An Extended Mathematical Programming Model to Optimize the Cable Trench Route of Power Transmission in a Metro Depot

Amin Jamili[1], Farshad Ramezankhani [2]
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Abstract
The necessary electricity of the workshops and buildings (W&Bs) located at the metro depot are provided by lighting power substation (LPS). To transmit electricity between LPS and W&Bs, some trenches should be dig and the requisite cables should be located in the trenches. This paper presents a new mixed integer linear programming (MILP) long-term decision model to find the best cable trench route between LPS and W&Bs and also the location of all W&Bs and LPS are fixed. In this problem, the objective is to minimize (1) used cables cost, and (2) trench digging cost. It should be considered that there exist many cases in which the minimum either used cables or trench digging does not result in minimum total cost. Therefore, in optimum solution, a tradeoff between these objectives should be achieved. Finally, the proposed model is applied to a real case study at the metro depot in Iran and the optimum solution is analyzed.

Keywords: Cable trench problem, lighting power substation, metro depot, optimization

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Corresponding author E-mail: a_jamili@ut.ac.ir
1. Assistant Professor, School of Industrial Engineering, University of Tehran, Tehran, Iran
2. MSc. Student, School of Industrial Engineering, University of Tehran, Tehran, Iran

1. Introduction

An LPS is a part of an electrical transmission system on a metro depot which transforms the electricity from high voltage to low voltage (20 kV to 400 V). The electricity enters from the related route cable loop and supplies the required electricity of the W&Bs located at the metro depot. In this system, there are two different cabling routes (1) between input power location (IPL) and LPS, and (2) between LPS and W&Bs. The outline of this system and the cabling routes are shown in Figure1. In fact, in the case that the total number of W&Bs and LPS is N, there exist one cable from IPL to LPS and N-1 cables which leave the LPS toward W&Bs. It is necessary to mention that, each W&Bs requires one cable which comes from LPS.

Power transmission to depot requires two main activities including cabling and trench digging. Both cabling and trench digging impose substantial costs to the system. It should be noted that the trenches can be used for one or more than one cable. So, it is rational to find the best route for trench digging and cabling to use the trenches for more than one cable simultaneously.

Cable trench problem (CTP) is to find the best routes for trenching and cabling whit respect to the per unit cable cost (*cc*) and the per unit trench cost (*tc*). An application of this problem is to determine the connections between W&Bs and IPL with the LPS at a metro depot. The *cc* and the *tc* as two important influencing parameters are specific. It is assumed that at the first stage the location of IPL, LPS and W&Bs are fixed disregarding the possible effects on the cable trenching costs. In the second stage and by considering the outputs of the first stage as the input data of the second stage, the cable trenching is optimized by the proposed model in this paper. Therefore, according to the fixed location of the IPL, LPS and W&Bs, all distances between (1) IPL and LPS, (2) LPS and W&Bs, and (3) W&Bs with each other are determined. With respect to these input parameters, the best cable trench route is achieved by the proposed mathematical programming model presented in section 3. In this paper, it is assumed that the voltage drop on AC network is computed after finding the optimum trench cabling routes. In the case that the voltage drop is more than permissible levels, the size of the cables are increased. Similarly, the headway affects the size of the cables as well. In other words as the headway reduces the size of cables will be increased.

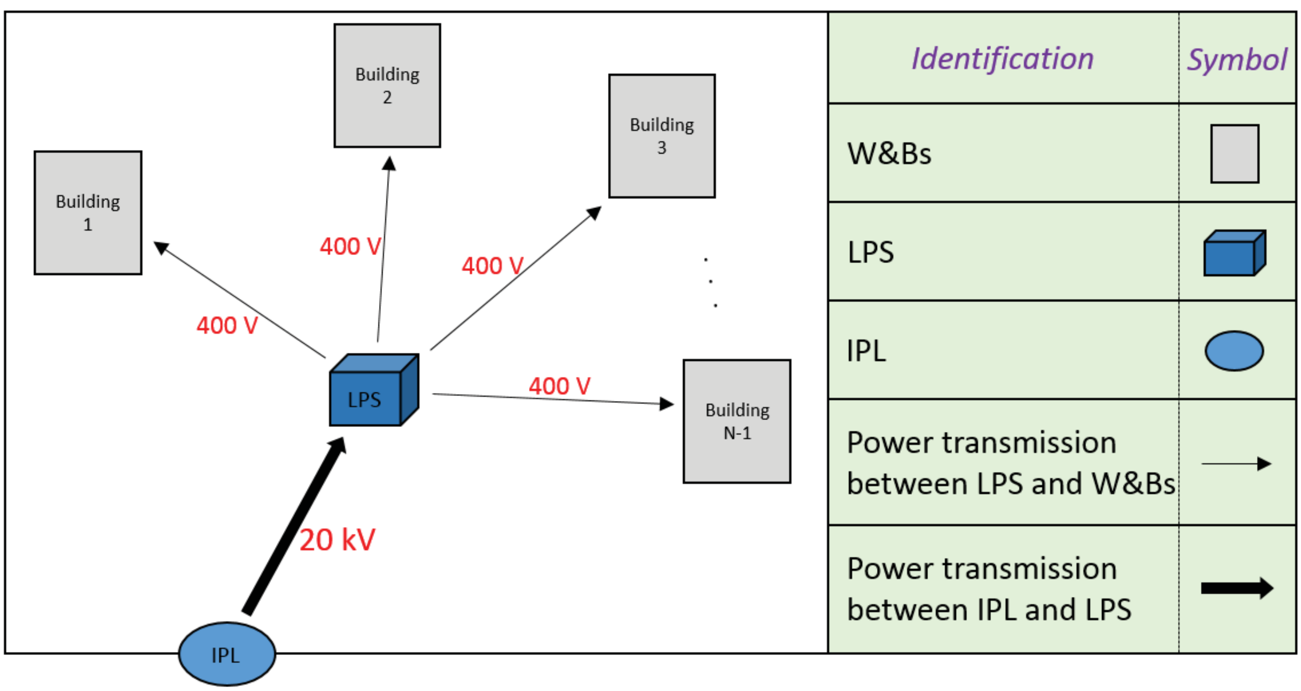

Figure 1. The outline of general metro depot

2. Literature Review

The CTP is considered as a combination of the shortest path (SP) and the minimum spanning tree (MST) problems. Also, both the SP and the MST problems are encompassed some subjects such as operational research and mathematical programming, integer programming, MILP, discrete mathematics, graph theory, combinatorial optimization and analysis of algorithms [Vasko et al, 2002][Jen g, Kim, and Watada, 2006]. MSTs have long been used in pattern recognition, machine learning and data mining [Zhong et al, 2015].

In this sake, Ralphs and Hartman [Ralphs and Hartman, 2001] presented several alternative integer programming formulation of the capacitated node routing problem. Capacitated node routing problem describes a branch, cut, and price algorithm for the solution of a class of network design problems with routing and packing constraints that can be shown to generalize a number of other important combinatorial models, including the vehicle routing problem (VRP), the capacitated spanning tree problem (CSTP), and the CTP. Marianov et al [Marianov et al, 2012] formulated an integer programming model of the p-cable-trench problem using multi-commodity flows that allows finding the solution to minimize the digging cost of the trenches, as well as the sum of the cable lengths between the customers and their assigned facilities for instances of up to 200 nodes. A linear algorithm for analyzing the MST and SP trees of planar graph was presented in a study by Booth and Westbrook [Booth and Westbrook, 1994]. Eppstein [Eppstein, 1999] surveyed results in geometric network design theory that include algorithms for constructing low-dilation graphs and MSTs. Verification under uncertainty for the MST problems for undirected weighted graphs, where each edge is associated with an uncertainty area and an assumed edge weight were considered in Erlebach and Hoffmann's work [Erlebach and Hoffmann, 2014].

In addition, Gouveia et al [Gouveia, Leitner, and Ljubi, 2014] provided a study of integer linear programming formulations for the diameter constrained MST problem in the natural space of edge design variables. Pérez-galarce et al [Pérez-galarce et al, 2014] considered uncertainty in the cost function of the MST problems. An SP tree based algorithm for designing relay placement in a wireless sensor network was proposed [Bhattacharya and Kumar, 2014]. Khuller et al [Khuller, Raghavachari, and Young, 1995] gave a simple algorithm to find a spanning tree that simultaneously approximates an SP tree and an MST. The NP-hard problem of approximating a minimum routing cost spanning tree in the message passing model with limited bandwidth that find an SP tree of a graph G over n nodes that minimizes the sum of distances between all vertices was presented in study of Hochuli et al [Hochuli, Holzer, and Wattenhofer, 2014]. Application of MST algorithm for network reduction of distribution systems was proposed in Nagarajan and Ayyanar's work [Nagarajan and Ayyanar, 2014]. Arkin et al [Arkin et al, 2014] gave approximation algorithm for minimizing the number of charging stations to facilitate almost SP. A hybrid optimized weighted MST for the shortest intrapath selection in wireless sensor network which consists of sensor nodes and storage resources was proposed by Saravanan and Madheswaran [Saravanan and Madheswaran, 2014].

In this respect, Trudeau [Trudeau, 2014] gave an extension of MST problems where some agents do not need to be connected to the source, but might decrease the cost of others to do so. Zhou et al [Zhou, He and Wang, 2014] introduced a quadratic MST problem to find a spanning tree on a graph which minimizes a quadratic objective function of the edge weights. Baxter et al [Baxter et al, 2014] focused on an incremental network design problem with SP to investigate the optimal choice and timing of network expansions. Ren [Ren, 2014] used SP principle to locate humans in mine personnel RFID positioning system based for improving the production safety situation and providing guarantee for relief and disaster prevention. The spanning tree and position based routing algorithm which finds the best path from source to destination was presented [Kumar and Kumar, 2014]. Liu et al [Liu et al, 2014] proposed K shortest path algorithm from a source node to each other node. Ralphs et al [Ralphs, Saltzman, and Wiecek, 2004] proposed an algorithm for bi-objective integer programming and its application to network routing problems in which examining the tradeoff between the fixed and variable costs.

As can be seen above, there are so many studies in the topics of CTP, MST, and SP in different cases. All previous studies have considered that the trenches can be only dig between two locations which require electricity or other requirements, but in reality, the trenches can be branched in the midway of two locations. The concept of branching the cables in the midway of two W&Bs (or LPS and W&Bs) is presented as the main contribution of this paper which lead to save money. The location of the candidate places for branching are determined by virtual vertices which are obtained from intersection of all x-coordinates and all y-coordinates of real vertices. In fact, the real vertices are the location of the LPS and the W&Bs. According to circumstances of the problem, the proposed model might opt some of the virtual vertices as branches in the final cable trench route. The importance of this contribution is clarified in section 4.1 by introducing a hypothetical case study.

The remainder of the paper is organized as follows: Section 3 represents a mathematical programing to minimize the total cost of cabling and trenching. In section 4, reliability and validity of the proposed model are investigated by two subsection that are (1) introducing a hypothetical case study which compare the results of proposed model with the model that Vasko et al [Vasko et al. 2002] introduced and (2) sensitivity analysis on the solution and the number of used virtual vertices. In section 5, the proposed model is implemented on a real case study at the depot of an Iranian urban railway. Results and discussions presents in section 6. The paper ends with main findings in the conclusion and future direction section which recaps our conclusions.

3. An Extended Mathematical Programming Model

3.1 Problem Description

The area of the depot is depicted by imaginary Cartesian coordinate in Figure 2. The depot consists of some real vertices (location of LPS and W&Bs) and virtual vertices (location of the candidate places for branching). As mentioned in the previous section, virtual vertices are obtained from intersection of all x-coordinates and all y-coordinates of real vertices. Figure 2 is to illustrate the visual description for a better understanding of intersecting. In Figure 2, the coordinates of real vertices are specified by $A_1=(x_1, y_1)$, $A_2=(x_2, y_2)$, $A_3=(x_3, y_3)$ and $A_4=(x_4, y_4)$. By intersecting of all x-coordinates and all y-coordinates of real vertices, the virtual vertices are generated which are specified by $a_1=(x_1, y_3)$, $a_2=(x_1, y_4)$, $a_3=(x_1, y_2)$, $a_4=(x_2, y_3)$, $a_5=(x_2, y_1)$, $a_6=(x_2, y_4)$, $a_7=(x_3, y_1)$, $a_8=(x_3, y_4)$, $a_9=(x_3, y_2)$, $a_{10}=(x_4, y_3)$, $a_{11}=(x_4, y_1)$ and $a_{12}=(x_4, y_2)$.

3.2 Assumptions

Some assumptions are provided to extend the mathematical model as follows:

1. The location of all W&Bs and LPS are fixed.
2. LPS is located at real vertex 1 and other W&Bs are located at real vertex 2 to the last real vertex.
3. To specify the route of trenching and cabling, four kinds of connections between the vertices are defined as: connection between (1) real vertex and real vertex, (2) real vertex and virtual vertex, (3) virtual vertex and real vertex, and (4) virtual vertex and virtual vertex.
4. All the distances between (1) real vertex and real vertex, (2) real vertex and virtual vertex, (3) virtual vertex and real vertex, (4) virtual vertex and virtual vertex, and (5) LPS and IPL are Euclidean and deterministic.
5. IPL and LPS connected by a straight trench belong its 20 kV cable. Also, LPS and W&Bs connected by 400 V cables.
6. The *cc*, *tc*, and input power cable cost (*ipcc*) are deterministic.
7. The number of real vertices and virtual vertices are specified.
8. Each W&Bs requires one cable to receive requisite electricity.
9. LPS supplies the required electricity of the W&Bs.
10. Trenches can be used for one or more than one cable.

3.3 Notations

Before bring forward to model, integer variables, binary variables, parameters of the proposed model are introduced as follows:

Indices

$i, j \in R$ real vertices $R = 1, 2, \ldots, nr$

$v, w \epsilon\ V$ virtual vertices $V = 1, 2, \ldots, nv$

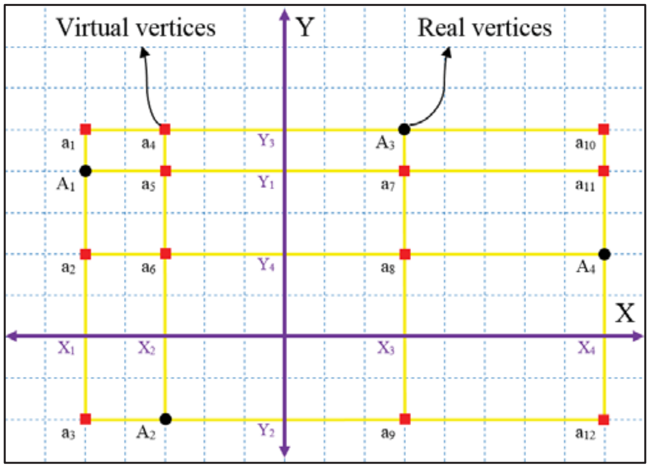

Figure 2. Real vertices and virtual vertices

Parameters

drr_{ij} the distance from real vertex i to real vertex j

drv_{iv} the distance from real vertex i to virtual vertex v

dvr_{vj} the distance from virtual vertex v to real vertex j

dvv_{vw} the distance from virtual vertex v to virtual vertex w

$dlip$ the distance between LPS and input power vertex

cc the per unit cable cost

tc the per unit trench cost

$ipcc$ the per unit input power cable cost

nr the number of real vertices (LPS and W&Bs)

nv the number of virtual vertices

Integer Variables

xrr_{ij} the number of cables from real vertex i to real vertex j

xrv_{iv} the number of cables from real vertex i to virtual vertex v

xvr_{vj} the number of cables from virtual vertex v to real vertex j

xvv_{vw} he number of cables from virtual vertex v to virtual vertex w

Binary Variables

yrr_{ij} 1 if a trench is dug between real vertex i and real vertex j

yrv_{iv} 1 if a trench is dug between real vertex i and virtual vertex v

yvr_{vj} 1 if a trench is dug between virtual vertex v and real vertex j

yvv_{vw} 1 if a trench is dug between virtual vertex v and virtual vertex w

3.4 Proposed Approach

In this section, a new MILP model is developed to optimize the cable trench route by considering the branching at the virtual vertices.

$$\begin{aligned} Min\ \ Z = & \ cc\sum_{j=1}^{nr}\sum_{i=1}^{nr} drr_{ij}\, xrr_{ij} + tc\sum_{j=1}^{nr}\sum_{i=1}^{nr} drr_{ij}\, yrr_{ij} \\ & + cc\sum_{v=1}^{nv}\sum_{i=1}^{nr} drv_{iv}\, xrv_{iv} + tc\sum_{v=1}^{nv}\sum_{i=1}^{nr} drv_{iv}\, yrv_{iv} \\ & + cc\sum_{v=1}^{nv}\sum_{i=1}^{nr} dvr_{vj}\, xvr_{vj} + tc\sum_{v=1}^{nv}\sum_{i=1}^{nr} dvr_{vj}\, yvr_{vj} \end{aligned} \quad (1)$$

$$+cc\sum_{w=1}^{nv}\sum_{v=1}^{nv} dvv_{vw}\,xvv_{vw} + tc\sum_{w=1}^{nv}\sum_{v=1}^{nv} dvv_{vw}\,yvv_{vw} + dlip(ipcc + tc)$$

Subject to:

$$\sum_{j=2}^{nr} xrr_{1j} + \sum_{v=1}^{nv} xrv_{1v} = nr - 1 \qquad (2)$$

$$\sum_{i<j} yrr_{ij} = nr - 1 \qquad \forall i, j \in R \qquad (3)$$

$$\sum_{j=1}^{nr} xrr_{ij} + \sum_{v=1}^{nv} xrv_{iv} = \sum_{j=1}^{nr} xrr_{ji} + \sum_{v=1}^{nv} xvr_{vi} - 1 \qquad \forall i = 2,3,\ldots,nr \qquad (4)$$

$$\sum_{i=1}^{nr} xrv_{iv} + \sum_{w=1}^{nv} xvv_{wv} = \sum_{j=1}^{nr} xvr_{vj} + \sum_{w=1}^{nv} xvv_{vw} \qquad \forall v \in V \qquad (5)$$

$$(nr-1)yrr_{ij} \geq xrr_{ij} + xrr_{ji} \qquad \forall i, j \in R \qquad (6)$$

$$(nr-1)yrv_{iv} \geq xrv_{iv} \qquad \forall i \in R, v \in V \qquad (7)$$

$$(nr-1)yvr_{vj} \geq xvr_{vj} \qquad \forall j \in R, v \in V \qquad (8)$$

$$(nr-1)yvv_{vw} \geq xvv_{vw} \qquad \forall v, w \in V \qquad (9)$$

$$yvr_{iv} + yvr_{vi} \leq 1 \qquad \forall i \in R, v \in V \qquad (10)$$

$$yvv_{vw} + yvv_{wv} \leq 1 \qquad \forall v, w \in V \qquad (11)$$

$$xrr_{ij} \geq 0 \qquad \forall i, j \in R \qquad (12)$$

$$xrv_{iv} \geq 0 \qquad \forall i \in R, v \in V \qquad (13)$$

$$xvr_{vj} \geq 0 \qquad \forall j \in R, v \in V \qquad (14)$$

$$xvv_{vw} \geq 0 \qquad \forall v, w \in V \qquad (15)$$

$$yrr_{ij} \in \{0,1\} \qquad \forall i < j \in R \qquad (16)$$

$$yrv_{iv} \in \{0,1\} \qquad \forall i \in R, v \in V \qquad (17)$$

$$yvr_{vj} \in \{0,1\} \qquad \forall j \in R, v \in V \qquad (18)$$

$$yvv_{vw} \in \{0,1\} \qquad \forall v, w \in V \qquad (19)$$

Equation (1) represents the objective function and the remained ones, i.e. (2) - (19), are the constraints of the model.

The objective function shown in Eq. (1) consists of five parts and computes the total cost. The total cost includes used cables cost and trench digging cost between all four kinds of connections and between IPL and LPS. Eq. (2) ensures that *nr*-1 cables leave the LPS. On the one hand, *nr*-1 is the number of required cables which obtained from the number of W&Bs. On the other hand, it is the summation of the number of cables from real vertex 1 (LPS) to other real vertices and all virtual vertices. Eq. (3) ensures that *nr*-1 trenches must be exactly dig. Eq. (4) ensures that each of the *nr*-1 W&Bs use one cable. Moreover, it indicates that each of the *nr*-1 W&Bs are connected by exactly one cable. The left and right side of this equation respectively specifies the total number of cables that leaves from each real vertices and the total number of cables that enter to each real vertices minus 1. Eq. (5) ensures that it is not necessary to assign trenches as well as cables between virtual vertices. In the other words, input and output cables of each virtual vertices must be equal. Inequality (6) ensures that cables between two real vertices are not laid unless a trench between them is dug. Moreover, it is an upper bound for the number of cables that might be laid. The concepts of inequalities (7) - (9) are the same as inequality (6). Inequality (10) determines that for one connection between real and virtual vertices only zero or one trench must be calculated. Also, the concept of inequality (11) is the same as inequality (10). Inequalities (12) – (15) guarantee that the number of cables between vertices could not be negative. At the end, constraints (16) – (19) specify that trench variables must be either zero or one.

4. Reliability and Validity of the Proposed Model

4.1. Computational Results

In this section, a numerical example of the model presents to clarify the applicability of the model. We consider hypothetical case study to validate the effectiveness of the proposed model in comparison with the model proposed by Vasko et al (Vasko et al. 2002). In this case, the *tc* and the *cc* are considered 200 US Dollar (USD). Also, the number of W&Bs is considered 5. The layout of this hypothetical case study is shown in figure 3. The LPS and W&Bs are specified by black solid circle in Figure 3. By intersecting of all x-coordinates and all y-coordinates of real vertices, the virtual vertices are generated which are specified by red solid circle in Figure 3.

This case is solved by the proposed model using the General Algebraic Modeling System (GAMS) software packages. The optimum results of cabling and trenching of hypothetical case study are shown in Figure 4. Total cost of proposed model in this study equals Z= 25214.433 USD.

Moreover, this case is solved by the method proposed by Vasko et al [Vasko et al. 2002] as the results are shown in Figure 5. Total cost of method proposed by Vasko et al [Vasko et al. 2002] equals Z= 25343.013 USD. By comparing the results, it is obtained that the proposed model results better solution in comparison with Vasko et al [Vasko et al. 2002] method.

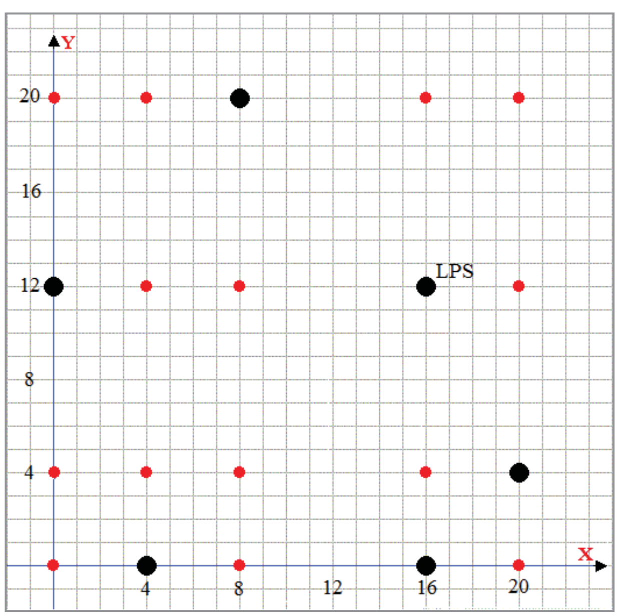

Figure 3. The position of real vertices and virtual vertices in the hypothetical case study

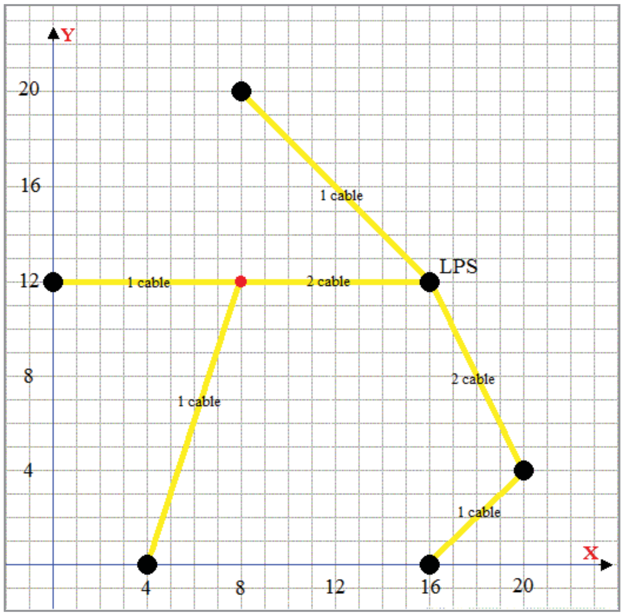

Figure 4. Results of cabling and trenching of the proposed model

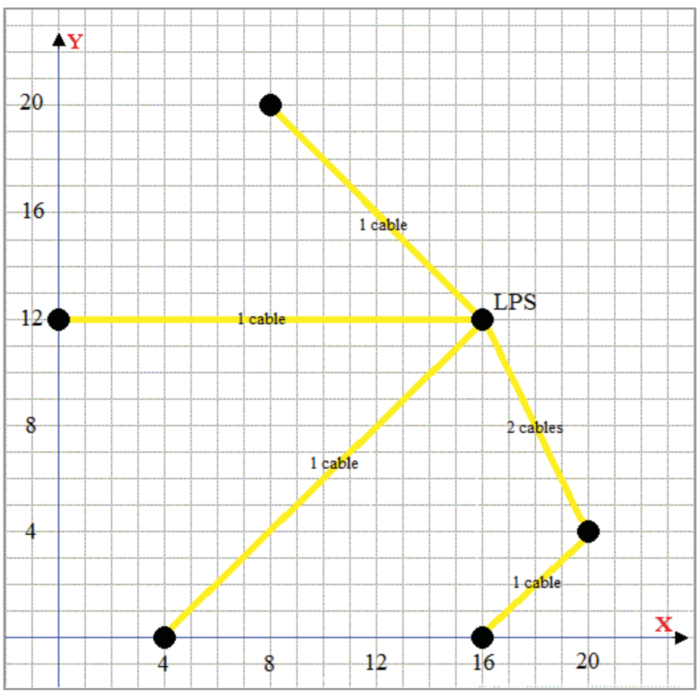

Figure 5. Results of cabling and trenching by the model proposed by Vasko et al [Vasko et al. 2002]

Sensitivity Analysis

In building the scenarios for a depot, we will analyze the impact on the solution and the number of used branches in three elements. The first element is the *cc*, the second element is the *tc*, and the third element is the *ipcc*. Due to (1) the input power cable only used between IPL and LPS, and (2) the IPL and LPS connected by a straight trench belong its cable, the number of used branches is not affected by *ipcc* changes. However, *cc* and *tc* are the two affecting elements on the number of the branches and *ipcc* is fixed. We look for the effect of setting *tc* equal to zero in the model. Note that if *tc* equals zero, the model contains only the *cc*. In that case, we focus solely on the components related to the cabling.

As it is clear in Table 1, whatever the differences between values of *cc* and *tc* be high, the number of branches became less and whatever the differences between values of *cc* and *tc* be low, the number of branches became more.
Finally, According to the two previous subsections, the reliability and validity of the proposed model in this study is proved.

5. Case Study

The depot of studied Iranian urban railway lines shown in Figure 6. This depot contains thirteen substantial W&Bs which require electricity (depicted in Figure 6 by 3-15) and two places (IPL and LPS) which exchange electricity among the W&Bs (depicted in Figure 6 by 1 and 2). To satisfy the demand of these W&Bs for electricity, required trenches must be dig among them. Each of the main areas (IPL, LPS and W&Bs) of the studied depot has not a specific coordination in the layout of Figure 6, but each of them has a specific point for entering power that shown in Figure 7. The coordination of main areas at the studied depot are shown in Table 2. The exact location of entering power for each W&Bs at the studied depot are separated to simplify the show of the exist vertices. Indeed, the vertices are shown in Figure 8 are the real vertices of the model. By intersecting of all x-coordinates and all y-coordinates of real vertices in Figure 8, the virtual vertices are generated which are specified by black solid circle in Figure. 9.

Table 1. The different scenarios for the studied depot

Scenario	The per unit trench cost (USD)	The per unit cable cost (USD)	The number of used branches	Scenario	The per unit trench cost (USD)	The per unit cable cost (USD)	The number of used branches
1	0	> 0	0	21	100	100	2
2	10	100000	0	22	200	200	2
3	10	40000	1	23	1000	1000	2
4	10	10000	2	24	12	10	7
5	10	1000	3	25	20	10	7
6	10	500	3	26	100	10	6
7	10	200	5	27	200	10	5
8	10	100	7	28	500	10	3
9	10	20	7	29	1000	10	2
10	10	12	7	30	10000	10	1
11	20	200000	0	31	100000	10	0
12	20	80000	1	32	24	20	7
13	20	20000	2	33	40	20	7
14	20	2000	3	34	200	20	6
15	20	1000	3	35	400	20	5
16	20	400	5	36	1000	20	3
17	20	200	7	37	2000	20	2
18	20	40	7	38	20000	20	1
19	20	24	7	39	100000	10	0
20	20	20	2	40	20000	20	0

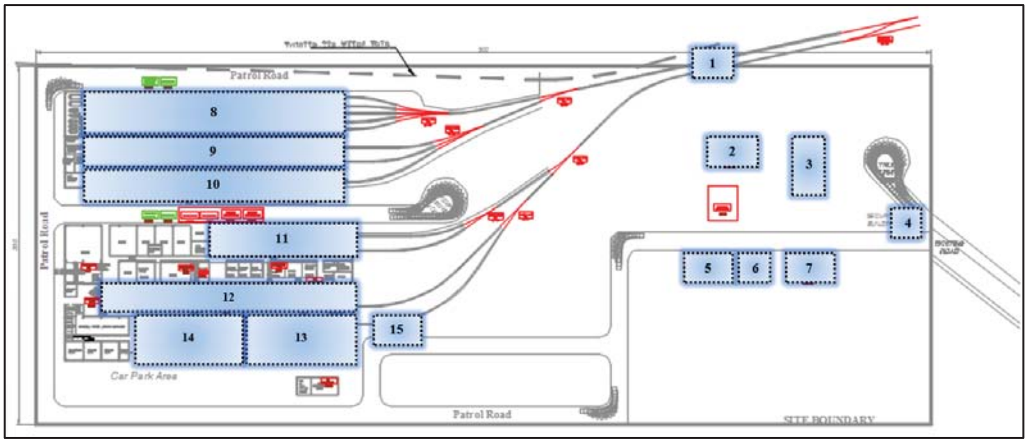

Figure 6. Depiction of W&Bs at the studied depot

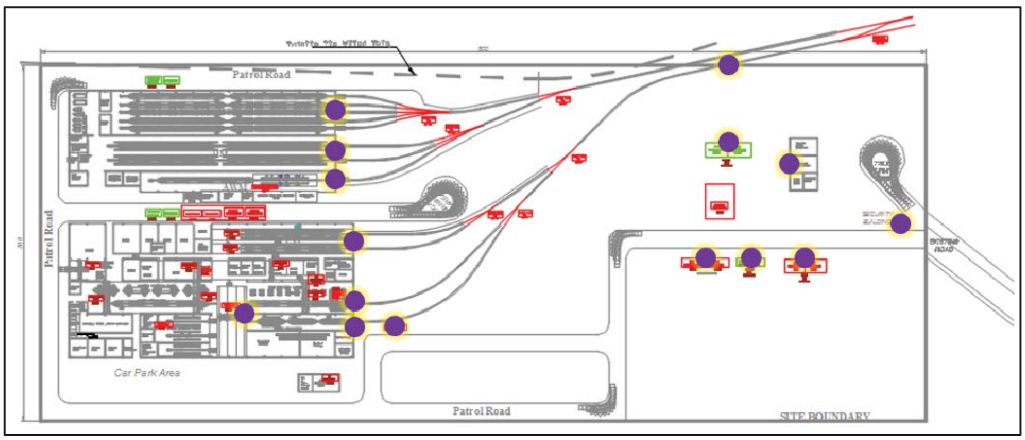

Figure 7. Specify the exact location of entering power for each W&Bs at the studied depot

Table 2. Coordination of main areas at the studied depot

Item	Area Identification	Main Area	Coordination (Meter)	
			x	y
1	IP	Input Power	388.36	201.00
2	LPS	Lighting Power Substation	388.36	157.60
3	TPSB	Traction Power Substation Building	423.31	150.01
4	SB	Security Building	483.50	113.68
5	UPSDL	Uninterruptible power supply Dairy Load	375.43	92.10
6	UPSCL	Uninterruptible power supply Clean Load	400.66	92.10
7	OCC	Operations Control Center	431.93	92.10
8	ST	Stabling	166.40	174.61
9	DM	Daily Maintenance	166.40	153.36
10	AWM	Automatic Washing Machine	166.40	136.16
11	LM	Light Maintenance	176.78	103.60
12	HM	Heavy Maintenance Hall	176.78	66.85
13	AV	Auxiliary Vehicles Stabling & workshop	176.78	56.40
14	PB	Painting Booth	115.98	61.42
15	FS	Fueling Station	193.72	56.40

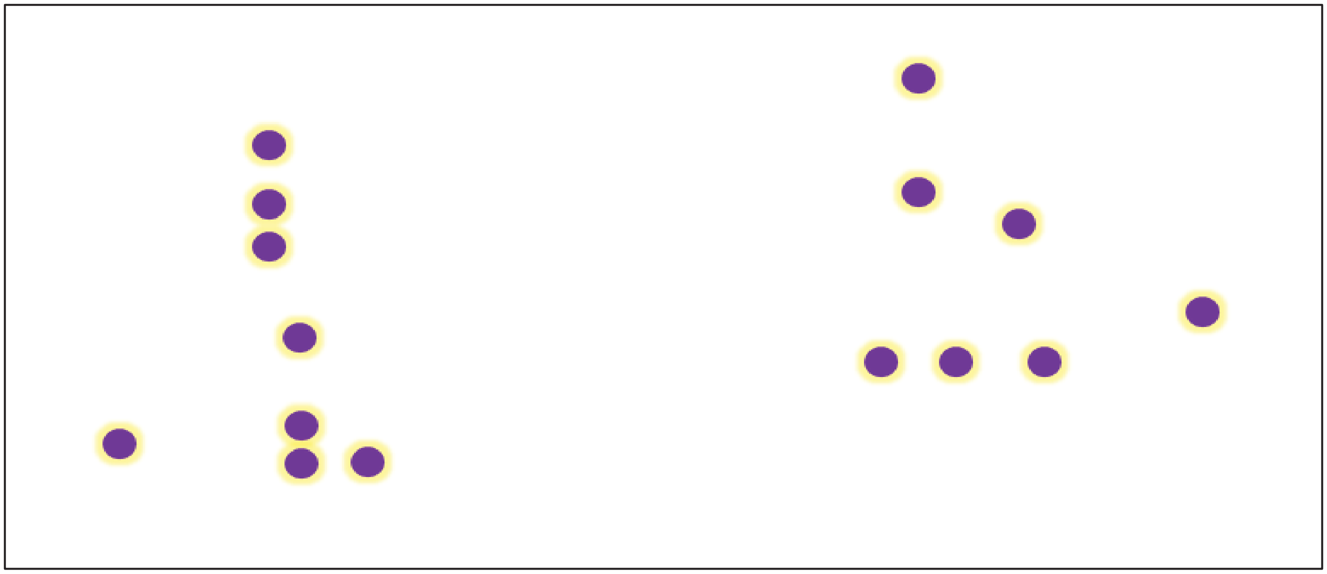

Figure 8. Separation of solid circles from the layout of studied depot

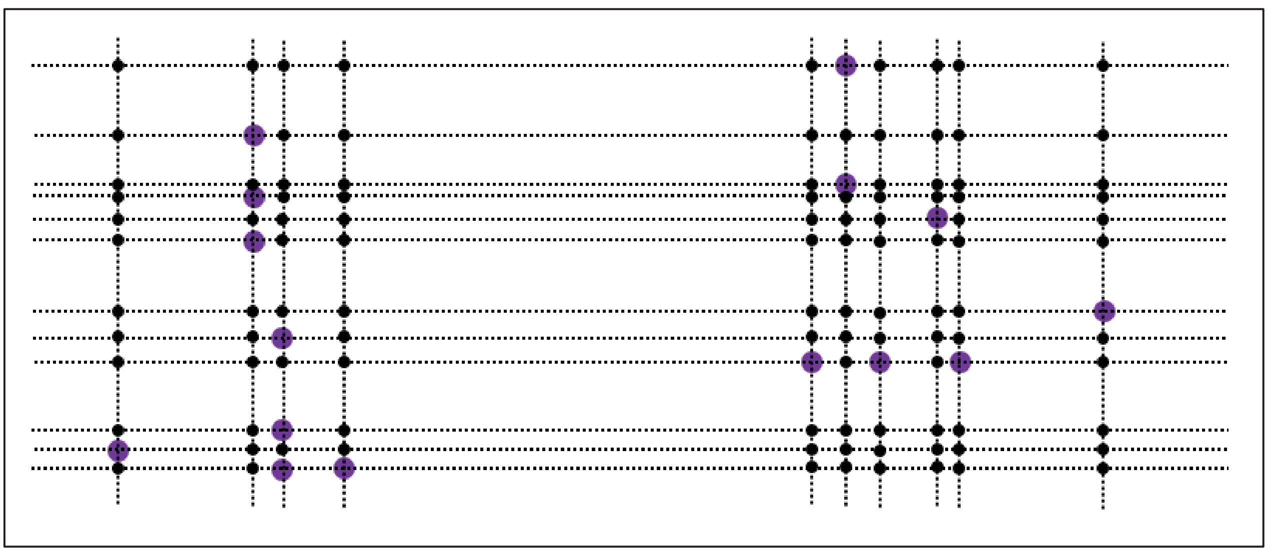

Figure 9. Specify the location of virtual vertices

In this case, all the distances between real to real (drr_{ij}), real to virtual (drv_{iv}), virtual to real (dvr_{vj}), and virtual to virtual (dvv_{vw}) vertices are calculated by the coordination's values of real and virtual vertices. The number of real vertices (*nr*) and the number of virtual vertices (*nv*) are respectively 14 and 96. Also, the *cc*, the *tc*, and the *ipcc* are respectively 55, 60, and 130 USD. This case is solved by the proposed model using the GAMS software packages. The optimum results of cabling and trenching of the depot of Iranian urban railway lines case study are shown in Figure 10. Total cost of proposed model in this study equals Z= 183,500 USD.

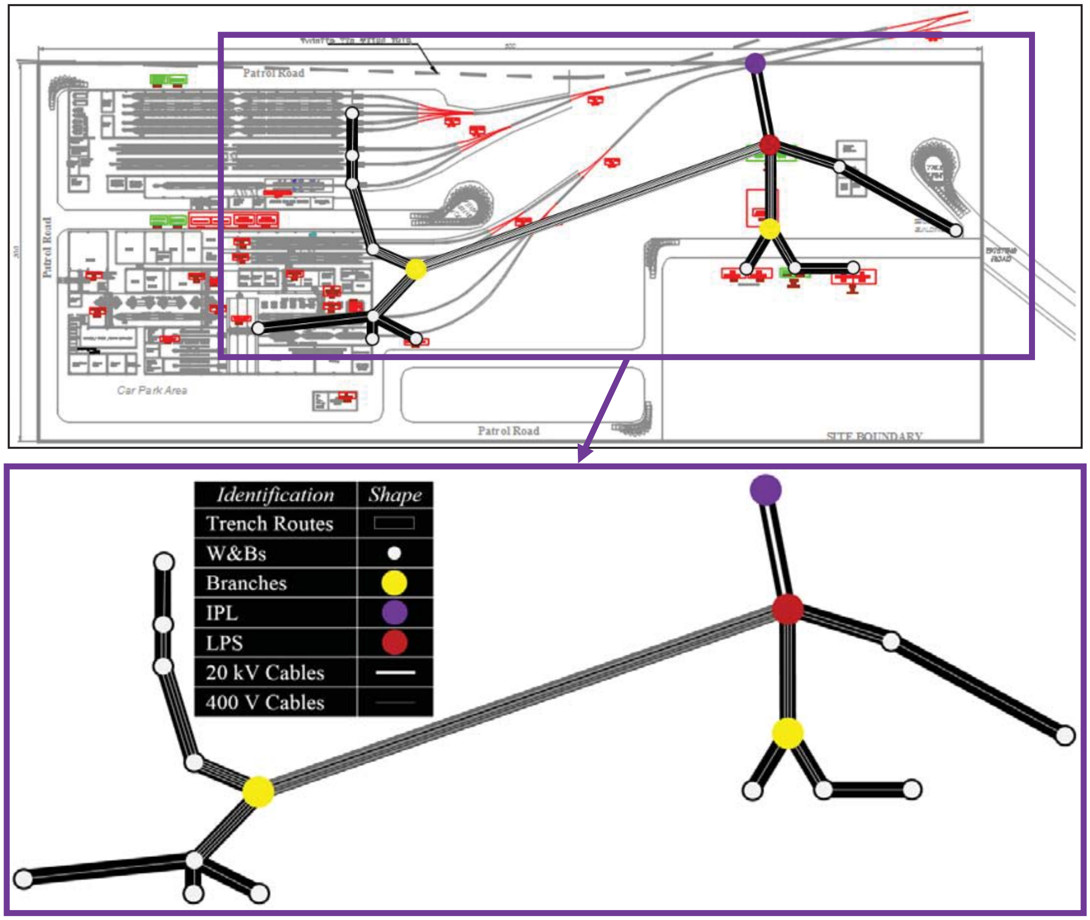

Figure 10. Results of cabling and trenching of the proposed model

Moreover, this case is solved by the method proposed by Vasko et al [Vasko et al. 2002] as the results are shown in Figure 11. Total cost of method proposed by Vasko et al [Vasko et al. 2002] equals Z= 183,812 USD. By comparing the results, it is obtained that the proposed model results better solution in comparison with Vasko et al [Vasko et al. 2002] method.

In this case, the difference between the total cost of proposed model and Vasko et al [Vasko et al. 2002] method is about 0.17%. Although, this difference is not considerable, but there are some cases in which the improvement of total cost is more considerable. To that end, some random examples are generated and analyzed. Table 3 depicts three hypothetical case studies in 6 different scenarios for each of them. Moreover, the coordination of W&Bs and LPS at three hypothetical case studies are illustrated in Table 4. The results show that the difference between total costs varies on different case studies. It is illusterated that our proposed model always generates equal or better solutions in comparison with the model introduced by Vasko et al (Vasko et al. 2002) for any different kind of cases.

According to the fact that the Vasko et al [Vasko et al. 2002] method is easier than the proposed method, but as the cable trench route of power transmission in a metro depot is a long-term decision making and is not a short-term or dynamic decision making, finding cost saving solutions is a more attractive criterion in comparison with the complexity or run time of the model.

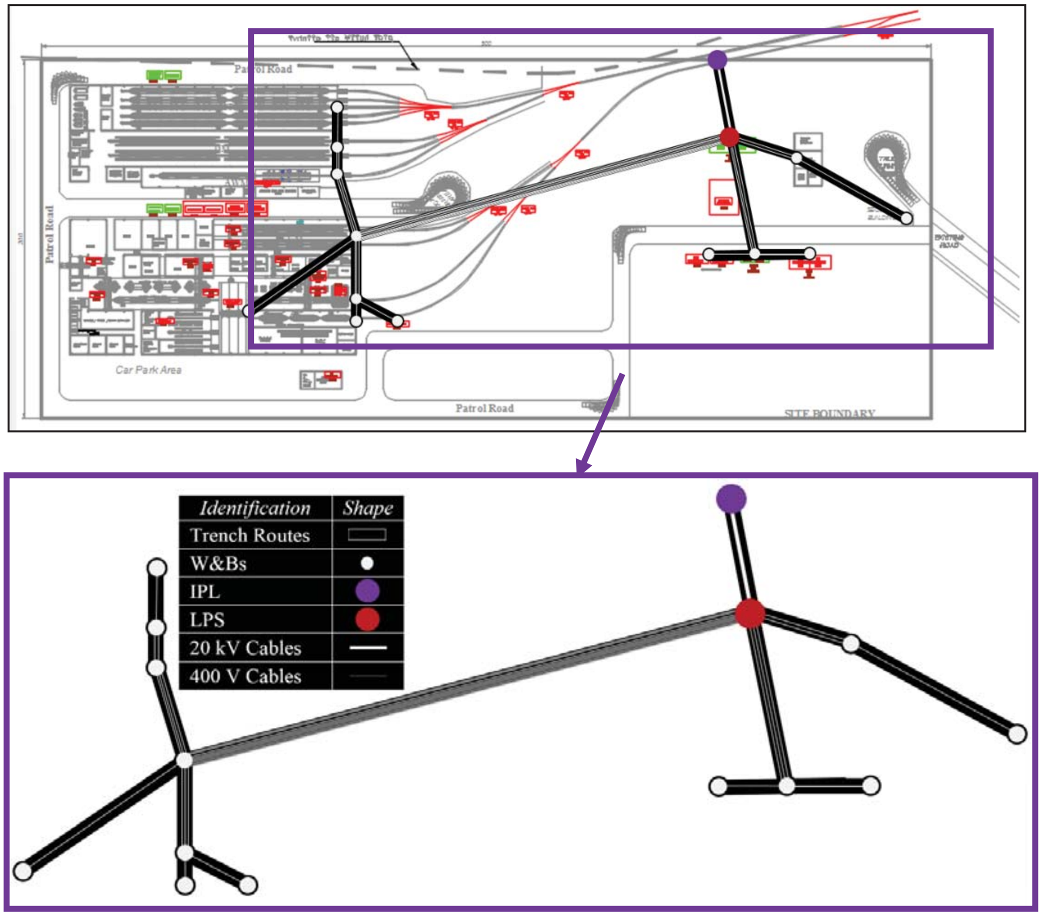

Figure 11. Results of cabling and trenching by the model proposed by [Vasko et al. 2002]

6. Results and Discussions

The concept of branching the cables in the midway of two W&Bs (or LPS and W&Bs) led to save money for cable trench problem. According to achieved values of Table 1, if the differences between values of *cc* and *tc* be very much, the proposed model became similar to the model proposed by Vasko et al [Vasko et al. 2002]. In some models which the values of *cc* and *tc* are almost the same, the proposed model in this study is more efficient than the model proposed by Vasko et al [Vasko et al. 2002].

6.1 Conclusion and Future Direction

In this article, a new MILP long-term decision model has been presented to find the best cable trench route between LPS and W&Bs at the metro depot. The LPS provides the necessary electricity of the W&Bs located at the metro depot. The location of all W&Bs and LPS have been considered fixed. In this problem, the proposed model has minimized (1) used cables cost, and (2) trench digging cost by finding the location of branches in the midway of trenches. The proposed model has been applied to the real case study at the metro depot in Iranian urban railway lines. Finally, the optimum solution has been analyzed to prove the reliability and validity of the proposed model.

In this paper it was assumed that the voltage drop on AC network is computed after finding the optimum trench cabling routes. As considering drop voltage limits in the model results in more cost saving solutions, it is proposed to consider it as the future works. The proposed model can be enhanced by considering the electric potential difference across the cables and amount of electricity demand by each W&Bs. Also, in addition to CTP, the proposed model can be used for Pipe Laying Problem (PLP), Sewer Trench Problem (STP), Drainage Installation Problem (DIP), and any problems that need trenching.

Table 3. Three hypothetical case studies in 6 different scenario

Hypothetical case study	Number of W&Bs	Scenario	The per unit trench cost (USD)	The per unit cable cost (USD)	Total cost of method proposed by Vasko et al (Vasko et al. 2002) (USD)	Total cost of proposed model (USD)	The difference between total cost of proposed and Vasko et al (Vasko et al. 2002) methods (percent %)
1	3	1	60	55	465463.5	455000	2.300
		2	100	20	408617	400000	2.154
		3	1000	10	3112781	3050000	2.058
		4	10	1000	4164331	4164331	0.000
		5	100	100	818465	800000	2.308
		6	10	10	81846.5	80000	2.308
2	3	1	60	55	1861858.87	1820000	2.300
		2	100	20	1634469	1600000	2.154
		3	1000	10	12451135.22	12200000	2.058
		4	10	1000	16657346.22	16657346.22	0.000
		5	100	100	3273863.3	3200000	2.308
		6	10	10	327386.33	320000	2.308
3	5	1	60	55	1861855.91	1820000	2.300
		2	100	20	1634469.58	1600000	2.154
		3	1000	10	12451135.49	12200000	2.058
		4	10	1000	16657346.75	16657346.75	0.000
		5	100	100	3273863.38	3200000	2.308
		6	10	10	327386.34	320000	2.308

Table 4. Co-ordination of W&Bs and LPS at three hypothetical case studies

Hypothetical case study	W&Bs and LPS of the hypothetical cases	Coordination (Meter)	
		x	y
1	LPS	2000	500
	W&Bs 1	0	0
	W&Bs 2	0	1000
2	LPS	8000	2000
	W&Bs 1	0	0
	W&Bs 2	0	4000
3	LPS	4000	2000
	W&Bs 1	0	0
	W&Bs 2	0	4000
	W&Bs 3	8000	0
	W&Bs 4	8000	4000

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