Evaluation of Turbulence Models in Predicting Turbulence influence from Flow Field to Boundary Layer

Shafahi, M.[1] and **Kahrom, M.**

Mechanical Engineering Department, Ferdowsi University of Mashhad,

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ABSTRACT

Turbulence models have long been developed and examined for their accuracy and stability in variety of environments. While many flows work with excited turbulence intensity, models have rarely been tested to explore whether their accuracy withstands with augmented free stream turbulence intensity or decline in reasonable solutions. In present study the turbulent intensity of the air, moving parallel to a flat plate is increased from 0.4 to 6.6% for the whole flow, downstream to the screen. Three popular turbulence models are examined by investigating the turbulence penetration into flow field as well as into turbulent boundary layers over the flat plate. Results of numerical solutions for Standard k-ε, Realizable k-ε and finally two equations Shear Stress Transport k-ω model are compared to experimental measurements and results are discussed. Results of variation of free stream turbulence intensity from flow field out of boundary layer, in addition, streamwise mean velocity, streamwise rms velocity and skin friction coefficient from boundary layer are investigated. Conclusion is made that despite restrictions of these turbulent models specially in predicting flow near a turbulent/non-turbulent interface, they have acceptable performance in both low and high intensity turbulent flows.

Keywords: Free Stream Turbulence (FST), Turbulence Model, Transition, Skin Friction.

ارزیابی مدلهای آشفتگی در پیشبینی تأثیرات جریان آزاد روی لایه مرزی

م. شفاهی و م. کهرم

دانشکده مهندسی مکانیک

دانشگاه فردوسی مشهد

چکیده

مدلهای آشفتگی سالهای سال در جریانهای مختلف مورد ارزیابی قرار گرفته اند. خیلی از جریانهای آزاد همراه با شدت آشفتگی غیر یکنواخت میباشند، در صورتی که کمتر بررسی شده که آیا مدلهای آشفتگی قابلیت این یکنواختی ها را دارد یا خیر. در مطالعه حاضر شدت آشفتگی در ورودی یک جریان آشفته لایه مرزی از ۰/۴ تا ۶/۶ درصد تغییر یافته است. سه مدل آشفتگی معروف در نظر گرفته شده و میزان نفوذ جریان در جهت عمود بر دیواره و در امتداد صفحه تخت مورد مطالعه قرار گرفته است. نتایج مربوط به این سه مدل (k-ε استاندارد، k-ε قابل مشاهده و SST - k-ω) با نتایج تجربی مقایسه و مورد بحث قرار گرفته اند. تأثیرات تغییرات شدت آشفتگی روی $\bar{u}$ ، $\sqrt{\overline{u}^2}$ و ضریب اصطکاک پوسته ای بررسی شده است. نتیجه اصلی این تحقیق این است که علیرغم محدودیت های مدلهای آشفتگی فوق (بخصوص جهت پیش بینی جریان در مرز بین جریانهای آرام و آشفته)، این مدلها پیش بینی نسبتاً خوبی در حالتهای مختلف شدت آشفتگی کم و زیاد از جریان دارند.

واژه های کلیدی: جریان آشفته آزاد، مدل آشفتگی، جریان انتقالی، اصطکاک پوسته ای

[1] Corresponding author, mshafahy@yahoo.com

Introduction

Flows in engineering applications are mostly three-dimensional, unsteady, highly chaotic and turbulent, which span over a wide range of length and time scales. One measure of unsteadiness is the turbulence intensity that is the cause of apparent shear stresses. In high intensity turbulent flows, such as those streaming through turbomachinery blade rows or flows through burners, the turbulence intensity is reported to be as high as 5 to 25 percent [1]. This order of intensity disrupts downstream flow field and penetrates into boundary layer and modifies the shear flow structure. The subject hasn't been attended in classical developments of turbulence modeling yet and elaboration on modeling techniques seems to be demanded.

Another example of complexity of such a flow field is the effect of free stream turbulence on the onset of transitional boundary layer, which has recently received great attention in flow modeling, [2-4]. Frequently, in flows passing through turbo machinery blade rows or in the case of promoted boundary layers, bypass transition occurs by which laminar boundary layer turns to turbulent, abruptly. For transitional boundary layers, attempts have been made to establish empirical correlations through experimental data between free stream turbulence intensity and transitional point Reynolds numbers, [5]. Though very helpful, application is limited. Putting these all cases together, a general question arises that, how reliable present turbulence models are in treating such a complex viscous flow field.

While, the numerical techniques are of great help in predicting flow developments, many of these models are based on some simplifications and assumptions. In general, the numerical modeling of turbulent is based on Navier-Stokes (N-S) equations that can be classified into two categories: 1) Direct Numerical Simulations, namely the DNS models, 2) all other models which are based on understanding of physics of turbulence, the way flow develops and then it"s modeling.

In many numerical techniques, the turbulence convection, it"s generation, dissipation and diffusion are all modeled by large scale grids and correlation is imposed to find out coefficients of these equations, so that to adjust final solution to cover particular case of fluid flow. Frequently, some simplification into modeling is imposed, such as assuming isotropy and homogeneity for the flow field. Amongst these numerical techniques, simplifications to DNS models are minor, [6]. However, the DNS model is in most cases impracticable, LES and DES models are very elaborate and expensive for engineering calculations. The most widely used turbulence model by engineers, are the Eddy-Viscosity Models (EVM). Despite attraction these models between engineers have, rarely have attempted to explore ability of these models in predicting turbulence penetration into flow field as well as into the boundary layers which has been taken as the main task of present research work.

In this paper, attempt is made to explore capability of some of most widely used eddy viscosity models together in predicting the turbulence penetration into a flow field out of boundary layer over a flat plate. The case of a flat plate is simple, common to many engineering problems and one should expect to achieve most accurate results for this geometry. The numerical results are compared to experimental data of wind tunnel measurements, [7].

Flow Field Description

Physical domain has previously employed for experimental measurements by Sohn and Roshotko [7] in a wind tunnel. The study was conducted on flow of air parallel to a flat plate at zero pressure gradients. The main stream flows with a constant velocity of V=33.3 m/s as a result laminar, transitional and turbulent boundary layers form over the flat plate. In experimental setup, before and far from leading edge, a grid screens is placed to impose desired turbulence intensity by value of 0.4% to 6.7% into air stream. In this research variety of turbulence intensity are imposed to flow field to study how successful is each model in predicting turbulence penetration into the main field as well as into the turbulent boundary layer.

In numerical study, each grid screen is meant by its equivalent turbulence kinetic energy that is related to turbulence intensity by the following relations:

$k=\frac{1}{2}\left(\overline{u'^2+v'^2+w'^2}\right)$, while , $Ti=\sqrt{\frac{1}{3}(\overline{u'^2}+\overline{v'^2}+\overline{w'^2})}/U_\infty$, then; $Ti=\frac{1}{U_\infty}\sqrt{\frac{2}{3}k}$.	(1)

In general in a wind tunnel there is so called isotropic flow [1]. For an isotropic flow the average fluctuation velocity is the same in all three coordinate directions. In this case the longitudinal velocity alone is used for the turbulence intensity.

Turbulence Models and Governing Equations

The flow field is assumed to be steady, two dimensional and turbulent. The velocity components and pressure are governed by equations of continuity and momentum:

$$\frac{\partial U_i}{\partial x_i}=0, \quad (2)$$

$$\frac{\partial U_i U_j}{\partial x_j}=-\frac{\partial p}{\partial x_i}+\frac{1}{Re}\frac{\partial^2 U_i}{\partial x_j^2}. \quad (3)$$

Amongst many turbulence models, some of most popular models are selected for evaluation. As a general trend, for all turbulence models, flow variables are decomposed into one average and one fluctuating part[8].

If the fluctuating part is directly employed to estimate the mass and momentum transfer, the model is categorized as Direct Numerical Simulation, namely the DNS method. If the average of fluctuations are used to define an apparent viscosity (υ_t), the model is recognized one of Bosinesque turbulence models. Most of Bosinesque hypotheses based models employ isotropy assumption and are referred to as eddy viscosity models (EVMs)[9].

The first selected EVM, in this study, is the widely used for engineering estimation the standard k-ε model. The high Reynolds number version of the model uses the law of the wall to estimate velocity profile near the wall. Much less grid points are then employed in each calculation. Simple implementation, stability, easily convergence and reasonable prediction of many engineering flows are the main advantages of the scheme. However, poor prediction for rotating flows, strong separation, axis-symmetric jets and fully developed flows in non-circular ducts are disadvantages conveyed with the scheme.

By some modifications to its coefficients, the k equation is nearly common to all two equations turbulent EVMs and constantly is assumed to be as, [10]:

$$\frac{\partial}{\partial t}(\rho k)+\frac{\partial}{\partial x_i}(\rho k u_i)=\frac{\partial}{\partial x_j}\left(\mu_{effect\ k}\frac{\partial k}{\partial x_j}\right) +G_k+S_k+[Terms\ Added\ to\ k\ Eqn.]. \quad (4)$$

The term in bracket is variously defined for different models and shall be defined for each model separately in nomenclature.

For k-ε models the turbulent viscosity is assumed as $\mu_t=\rho\zeta\frac{k^2}{\varepsilon}$ and in k-ω models, is defined as:

$$\mu_t=\rho\zeta\frac{k}{\omega}. \quad (5)$$

An extra equation is then needed to close two equation models for a locally isotropic flow fields. In general, this equation is presented as, [10]:

$$\frac{\partial}{\partial t}(\rho\gamma)+\frac{\partial}{\partial x_i}(\rho\gamma u_i)=\frac{\partial}{\partial x_j}\left(\mu_{effect\ \varepsilon}\frac{\partial\gamma}{\partial x_j}\right) +S_\gamma+[terms\ added\ to\ \gamma\ equ.]. \quad (6)$$

For Standard k-ε models assumptions are made as follows:

$$\zeta=C_\mu=0.09\ \ Const.\ , \quad \gamma=\varepsilon\ , \quad \mu_{effect\ K}=\mu+\frac{\mu_t}{\sigma_K}\ , \quad \mu_{effect\ \varepsilon}=\mu+\frac{\mu_t}{\sigma_\varepsilon}\ , \quad (7)$$

$$added\ terms\ to\ k\ equ.=\ G_b-\rho\varepsilon-Y_M\ , \quad added\ terms\ to\ \gamma\ equ.=\ C_{1\varepsilon}\frac{\varepsilon}{K}(G_K+C_{3\varepsilon}G_b)-C_{2\varepsilon}\rho\frac{\varepsilon^2}{K}\ . \quad (8)$$

Realizable k-ε is second chosen model, the term "realizable" means that the model satisfies certain mathematical constraints on the Reynolds stresses, consistent with the physics of turbulent flows. Neither the Standard k-ε model nor the RNG *k*-ε model is realizable. It is likely to provide superior performance for flows involving rotation, boundary layers under strong adverse pressure gradients, separation, and recirculation. This model contains a new formulation for the turbulent viscosity (C_μ) Instead of a constant value, and a new transport equation for the dissipation rate, ε, has been derived from an exact equation for the transport of the mean-square vorticity fluctuation.

Added Term to γ Equation:

$$\rho C_1 S_\varepsilon - \rho C_2 \frac{\varepsilon^2}{k+\sqrt{\varepsilon\nu}} + C_{1\varepsilon}\frac{\varepsilon}{k} C_{3\varepsilon} G_b \qquad (9)$$

$$\text{and,}\ \zeta = C_\mu = 1/(A_0 + A_S \frac{kU^*}{\varepsilon}) \qquad (10)$$

It can be seen that C_μ is a function of the mean strain and rotation rates, the angular velocity of the system rotation, and the turbulence fields (k and ε). C_μ in Equation (10) can be shown to recover the standard value of 0.09 for an inertial sublayer in an equilibrium boundary layer.

To end with two equations EVM�///s, we employ the Shear Stress Transport k-ω model, SST k-ω, Menter [11]. The method combines best of two k-ε and LRN k-ω model. In the inner parts of the boundary layer, down to the sub layer and wall, the SST employs the LRN k-ω model, gaining benefits of excellent near wall estimation of this model. By approaching to main stream SST model gradually switches to HRN k-ε to avoid sensitivity of k-ω model in compatibility with surrounding boundaries. These all together, the model is merit for its good behavior in adverse pressure gradients flows with separation.

For SST k-ω model some parameters read again, [10]:

$$\zeta = \frac{1}{\max\left[\frac{1}{\alpha^*}, \frac{\sqrt{2\Omega_{ij}\Omega_{ij}}\,F_2}{a_1\,\omega}\right]},$$

$$F_2 = \tanh\left(\max\left[\frac{2\sqrt{k}}{0.09\,\omega y}, \frac{500\,\mu}{\rho y^2 \omega}\right]\right)^2, \qquad (11)$$

$$a_1 = 0.31 \qquad \gamma = \omega$$

Ω is the mean of rotational tensor and:

$$\mu_{effect\ K} = \mu + \frac{\mu_t}{\sigma_K},$$

$$\mu_{effect\ \varepsilon} = \mu + \frac{\mu_t}{\sigma_\omega}.$$

Added terms in k equation, in brackets, are:

$$Added\ Terms\ to\ \gamma\ Equation = G_\omega - Y_\omega + D_\omega .$$

Amongst highlighted advantages of two equation models, one can say they are simple due to isotropy assumption, stable and short CPU time consumption. let‵s see how good and accurate these all discussed models are in predicting turbulence penetration into main flow as well as into boundary layer.

Numerical Method

The present steady and incompressible fluid flow calculations were performed with the finite-volume code Teach-t [12].in this code pressure field is linked to that of velocity through the well-known SIMPLE pressure correction algorithm [12]. This two-dimensional simulation program has been developed for the numerical calculation of laminar and turbulent flows over a flat plate. Fig. **1** shows one sketch of two dimensional test case added with boundary condition.

This code has STD k-ε turbulence model as a default model, therefore Realizable k-ε and SST k-ω added and validated with experimental results of flow over a flat plate.

It is worth to mention that during run with SST k-ω the convergence pattern has its lowest speed. It was predictable because of combination k-ε and k-ω base models far and near the wall causes to delay convergence.

To capture flow behavior close to wall several non-unified meshes which are fined close to the wall, have been investigated in so far as the solution is grid independence. Fig. **2** shows one of the suitable mesh whit 63*60 grid nodes in the stream-wise (x) and cross-stream (y) directions respectively and Fig. **3** exhibits turbulent viscosity contour.

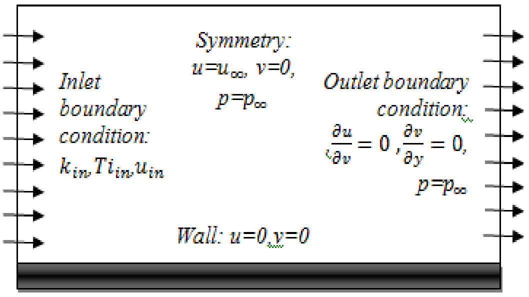

Fig. 1. Two-dimensional test case.

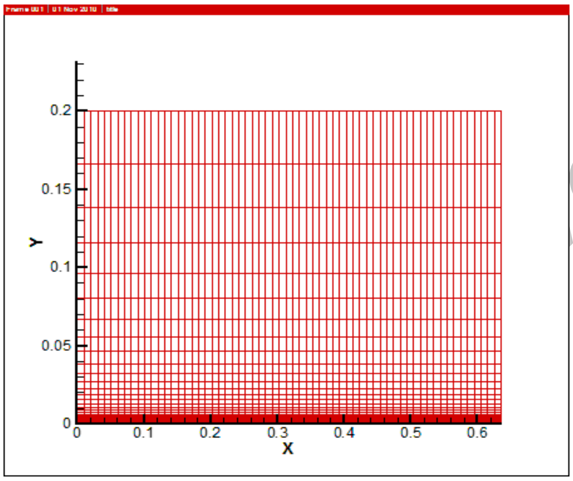

Fig. 2. Computational grid whit 63*60 nodes.

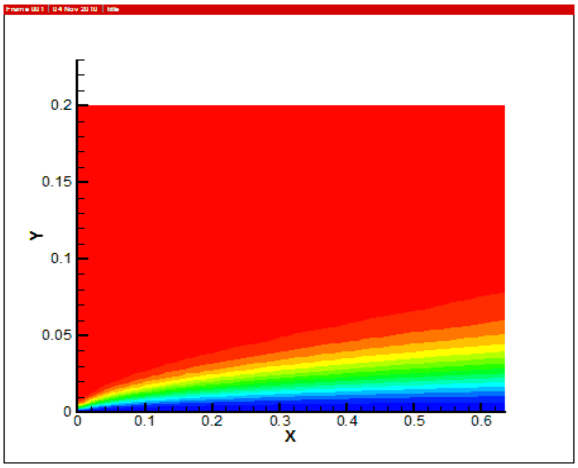

Fig. 3. Turbulent viscosity contour.

Results and Discussion

- **Free Stream Turbulence Intensity**

Development of T_i inside the free stream and out of the boundary layer is very important as at each section it is the source of penetration turbulence to boundary layers [13-14]. The experimental results show that up to T_i<1.1% (k=2.7%) the turbulence intensity decays slowly along the flow field and is rather constant over this short field of study. Also if the size of the turbulent motions in the free stream is too small, FST (Free Stream Turbulence) can not influence the near-wall turbulent regime. As imposed value of T_i increases, balance between production, dissipation, and convection of k disturbs and k dissipates faster on the other word, too large structured FST is damped by the wall. Fig. **4** and Fig. **5** show stream turbulence intensity as a function of streamwise location for T_i=5.6% and T_i=6.7% respectively. It can be seen that the rate of dissipation depends upon the values of T_i , as T_i increases it dissipates faster along the plate. All models predict decay of kinetic energy, but Realizable has best prediction of 4% error compared to experiment along the plate. Too large structured FST is damped by the wall, in fact to model the effects of FST a numerical approach has to be sensitive to both parameters, the turbulence intensity as well as its structure. However in this part, numerical error between models themselves, are not more than 3% for present domain of solution.

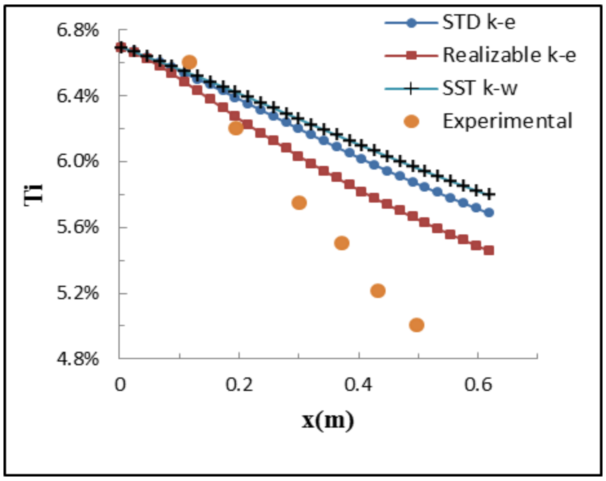

Fig. 4. Streamwise free stream turbulenc intensity through the plate, inlet Ti=5%.

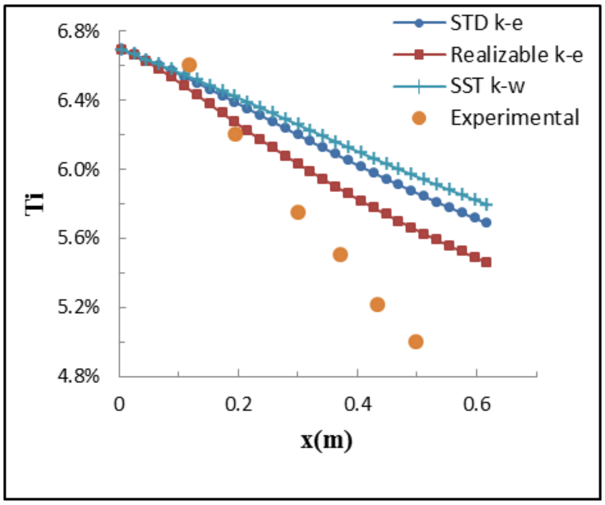

Fig. 5. Streamwise free stream turbulence intensity through the plate, inlet Ti=6.7%.

- **Streamwise Mean Velocity Profile**

There are many factors that influence the onset of transition. These include disturbances in external flow in the form of turbulence and noise, heat transfer, pressure gradient, suction, roughness and surface curvature [15]. however understanding and prediction of these effects on transition is steel very limited [16,17]. In this part ability of turbulence model to find beginning and end of transient region and effect of free stream turbulence on transition are examined. Due to the similarity of the laminar boundary layers, Comparison of mean velocity profile which is normalized whit Hartree similarity variables, η, whit Blasius one could be helpful to find initiate of transition. According to experimental results for Ti=0.8% in x=0.229m mean velocity profile begins to deviate from Blasius profile, which indicates the start of boundary layer transition. Fig. **6** compares numerical result from turbulence models whit experimental ones. Realizable and STD k-e models approve each other and they have less fault compared whit SST k-ω model. In fact, none of these models – along with many other widely used models– were not designed to predict transition. Nevertheless, as it shown in Fig. **6** these models could predict beginning of transition somehow. It is important to recognize that, even when run in "fully-turbulent" mode, turbulence models do not necessarily yield a fully-turbulent solution everywhere in the boundary layer. There is often a region near the leading edge of aerodynamic bodies where the flow is effectively laminar because the eddy viscosity produced by the turbulence model is low. The low values of eddy viscosity are a consequence of the turbulence model not having sufficient turbulence-production strength from the mean shear flow.

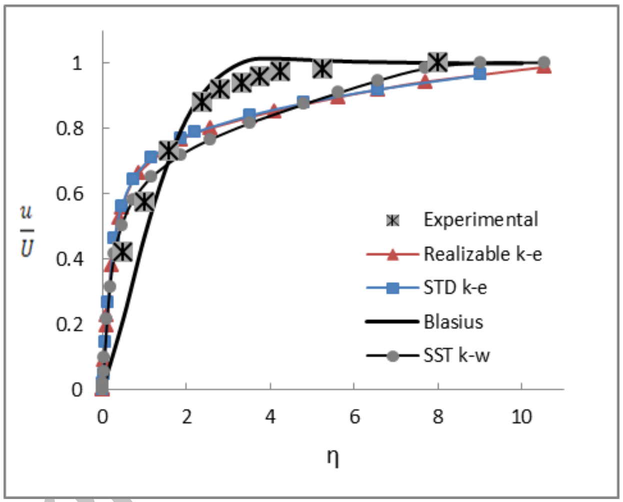

Fig. 6. Streamwise mean velocity profile for Ti=0.8%, x=0.229m.

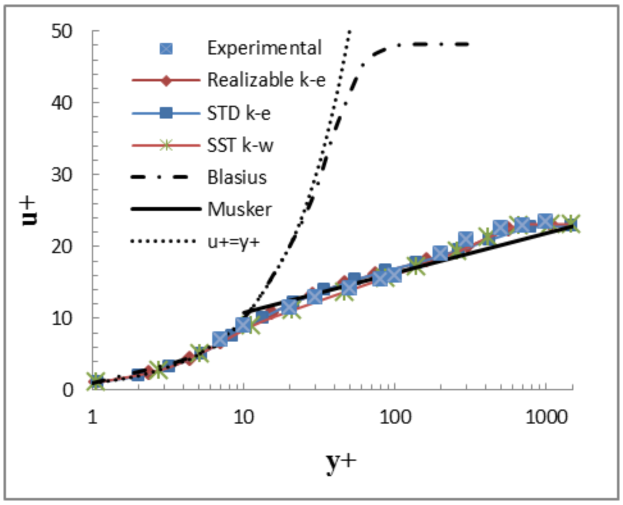

Fig. 7. Streamwise mean velocity profile for Ti=2.4%, x=0.38m.

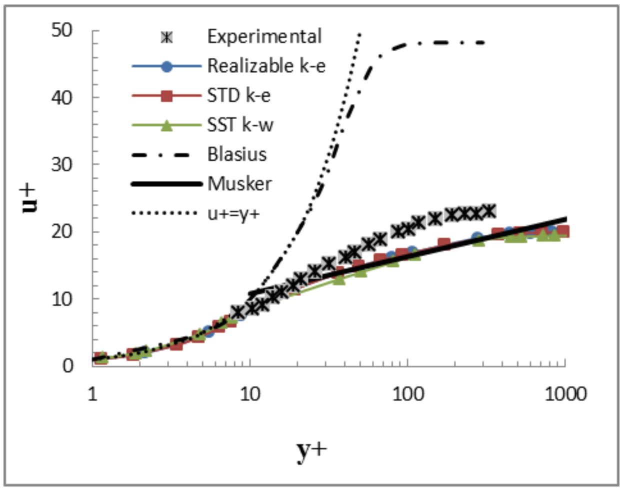

Fig. 8. Streamwise mean velocity profile for Ti=6.6%, x=0.127m.

To capture the end of the transition region the same mean velocity profiles were normalized in terms of wall units, u^+ and y^+ and plotted using a logarithmic scale for y^+ axis. These profile compared to refrence curves: 1) $u^+=y^+$, 2) the Blasius solution, and 3) the Musker curve for a fully turbulent boundary layer [18]. Once the profile falls close to the Musker curve especially in the log-linear region, it can be said that this might be the location of the end of transition. For Ti=2.4% experimental results indicate that the end of transition occurs at x=0.38m, Fig. 7. shows turbulence model numerical results compared whit experimental ones. As it shown, in this part all models are successful to capture the end of transition similar to experimental results. As the level of free stream turbulence increases, almost turbulent boundary layer begins close to leading edge of plate. Fig. 8. shows result of flow whit Ti=5.6% at x=0.127m. Experimental results show that turbulent boundary layer already has begun. In this part also models predictions are very well. In fact, when free stream turbulence increases, the wake strenght which is the amount of deviation of mean velocity profile from fully turbulent profile is getting diminished.

- **Streamwise rms Velocity**

Variation of u' in boundary layer is highly dependent on free stream turbulence intensity, but turbulence models are not able to estimate fluctutions parameters directly. In this part, u' is calculated based on the following empirical equation [18]:

$$u' = \sqrt{\frac{3k}{2}} \quad . \qquad (12)$$

The profile of overall streamwise velocity fluctuations normalized whit respect to u_τ across the boundary layer. Two cases are selected for comparison. First low turbulence intensity, Ti=0.8% in x=0.127m and x=0.38m from laminar boundary layer and second Ti=6.7% at x=0.38m from fully turbulent boundary layer.

The peak value of $\frac{u'}{u_\tau}$ in the laminar boundary layer occures at y^+=30 (x=0.127m). As it shown in Fig.9. there is a gap between experimental resuls and numerical ones from models, nonetheless all models capture the maximum relatively in correct position.

As the flow develops downstream, a double peak appears (x=15in), however models are successful to capture two maximum but still dissimilarity exists on quantity of distribution of $\frac{u'}{u_\tau}$ compared to experimental results. The magnitude of $\frac{u'}{u_\tau}$ in the turbulent boundary layer is relatively constant at 2 in region of 20<y^+<200 for Ti=6.7% and finally drops off to the free stream value (Fig. 11.). As the boundary layer is fully turbulent, good qualitative approach can be seen by all models, however at y^+<100 the SST k-ω has smallest quantative error.

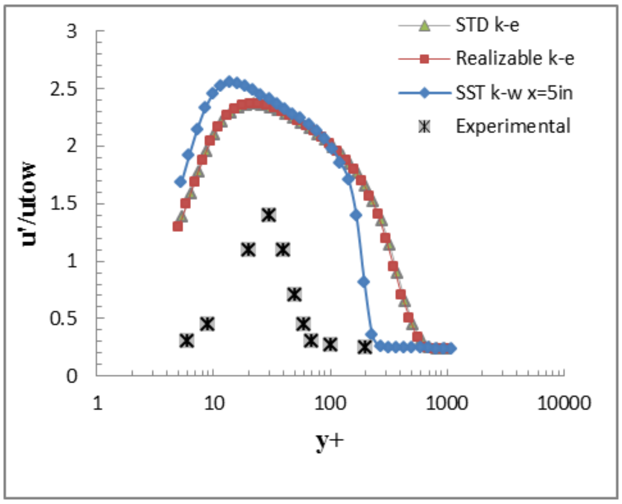

Fig. 9. Overal streamwise rms velocity profile in wall units, Ti=0.8%, x=0.128m.

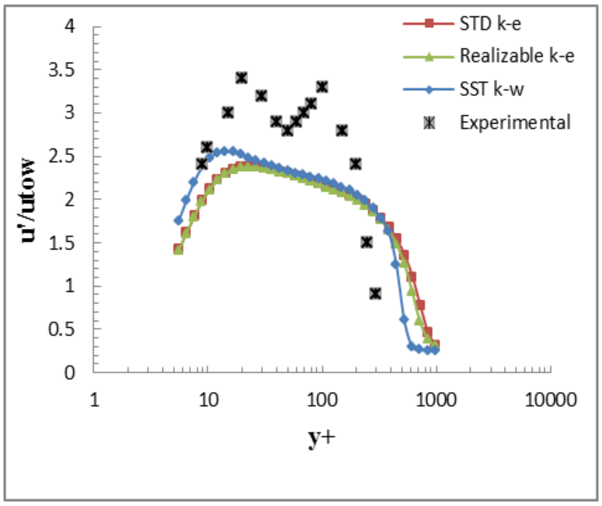

Fig. 10. Overall streamwise rms velocity profile in wall units, Ti=0.8%, x=0.38m.

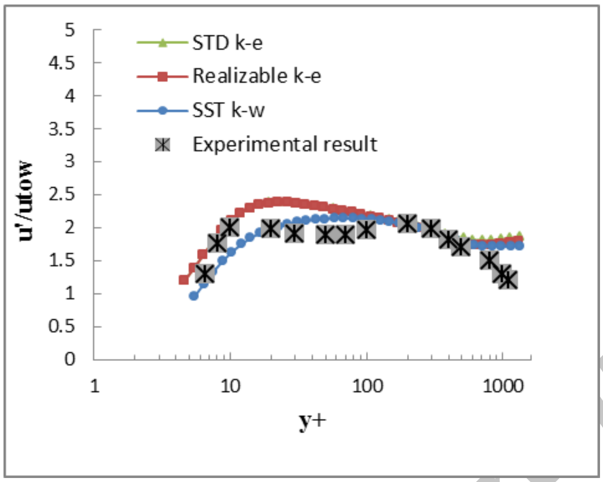

Fig. 11. Overal streamwise rms velocity profile in wall units, Ti=0.8%, x=0.38m.

- **Skin Friction Coefficient**

The effects of free stream turbulence intensity on the drag coefficient has been investigated experimentally and numerically [19.20].

For turbulent boundary layer some empirical relation of C_f suggested [15] which no effect of wake strenght due to the variation of freestream turbulence level was considered. Experimental results indicate that as free stream turbulence increases the amount of skin friction increases too. Fig. 12. and Fig. 13. exhibit experimental results compared to numerical results for Ti=5.6% and Ti=6.7% respectively. However all models predict good tendency of C_f , but Realizable k-ε has best prediction. As it is disscused, boundary layer transition occures at incrasingly lower value of reynolds number as the free stream turbulence level increases, and then whit earlier turbulent boundary layer, amount of C_f increases.

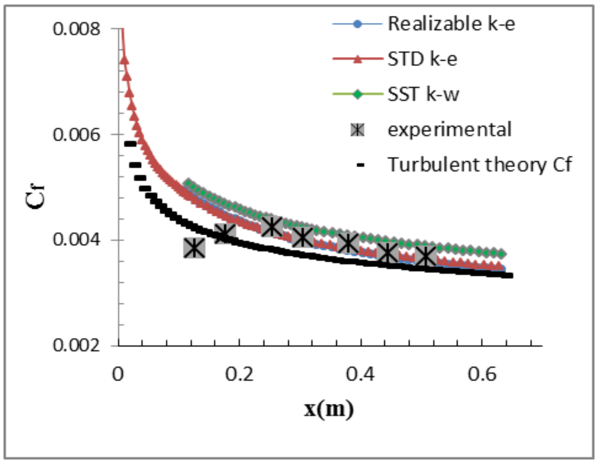

Fig. 12. Skin friction coefficient, Ti=5.6%.

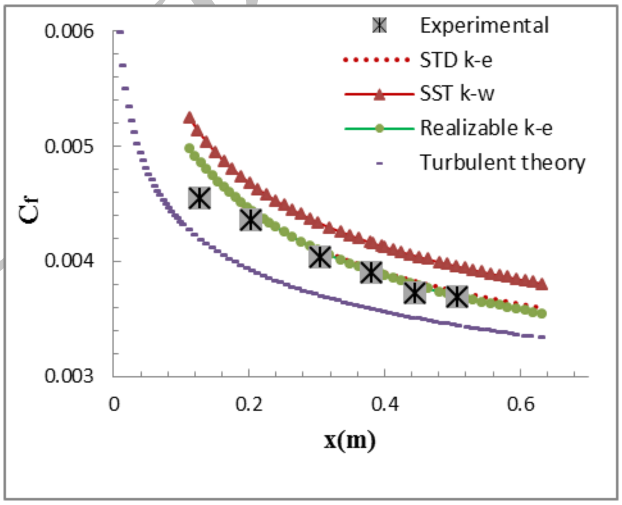

Fig. 13. Skin friction coefficient, Ti=6.7%.

Conclusion

Three reputable two-equation turbulence models have been evaluated in terms of their ability to predict influence of free stream turbulence intensity on flow field as well as penetration into turbulent boundary layers. Results are compared with similar case of experimental measurements. and show that: 1) descending trend of free stream turbulence along the plate has been predicted by all models, however, dissipation of turbulent kinetic energy is less than that for experiments. Realizable k-ε model has minimum fault compared to experimental results. 2) onset of boundary layer transition which is influenced by FST could be predicted whit all tested models relatively correct, specially in low turbulence intensity and near the leading edge, where the amount of turbulent viscosity is close to zero. 3) at high turbulence intensity where the boundary layer is fully turbulent all models have acceptable results and they predict effect of free stream turbulence into boundary layer correctly. 4) descending variation of C_f in high FST could be predicted accurately whit all models, particularly Realizable k-ε model which covers correctly experimental results, in addition, while increasing free stream turbulence, skin friction coefficient increases too, this trend also has been estimated by models.

Abbreviations

DNS	Direct Numerical Simulation
LES	Large Eddy Simulation
RSM	Reynolds Stress Model
RNG	Re-Normalization Group Theory
H.R.N.	High Reynolds Number
L.R.N.	Low Reynolds Number
SST	Shear Stress Transport Model
EVM	Eddy-Viscosity-Models

Nomenclature

C_{ij}	Convection-Terms
$,C_{\varepsilon 2}\ C_{\varepsilon 1}$ $C_{\varepsilon 3}$	A Constant
C_{b2}	0.662
C_{v1}	A constant, =7.1
$D_{T,ij}$	Turbulent Diffusion
$D_{L,ij}$	Molecular Diffusion
F_{ij}	Production by System Rotation
G_k	Effect of buoyancy on turbulence kinetics energy
$G_v, Y\upsilon$	Functions defined in Spalart-Allmaras equation
$S_{\bar{\upsilon}}$	
k	Turbulent Kinetic Energy per Unit Mass
P_{ij}	Stress Production
S_k	Source Term for k Equation/ Mean Rate of Stress Tensor
T_i	Turbulence Intensity of Flow
u_i	Averaged Stream Velocity in i Direction
u'	Fluctuating Part of Velocity Component
u_τ	Shear Velocity
y^+	$= y u_\tau / \nu_w$
x	Distance from Leading Edge
x_j	Distance in j Direction

Greek symbols

α_ε , α_k	Ratios of Effective to Molecular Viscosity in k and ε Equations
ε	Dissipation Rate
ϕ_{ij}	Pressure Strain
γ	A dissipation variable, e.g.: $\varepsilon, \ \omega$,
μ	Molecular viscosity
μ_t	Turbulent eddy viscosity
$\mu_{effec\ k}$	Effective Viscosity in k Equation
$\mu_{effec\ \varepsilon}$	Effective Viscosity in ε Equation

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