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Uncertainty Analysis of Routed Outflow in Rockfill Dams

J. M.V. Samani[1]* and A. Solimani[1]

ABSTRACT

Detention rockfill dams are an easy and common tool for flood control. Due to their coarse pores, the flow in void spaces is turbulent and non-Darcy. Different relationships introduced by researchers are used to define the hydraulics of the flow within the rockfill materials. The present research is aimed at gaining a better understanding of the difference among these relationships and the sources of uncertainty associated with the different parameters of each of the relationships. To examine the importance of various factors on the uncertainty of the outflow hydrograph, sensitivity analysis was conducted. For this purpose, a rockfill mass was provided, fifteen random samples of the mass selected, and then the physical characteristics of the material were measured or estimated. Also, some flood routing tests have been conducted. In these tests a physical model of a dam was installed and downstream water level was measured for different outflow rates. While the downstream water level was considered as certain variable but other parameters were seen as stochastic (stochastic parameters are considered as random variables) and outflow discharge as an output uncertain parameter. Uncertainty analysis has been conducted for different points of the outflow hydrograph by employing available methods. The results show that the Samani *et al.* and McCorocoudale *et al.* relationships have the lowest and highest uncertainty, respectively. The sensitivity analysis demonstrates different levels of sensitivity accompanied each of the relationship parameters which results in different effects on the total uncertainty of the relationships.

Keywords: First-Order Variance Estimation (FOVE) Method, Flood Routing, Harr's Method, Latin Hypercube Sampling (LHS) Method, Rockfill Dam.

INTRODUCTION

Uncertainties may arise due to natural variations in the phenomenon being considered, or to an incompleteness of our understanding. Uncertainties may also arise from the inaccurate characterization of important parameters or variables. Hence, engineering practice is frequently associated with decision making under uncertainty. The physical or numerical models, developed and used to simulate natural phenomena, are often in reality probabilistic, and hence, subject to analysis by rules of probability theory. Identifying the components of uncertainty related the physical phenomenon and quantifying them, can therefore improve decision making and the results (Haung, 1986; Mercer, 1975).

One of the common and most economic methods for flood mitigation used in watershed management is rockfill dams. The fact that flood mitigation through rockfill dams is an uncertain phenomenon, raises questions about the reliability and credibility of the relationships involved. As this type of dam consists of coarse particles, the flow deviates from Darcy's law resulting in turbulence in the void spaces. This means that the relationship between the flow velocity, V, and its hydraulic gradient, i, is a nonlinear one. Different researchers have proposed different nonlinear relationships that give various outputs.

1. Department of Water Structure Engineering, Tarbiat Modares University, Tehran, Islamic Republic of Iran.
* Corresponding author, e-mail: samani_j@modares.ac.ir

McCorquodale *et al*. (1978) introduced the following equations:

$$i=\frac{4.6v}{gnm^2}V+\frac{0.79}{gn^{1/2}m}V^2 \quad \text{for} \quad R_W \le 125 \quad \text{or} \quad R_P > 500 \quad \text{and}$$

$$i=\frac{70v}{gnm'^2}V+\frac{0.27(1+f_e/f_0)}{gn^{1/2}m'}V \qquad (1$$

for $R_W > 125$ or $R_P > 500$

where R_W and R_P, Reynolds numbers, are defined as below:

$$R_W=\frac{V\,mn^{1/2}}{v} \qquad (2$$

$$R_P=\frac{V\,m}{vn}$$

Based on the data collected by McCorquodale *et al*. the specific Reynolds numbers (R_W and R_P) are dimensionless variables selected by them to define the constrains of his defined relationships.

Stephenson's (1979) relationship is

$$i=\frac{800v}{gnd^2}V+\frac{K_t}{n^2gd}V^2 \qquad (3$$

and Adel's relationship is (Ahmed and Sunada, 1969)

$$i=\frac{160v(1-n)^2}{gn^3d_{15}^2}V+\frac{2.2}{gn^2d_{15}}V^2 \qquad (4$$

In the above equations, i is the hydraulic gradient, V is flow velocity, d is the average diameter of rock, g is acceleration due to gravity, K_t is the friction coefficient in the turbulent flow region, n is porosity, f is the friction factor, d_{15} is particle diameter where 15% of the total particles weight are smaller, f_e is the friction factor between large particles and the instrument wall, f_o is the Darcy-Weisbach coefficient, m´ is the pores effective hydraulic radius, and m refers to the hydraulic radius of pores.

Samani *et al*. (2003) introduced the following equation:

$$i=\left(\frac{V}{\alpha}\right)^{b+2} \qquad (5$$

in which

$$\alpha=\left(\frac{2gv^b}{a(d_{50}-\sigma)^{b-1}}\right)^{\frac{1}{b+2}} \qquad (6$$

where d_{50} is particle diameter size where 50% of the total particles' weight is smaller, a and b are empirical coefficients of the equation related to the flow and particles characteristics, and σ is the standard deviation of particles.

Several methods of uncertainty analysis have been developed and applied in water resource engineering. The most widely used methods are first-order variables estimating (FOVE), Harr's Probabilistic Point Estimation method and Monte Carlo Simulation (MCS) (Ang and Tang, 1984). FOVE is based on linearizing the functional relationship that relates a dependent random variable and a set of independent random variables by Taylor series expansion (Yen *et al*., 1986). This method has been applied in several water resource and environmental engineering problems involving uncertainty. Examples include storm sewer design (Tang and Yen, 1972), ground-water flow estimation (Dettinger and Wilson, 1981), prediction of dissolved oxygen (Burges and Lettenmaier, 1975; Chadderton *et al*., 1982), subsurface flow and contaminant transport estimation (Sitar *et al*., 1987), and water surface profile of a buried stream flowing under coarse material (Hansen and Bari, 2002). In Harr's method, the average and variance of probabilistic variables and their correlations are used (more details are introduced in Tung, 1993). If there are N variables, the number of cases (points) will be 2N which is considered an important advantage compared to the point estimate method proposed by Rosenblueth (1981). In cases when obtaining the derivatives is too complicated, Harr's method is considered as a good substitute for the FOVE method. Herr's method has been used in studying the spatial variation of river bed scouring (Yeh and Tung, 1993) and for uncertainty analysis incorporating marginal distribution (Chang and Yang, 1997). In MCS, stochastic inputs are generated from their probability distributions and are then entered into empirical or analytical models of underlying physical process involved in generating stochastic outputs. Then, the generated outputs are analyzed statistically to quantify the uncertainty of the output. Several examples of uncertainty analysis by MCS can be found in wa-

ter resource and environmental engineering (Salas, 1993; Hipel and Mcleod, 1994; Melching, 1995), in ground-water flow estimation (Smith and Freeze, 1979; Freeze, 1975; Jones, 1989), water quality modeling (Warwick and Cale, 1986; Brutsaert, 1975) and in studying the spatiotemporal stochastic open-channel flow (Gates and Al-Zahrani, 1996). Scavia *et al*. (1981) made a comparison of MCS and FOVE for determining uncertainties associated with eutrophication model outputs such as plankton, zooplankton, and nitrogen forms. They concluded that both FOVE and MCS agree well in estimating the mean and variance of model estimates. However, MCS has the advantage of providing better information about the frequency distribution. Latin hypercube sampling (LHS) is used to generate random stochastic inputs in a stratified manner from the probability distributions. In this way the number of generated inputs can be considerably reduced as compared to MCS (see McKay *et al*., 1979). Chang *et al*. (1993) used LHS to perform sensitivity and uncertainty analysis in his research. Yeh and Tung (1993) applied FOVE, the point estimate method proposed by many studies, and LHS to analyze the uncertainty of migration of a pit. They pointed out that the point estimate method yields a larger mean and variance than those obtained by the FOVE and LHS methods. Furthermore, in studying the importance of stochastic inputs on the output by sensitivity analysis, LHS yields more information than the other two methods.

In this study, uncertainty analysis based on FOVE, Harr's method, and LHS methods are conducted to obtain a deep insight into the scattered results of different flow relationships used for non-Darcy water flow through rockfill dams and to show the contribution of input parameters on total uncertainty of the outflow routed hydrograph. In addition, sensitivity analysis is performed to see the relative importance of input parameters in estimating the variability of the outflow routed hydrograph.

MATERIALS AND METHODS

The uncertainty of predicted routed flow hydrographs depends on the physical and hydraulic parameters of the flow relationships. The physical parameters can be measured or estimated by providing a big mass of rockfill material. The hydraulic parameters, however, can not be measured unless an experiment is conducted. For this purpose, a small rockfill material, from the big mass, has been used to build a small physical dam model to be used in flow rating relationship measurements.

Flow Rating Relationships

The objective of this study is to determine the uncertainty of outflow routed hydrographs resulting from different relationships, i.e. equations 1, 2, 3, 4, and 5. For this purpose, by considering $i = \frac{-dh}{dx}$ and integrating each relationship for the limits x = 0 to D and H = H_1 to H_2, the following relationship among Q, H_1 (dam upstream water level) and H_2 (dam downstream water level) is obtained:

$$P\left(\begin{array}{l}\frac{M^2}{2}\left(H_2^2-H_1^2\right)-BQ^2\,M\left(H_2-H_1\right)+\\ B^2Q^4\ln\left(\frac{BQ^2+MH_2}{BQ^2+MH_1}\right)\end{array}\right)-L=0 \qquad (7$$

where Q is the flow rate, P is $-\frac{w^2}{M^3}$, M is CQw, w is dam width and D is calculated according to Sharma (1991). The amount of D is less than L for Trapezoidal rockfill dams and equal to L for rectangular ones. According to McCorquodale *et al's*., Stephenson's and Adel's relationships introduced above, equation (7) would have a general form providing that B and C are defined as the following:

According to the McCorquodale *et al*. relationship:

$$C=\frac{70v}{gnm'^2} \text{ and } B=\frac{0.27\left(1+\left(f_e/f_o\right)\right)}{gn^{1/2}m'} \quad (7a$$

According to the Stephenson relationship:

$$C=\frac{800v}{gnd^2} \quad \text{and} \quad B=\frac{K_t}{gn^2d} \quad (7b$$

According to the Adel relationship:

$$C=\frac{160v(1-n)^2}{gn^3d_{15}^2} \text{ and } B=\frac{2.2}{gn^2d_{15}} \quad (7c$$

The flow rating relationship for Samani *et al*. is different from the other relationships where it is as follows (Samani *et al*., 2003):

$$Q=\left(\frac{1}{D}\right)^{\frac{1}{b+2}}\frac{\alpha w}{(3+b)^{1/b+2}}\left(H_1^{3+b}+H_2^{3+b}\right)^{\frac{1}{b+2}} \quad (8$$

in which D and L are equal in rectangular rockfill dams.

Hydraulic Parameters

In this analysis, H_2 and Q are considered as certain and resultant uncertain variables, respectively. Defining H_2 would mean a certain H_1 for a specific Q. The physical rockfill dam model of 66 cm length, 30 cm width and 33 cm height has been installed in a same width of 9 m flume. Table 1 shows the range of hydraulic parameters measurements used in the experiment that would be used in the routing calculation.

Physical Parameters

In order to evaluate the physical parameters of the different relationships, 15 random samples from the mass provided were selected and used. Table 2 shows the grain size distribution of the rockfill material. The size distribution curve, d_m, d_{60}, d_{50}, d_{15}, d_{10}, d, n, e, m, m′ and σ of each sample have been determined, where d_m is the average size diameter and d is the harmonic average size. Different subscripts for notation d refer to the percentages of total particle weight that are smaller than the related d. Table 3 shows the average size, standard deviation, and coefficient of variation of the parameters of the 15 samples.

Table 1. H_2 and Corresponding Q of experiment.

Q ($m^3 s^{-1}$)	H_2 (m)
0.00046	0.029
0.0006	0.032
0.00105	0.044
0.0015	0.059
0.0026	0.084
0.0015	0.076
0.00098	0.049

It is necessary to say that the measurement of parameters was carried out by several persons and several times in order tat the human error can be assumed to be minimized.

In this research, the parameters K_t, fe/fo, a, and b for each sample were estimated by taking into account the suggestions given by relationship developers. Table 4 shows the estimated values of the parameters.

Routing and Uncertainty Analysis

In this research the following storage routing equation has been employed:

$$I-O=\frac{\Delta s}{\Delta t} \quad (9$$

where I and O indicate the flow rates of the inflow and outflow hydrographs, respectively, and Δs is the storage within the time Δt. According to equation (9), the inflow hydrograph will be routed in the reservoir

Table 2. Particle size distribution of rockfill mass.

%	Sieve size(mm)
100	32
96.8	24
68.8	18
26.5	12.7
3.5	10
0	4

Table 3. Statistical characteristics of different parameters.

d_{10} mm	d_{15} mm	d_{50} mm	d_{60} mm	d_m mm	d mm	e	σ	*m*	*m′*	n	
10	11	13.74	15	15	13	0.67	0.00039	0.0034	0.0025	0.40	Aver.
0.49	0.38	0.83	0.75	0.60	0.77	0.034	0.000008	0.000199	0.000236	0.013	St. Dev.
0.05	0.04	0.06	0.05	0.04	0.06	0.05	0.02	0.06	0.11	0.03	Vari.
0.22	0.45	0.69	0.79	0.60	0.45	-1.053	-0.37	0.03	0.06	-1.114	Skew.

upstream the rockfill dam (the physical model) and then a routed outflow hydrograph is introduced downstream of the dam. The outflow hydrograph is accompanied with uncertainty which is the final objective of this procedure.

The methods selected for the analysis are FOVE, Harr, and LHS. In this analysis, d_m, d_{60}, d_{50}, d_{15}, d_{10}, d, n, e, m, m', σ, a, b, fe/fo, and K_t are considered as stochastic inputs, H_2 as a certain input and Q as an uncertain output.

FOVE is a simple, effective, and precise method especially when the relationship of input and output variables is linear. This method does not take into account the probability distribution of variables which might be considered as a disadvantage. This method uses a Taylor series to linearize the relationship between output and input variables. The following shows the function:

$$Y = g(X) = g(X_1, X_2, ..., X_N) \quad (10$$

where Y is the function of N stochasic variables, X The Taylor series is written as

$$Y = g(x_0) + \sum_{i=1}^{N}\left(\frac{\partial g}{\partial X_i}\right)_{x_0}(X_i + X_0) + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\left(\frac{\partial^2 g}{\partial X_i \partial X_j}\right)_{x_0}(X_i - x_0)^2 \quad (11$$

and, considering the first two terms and neglecting higher order terms of the above series, gives the following:

$$Y \approx g(x_0) + S_0^T(X - x_0) \quad (12$$

where, $S_0 = \left(\frac{\partial g}{\partial X_i}\right)_{x_0}$ is the sensitivity coefficient vector. According to equation (12), the average is $E[Y] \approx g(x_0) + S_0^T(\mu - x_0)$ and the variance is $Var[Y] \approx S_0^T C(X) S_0$, where μ is average matrix and C(X) is the matrix of stochastic variables, X Assuming $x_0 = \mu$, then:

$$E[Y] \approx g(\mu) \text{ and } Var[Y] \approx S^T C(X) S \quad (12a$$

where S is the vector of sensitivity coefficients at $x_0 = \mu$. When the stochastic variables are all independent, the variance can be calculated from $Var[Y] \approx S^T DS = \sum S_i^2 \sigma_i^2$ where D is the diagonal matrix of stochastic variables variance, i.e. $D = diag(\sigma_1^2, \sigma_2^2, ..., \sigma_N^2)$. The different stages of the FOVE method can be summarized as:

- Identifying input stochastic (physical) parameters,
- Applying the Taylor series, and,
- Calculating the average and variance of flow rates of different relationships (models).

Harr's method is similar to the FOVE method because it uses the two first order moments of stochastic variables and not the probability distribution, but it is easier in terms of calculations. It is considered as a good substitute for FOVE when it is dealing with complex derivatives. The different stages of Harr's method can be summarized as:

Table 4. Statistical Characteristics of Estimated Parameters.

	A	b	fe/fo	K_t
54	-0.074	1.54	3.1	Average
4.47	0.014	0.036	0.139	St. Deviation.
0.083	-0.194	0.023	0.045	Variance

- Identifying input physical parameters of each of the relationships and calculating its correlation matrix,

- Decomposition of the correlation matrix (CO) to the eigen vectors matrix and eigen values matrix (with MATLAB software)

$$CO = VLV' \quad (13$$

where $V = (v_1, v_2, \Lambda, v_n)$ is the eigen vectors matrix and L = $\lambda_1, \lambda_2 ....., \lambda_n$ is the eigen value diagonal matrix,

- Calculating 2N intersection points where this couple of points are calculated from the following equation:

$$X_{i\pm} = \mu \pm \sqrt{N} \begin{bmatrix} \sigma_1 ..... \\ \sigma_2 .....0 \\ ....-.... \\ 0...-... \\ .....\sigma_N \end{bmatrix} \quad (14$$

where μ= Mean; σ_i= Standard deviation of *i*th stochastic input; N= Number of inputs; V_i= Eigen vectors matrix,

-calculating $Y_{i\pm}$= g (X_{ith}) and $Y_{i\pm}^2$= g^2 ($X_{i\pm}$) for (i=1, 2,…, N) where Y_i= Model output and then calculating $\overline{Y}_i = \frac{Y_{i+} + Y_{i-}}{2}$ and

$$Y_i^2 = \frac{Y_i^2 + Y_{i-}^2}{2}$$

-Calculating the average and variance of different model outputs:

$$E(Y) = \frac{\sum_{i=1}^{N} \overline{Y}_i \lambda_i}{\sum_{i=1}^{N} \lambda_i} = \frac{\sum_{i=1}^{N} \overline{Y}_i \lambda_i}{N} \quad (15$$

$$E(Y^2) \frac{\sum_{i=1}^{N} \overline{Y}_i i \lambda_i}{N} \quad (16$$

$$Var(Y) = E(Y^2) - E^2(Y) \quad (17$$

-Computing model uncertainty with the coefficient of variation. For an elaborate discussion on the FOVE and Harr's methods the reader is referred to Hosseini (2000).

LHS is an effective method especially in dealing with nonlinear relationships. Its main disadvantage is the need for a probability distribution of variables. The probability distribution of Q can be estimated as follows: (1) obtain a random set of size n of the stochastic inputs from the corresponding probability distribution using LHS; (2) follow the necessary steps to route the inflow hydrograph and obtain the outflow hydrograph. For ease of calculation, just seven points of the routed outflow hydrograph are used in this analysis; (3) analyze statistically the outflows (7 points) to determine its probability distribution and its basic statistics such as mean, standard deviation, coefficient of variation, and coefficient of skewness.

In using MCS to generate the stochastic inputs referred to in Step 1 above, normally a large data set, for instance, n = 1000, is generated from the probability distribution of each input. The probability distribution of each parameter was normal, log-normal, uniform, and log-Pierson. For more details on MCS the reader is referred to Rief (1988).

An alternative to MCS sampling that reduces the number of sets of generated inputs and consequently the number of generated outputs is the LHS method. The basic concept of LHS lies in generating random numbers of a stochastic input over its range in a stratified manner, such as the overall variability of the given stochastic input can reasonably be delineated by limited sample size. The properties of LHS are discussed by McKay (1988) and McKay *et al*. (1979). In the MCS or LHS procedures, all stochastic inputs are assumed to be independent.

Sometimes, when a large number of stochastic inputs are involved in determining the output, sensitivity analysis may be carried out to determine the degree of influence of each stochastic input on the output uncertainty, C_i. In FOVE, C_i is calculated by the following:

$$C_i = \frac{S_i^2 \sigma_i^2}{\sigma_Y^2} \quad \text{i=1, 2…, N} \quad (18$$

Table 5. Uncertainty analysis results using FOVE for independent input parameters.

H_2 (m)	McCorquodale *et al*.			Stephenson			Adel			Samani *et al*.		
	E (Q)	Var (Q)	C.V (Q)	E (Q)	Var (Q)	C.V (Q)	E (Q)	Var (Q)	C.V (Q)	E (Q)	Var (Q)	C.V (Q)
0.032	0.00056	2.5×10^{-10}	0.03	0.00053	2.6×10^{-9}	0.10	0.00053	7.2×10^{-10}	0.05	0.00055	0.0	0.0
0.044	0.00092	5.8×10^{-10}	0.03	0.00086	4.1×10^{-9}	0.07	0.00086	1.1×10^{-9}	0.04	0.00089	0.0	0.0
0.059	0.00126	8.8×10^{-10}	0.02	0.00135	6.2×10^{-9}	0.06	0.00137	1.7×10^{-9}	0.03	0.0014	0.0	0.0
0.084	0.00232	2.2×10^{-8}	0.07	0.00198	9×10^{-9}	0.05	0.0023	2.8×10^{-9}	0.02	0.00231	0.0	0.0
0.076	0.00192	1.2×10^{-9}	0.02	0.00102	5.2×10^{-9}	0.07	0.0020	2.5×10^{-9}	0.02	0.00202	0.0	0.0
0.049	0.00097	5.9×10^{-10}	0.03	0.00085	4×10^{-9}	0.08	0.00104	1.3×10^{-9}	0.04	0.00106	0.0	0.0
0.043	0.00084	5.2×10^{-10}	0.03	0.0008	4×10^{-9}	0.08	0.00083	1.1×10^{-9}	0.04	0.00085	0.0	0.0

where N is the number of stochastic input parameters, σ^2_i is variance of ith input parameter, σ^2_Y is variance of output parameter, S_i is parameter sensitivity coefficient, and $Y = f(x_1, x_2, ..., x_N)$.

In Harr's method and LHS, a linear regression relationship between x's, input parameters, and output, Y, can be considered, as the following

$$Y = a_0 + \sum_{i=1}^{N} a_i x_i + e \qquad (19$$

where a_0 is the interception value of the line with the y axis, a_i refers to regression coefficients that show the sensitivity coefficients, and e indicating the model error. Due to the dimensional problems, it is recommended to centralize the output parameter and then, by standardizing (Y-Y) and input parameters, the regression can be conducted. In this case, coefficients will indicate the output variation for a variation of input parameter equal to one standard deviation. Then C_i values which indicates the uncertainty of input parameter can be calculated from the following relationship:

$$C_i = \frac{SSR_i}{SSR} R^2 \quad \text{for} \quad i=1,2\ldots, m \qquad (20$$

In which SSR_i is the summation of square values of the ith input stochastic parameter from the regressed line and SSR is the summation of SSR_i for independent input parameters. For more detailed the reader is referred to McKay (1988).

Steps in the Procedure

The uncertainty in predicting routed flow hydrographs depends on the stochastic input parameters of the flow relationships; in this research four of the relationships, i.e. McCorquodale *et al*., Stephenson, Adel, and Samani *et al*., are investigated. The different stochastic inputs have been obtained by conducting an experimental procedure. For this a big mass of rockfill material was provided. Fifteen random samples of the mass have been selected and the related stochastic input parameters were measured or estimated. The data collected was employed for the uncertainty and sensitivity analyses.

From the same rockfill mass, a small

Table 6. Uncertainty analysis results using Harr for independent input parameters.

H_2 (m)	McCorquodale *et al*.			Stephenson			Adel			Samani *et al*.		
	E (Q)	Var (Q)	C.V (Q)	E (Q)	Var (Q)	C.V (Q)	E (Q)	Var (Q)	C.V (Q)	E (Q)	Var (Q)	C.V (Q)
0.032	0.00056	5×10^{-10}	0.04	0.00053	3×10^{-9}	0.10	0.00053	8×10^{-10}	0.10	0.00054	7×10^{-10}	0.05
0.044	0.00094	1×10^{-9}	0.03	0.00086	6×10^{-9}	0.10	0.00086	2×10^{-9}	0.10	0.00086	2×10^{-9}	0.05
0.059	0.00146	3×10^{-9}	0.04	0.00134	1×10^{-8}	0.09	0.00137	5×10^{-9}	0.09	0.00135	5×10^{-9}	0.05
0.084	0.00232	1×10^{-7}	0.15	0.00198	4×10^{-8}	0.09	0.00225	1×10^{-8}	0.09	0.00224	1×10^{-8}	0.05
.076	0.00193	5×10^{-9}	0.04	0.00102	1×10^{-8}	0.09	0.00199	1×10^{-8}	0.09	0.00196	1×10^{-8}	0.05
0.049	0.00097	1×10^{-9}	0.04	0.00083	6×10^{-9}	0.09	0.00104	3×10^{-8}	0.10	0.00103	3×10^{-9}	0.05
0.043	0.00056	5×10^{-10}	0.04	0.00053	3×10^{-9}	0.10	0.00053	8×10^{-10}	0.10	0.00054	7×10^{-10}	0.05

physical dam was built and installed in a laboratory flume and then, by introducing different inflows, different corresponding downstream water levels, H_2, were identified. By introducing a hypothetical inflow hydrograph and conducting the routing calculation, routed outflow hydrograph was obtained. For the uncertainty analysis, H_2 has been regarded as certain input parameter and other parameters as stochastic. The analysis was conducted for seven H_2 values corresponding to flow rates covering the useful range of the outflow hydrograph, 0.032, 0.044, 0.095, 0.084, 0.076, 0.049, and 0.043 m. After gathering the certain and stochastic inputs, FOVE, Harr and LHS were employed for determining the uncertainty of the routed outflow hydrograph and then sensitivity analysis was applied to the different flow relationships to see the relative importance of stochastic inputs in estimating the variability of the output, routed outflow rate.

RESULTS

The results of calculating the outflow discharge uncertainty considering the input parameters dependent and independent parameters were very close to each other, therefore just the independent ones are introduced. The results of uncertainty analysis for different relationships are shown in Tables 5, 6, and 7 and Figure1 (a, b and c). It can be concluded that:

If the Coefficient of Variation (C.V.) is considered as the indicator of uncertainty, LHS sampling and FOVE show the highest and the lowest uncertainty results, respectively, and Harr's method falls in between. As an example, the average C.V.'s of depth 0.084 for the different methods are 0.12, 0.9, and 0.04, respectively. Therefore, applying LHS sampling would mean a more uncertain environment and more reliable design.

By averaging the results of the three methods, the routed outflow hydrograph calculated by Samani *et al*., McCorquodale *et al*., Adel, and Stephenson, relationships show uncertainty of 0.05, 0.5, 0.6, and 0.08, respectively. The results of the first three relationships are almost the same which means using any of those relationships would make no significant difference.

Results of McCorquodale *et al*.'s relationship for R_P>500 show high uncertainty; for instance for H_2= 0.084, the uncertainty of outflow using LHS is 0.21 which is 3.5 times the average C.V. of the outflow hydrograph. In case of R_P<500 it gives the less uncertainty among the other relationships.

The sensitivity analysis shows that the parameters m′ and n in McCorquodale *et al*.'s relationship introduce the greatest uncertainty, at 0.72 and 0.25, respectively, among the other parameters where in Stephenson's, the parameters d and n expose the highest sensitivity of 0.85 and 0.8, respectively. In Adel's relationship, d_{15} with 0.88 and n with 0.12 show the highest influence on the routed outflows. Finally, in Samani *et al*.'s a and d_{50} as the most important parameters, shows the influence of 0.77 and 0.12, respectively.

Table 7. Uncertainty analysis results using MCS with LHS.

H_2 (m)	McCorquodale *et al*.			Stephenson			Adel			Samani *et al*.		
	E (Q)	Var (Q)	C.V (Q)	E (Q)	Var (Q)	C.V (Q)	E (Q)	Var (Q)	C.V (Q)	E (Q)	Var (Q)	C.V (Q)
0.032	0.00056	3×10^{-10}	0.03	0.00053	3×10^{-9}	0.10	0.00056	3×10^{-9}	0.10	0.00056	3×10^{-9}	0.09
0.044	0.00094	8×10^{-10}	0.03	0.00085	6×10^{-9}	0.09	0.0009	8×10^{-9}	0.10	0.00092	7×10^{-9}	0.09
0.059	0.00145	2×10^{-9}	0.03	0.00134	1×10^{-8}	0.09	0.00143	2×10^{-8}	0.09	0.00144	2×10^{-8}	0.09
0.084	0.00232	2×10^{-7}	0.21	0.00198	3×10^{-8}	0.09	0.00235	5×10^{-8}	0.09	0.00239	5×10^{-8}	0.09
0.076	0.00192	3×10^{-9}	0.03	0.00102	9×10^{-9}	0.09	0.00208	4×10^{-9}	0.09	0.00209	4×10^{-9}	0.09
0.049	0.00097	8×10^{-10}	0.03	0.00085	6×10^{-9}	0.09	0.00109	1×10^{-8}	0.10	0.00109	9×10^{-9}	0.09
0.043	0.00084	7×10^{-10}	0.03	0.00081	6×10^{-9}	0.09	0.00088	7×10^{-9}	0.10	0.00088	6×10^{-9}	0.09

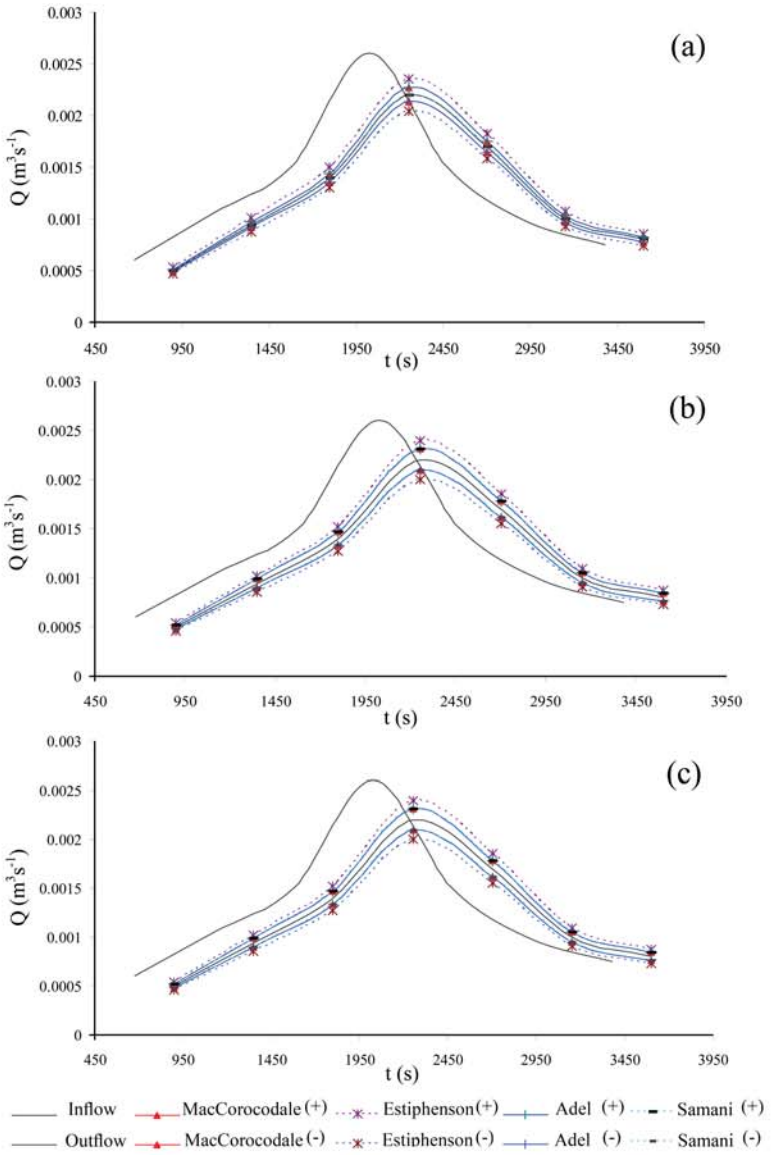

Figure1. Uncertainty analysis results using (a): FOVE, (b): Harr's method and (c): MSC with LHS.

CONCLUSION

In flood routing, routed hydrographs have an uncertainty that depends on different physical stochastic parameters. Four of the relationships, namely of Samani *et al*., Stephenson, Adel, and McCorquodale *et al*. are employed to reflect the hydraulics of the flow through rockfill dams. In this analysis, FOVE, Harr's method and the LHS have been used to evaluate the uncertainty of the routed hydrograph. The results of the analysis show that Samani *et al*.'s Stephenson's and Adel's relationships are almost the same and this means using any of those relation-

ships make no significant difference. Also, applying LHS sampling means more reliable design. The sensitivity analysis shows that the parameters m and n in McCorquodale *et al*.'s relationship, d and n in Stephenson's, d_{15} and n in Adel's, and a and d_{50} in Samani *et al*.'s show the highest influence on the routed outflows among other parameters.

Notations

a = Empirical coefficient of the equation related to flow and particle characteristics;
b = Empirical coefficient of the equation related to flow and particle characteristics;
C_i = Indicates the uncertainty of the input parameter;
d = Average diameter of rock;
d_{15} = Particle diameter where 15% of the total particles' weight is smaller;
d_{50} = Particle diameter size where 50% of the total particles' weight is smaller;
f = Friction factor;
f_e = Friction factor between large particles and the instrument wall;
f_o = Darcy-Weisbach coefficient;
g = Acceleration due to gravity;
H_1 = Dam upstream water level;
H_2 = Dam downstream water level;
i = Hydraulic gradient;
I = Inflow rate;
K_t = Friction coefficient in the turbulent flow region;
L = Length of dam base;
m = Refers to pores hydraulic radius;
m′ = Pores effective hydraulic radius;
n = Porosity;
O = Outflow rate;
s = Storage;
t = Time;
V = Flow velocity;
σ = The standard deviation of particle;
Q = Flow rate;
SSR = The summation of SSR_i;
SSR_i = The summation of square values of the ith input stochastic parameter;
w = Dam width.

REFERENCES

1. Ahmed, N. and Sunada, D. K. 1969. Nonlinear Flow in Porous Media. *J. Hydr. Div., ASCE*, **91(6)**:
2. Ang, A. H. S. and Tang, W. H. 1984. *Probability Concepts in Engineering Planning and Design. Vol.2: Decision, Risk and Reliability*, John Wiley, New York.
3. Brutsaert, W. F. 1975. Water Quality Modeling by Monte Carlo Simulation. *Water Resour. Bull.*, **11**: 175-186.
4. Burges, S. J. and Lettenmaier, D. P. 1975. Probabilistic Methods in Stream Quality Management. *Water Resour. Bull.*, **11(1)**: 115-130.
5. Chadderton. R. A., Miller, A. C. and McDonnell, A. J. 1982. Uncertainty Analysis of Dissolved Oxygen Model. *J. Envir. Engrg. Div., ASCE*, **108(5)**: 1003-1013.
6. Chang, C. H., Yang, J. C. and Tung, Y. K. 1993. Sensitivity and Uncertainty Analysis of a Sediment Transport Model: A global approach. *Stoch. Hydrol. Hydroul*., **7**: 299-314
7. Chang, C. H., Yang, J. C., and Tung, Y. K. 1997. Uncertainty analysis by point estimate methods incorporating marginal distribution. *J. Hydraul. Eng. ASCE*, **123(3)**: 244-250.
8. Dettinger, M. D. and Wilson, J. L. 1981. First Order Analysis of Uncertainty in Numerical Models of Groundwater Flow, Part 1. Mathematical Development. *Water Resour. Res*., **17(1)**: 149-161.
9. Freeze, R. A. 1979. A Stochastic Conceptual Analysis of One–Dimensional Groundwater Flow in Nonuniform Homogeneous Media. *Water Resour. Res*., 11(5): 725-741.
10. Gates, K. and Al-Zahrani, M. A. 1996. Spatiotemporal Stochastic Open-channel Flow. *J. Hydraul. Eng, ASCE*, **122(11)**: 641-661.
11. Hansen, D., and Bari R. 2002. Uncertainty in Water Profile of Buried Stream Flowing under Coarse Material. *J. Hydraul. Eng., ASCE*, **128(8)**: 761-773.
12. Hipel, K. W. and McLeod, A. I. 1994. *Time Series Modeling of Water Resources and Environmental Systems*. Development in Water Science, 45 Elsevier Science, New York.
13. Hosseni, S. M. 2000. Statistical Evaluation of the Empirical Equation that Estimate Hydraulic Parameters Flow through Rockfill. In: "*Stoch. Hydraulics*" (Eds.) Wang, Z. Y.

and Hu, S. X. Balkema, Rotterdam.
14. Huang, K. Z. 1986. Reliability Analysis of Hydraulic Design of Open Channel. Stochastic and Risk Analysis in Hydraulic Engineering. In: "*Water Resources Publication*", (Ed.) Yen, B. C., Littleton, Co.
15. Jones, L. 1989. Some Results Comparing Monte Carlo Simulation and First Order Taylor Series Approximation for Steady Groundwater Flow. *Stochastic Hydrol. Hydraul*., **3**: 179-190.
16. McKay, M. D. 1988. Sensitivity and Uncertainty Analysis Using Statistical Sample of Input Values. In: "*Uncertainty Analysis*"(Ed.). Y. Ronen, CRC, Boca Raton, Fla., pp. 145-185.
17. McKay, M. D., Beckman, R. J. and Conover, W. J. 1979. A Comparison of Three Methods for Selecting Values of Input Variables in the Analysis of Output from a Computer Code. *Technometrics*., **21(2)**: 239-245.
18. McCorquodale, J. A., Hannoura, A. and Nasser, M. S. 1978. Hydraulic Conductivity of Rock Fill. *J. Hydr. Res*., **16(2)**: IAHR.
19. Melching, C. S. 1995. Reliability Estimation. In: "*Computer Models of Watershed Hydrology*", (Ed.) Singh, V. P., Water Resources Publications, Littleton, Colo., pp. 69-118.
20. Mercer, L. J. and Morgan, W. D. 1975. Evaluation of a Probability Approach to Uncertainty in Benefit-cost Analysis. Technical Report, Contribution No: 149, California Water Resources Centre, University of California, Davis.
21. Rief, H. 1988. Monte Carlo Uncertainty Analysis. In: "Uncertainty *Analysis*", (Ed.) Ronen, Y., CRC, Boca Raton, Fla., 187-215.
22. Rosenblueth, E. 1981. Two-point Estimates in Probabilities. *Appl. Math. Modeling*, **5**: 329-335.
23. Salas, J. D. 1993. Analysis and Modeling of Hydrologic Time Series. In: "*Handbook of Hydrology*", (Ed.) Maidment, D. R., Chapter 19, McGraw-Hill, New York.
24. Samani H. M. V., Samani, J. M. V. andShayan Nejad, M. 2003. Reservoir Routing Using Steady and Unsteady Flow through Rockfill Dams. *J. Hydr. Eng., ASCE*, **129(6)**: 448-458.
25. Scavia, D., Powers, W. F., Canale, R. P. and Moody, J. L. 1981. Comparison of First-order Error Analysis and Monte Carlo simulation in Time Dependent Lake Eutrophication Models. *Water Resour. Res*., **17(4)**: 1051-1059.
26. Sharma, H. D. 1991. Earth Embankment Dams. Oxford and IBH Publishing Co. New Delhi, India.
27. Sitar, N., Cawlfield, J. D. and der Kiureghian, A. 1987. First-order Reliability Approach to Stochastic Analysis of Subsurface Flow and Contaminant Transport. *Water Resour. Re.*, **23(5)**: 794-804.
28. Smith, L., and Freeze, R. A. 1979. Stochastic Analysis of Steady State Groundwater Flow in as Bounded Domain. 2. Two-dimensional Simulations. *Water Resour. Res*., **15(6)**: 1543-1559.
29. Stephenson, D. 1979. *Rockfill in Hydraulic Engineering*. Developments in Geotechnical Engineering 27, Elsevier Scientific Publishing Co., Amsterdam, The Netherlands.
30. Tang, W. H., and Yen, B. C. 1972. Hydrologic and Hydraulic Design under Uncertainties. *Proc. Int. Symp. On Uncertainties in Hydrology and Water Resour. Sys*., Vol. 2, University of Arizona, Tucson, 868-882.
31. Tung, Y. K. 1993. Uncertainty and Reliability Analysis. In: "*Water Resources Handbook*", Chapter 7, (Ed.) Mays, L. W., McGraw Hill Book Co.
32. Warwick, J. J., and Cale, W. G. 1986. Effects of Parameter Uncertainty in Stream Modeling. *J. Envir. Eng., ASCE*, **112(3)**: 479-489.
33. Yeh, K. C. and Tung, Y. K. 1993. Uncertainty and Sensitivity Analysis of Pit-Migration Model. *J. Hydraul. Eng., ASCE*, **119(2)**: 262-283.
34. Yen, B. C., Cheng, S. T. and Vielching, C. S. 1986. First-order Reliability Analysis. In: "*Stochastic and Risk Analysis in Hydraulic Engineering*", (Ed.) Yen, B. C., Water Resources Publication, Littleton, Colo., pp.1-36.

آنالیز عدم قطعیت هیدروگراف خروجی روند یابی سیل در سد پاره‌سنگی

ج. م. و. سامانی و ع. سلیمانی

چکیده

سدهای پاره سنگی تأخیری یکی از روشهای ارزان برای کاهش خسارات سیل هستند که از مواد سنگی ساخته شده‌اند. هیدرولیک این سدها به دلیل وجود خلل و فرج بزرگ از قانون دارسی تبعیت نمی‌کند. تحقیقات قبلی انجام گرفته نشان می دهد که چهار رابطه مک کور کودل و همکاران، استیفنس، آدل و سامانی و همکاران پیش‌بینی خوبی از رفتار جریان در محیط سنگریزه دارند. پس ضروری به نظر می‌رسید که تجزیه و تحلیل عدم قطعیت و تجزیه و تحلیل حساسیت بر روی این روابط صورت گیرد تا تصویر بهتری از میزان پراکندگی نتایج حاصل از استفاده از این روابط بدست آید و سهم پارامترهای ورودی در عدم قطعیت خروجی این روابط مشخص گردد. بدین منظور یک توده سنگریزه‌ای تهیه و پانزده نمونه تصادفی انتخاب شد و مشخصات فیزیکی مرتبط با هریک از روابط تجربی برای این پانزده نمونه اندازه‌گیری یا تخمین زده شد. همچنین از این توده یک سد پاره‌سنگی در فلوم آزمایشگاه ساخته شد و ارتفاع آب در بالا دست ($H1$) و پایین دست ($H2$) سد برای دبی های مختلف اندازه‌گیری گردید و هیدروگراف خروجی بدست آمد. در تجزیه و تحلیل عدم قطعیت $H2$ به عنوان متغیر قطعی، سایر پارامترها به عنوان متغیر آماری تصادفی و دبی خروجی به عنوان پارامتر خروجی در نظر گرفته شده‌اند. تجزیه و تحلیل عدم قطعیت برای هفت نقطه از هیدروگراف خروجی انجام گردید. سه روش تخمین مرتبه اول تغییرات ($FOVE$) روش تخمین نقطه‌ای هار ($Harr$) و روش شبیه‌سازی مونت کارلو با نمونه‌گیر LHS در رابطه با تجزیه و تحلیل عدم قطعیت به کار گرفته شدند. بطور کلی نتایج نشان می‌دهند که رابطه سامانی و همکاران و رابطه مک کوردل و همکاران به ترتیب دارای کمترین و بیشترین عدم قطعیت می‌باشند. همچنین نتایج تجزیه و تحلیل حساسیت نشان می‌دهد که پارامترهای مختلف در تعیین عدم قطعیت دبی خروی تأثیر دارند.