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On Prime and Semiprime Ideals in Ordered AG-Groupoids

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Abstract. The aim of this short note is to introduce the concepts of prime and semiprime ideals in ordered AG-groupoids with left identity. These concepts are related to the concepts of quasi-prime and quasi-semiprime ideals, play an important role in studying the structure of ordered AG-groupoids, so it seems to be interesting to study them.

Keywords: Ordered AG-groupoid, Prime, Semiprime, Quasi-prime, Quasi-semiprime.

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1. Introduction

A groupoid S is called an Abel-Grassmann's groupoid, abbreviated as an AG-groupoid, if its elements satisfy the left invertive law [4, 5], that is: for all Several examples and interesting properties of AG-groupoids can be found in [6], [11], [12] and [13]. It has been shown in [6] that if an AG-groupoid contains a left identity then it is unique. It has been proved also that an AG-groupoid with right identity is a commutative monoid, that is, a semigroup with identity element. It is also known [5] that in an AG-groupoid , the medial law, that is,

$$(ab)(cd) = (ac)(bd)$$

for all $a, b, c, d \in S$ holds. Now we define the concepts that we will used. Let S be an AG-groupoid. By an AG-subgroupoid of [15], we means a non-empty subset A of S such that $A^2 \subseteq A$. A non-empty subset A of an AG-groupoid S

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is called a left (right) ideal of [14] if $SA \subseteq A(AS \subseteq A)$. By two-sided ideal or simply ideal, we mean a non-empty subset of an AG-groupoid S which is both a left and a right ideal of S.

The concept of an ordered AG-groupoid was first given by Khan and Faisal in [8] which is infect the generalization of an ordered semigroup.

In this paper we characterize the ordered AG-groupoid. We study prime, semiprime, quasi-prime and quasi-semiprime in ordered AG-groupoids with left identity are introduced and described.

2. Basic Results

In this section, we refer to [17, 18] for some elementary aspects and quote few definitions and essential examples which are essential to step up this study. For more details, we refer to the papers in the references.

Definition 2.1. [17, 18] Let S be a nonempty set, $\cdot$ a binary operation on S and $\leq$ a relation on $S.(S, \cdot, \leq)$ is called an ordered AG-groupoid if $(S, \cdot)$ is a AG-groupoid, $(S, \leq)$ is a partially ordered set and for all $a, b, c \in S, a \leq b$ implies that $ac \leq bc$ and $ca \leq cb$.

Lemma 2.2. [17, 18] *An ordered AG-groupoid $(S, \cdot, \leq)$ is an ordered semigroup if and only if $a(bc) = (cb)a$, for all $a, b, c \in S$.*

Proof. See [17, 18]. □

Lemma 2.3. [17, 18] *Let $(S, \cdot, \leq)$ be an ordered AG-groupoid and let A, B subsets of S. The following statements hold:*

(1) *If $A \subseteq B$, then $(A] \subseteq (B]$.*
(2) $(A](B] \subseteq (AB]$.
(3) $((A](B]] \subseteq (AB]$.

Proof. See [17, 18]. □

Lemma 2.4. *Let $(S, \cdot, \leq)$ be an ordered AG-groupoid and let A, B subsets of S. The following statements hold:*

(1) $A \subseteq (A]$
(2) $((A]] \subseteq (A]$.

Proof. The proof is obvious. □

Definition 2.5. [17, 18] A nonempty subset A of an ordered AG-groupoid $(S, \cdot, \leq)$ is called an AG-subgroupoid of S if $AA \subseteq A$.

Definition 2.6. [17, 18] A nonempty subset A of an ordered AG-groupoid $(S, \cdot, \leq)$ is called a left ideal of S if $(A] \subseteq A$ and $SA \subseteq A$ and called a right ideal of S if $(A] \subseteq A$ and $AS \subseteq A$. A nonempty subset A of S is called an ideal of S if A is both left and right ideal of S.

Lemma 2.7. [17, 18] *Let $(S,\cdot,\leq)$ be an ordered AG-groupoid with left identity. Then every right ideal of $(S,\cdot,\leq)$ is a left ideal of S.*

Proof. See [17, 18]. □

Lemma 2.8. [17, 18] *Let $(S,\cdot,\leq)$ be an ordered AG-groupoid with left identity and $A\subseteq S$. Then $S(SA)=SA$ and $S(SA]\subseteq(SA]$.*

Proof. See [17, 18]. □

Lemma 2.9. [17, 18] *Let $(S,\cdot,\leq)$ be an ordered AG-groupoid with left identity and $a\in S$. Then $<a>=(Sa]$.*

Proof. See [17, 18]. □

3. Ideals in Ordered AG-groupoids

The results of the following lemmas seem to play an important role to study ordered AG-groupoids; these facts will be used frequently and normally we shall make no reference to this lemma.

Lemma 3.1. *If $(S,\cdot,\leq)$ is an ordered AG-groupoid with left identity and let $a\in S$, then $(aS]$ is a left ideal of S.*

Proof. Assume that $(S,\cdot,\leq)$ is an ordered AG-groupoid with left identity. By Lemma 2.4, we have $(aS]=((aS]]$. Then

$$\begin{aligned} S(aS] &= (S](aS] \\ &\subseteq (S(aS)] \\ &= (a(SS)] \\ &= (aS]. \end{aligned}$$

Therefore $(aS]$ is a left ideal of S. □

Proposition 3.2. *If $(S,\cdot,\leq)$ is an ordered AG-groupoid with left identity and let $a\in S$, then $(a^2S]$ is an ideal of S.*

Proof. Assume that $(S,\cdot,\leq)$ is an ordered AG-groupoid with left identity. By Lemma 3.1, we have $(a^2S]$ left ideal of S. Then

$$\begin{aligned} (a^2S]S &= (a^2S](S] \\ &\subseteq ((a^2S)S] \\ &= ((SS)a^2] \\ &= (a((SS)a)] \\ &= (a((aS)S)] \\ &= ((aS)(aS)] \\ &= ((aa)(SS)] \\ &= (a^2S]. \end{aligned}$$

Therefore $(a^2S]$ is an ideal of S. □

Lemma 3.3. *If $(S, \cdot, \leq)$ is an ordered AG-groupoid with left identity and let $a \in S$, then $(a \cup Sa]$ is a left ideal of S.*

Proof. Assume that $(S, \cdot, \leq)$ is an ordered AG-groupoid with left identity. By Lemma 2.4, we get $(a \cup Sa] = ((a \cup Sa]]$. Then

$$\begin{aligned} S(a \cup Sa] &= (S](a \cup Sa] \\ &\subseteq (S(a \cup Sa)] \\ &= (Sa \cup S(Sa)] \\ &= (Sa \cup (SS)(Sa)] \\ &= (Sa \cup (aS)(SS)] \\ &= (Sa \cup (aS)S] \\ &= (Sa \cup (SS)a] \\ &= (Sa \cup Sa] \\ &= (Sa] \\ &\subseteq (a \cup Sa]. \end{aligned}$$

Therefore $(a \cup Sa]$ is a left ideal of S. □

Proposition 3.4. *If $(S, \cdot, \leq)$ is an ordered AG-groupoid with left identity and let $a \in S$, then $(Sa \cup aS]$ is an ideal of S.*

Proof. Assume that $(S, \cdot, \leq)$ is an ordered AG-groupoid with left identity. By Lemma 2.4, we have $(aS \cup aS] = ((aS \cup aS]]$. Then

$$\begin{aligned} (aS \cup Sa]S &= (aS \cup Sa)(S] \\ &\subseteq ((aS \cup Sa)S] \\ &= ((aS)S \cup (Sa)S] \\ &= ((SS)a \cup (Sa)(SS)] \\ &= (Sa \cup (SS)(aS)] \\ &= (Sa \cup S(aS)] \\ &= (Sa \cup a(SS)] \\ &= (Sa \cup aS]. \end{aligned}$$

Therefore $(Sa \cup aS]$ is a right ideal of S. By Lemma 2.7, we have $(Sa \cup aS]$ is an ideal of S. □

Lemma 3.5. *If $(S, \cdot, \leq)$ is an ordered AG-groupoid with left identity and let $a \in S$, then $(a \cup Sa \cup aS]$ is an ideal of S.*

Proof. Assume that $(S, \cdot, \leq)$ is an ordered AG-groupoid with left identity. By Lemma 2.4, we have $(a \cup Sa \cup aS] = ((a \cup Sa \cup aS]]$. Then

$$\begin{aligned}
(a \cup Sa \cup aS]S &= (a \cup Sa \cup aS](S] \\
&\subseteq ((a \cup Sa \cup aS)S] \\
&= (aS \cup (Sa)S \cup (aS)S] \\
&= (aS \cup (Sa)(SS) \cup (SS)a] \\
&= (aS \cup (SS)(aS) \cup Sa] \\
&= (aS \cup S(aS) \cup Sa] \\
&= (aS \cup a(SS) \cup Sa] \\
&= (Sa \cup aS \cup Sa] \\
&= (Sa \cup aS] \\
&\subseteq (a \cup Sa \cup aS].
\end{aligned}$$

Therefore $(a \cup Sa \cup aS]$ is a right ideal of S. By Lemma 2.7, we have $(a \cup Sa \cup aS]$ is an ideal of S. □

Proposition 3.6. *If $(S, \cdot, \leq)$ is an ordered AG-groupoid with left identity and let $a \in S$, then $(a^2 \cup a^2S]$ is an ideal of S.*

Proof. Assume that $(S, \cdot, \leq)$ is an ordered AG-groupoid with left identity. By Lemma 2.4, we have $(a^2 \cup a^2S] = ((a^2 \cup a^2S]]$. Then

$$\begin{aligned}
(a^2 \cup a^2S]S &= (a^2 \cup a^2S](S] \\
&\subseteq ((a^2 \cup a^2S)S] \\
&= (a^2S \cup (a^2S)S] \\
&= (a^2(SS) \cup (SS)a^2] \\
&= (S(a^2S) \cup Sa^2] \\
&= (S((aa)S) \cup Sa^2] \\
&= (S((Sa)a) \cup Sa^2] \\
&= ((Sa)(Sa) \cup Sa^2] \\
&= ((SS)(aa) \cup Sa^2] \\
&= (Sa^2 \cup Sa^2] \\
&= (Sa^2] \\
&\subseteq (a^2 \cup a^2S].
\end{aligned}$$

Therefore $(a^2 \cup a^2S]$ is a right ideal of S. By Lemma 2.7, we have $(a^2 \cup a^2S]$ is an ideal of S. □

Corollary 3.7. *If $(S, \cdot, \leq)$ is an ordered AG-groupoid with left identity and let $a \in S$, then $(a^2 \cup Sa^2]$ is an ideal of S.*

Proof. By Proposition 3.6. □

4. Prime and Semiprime Ideals in Ordered AG-Groupoids

We start with the following theorem that gives a relation between semiprime and quasi-semiprime ideal in ordered AG-groupoid. Our starting points are the following definitions:

Definition 4.1. Let $(S, \cdot, \leq)$ be an ordered AG-groupoid. An ideal P of S is called semiprime if every ideal A of S such that $AA \subseteq P$, then $A \subseteq P$. A left ideal P of S is called quasi-semiprime if every left ideal A of S such that $AA \subseteq P$, then $A \subseteq P$.

It is easy to see that every quasi-semiprime ideal is semiprime.

Lemma 4.2. *Let $(S, \cdot, \leq)$ be an ordered AG-groupoid with left identity. Then an ideal P of S is quasi-semiprime if and only if $a^2 \in P$ implies that $a \in P$, where $a \in S$.*

Proof. $(\Rightarrow)$ Assume that $(S, \cdot, \leq)$ is an ordered AG-groupoid with left identity. Then by hypothesis, we get $a^2 \in P$, for any $a \in S$. Then

$$\begin{aligned}
(a \cup Sa](a \cup Sa] &\subseteq ((a \cup Sa)(a \cup Sa)] \\
&= (a(a \cup Sa) \cup (Sa)(a \cup Sa)] \\
&= (a^2 \cup a(Sa) \cup (Sa)a \cup (Sa)(Sa)] \\
&\subseteq (P \cup S(aa) \cup a^2 S \cup S((Sa)a)] \\
&\subseteq (P \cup SP \cup PS \cup S(a^2 S)] \\
&= (P \cup P \cup P \cup S(PS)] \\
&\subseteq (P \cup SP] \\
&\subseteq (P] \\
&= P.
\end{aligned}$$

By Definition 4.1, we get $a \in (a \cup Sa] \subseteq P$ so that $a \in P$.
$(\Leftarrow)$ Assume that if $a^2 \in P$, then $a \in P$, where $a \in S$. Suppose that $AA \subseteq P$, where A is a left ideal of S such that $A \nsubseteq P$. Then there exists $a \in A$ such that $a \notin P$. Now $aa \in AA \subseteq P$, for all $a \in A$. So by hypothesis, we get $a \in P$ which is a contradiction. Hence P is quasi-semiprime ideal in S. □

Definition 4.3. Let $(S, \cdot, \leq)$ be an ordered AG-groupoid, $a \in S$ arbitrary element if $(aa)a = a(aa) = a$ we say that a is a 3-potent. Ordered AG-groupoid S is a 3-band (or ordered AG-3-band) if all of its elements are 3-potents.

Lemma 4.4. *Let $(S, \cdot, \leq)$ be an ordered AG-3-band with left identity. Then an ideal P of S is semiprime if and only if $a^2 \in P$ implies that $a \in P$, where $a \in S$.*

Proof. ($\Rightarrow$) Assume that $(S, \cdot, \leq)$ is an ordered AG-groupoid with left identity. Then by hypothesis, we get $a^2 \in P$, for any $a \in S$. Then

$$\begin{aligned}
(a \cup Sa \cup aS](a \cup Sa \cup aS] &\subseteq ((a \cup Sa \cup aS)(a \cup Sa \cup aS)] \\
&= (a(a \cup Sa \cup aS) \cup (Sa)(a \cup Sa \cup aS) \cup \\
&\quad (aS)(a \cup Sa \cup aS)] \\
&= (aa \cup a(Sa) \cup a(aS) \cup (Sa)a \cup (Sa)(Sa) \cup \\
&\quad (Sa)(aS) \cup (aS)a \cup (aS)(Sa) \cup (aS)(aS)] \\
&\subseteq (P \cup (Sa)(Sa) \cup (Sa)(aS) \cup (Sa)(Sa) \\
&\quad \cup (Sa)(Sa) \cup (Sa)(aS) \cup (aS)(Sa) \\
&\quad \cup (aS)(Sa) \cup (aS)(aS)] \\
&= (P \cup (Sa)(Sa) \cup (Sa)(aS) \\
&\quad \cup (aS)(Sa) \cup (aS)(aS)] \\
&= (P \cup ((Sa)a)S \cup ((aS)a)S \\
&\quad \cup S((aS)a) \cup ((aS)S)a] \\
&= (P \cup ((aa)S)S \cup ((aS)a)(SS) \\
&\quad \cup (SS)((aS)a) \cup ((SS)a)a] \\
&\subseteq (P \cup (PS)S \cup (SS)(a(aS)) \cup \\
&\quad (a(aS))(SS) \cup (Sa)a] \\
&\subseteq (P \cup PS \cup S((a(aa))(aS)) \cup \\
&\quad ((a(aa))(aS))S \cup (aa)S] \\
&\subseteq (P \cup P \cup S((aa)((aa)S)) \cup \\
&\quad ((aa)((aa)S))S \cup PS] \\
&\subseteq (P \cup S(P(PS)) \cup (P(PS))S \cup P] \\
&\subseteq (P \cup SP \cup PS] \\
&\subseteq (P] \\
&= P.
\end{aligned}$$

By Definition 4.1, we get $a \in (a \cup Sa \cup aS] \subseteq P$ and so that $a \in P$.
($\Leftarrow$) Assume that if $a^2 \in P$, then $a \in P$, where $a \in S$. Suppose that $AA \subseteq P$, where A is an ideal of S such that $A \nsubseteq P$. Then there exists $a \in A$ such that $a \notin P$. Now $aa \in AA \subseteq P$, for all $a \in A$. So by hypothesis, we get $a \in P$ which is a contradiction. Hence P is semiprime ideal in S. □

Lemma 4.5. *Let $(S, \cdot, \leq)$ be an ordered AG-3-band with left identity. Then an ideal P of S is semiprime if and only if P is quasi-semiprime.*

Proof. This follows from Lemma 4.2 and Lemma 4.4. □

Theorem 4.6. *Let $(S, \cdot, \leq)$ be an ordered AG-3-band with left identity and let P be an ideal of S. Then an ideal P of S is semiprime if and only if $(a(Sa)] \subseteq P$ implies that $a \in P$, where $a \in S$.*

Proof. ($\Rightarrow$) Assume that $(S,\cdot,\leq)$ is an ordered AG-groupoid with left identity. Then

$$\begin{aligned} aa \in (Sa)(Sa) &= ((Sa)a)S \\ &= ((Sa)a)(SS) \\ &= (SS)(a(Sa)) \\ &= S(a(Sa)) \\ &\subseteq S(a(Sa))] \\ &\subseteq SP \\ &\subseteq P. \end{aligned}$$

By Lemma 4.4, we have $a \in P$.
($\Leftarrow$) Assume that if $(a(Sa)] \subseteq P$, then $a \in P$, where $a \in S$. Suppose that $AA \subseteq P$, where A is an ideal of S such that $A \nsubseteq P$. Then there exists $a \in A$ such that $a \notin P$. Now

$$aa = (a(ea)) \in A(SA) \subseteq AA \subseteq P,$$

for all $a \in A$. So by hypothesis, we get $a \in P$ which is a contradiction. Hence P is semiprime ideal in S. □

Definition 4.7. Let $(S,\cdot,\leq)$ be an ordered AG-groupoid. An ideal P of S is called prime if every ideals A, B of S such that $AB \subseteq P$, then $A \subseteq P$ or $B \subseteq P$. A left ideal P of S is called quasi-prime if every left ideals A, B of S such that $AB \subseteq P$, we have $A \subseteq P$ or $B \subseteq P$.

It is easy to see that every quasi-prime ideal is prime.

Lemma 4.8. *Let $(S,\cdot,\leq)$ be an ordered AG-groupoid with left identity. Then an ideal P of S is quasi-prime if and only if $ab \in P$ implies that $a \in P$ or $b \in P$, where $a, b \in S$.*

Proof. ($\Rightarrow$) Assume that $(S,\cdot,\leq)$ is an ordered AG-groupoid with left identity. Then by hypothesis, we get $ab \in P$, for any $a, b \in S$. Then

$$\begin{aligned} (a \cup Sa](b \cup Sb] &\subseteq ((a \cup Sa)(b \cup Sb)] \\ &= (a(b \cup Sb) \cup (Sa)(b \cup Sb)] \\ &= (ab \cup a(Sb) \cup (Sa)b \cup (Sa)(Sb)] \\ &\subseteq (P \cup S(ab) \cup (ba)S \cup (SS)(ab)] \\ &\subseteq (P \cup SP \cup (ba)(SS) \cup (SS)P] \\ &\subseteq (P \cup P \cup (SS)(ab) \cup SP] \\ &\subseteq (P \cup SP \cup P] \\ &\subseteq (P \cup P] \\ &\subseteq (P] \\ &= P. \end{aligned}$$

By Definition 4.7, we get $a(a \cup Sa] \subseteq P$ or $b \in (b \cup Sb] \subseteq P$ so that $a \in P$ or $b \in P$.
($\Leftarrow$) Assume that if $ab \in P$, then $a \in P$ or $b \in P$, where $a, b \in S$. Suppose

that $AB \subseteq P$, where A and B are left ideals of S such that $A \nsubseteq P$. Then there exists $a \in A$ such that $a \notin P$. Now $ab \in AB \subseteq P$, for all $b \in B$. So by hypothesis, we get $b \in P$, for all $b \in B$ implies that $B \subseteq P$. Hence P is quasi prime ideal in S. □

Lemma 4.9. *Let $(S, \cdot, \leq)$ be an ordered AG-3-band with left identity. Then an ideal P of S is prime if and only if $ab \in P$ implies that $a \in P$ or $b \in P$, where $a, b \in S$.*

Proof. $(\Rightarrow)$ Assume that $(S, \cdot, \leq)$ is an ordered AG-groupoid with left identity. Then by hypothesis, we get $ab \in P$, for any $a, b \in S$. Then

$$\begin{aligned}
(a \cup Sa \cup aS](b \cup Sb \cup bS] &\subseteq ((a \cup Sa \cup aS)(b \cup Sb \cup bS)] \\
&= (a(b \cup Sb \cup bS) \cup (Sa)(b \cup Sb \cup bS) \cup \\
&\quad (aS)(b \cup Sb \cup bS)] \\
&= (ab \cup a(Sb) \cup a(bS) \cup (Sa)b \cup (Sa)(Sb) \cup \\
&\quad (Sa)(bS) \cup (aS)b \cup (aS)(Sb) \cup (aS)(bS)] \\
&\subseteq (P \cup (Sa)(Sb) \cup (Sa)(bS) \cup (Sa)(Sb) \\
&\quad \cup (Sa)(Sb) \cup (Sa)(bS) \cup (aS)(Sb) \\
&\quad \cup (aS)(Sb) \cup (aS)(bS)] \\
&= (P \cup (Sa)(Sb) \cup (Sa)(bS) \\
&\quad \cup (aS)(Sb) \cup (aS)(bS)] \\
&= (P \cup (SS)(ab) \cup ((bS)a)S \\
&\quad \cup S((bS)a) \cup (ab)(SS)] \\
&\subseteq (P \cup SP \cup ((bS)a)(SS) \cup (SS)((bS)a) \cup PS] \\
&\subseteq (P \cup P \cup (SS)(a(bS)) \cup (a(bS))(SS) \cup P] \\
&= (P \cup S((a(aa))(bS)) \cup ((a(aa))(b)S] \\
&= (P \cup S((ab)((aa)S)) \cup ((ab)((aa)S))S] \\
&= (P \cup S((a((bb)b))((aa)S)) \cup ((a((bb)b))((aa)S))S] \\
&= (P \cup S(((bb)(ab))((aa)S)) \cup (((bb)(ab))((aa)S))S] \\
&= (P \cup S(((bb)(aa))((ab)S)) \cup (((bb)(aa))((ab)S))S] \\
&= (P \cup S(((aa)(bb))((ab)S)) \cup (((aa)(bb))((ab)S))S] \\
&= (P \cup S(((ab)(ab))((ab)S)) \cup (((ab)(ab))((ab)S))S] \\
&\subseteq (P \cup S((PP)(PS)) \cup S((PP)(PS))S] \\
&\subseteq (P \cup P \cup P] \\
&= (P] \\
&= P.
\end{aligned}$$

By Definition 4.7, we get $a \in (a \cup Sa \cup aS] \subseteq P$ or $a \in (a \cup Sb \cup bS] \subseteq P$ so that $a \in P$ or $b \in P$.
$(\Leftarrow)$ It is obvious. □

Lemma 4.10. *Let $(S, \cdot, \leq)$ be an ordered AG-3-band with left identity. Then an ideal P of S is prime if and only if P is quasi-prime.*

Proof. This follows from Lemma 4.8 and Lemma 4.9. □

Theorem 4.11. *Let* $(S, \cdot, \leq)$ *be an ordered AG-3-band with left identity and let* P *be an ideal of* S. *Then an ideal* P *of* S *is semiprime if and only if* $(a(Sb)] \subseteq P$ *implies that* $a \in P$ *or* $b \in P$, *where* $a, b \in S$.

Proof. $(\Rightarrow)$ Assume that $(S, \cdot, \leq)$ is an ordered AG-groupoid with left identity. Then

$$\begin{aligned} ab \in (Sa)(Sb) &= ((Sb)a)S \\ &= ((Sb)a)(SS) \\ &= (SS)(a(Sb)) \\ &= S(a(Sb)) \\ &\subseteq S(a(Sb))] \\ &\subseteq SP \\ &\subseteq P. \end{aligned}$$

By Lemma 4.9, we have $a \in P$ or $b \in P$.
$(\Leftarrow)$ It is obvious. □

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