

# Evaluation of current distribution induced on perfect electrically conducting scatterers

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## Abstract

This article deals with the use of an appropriate model in determining the induced current distribution on finite-width strip electromagnetic scatterer. For this purpose, electric field integral equation (EFIE) as a mathematical model is surveyed and used for modeling of the problem. An appropriate numerical method is then proposed to obtain the approximate numerical results for the EFIE. The results are given in both TM and TE polarizations.

*Keywords* : Current density; EFIE; Mathematical modeling; Approximate solution; Finite-width strip scatterer.

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## 1 Introduction

Valuable efforts have been spent, by researchers, on introducing novel ideas for the solution of various functional equations (for example, see [3, 4, 10, 16, 17]). Integral equation technique is a well-known approach for modeling of scattering problems. Traditionally, most of the numerical methods for the solution of these models use basis functions including, for example, characteristic basis functions [18], Rao, Wilton, and Glisson (see references of [13]), radial basis functions [11], Fourier series [12], wavelets [8, 9], etc. Some methods solve the integral equation model using entire domain basis functions [14] and some of them use the singular integral equation approach [19, 15]. Some researchers have proposed modified or hybrid methods to increase the computational efficiency of the traditional ap-

proaches. A regularized combined field integral equation (CFIE) pertinent to the analysis of scattering from 2-D perfect electrically conducting objects is presented in [2], in which the regularization is achieved via analytical inversion of the hypersingular part of the CFIE. An alternate form of the CFIE is introduced in [1] which is free of many of the deficiencies of traditional formulations for smooth, convex geometries. A mixed-domain Galerkin expansion is shown in [7] to lead to a computationally efficient formulation for classes of scattering problems involving electrically large two-dimensional scatterers with localized discontinuities.

This article focuses on using the integral equation model to survey the problem of electromagnetic scattering from finite-width strips. Firstly, the EFIE is surveyed as a mathematical model for electromagnetic scattering from arbitrary bodies and then it is simplified to be matched with the main problem. Since the obtained model has no analytical solution, hence an approximation method is presented for solution of the problem. For determining the scattered field we must know

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the current density on the strip, therefore the plots of surface current are given for different conditions.

## 2 EFIE model

The EFIE is based on the boundary condition that the total tangential electric field on a perfectly electric conducting (PEC) surface of scatterer is zero [5]. This can be expressed as

$$\begin{aligned} \mathbf{E}_t^t(\mathbf{r} = \mathbf{r}_s) &= \mathbf{E}_t^i(\mathbf{r} = \mathbf{r}_s) + \mathbf{E}_t^s(\mathbf{r} = \mathbf{r}_s) \\ &= 0, \quad \text{on } S, \end{aligned} \quad (2.1)$$

or

$$\mathbf{E}_t^s(\mathbf{r} = \mathbf{r}_s) = -\mathbf{E}_t^i(\mathbf{r} = \mathbf{r}_s), \quad \text{on } S, \quad (2.2)$$

where  $S$  is the conducting surface of the scatterer,  $\mathbf{r} = \mathbf{r}_s$  is the position vector of any point on the surface of the scatterer, and  $\mathbf{E}^i$ ,  $\mathbf{E}^s$ , and  $\mathbf{E}^t$  are respectively the incident, scattered, and total electric fields. Also, subscript  $t$  indicates tangential components.

The incident field that impinges on the surface of the scatterer induces on it an electric current density  $\mathbf{J}_s$  which in turn radiates the scattered field. The scattered field everywhere can be found using the following equation [5]:

$$\begin{aligned} \mathbf{E}^s(\mathbf{r}) &= -j\omega\mathbf{A} - j\frac{1}{\omega\mu\epsilon}\nabla(\nabla\cdot\mathbf{A}) \\ &= -j\frac{1}{\omega\mu\epsilon}[\omega^2\mu\epsilon\mathbf{A} + \nabla(\nabla\cdot\mathbf{A})], \end{aligned} \quad (2.3)$$

where  $\epsilon$  is the permittivity of the medium,  $\mu$  is the permeability of the medium,  $\omega$  is the angle frequency of the incident field,  $\nabla$  is the gradient operator,  $j$  is imaginary unit, and  $\mathbf{A}$  is the magnetic vector potential defined as

$$\mathbf{A}(\mathbf{r}) = \mu \iint_S \mathbf{J}_s(\mathbf{r}') \frac{e^{-j\beta R}}{4\pi R} ds', \quad (2.4)$$

where  $R$  is the distance from source point to the observation point.

Equations (2.3) and (2.4) can also be expressed as [5]

$$\mathbf{E}^s(\mathbf{r}) = -j\frac{\eta}{\beta} \left[ \beta^2 \iint_S \mathbf{J}_s(\mathbf{r}') G(\mathbf{r}, \mathbf{r}') ds' + \nabla \iint_S \nabla' \cdot \mathbf{J}_s(\mathbf{r}') G(\mathbf{r}, \mathbf{r}') ds' \right] \quad (2.5)$$

where  $\eta$  is the intrinsic impedance of the medium,  $\beta$  is the phase constant,  $\mathbf{r}$  and  $\mathbf{r}'$  are position vectors of the observation point and source point respectively, and

$$G(\mathbf{r}, \mathbf{r}') = \frac{e^{-j\beta R}}{4\pi R}, \quad (2.6)$$

and

$$R = |\mathbf{r} - \mathbf{r}'|. \quad (2.7)$$

In Eq. (2.5),  $\nabla$  and  $\nabla'$  are respectively the gradients with respect to the observation and source coordinates, and  $G(\mathbf{r}, \mathbf{r}')$  is referred to as Green's function for a three-dimensional scatterer.

If the observations are restricted on the surface of the scatterer ( $\mathbf{r} = \mathbf{r}_s$ ), then Eq. (2.5) through Eqs. (2.6) and (2.7) can be expressed using Eq. (2.2) as

$$\begin{aligned} j\frac{\eta}{\beta} [\beta^2 \iint_S \mathbf{J}_s(\mathbf{r}') G(\mathbf{r}_s, \mathbf{r}') ds' + \\ \nabla \iint_S \nabla' \cdot \mathbf{J}_s(\mathbf{r}') G(\mathbf{r}_s, \mathbf{r}') ds'] = \mathbf{E}_t^i(\mathbf{r} = \mathbf{r}_s). \end{aligned} \quad (2.8)$$

Because the right side of Eq. (2.8) is expressed in terms of the known incident electric field, it is referred to as the electric field integral equation. It can be used to find the current density  $\mathbf{J}_s(\mathbf{r}')$  at any point  $\mathbf{r} = \mathbf{r}'$  on the scatterer. It should be noted that Eq. (2.8) is actually an integro-differential equation, but usually it is referred to as an integral equation.

Equation (2.8) is a general surface EFIE for three-dimensional problems and its form can be simplified for two-dimensional geometries. Considering cylindrical coordinates system, it is assumed that the scatterer is very long in the  $\pm z$  direction and is parallel to the  $z$ -axis. After several steps of mathematical operations, the two-dimensional EFIE for the case of TM polarization can be concluded form (2.8) as [5]

$$\frac{\beta\eta}{4} \int_C I_z(\rho') H_0^{(2)}(\beta|\rho_m - \rho'|) dc' = E_z^i(\rho_m), \quad (2.9)$$

where  $H_0^{(2)}$  is Hankel function of the second kind of zero order,  $\rho_m$  is position vector of any observation point on the scatterer,  $\rho'$  is position vector of any source point on the scatterer,  $C$  is perimeter of the scatterer,  $I$  is the surface current distribution on the scatterer,  $E^i$  is the incident electric field,  $\beta$  is the phase constant, and  $\eta$  is intrinsic

impedance of the medium. For the TE polarization we have [5]

$$\begin{aligned} & \frac{\eta}{4\beta} \left\{ \beta^2 \int_C I_c(\rho') \left[ \hat{c}_m \cdot \hat{c}' H_0^{(2)}(\beta|\rho_m - \rho'|) \right] dc' \right. \\ & \left. + \frac{d}{dc} \left[ \nabla \cdot \int_C I_c(\rho') \left[ \hat{c}' H_0^{(2)}(\beta|\rho_m - \rho'|) \right] dc' \right] \right\} \\ & = -E_c^i(\rho_m), \end{aligned} \tag{2.10}$$

where  $\hat{c}_m$  and  $\hat{c}'$  are unit vectors tangent to the scatterer perimeter at the observation and source points, respectively. We see that the EFIE, for both TM and TE polarizations, has the form of Fredholm integral equation of the first kind.

In general, (2.9) and (2.10) do not have analytical solutions. Therefore, we have to obtain approximate solutions for them by using a numerical method.

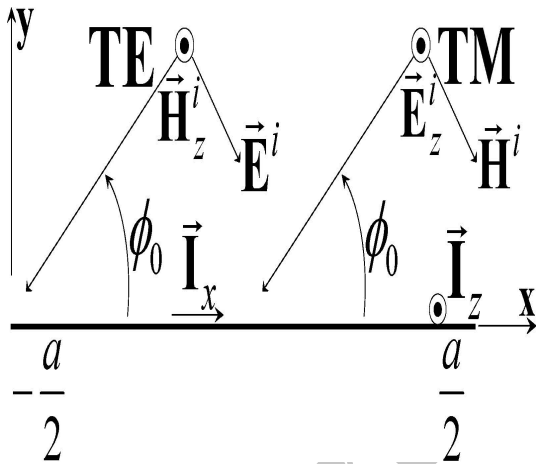


Figure 1: Finite-width strip encountered by an incoming plane wave.

### 3 Approximate solution of EFIE for finite-width conducting strip

Now, the problem of electromagnetic scattering from a strip is solved. In Figure 1, there is a perfect electrically conducting strip that is very long in the  $\pm z$  direction. This strip is encountered by an incoming plane wave. Both TM and TE polarizations are analyzed here. If the incoming plane wave has a polarization with its electric field parallel to the  $z$ -axis and the magnetic field of this

wave is entirely in the  $x - y$  plane (and is therefore transverse to the  $z$ -axis), then such a wave is called transverse magnetic (TM) polarized wave. This polarization therefore produces a current on the strip that flows along the  $z$ -axis [6]. If it has a polarization with its magnetic field parallel to the  $z$ -axis and the electric field of this wave is entirely in the  $x - y$  plane, then it is called transverse electric (TE) polarized wave. This polarization produces a current on the strip that flows along the  $x$ -axis.

For determining the current density in TM or TE polarization, we must find an approximate solution for (2.9) or (2.10), respectively. For this purpose, we use an appropriate set of basis functions surveyed in the following subsection.

#### 3.1 Basis functions

Let us consider an  $m$ -set of truncated cosines for any positive integer  $m$  over real interval  $[a, b]$  as

$$T_i(t) = \begin{cases} \cos(\gamma(t - a - ih - \frac{h}{2})), & a + ih \leq t, \\ & t < a + (i + 1)h, \\ 0, & \text{otherwise,} \end{cases} \tag{3.11}$$

where  $h = \frac{b-a}{m}$ ,  $i = 0, 1, \dots, m - 1$ , and  $\gamma$  has a real value and may be considered as a regularization factor.

The above definition can obviously make a set of disjoint and orthogonal basis functions. For arbitrary  $i$  and  $j$ , such that  $i = 0, 1, \dots, m - 1$  and  $j = 0, 1, \dots, m - 1$ , we have

$$\begin{aligned} \langle T_i, T_j \rangle &= \int_a^b T_i(t) T_j(t) dt \\ &= \begin{cases} \frac{h}{2} + \frac{1}{2\gamma} \sin(\gamma h), & i = j, \\ 0, & i \neq j, \end{cases} \end{aligned} \tag{3.12}$$

in which  $\langle \cdot, \cdot \rangle$  indicates the inner product.

Moreover, it is clear that function  $T_i$  may be considered as follows:

$$T_i(t) = \varphi_i(t) \cos(\gamma(t - a - ih - \frac{h}{2})), \tag{3.13}$$

where  $\varphi_i$  is  $i$ th block-pulse function (BPF) defined as

$$\varphi_i(t) = \begin{cases} 1, & a + ih \leq t < a + (i + 1)h, \\ 0, & \text{otherwise.} \end{cases} \tag{3.14}$$

The disjointness and orthogonality properties of  $T_i$ 's can make them very efficient for approximation of functions. The expansion of a function  $f$  over  $[a, b)$  with respect to  $T_i, i = 0, 1, \dots, m-1$ , may be compactly written as

$$f(t) \simeq \sum_{i=0}^{m-1} f_i T_i(t), \quad (3.15)$$

where  $f_i$ 's, the expansion coefficients, may be computed by

$$f_i = \langle f, T_i \rangle = \int_a^b f(t) T_i(t) dt. \quad (3.16)$$

In the next subsection, we will formulate a numerical method based on these functions for solving the EFIE for the strip problem.

### 3.2 Formulation of numerical method

Let us consider first kind Fredholm integral equation of the form

$$\int_a^b k(s, t) x(t) dt = f(s), \quad a \leq s < b, \quad (3.17)$$

where the functions  $k$  and  $f$  are known but  $x$  is the unknown function to be determined. Also,  $k \in L^2([a, b) \times [a, b))$  and  $f \in L^2([a, b))$ .

Approximating the unknown function  $x$  with respect to the truncated cosines using (3.15) gives

$$x(t) \simeq \sum_{i=0}^{m-1} x_i T_i(t), \quad (3.18)$$

where  $x_i$ 's are defined as in (3.16). Substituting (3.18) into (3.17) results in

$$\int_a^b k(s, t) \left( \sum_{i=0}^{m-1} x_i T_i(t) \right) dt = f(s), \quad (3.19)$$

or

$$\sum_{i=0}^{m-1} x_i \int_a^b k(s, t) T_i(t) dt = f(s). \quad (3.20)$$

Now, choosing  $m$  appropriate points  $s_j, j = 0, 1, \dots, m-1$ , we obtain

$$\sum_{i=0}^{m-1} x_i \int_a^b k(s_j, t) T_i(t) dt = f(s_j), \quad j = 0, 1, \dots, m-1. \quad (3.21)$$

(3.21) is a linear system of  $m$  algebraic equations in terms of  $m$  unknown coefficients  $x_i$ . Solution of this systems gives  $x_i$ 's, and then we obtain an approximate solution  $x(s) \simeq \sum_{i=0}^{m-1} x_i T_i(s)$  for (3.17).

### 3.3 Numerical results

The results of applying the proposed numerical method in solving the EFIE for the finite-width strip are given here. The medium is assumed to be free space. Figure 2 shows the current distribution across the strip obtained by the method for  $\beta = 2\pi, \phi_0 = \frac{\pi}{2}, a = \frac{3\lambda}{2}$ , and  $\gamma = 1$ , in TM polarization. Also, Figure 3 gives the results in TE polarization. For  $a = 6\lambda$ , the results in TM and TE polarizations are respectively given in Figures 4 and 5.

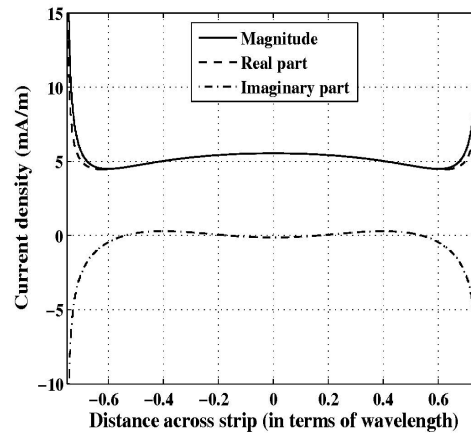


Figure 2: Current distribution across the strip for  $\beta = 2\pi, \phi_0 = \frac{\pi}{2}, a = \frac{3\lambda}{2}$ , and  $\gamma = 1$ , in TM polarization.

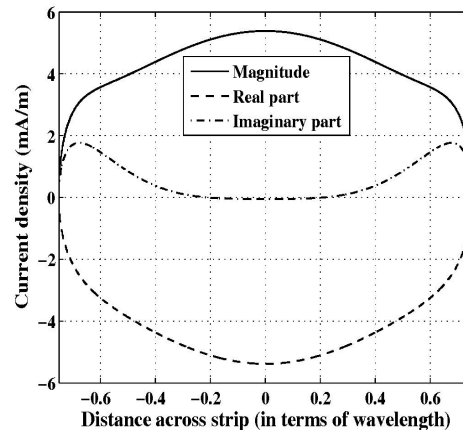
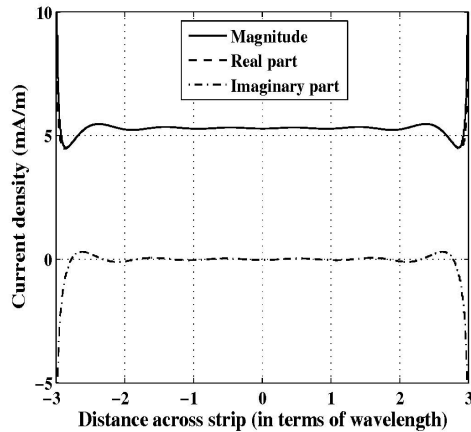
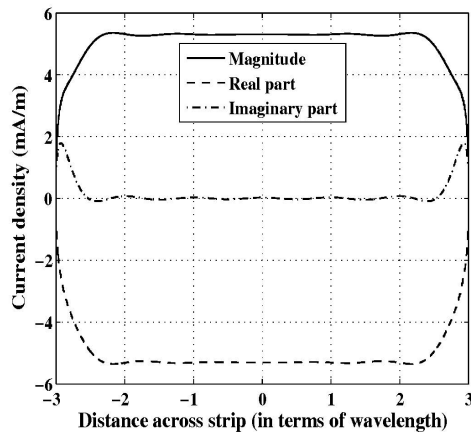


Figure 3: The results in TE polarization.



**Figure 4:** Current distribution across the strip for  $\beta = 2\pi$ ,  $\phi_0 = \frac{\pi}{2}$ ,  $a = 6\lambda$ , and  $\gamma = 1$ , in TM polarization.



**Figure 5:** The results in TE polarization.

## 4 Conclusion

We surveyed, in this article, an appropriate mathematical model for calculating the current density on a finite-width strip scatterer. We saw that the analysis of the problem in TM and TE polarizations needs different models. Two integral equations were mentioned for modeling of the problem in both polarizations. A numerical method was proposed for the solution of the two integral equations because they have no analytical solution. Finally the results were given for both polarizations.

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