdot \frac{1}{\prod_{t=1}^{\alpha} a_{t,n} e_t} \cdot e_k\right)\right\| < \frac{\varepsilon}{m_n} \cdot \dots \cdot \left\|\sum_{n=M_n+1}^{\infty} \left(\varepsilon_{k,n} \cdot \frac{1}{\prod_{t=1}^{\alpha} a_{t,n} e_t} \cdot e_k\right)\right\| < \frac{\varepsilon}{m_n} \cdot \dots \cdot \left\|\sum_{n=M_n+1}^{\infty} \left(\varepsilon_{k,n} \cdot \frac{1}{\prod_{t=1}^{\alpha} a_{t,n} e_t} \cdot e_k\right)\right\| < \frac{\varepsilon}{m_n} \cdot \dots \cdot \left\|\sum_{n=M_n+1}^{\infty} \left(\varepsilon_{k,n} \cdot \frac{1}{\prod_{t=1}^{\alpha} a_{t,n} e_t} \cdot e_k\right)\right\| < \frac{\varepsilon}{m_n} \cdot \dots \cdot \left\|\sum_{n=M_n+1}^{\infty} \left(\varepsilon_{k,n} \cdot \frac{1}{\prod_{t=1}^{\alpha} a_{t,n} e_t} \cdot e_k\right)\right\| < \frac{\varepsilon}{m_n} \cdot \dots \cdot \left\|\sum_{n=M_n+1}^{\infty} \left(\varepsilon_{k,n} \cdot \frac{1}{\prod_{t=1}^{\alpha} a_{t,n} e_t} \cdot e_k\right)\right\| < \frac{\varepsilon}{m_n} \cdot \dots \cdot \left\|\sum_{n=M_n+1}^{\infty} \left(\varepsilon_{k,n} \cdot \frac{1}{\prod_{t=1}^{\alpha} a_{t,n} e_t} \cdot e_k\right)\right\| < \frac{\varepsilon}{m_n} \cdot \dots \cdot \left\|\sum_{n=M_n+1}^{\infty} \left(\varepsilon_{k,n} \cdot \frac{1}{\prod_{t=1}^{\alpha} a_{t,n} e_t} \cdot e_k\right)\right\| < \frac{\varepsilon}{m_n} \cdot \dots \cdot \left\|\sum_{n=M_n+1}^{\infty} \left(\varepsilon_{k,n} \cdot \frac{1}{\prod_{t=1}^{\alpha} a_{t,n} e_t} \cdot e_k\right)\right\| < \frac{\varepsilon}{m_n} \cdot \dots \cdot \left\|\sum_{n=M_n+1}^{\infty} \left(\varepsilon_{k,n} \cdot \frac{1}{\prod_{t=1}^{\alpha} a_{t,n} e_t} \cdot e_k\right)\right\| < \frac{\varepsilon}{m_n} \cdot \dots \cdot \left\|\sum_{n=M_n+1}^{\infty} \left(\varepsilon_{k,n} \cdot \frac{1}{\prod_{t=1}^{\alpha} a_{t,n} e_t} \cdot e_k\right)\right\| < \frac{\varepsilon}{m_n} \cdot \dots \cdot \left\|\sum_{n=M_n+1}^{\infty} \left(\varepsilon_{k,n} \cdot \frac{1}{\prod_{t=1}^{\alpha} a_{t,n} e_t} \cdot e_k\right)\right\| < \frac{\varepsilon}{m_n} \cdot \dots \cdot \left\|\sum_{n=M_n+1}^{\infty} \left(\varepsilon_{k,n} \cdot \frac{1}{\prod_{t=1}^{\alpha} a_{t,n} e_t} \cdot e_k\right)\right\| < \frac{\varepsilon}{m_n} \cdot \dots \cdot \left\|\sum_{n=M_n+1}^{\infty} \left(\varepsilon_{k,n} \cdot \frac{1}{\prod_{t=1}^{\alpha} a_{t,n} e_t} \cdot e_k\right)\right\| < \frac{\varepsilon}{m_n} \cdot \dots \cdot \left\|\sum_{n=M_n+1}^{\infty} \left(\varepsilon_{k,n} \cdot \frac{1}{\prod_{t=1}^{\alpha} a_{t,n} e_t} \cdot e_k\right\right\| < \frac{\varepsilon}{m_n} \cdot \dots \cdot \left\|\sum_{t=M_n+1}^{\infty} \left(\varepsilon_{k,n} \cdot \frac{1}{\prod_{t=1}^{\alpha} a_{t,n} e_t} \cdot e_k\right\right\| < \frac{\varepsilon}{m_n} \cdot \dots \cdot \left\|\sum_{t=M_n+1}^{\infty} \left(\varepsilon_{k,n} \cdot \frac{1}{\prod_{t=1}^{\alpha} a_{t,n} e_t} \cdot e_k\right\right\| < \frac{\varepsilon}{m_n} \cdot \dots \cdot \left\|\sum_{t=M_n+1}^{\infty} \left(\varepsilon_{k,n} \cdot \frac{1}{\prod_{t=M_n+1}^{\infty} \left(\varepsilon_{k,n} \cdot \frac{1}{\prod_{$$

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for every k = 1, 2, ..., n sequences  $\{\varphi_{\alpha,i}\}$  taking values 0 or 1. By conditions 1 and 2, for l = 1, 2, ..., n

the elements  $y_l = \sum_{i=1}^{m_l} x_i$  of X is a periodic point for T, and we have

$$\begin{split} \|y_{\lambda} - x\| &= \left\|\sum_{i=1}^{m_{\lambda}} \left(x_{i} \cdot \psi(i, M_{\varphi}) - e_{i}\right)_{i}\right\| = \left\|\sum_{i=1}^{m_{\lambda}} \left(x_{i} \cdot \prod_{t=1}^{i} d_{t}, M_{\lambda}\right) \sum_{k=1}^{\infty} \left(\frac{1}{\prod_{t=1}^{i+kM_{\lambda}} a_{t}, M_{\lambda}} + e_{i} + M_{\lambda}\right)\right\| \\ &\leq \sum_{i=1}^{m_{\lambda}} \left\|\left(x_{i} \cdot \prod_{t=1}^{i} d_{t}, M_{\lambda}\right) \left(\sum_{k=1}^{\infty} \left(\frac{1}{\prod_{t=1}^{i+kM_{\lambda}} a_{t}, M_{\lambda}} + e_{i} + M_{\lambda}\right)\right)\right\| \\ &\leq \sum_{i=1}^{m_{\lambda}} \left\|\left(x_{i} \cdot \prod_{t=1}^{i} d_{t}, M_{\lambda}\right) \left(\sum_{k=1}^{\infty} \left(\frac{1}{\prod_{t=1}^{i+kM_{\lambda}} a_{t}, M_{\lambda}} + e_{i} + M_{\lambda}\right)\right)\right\| \\ &\leq \sum_{i=1}^{m_{\lambda}} \left\|\left(\sum_{k=1}^{\infty} \left(\frac{1}{\prod_{t=1}^{i+kM_{\lambda}} a_{t}, M_{\lambda}} + e_{i} + M_{\lambda}\right)\right)\right\| \\ &\leq E_{i} \left\|\left(\sum_{k=1}^{\infty} \left(\frac{1}{\prod_{t=1}^{i+kM_{\lambda}} a_{t}, M_{\lambda}} + e_{i} + M_{\lambda}\right)\right)\right\| \\ &\leq E_{i} \left\|\left(\sum_{k=1}^{\infty} \left(\frac{1}{\prod_{t=1}^{i+kM_{\lambda}} a_{t}, M_{\lambda}} + e_{i} + M_{\lambda}\right)\right)\right\| \\ &\leq E_{i} \left\|\left(\sum_{k=1}^{\infty} \left(\frac{1}{\prod_{t=1}^{i+kM_{\lambda}} a_{t}, M_{\lambda}} + e_{i} + M_{\lambda}\right)\right)\right\| \\ &\leq E_{i} \left\|\left(\sum_{k=1}^{\infty} \left(\frac{1}{\prod_{t=1}^{i+kM_{\lambda}} a_{t}, M_{\lambda}} + e_{i} + M_{\lambda}\right)\right)\right\| \\ &\leq E_{i} \left\|\left(\sum_{k=1}^{\infty} \left(\frac{1}{\prod_{t=1}^{i+kM_{\lambda}} a_{t}, M_{\lambda}} + e_{i} + M_{\lambda}\right)\right)\right\| \\ &\leq E_{i} \left\|\left(\sum_{k=1}^{\infty} \left(\sum_{k=1}^{\infty} \left(\frac{1}{\prod_{t=1}^{i+kM_{\lambda}} a_{t}, M_{\lambda}} + e_{i} + M_{\lambda}\right)\right)\right\| \\ &\leq E_{i} \left\|\left(\sum_{k=1}^{\infty} \left(\sum_{k=1}^{\infty} \left(\sum_{k=1}^{\infty}$$

So we find  $|y_{\lambda} - x| < \varepsilon$ . By this, the proof is complete.

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