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FUZZY HYPERIDEALS IN TERNARY SEMIHYPERRINGS

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Abstract. In a ternary semihyperring, addition is a hyperoperation and multiplication is a ternary operation. Indeed, the notion of ternary semihyperrings is a generalization of semirings. Our main purpose of this paper is to introduce the notions of fuzzy hyperideal and fuzzy bi-hyperideal in ternary semihyperrings. We give some characterizations of fuzzy hyperideals and investigate several kinds of them.

1. **Introduction**

The theory of semiring was first developed by H. S. Vandiver and he has obtained important results of the objects. Semiring constitute a fairly natural generalization of rings, with board applications in the mathematical foundation of computer science. Also, semiring theory has many applications to other branches. For example, automata theory, optimization theory, algebra of formal process, combinatorial optimization, Baysian networks and belief propagation.

The idea of investigations of n-ary algebras, i.e., sets with one n-ary operation, seems to be going back to Kasner's lecture [35] at the 53rd annual meeting of the American Association of the Advancement of Science in 1904. But the first paper concerning the theory of n-ary groups was written (under inspiration of Emmy Noether) by Dörnte in 1928 (see [30]). Since then many papers concerning various n-ary algebras have appeared in the literature, for example see [19, 31, 50, 53]. Ternary semigroups are universal algebras with one associative operation. A ternary semigroup is a particular case of an n-ary semigroup (n-semigroup) for $n = 3$ (cf, [5, 6, 29, 38, 47, 55]). The literature of ternary algebraic system was introduced by D.H. Lehmer [38] in 1932. He investigated certain ternary algebraic system called triplexes which turn out to be ternary groups. The notion of ternary semigroups was known to S. Banach. He showed by an example that a ternary semigroup does not necessarily reduce to an ordinary semigroup. (m, n)-rings studied by Crombez [9], Crombez and Timm [10] and Dudek [32]. In [41], W.G. Lister characterized those additive subgroups of rings which are closed under the triple ring product and he called this algebraic system a ternary ring. Dutta and Kar [33] introduced the notion of ternary semiring which is a generalization of the notion of ternary ring.

Algebraic hyperstructures which is based on the notion of hyperoperation was introduced by Marty [43] and studied extensively by many mathematicians. Several books have been written on hyperstructure theory, see [7, 8, 22, 56]. A recent

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book on hyperstructures [8] points out on their applications in fuzzy and rough set theory, cryptography, codes, automata, probability, geometry, lattices, binary relations, graphs and hypergraphs. Another book [22] is devoted especially to the study of hyperring theory. Several kinds of hyperrings are introduced and analyzed. The volume ends with an outline of applications in chemistry and physics, analyzing several special kinds of hyperstructures: e-hyperstructures and transposition hypergroups. The theory of suitable modified hyperstructures can serve as a mathematical background in the field of quantum communication systems. The notion of hyperrings was studied by many authors. Some principal new notions about hyperrring theory can be found in [2, 3, 24, 25, 48, 46].

The concept of fuzzy algebraic structures was started with the introduction of the concept of fuzzy subgroups by Rosenfeld [52], also see [59]. Liu [42] introduced the notion of fuzzy subrings and ideals of a ring, also see [4]. The study of fuzzy hyperstructures is an interesting research topic of fuzzy sets. There is a considerable amount of work on the connections between fuzzy sets and hyperstructures. Some of them concern the *fuzzy hyperalgebras*. This is a direct extension of the concept of fuzzy algebras (fuzzy (sub)groups, fuzzy lattices, fuzzy rings etc). This approach can be extended to fuzzy hypergroups. For example, given a crisp hypergroup $(H, \circ)$ and a fuzzy set μ, we say that μ is a fuzzy (sub)hypergroups of $(H, \circ)$ if every cut of μ, say μ_t, is a (crisp) subhypergroup of $(H, \circ)$. This was initiated by Zahedi and et. al. [60] and continued by Davvaz [12], [26] and Davvaz and et. al. [14, 15, 16, 17, 18].

In [26], Davvaz and Vougiouklis introduced the concept of n-ary hypergroups as a generalization of hypergroups in the sense of Marty. Then this concept studied Leoreanu-Fotea and Davvaz [39, 40], Davvaz, Dudek Mirvakili and Vougiouklis [27, 28] and others. Leoreanu-Fotea and Davvaz in [40] introduced and studied the notion of a partial n-ary hypergroupoid, associated with a binary relation. Some important results, concerning Rosenberg partial hypergroupoids, induced by relations, are generalized to the case of n-ary hypergroupoids. Davvaz and et. al. in [28] considered a class of algebraic hypersystems which represent a generalization of semigroups, semihypergroups and n-ary semigroups. Ternary semihypergroups are algebraic structures with one associative hyperoperation. A ternary semihypergroup is a particular case of an n-ary semihypergroup (n-semihypergroup) for $n = 3$ [23]. The main propose of [23] is the study of binary relations on ternary semihypergroups.

In a ternary semihyperring, addition is a hyperoperation and multiplication is a ternary operation. Indeed, the notion of ternary semihyperrings is a generalization of semirings. Our main purpose of this paper is to introduce the notions of fuzzy hyperideal and fuzzy bi-hyperideal in ternary semihyperrings. We give some characterizations of fuzzy hyperideals and investigate several kinds of them.

2. **Basic Facts about Semirings and Hyperstructures**

We consider the ring of integers $\mathbb{Z}$ which plays a vital role in the literature of ring. The subset $\mathbb{Z}^+$ of all positive integers of $\mathbb{Z}$ is an additive commutative semigroup which is closed under the binary product, i.e., $\mathbb{Z}^+$ forms a semiring.

A *semiring* is a system consisting of a set S together with two binary operations on S called *addition* and *multiplication* (denoted in the usual manner) such that

(1) S together with addition is a (commutative) semigroup,
(2) S together with multiplication is a semigroup,
(3) $a \cdot (b + c) = a \cdot b + a \cdot c$ and $(a + b) \cdot c = a \cdot c + b \cdot c$ for all $a, b, c \in S$.

In the following table we present some examples of semirings which occur in combinatorics.

S	$\oplus$	$\odot$	e
$\mathbb{R}^+$	$+$	$\cdot$	0
$\mathbb{R}^+$	max	$+$	0
$\mathbb{R}^+$	$(a^m + b^m)^{1/m}$	$\cdot$	0
$[a, b]$	max	min	b
$\mathbb{R} \cup \{+\infty\}$	min	$+$	$+\infty$
$\{0, 1\}$	and	or	0

A *left (right) ideal* of a semiring S is a subset I of S such that

(1) $a + b \in I$ for all $a, b \in I$,
(2) $r \cdot a \in I$ $(a \cdot r \in I)$ for all $r \in S$ and $a \in I$.

An ideal of a semiring S is a subset I of S such that I is both a left and a right ideal of S.

Let H be a non-empty set and let $\wp^*(H)$ be the set of all non-empty subsets of H. A *hyperoperation* on H is a map $\circ : H \times H \longrightarrow \wp^*(H)$ and the couple $(H, \circ)$ is called a *hypergroupoid*. If A and B are non-empty subsets of H, then we denote

$$A \circ B = \bigcup_{a \in A,\, b \in B} a \circ b, \quad x \circ A = \{x\} \circ A \ \text{ and } \ A \circ x = A \circ \{x\}.$$

A hypergroupoid $(H, \circ)$ is called a *semihypergroup* if for all x, y, z of H we have $(x \circ y) \circ z = x \circ (y \circ z)$, which means that

$$\bigcup_{u \in x \circ y} u \circ z = \bigcup_{v \in y \circ z} x \circ v.$$

Let $(S, \cdot)$ be an ordinary semigroup and let P be a subset of S. We define

$$x \circ y = x \cdot P \cdot y, \text{ for all } x, y \in S.$$

Then $(S, \circ)$ is a semihypergroup.

A *semihyperring* is essentially a semiring, with approximately modified axioms in which addition is a hyperoperation (i.e., $a + b$ is a set). This concept has been studied by Ameri and Hedayati [1]. Of course, early, Vougiouklis in [57] and Davvaz in [20] studied the notion of semihyperrings in a general form, i.e., both the sum and product are hyperoperations. For example, let $(R, +, \cdot)$ be a semiring. We define $x \oplus y =< x, y >$ (the ideal generated by x and y) and $x \odot y = x \cdot y$. Then $(R, \oplus, \odot)$ is a semihyperring.

3. Ternary Semihyperrings and Hyperideals

We know that $\mathbb{Z}^+$ forms a semiring. Now, if we consider the subset $\mathbb{Z}^-$ of all negative integers of $\mathbb{Z}$, then we see that $\mathbb{Z}^-$ is an additive commutative semigroup which is closed under the ternary product. However $\mathbb{Z}^-$ is not closed under the binary product, i.e., $\mathbb{Z}^-$ does not form a semiring. Moreover, we know that the addition of two elements can be a set. Taking these facts, Davvaz in [13] introduced the notion of ternary semihyperrings which is a generalization of semihyperrings [1] and a generalization of ternary semirings [33].

Definition 3.1. A set R together with a binary hyperoperation $+$ and a ternary multiplication f is said to be a *ternary semihyperring* if $(R,+)$ is a (commutative) semihypergroup satisfying the following conditions:

(1) $f(f(a,b,c),d,e) = f(a,f(b,c,d),e) = f(a,b,f(c,d,e))$,
(2) $f(a+b,c,d) = f(a,c,d)+f(b,c,d)$,
(3) $f(a,b+c,d) = f(a,b,d)+f(a,c,d)$,
(4) $f(a,b,c+d) = f(a,b,c)+f(a,b,d)$,

for all $a,b,c,d,e \in R$. If $(R,+)$ be a semigroup, i.e., $+$ be an ordinary operation, then $(R,+,f)$ is called a *ternary semiring*.

Example 1. Let $\mathbb{Z}$ be the set of all integers. We define a binary hyperoperation and a ternary multiplication on $\mathbb{Z}$ in the following way: $x \oplus y = \{x,y\}$ and $f(x,y,z) = x \cdot y \cdot z$. Then $(\mathbb{Z},\oplus,f)$ is a ternary semihyperring.

Example 2. Let $(R,+,\cdot)$ be a semiring. We define a binary hyperoperation and a ternary multiplication on R in the following way: $x \oplus y = < x,y >$ (the ideal generated by x,y) and $f(x,y,z) = x \cdot y \cdot z$. Then $(R,\oplus,f)$ is a ternary semihyperring.

Example 3. Let I be the real interval $[0,1]$. On I we define $x \oplus y = \{t \in I \mid \min\{x,y\} \leq t \leq \max\{x,y\}\}$ and $f(x,y,z) = \min\{x,y,z\}$. Then $(I,\oplus,f)$ is a ternary semihyperring.

Example 4. If $(L,\wedge,\vee)$ is a relatively complemented distributive lattice and if $\oplus$ and f are defined as: $a \oplus b = \{c \in L \mid a \wedge c = b \wedge c = a \wedge b,\ a,b \in L\}$ and $f(a,b,c) = a \vee b \vee c$. Then $(L,\oplus,f)$ is a ternary semihyperring.

Example 5. Let R be a semihyperring and $\mathbb{M}$ be the set of all $n \times n$ matrixes with entries from R. The hyperproduct of two matrixes $(a_{ij}),(b_{ij})$ is defined in the usual manner

$$(a_{ij}) \cdot (b_{ij}) = \{(c_{ij}) \mid c_{ij} \in \sum_{k=1}^{n} a_{ik} + b_{kj}\}.$$

We define a binary hyperoperation and a ternary multiplication on $\mathbb{M}$ as follows:

$$(a_{ij}) \oplus (b_{ij}) = \{(c_{ij}) \mid c_{ij} \in a_{ij} + b_{ij}\} \quad \text{and} \quad f((a_{ij}),(b_{ij}),(c_{ij})) = (a_{ij}) \cdot (b_{ij}) \cdot (c_{ij}).$$

Then $(M,\oplus,f)$ is a ternary semihyperring.

Example 6. Let S be the set of all continuous functions $f : X \longrightarrow \mathbb{R}^-$, where X is a topological space and $\mathbb{R}^-$ is the set of all negative real numbers. Now, we define a binary hyperoperation (operation) and a ternary multiplication on R in the following way:

(1) $(f \oplus g)(x) = \{f(x) + g(x)\} := f(x) + g(x)$,
(2) $F(f, g, h)(x) = f(x) \cdot g(x) \cdot h(x)$,

for all $f, g, h \in S$ and $x \in X$. Then $(S, \oplus, F)$ forms a ternary semihyperring (indeed, it is a ternary semiring).

Notice that the positive real valued continuous functions form a semiring where as the negative real valued continuous functions form a ternary semiring.

Let $(R_1, +, f)$ and $(R_2, +', g)$ be ternary semihyperrings. Then a map $\varphi : R_1 \longrightarrow R_2$ is called a *homomorphism* if

$$\varphi(a + b) = \varphi(a) +' \varphi(b) \quad \text{and} \quad \varphi(f(a, b, c)) = f(\varphi(a), \varphi(b), \varphi(c))$$

are satisfied for all $a, b, c \in R_1$. Moreover, if φ is onto and one to one, then φ is called an *isomorphism*, and in this case we write $R_1 \cong R_2$.

Definition 3.2. Let $(R, +, f)$ be a ternary semihyperring. An additive sub-semihypergroup A of R is called a *ternary sub-semihyperring* of R if $f(a, b, c) \in A$, for all $a, b, c \in A$. An additive sub-semihypergroup I of R is called

(1) a *left hyperideal* of R if $f(a, b, i) \in I$, for all $a, b \in R$ and $i \in I$,
(2) a *right hyperideal* of R if $f(i, a, b) \in I$, for all $a, b \in R$ and $i \in I$,
(3) a *lateral hyperideal* of R if $f(a, i, b) \in I$, for all $a, b \in R$ and $i \in I$.

If I is both left and right hyperideal of R, then I is called a *two-sided hyperideal* of R. If I is a left, a right and a lateral hyperideal of R, then I is called a *hyperideal* of R.

Example 7. Consider Example 1. It is easy to see that $I =< 2 >= \{2k \mid k \in \mathbb{Z}\}$ is a hyperideal of $\mathbb{Z}$.

4. **Fuuzzy Hyperideals**

In this section, we introduce the notion of fuzzy hyperideal in a ternary semihyperring.

Let X be a non-empty set. A map $\mu : X \longrightarrow [0, 1]$ is called a *fuzzy subset* of X. Let μ and λ are fuzzy subsets of X. Then $\mu \cap \lambda$ and $\mu \cup \lambda$ are defined as follows:

$$(\mu \cap \lambda) = \min\{\mu(x), \lambda(x)\} \text{ and } (\mu \cup \lambda)(x) = \max\{\mu(x), \lambda(x)\}.$$

If μ is a fuzzy subset of X, then for any $t \in [0, 1]$, the set $\mu_t = \{x \in X \mid \mu(x) \geq t\}$ is called a *level subset* of μ.

A *fuzzy ideal* of a semiring S is a function $\mu : S \longrightarrow [0, 1]$ satisfying the following conditions:

(1) $\min\{\mu(x), \mu(y)\} \leq \mu(x + y)$, for all $x, y \in S$,
(2) $\min\{\mu(x), \mu(y)\} \leq \mu(x \cdot y)$, for all $x, y, z \in S$.

Example 8. Let μ be a fuzzy subset of the semiring $\mathbb{N}$ of natural numbers defined by

$$\mu(x) = \begin{cases} 0 & \text{if } 0 \leq x < 10 \\ 0.6 & \text{if } 10 \leq x < 100 \\ 1 & \text{if } 100 \leq x. \end{cases}$$

Then it is easy to see that μ is a fuzzy ideal of $\mathbb{N}$.

Definition 4.1. A fuzzy subset μ of a ternary semihyperring $(R, +, f)$ is called a *fuzzy sub-semihyperring* of R if

(1) $\min\{\mu(x), \mu(y)\} \leq \inf\limits_{z \in x+y)} \{\mu(z)\}$, for all $x, y \in R$,
(2) $\min\{\mu(x), \mu(y), \mu(z)\} \leq \mu(f(x, y, z))$, for all $x, y, z \in R$.

Example 9. Consider the ternary semihyperring $(\mathbb{Z}, \oplus, f)$ defined in Example 1. Let $S = \mathbb{Z}^-$ be the set of all negative integers. We define a fuzzy subset $\mu : \mathbb{Z} \longrightarrow [0, 1]$ as follows:

$$\mu(x) = \begin{cases} 0.8 & \text{if } x \in S \\ 0.3 & \text{otherwise} \end{cases}$$

Then μ is a fuzzy ternary sub-semihyperring of $(\mathbb{Z}, \oplus, f)$.

Definition 4.2. A fuzzy subset μ of a ternary semihyperring $(R, +, f)$ is called a *fuzzy hyperideal* of R if

(1) $\min\{\mu(x), \mu(y)\} \leq \inf\limits_{z \in x+y)} \{\mu(z)\}$, for all $x, y \in R$,
(2) $\mu(x) \leq \mu(f(x, y, z))$, for all $x, y, z \in R$,
(3) $\mu(y) \leq \mu(f(x, y, z))$, for all $x, y, z \in R$,
(4) $\mu(z) \leq \mu(f(x, y, z))$, for all $x, y, z \in R$.

Example 10. Suppose that $R := \{0, 1, 2, 3\}$ and define a hyperoperation $+$ on R as follows:

$+$	0	1	2	3
0	0	1	2	3
1	1	$\{0, 1\}$	3	$\{2, 3\}$
2	2	3	0	1
3	3	$\{2, 3\}$	1	$\{0, 1\}$.

Let f be a ternary operation on R such that

$$f(x, y, z) = \begin{cases} 2 & \text{if } x, y, z \in \{2, 3\}, \\ 0 & \text{otherwise.} \end{cases}$$

Then, $(R, +, f)$ is a ternary semihyperring. Now, let $\alpha, \beta \in [0, 1]$ and $\beta \leq \alpha$. We define

$$\mu(x) = \begin{cases} \alpha & \text{if } x = 0, 1 \\ \beta & \text{if } x = 2, 3 \end{cases}$$

Then μ is a fuzzy hyperideal of R.

Lemma 4.3. *Any hyperideal of a ternary semihyperring $(R, +, f)$ can be realized as a level subset of some fuzzy hyperideal of R.*

Proof. Let I be a hyperideal of a given ternary semihyperring R and let μ_I be a fuzzy subset of R defined by

$$\mu_I(x) = \begin{cases} t & \text{if } x \in I \\ s & \text{if } x \notin I \end{cases}$$

where $0 \leq s < t \leq 1$ is fixed. It is not difficult to see that μ is a fuzzy hyperideal of R such that $\mu_t = I$. □

Notice that the characteristic function of a non-empty subset A of a ternary semihyperring R is a fuzzy hyperideal of R if and only if A is a hyperideal of R.

Theorem 4.4. *A fuzzy subset μ of a ternary semihyperring $(R, +, f)$ is a fuzzy hyperideal if and only if each its non-empty level subset is a hyperideal of R.*

Proof. Let μ be a fuzzy hyperideal of a ternary semihyperring $(R, +, f)$. If $x, y \in \mu_t$ for some $t \in [0, 1]$, then $\mu(x) \geq t$ and $\mu(y) \geq t$. Thus

$$t \leq \min\{\mu(x), \mu(y)\} \leq \inf_{z \in x+y} \{\mu(z)\},$$

which implies that $\mu(z) \geq t$ for every $z \in x + y$. Therefore, $x + y \subseteq \mu_t$. Moreover, suppose that $x, y, z \in R$ and $x \in \mu_t$. Then $\mu(x) \geq t$. So we have $t \leq \mu(x) \leq \mu(f(x, y, z))$, which implies that $f(\mu_t, y, z) \subseteq \mu_t$. Similarly, we obtain $f(x, \mu_t, z) \subseteq \mu_t$ and $f(x, y, \mu_t) \subseteq \mu_t$. Hence, μ_t is a hyperideal of R.

Conversely, assume that every non-empty level subset μ_t is a hyperideal of R. Let $t_0 = \min\{\mu(x), \mu(y)\}$ for $x, y \in R$. Then obviously $x, y \in \mu_{t_0}$, consequently, $x + y \subseteq \mu_{t_0}$. Thus

$$\min\{\mu(x), \mu(y)\} = t_0 \leq \inf_{z \in x+y} \{\mu(z)\}.$$

Now, let $\mu(x) = t_1$. Then $x \in \mu_{t_1}$. So we obtain $f(x, y, z) \in \mu_{t_1}$, which implies that $t_1 \leq \mu(f(x, y, z))$. Hence, $\mu(x) = t_1 \leq \mu(f(x, y, z))$. Similarly, we obtain $\mu(y) \leq \mu(f(x, y, z))$ and $\mu(z) \leq \mu(f(x, y, z))$. In this way all conditions of definition are verified. This completes the proof. □

A *strong level subset* $\mu_t^>$ of a fuzzy set μ in R is defined by $\mu_t^> = \{x \in R \mid \mu(x) > t\}$.

Corollary 4.5. Let μ be a fuzzy set with the upper bound t_0 of a ternary semihyperring R. Then the following are equivalent:

(1) μ is a fuzzy hyperideal of R.
(2) Each level subset μ_t, for $t \in [0, t_0]$ is a hyperideal of R.
(3) Each strong level subset $\mu_t^>$, for $t \in [0, t_0]$ is a hyperideal of R.
(4) Each level subset μ_t, for $t \in Im(\mu)$ is a hyperideal of R, where $Im(\mu)$ denotes the image of μ.
(5) Each strong level subset $\mu_t^>$, for $t \in Im(\mu) \setminus \{t_0\}$ is a hyperideal of R.
(6) Each non-empty level subset of μ is a hyperideal of R.
(7) Each non-empty strong level subset of μ is a hyperideal of R.

Let $\varphi : R_1 \longrightarrow R_2$ be a function and μ be a fuzzy subset of R_1. Then φ induces a fuzzy subset $\varphi(\mu)$ in $\varphi(\mu)$ in R_2 defined by:

$$\varphi(\mu)(y) = \begin{cases} \sup\limits_{x \in \varphi^{-1}(y)} \{\mu(x)\} & \text{if } y \in \varphi(R_1) \\ 0 & \text{otherwise.} \end{cases}$$

Here $\varphi(\mu)$ is called the *image* of μ under φ. Let λ be a fuzzy subset in R_2. Then φ induces a fuzzy subset $\varphi^{-1}(\lambda)$ in R_1 defined by $\varphi^{-1}(\lambda)(x) = \lambda(\varphi(x))$ for all $x \in R_1$. Here $\varphi^{-1}(\lambda)$ is called the *inverse image* of λ under μ.

Lemma 4.6. *Let R_1 and R_2 be two ternary semihyperrings. Let $\varphi : R_1 \longrightarrow R_2$ be a homomorphism.*

(1) *If μ is a fuzzy hyperideal of R_1, then $\varphi(\mu)$ is a fuzzy hyperideal of R_2.*
(2) *If λ is a fuzzy hyperideal of R_2, then $\varphi^{-1}(\lambda)$ is a fuzzy hyperideal of R_1.*

Proof. It is straightforward. □

Let $(R_1, +, f)$ and $(R_2, +', g)$ be two ternary semihyperrings. The *direct product* $R_1 \times R_2$ is a ternary semihyperring such that for $a_1, a_2, a_3 \in R_1$, $b_1, b_2, b_3 \in R_2$,

$$((a_1, b_1) \oplus (a_2, b_2)) = \{(a, b) \mid a \in a_1 + a_2, b \in b_1 +' b_2\},$$
$$(f \times g)((a_1, b_1), (a_2, b_2), (a_3, b_3)) = (f(a_1, a_2, a_3), g(b_1, b_2, b_3)))\,.$$

Let μ, λ be fuzzy hyperideals of R_1, R_2, respectively. Then the *product* of μ and λ is the fuzzy subset $\mu \times \lambda$ of $R_1 \times R_2$ where $(\mu \times \lambda)(x, y) = \min\{\mu(x), \lambda(y)\}$ for all $(x, y) \in R_1 \times R_2$.

Lemma 4.7. *Let $(R_1, +, f)$ and $(R_2, +', g)$ be two ternary hyperrings and μ, λ be fuzzy hyperideals of R_1, R_2, respectively. Then $\mu \times \lambda$ is a fuzzy hyperideal of $R_1 \times R_2$.*

Proof. It is straightforward. □

A fuzzy subset μ of R of the form

$$\mu(y) = \begin{cases} t \neq 0 & \text{if } \; y = x, \\ 0 & \text{if } \; y \neq x. \end{cases}$$

is called a *fuzzy point* with support x and value t and is denoted x_t. A fuzzy point x_t is said to be *belong* to (resp. be *quasi-coincident* with) a fuzzy set μ, written as $x_t \in \mu$ (resp. $x_t q \mu$) if $\mu(x) \geq t$ (resp. $\mu(x) + t > 1$). If $x_t \in \mu$ or $x_t q \mu$, then we write $x_t \in \vee q \mu$. The symbol $\overline{\in \vee q}$ means neither $\in$ nor q hold.

Definition 4.8. A fuzzy subset μ of a ternary semihyperring $(R, +, f)$ is called an $(\in, \in \vee q)$-*fuzzy hyperideal* of R if

(i) $x_t, y_s \in \mu$ implies $z_{t \wedge s} \in \vee q \mu$ for all $z \in x + y$, $t, s \in (0, 1]$ and $x, y \in R$,
(ii) $x_t \in \mu$ implies $f(x, y, z)_t \in \vee q \mu$, for all $t \in (0, 1]$ and $x, y, z \in R$,
(iii) $y_t \in \mu$ implies $f(x, y, z)_t \in \vee q \mu$, for all $t \in (0, 1]$ and $x, y, z \in R$,
(iv) $z_t \in \mu$ implies $f(x, y, z)_t \in \vee q \mu$, for all $t \in (0, 1]$ and $x, y, z \in R$.,

Proposition 4.9. *Conditions (i), (ii), (iii) and (iv) in Definition 4.8 are equivalent,respectively, to the following conditions.*

(1) $\mu(x) \wedge \mu(y) \wedge 0.5 \leq \bigwedge_{z \in x+y} \mu(z)$ *for all* $x, y \in R$,
(2) $\mu(x) \wedge 0.5 \leq \mu(f(x, y, z))$ *for all* $x, y, z \in R$,
(3) $\mu(y) \wedge 0.5 \leq \mu(f(x, y, z))$ *for all* $x, y, z \in R$,
(4) $\mu(z) \wedge 0.5 \leq \mu(f(x, y, z))$ *for all* $x, y, z \in R$.

Proof. (i $\Rightarrow$ 1): Suppose that $x, y \in R$. We consider the following cases:
(a) $\mu(x) \wedge \mu(y) < 0.5$
(b) $\mu(x) \wedge \mu(y) \geq 0.5$.

Case a: Assume that there exists $z \in x + y$ such that $\mu(z) < \mu(x) \wedge \mu(y) \wedge 0.5$, which implies that $\mu(z) < \mu(x) \wedge \mu(y)$. Choose t such that $\mu(z) < t < \mu(x) \wedge \mu(y)$. Then $x_t, y_t \in \mu$, but $z_t \overline{\in \vee q \mu}$, which contradicts (i).

Case b: Assume that $\mu(z) < 0.5$ for some $z \in x + y$. Then $x_{0.5}, y_{0.5} \in \mu$, but $z_{0.5} \overline{\in \vee q \mu}$, which is a contradiction. Therefore, (1) holds.

(ii $\Rightarrow$ 2): Suppose that $x, y, z \in R$. We consider the following cases:
(a) $\mu(x) < 0.5$
(b) $\mu(x) \geq 0.5$.

Case a: Assume that $\mu(f(x, y, z))) < \mu(x) \wedge 0.5$, which implies that $\mu(f(x, y, z))) < \mu(x)$. Choose t such that $\mu(f(x, y, z))) < t < \mu(x)$. Then $x_t \in \mu$, but $(f(x, y, z)))_t \overline{\in \vee q \mu}$, which contradicts (ii).

Case b: Assume that $\mu(f(x, y, z))) < 0.5$. Then $x_{0.5} \in \mu$, but $(\mu(f(x, y, z)))_{0.5} \overline{\in \vee q \mu}$, which is a contradiction. Therefore (2) holds.

The proofs of (iii $\Rightarrow$ 3) and (iv $\Rightarrow$ 4) are similar to (ii $\Rightarrow$ 2).

(1 $\Rightarrow$ i): Let $x_t, y_s \in \mu$. Then $\mu(x) \geq t$ and $\mu(y) \geq s$. For every $z \in x + y$, we have

$$\mu(z) \geq \mu(x) \wedge \mu(y) \wedge 0.5 \geq t \wedge s \wedge 0.5.$$

If $t \wedge s > 0.5$, then $\mu(z) \geq 0.5$ which implies that $\mu(z) + (t \wedge s) > 1$. If $t \wedge s \leq 0.5$, then $\mu(z) \geq t \wedge s$. Therefore $z_{t \wedge s} \in \vee q \mu$ for all $z \in x + y$.

(2 $\Rightarrow$ ii): Let $x_t \in \mu$. Then $\mu(x) \geq t$. We have $\mu(f(x, y, z)) \geq \mu(x) \wedge 0.5 \geq t \wedge 0.5$. If $t > 0.5$, then $\mu(f(x, y, z)) \geq 0.5$ which implies that $\mu(f(x, y, z)) + t > 1$. If $t \leq 0.5$, then $\mu(f(x, y, z)) \geq t$. Therefore, $(f(x, y, z))_t \in \vee q \mu$.

The proofs of (3 $\Rightarrow$ iii) and (4 $\Rightarrow$ iv) are similar to (2 $\Rightarrow$ ii). □

By Definition 4.8 and Proposition 4.9, we obtain immediately:

Corollary 4.10. A fuzzy subset μ of a ternary semihyperring $(R, +, f)$ is an $(\in, \in \vee q)$-*fuzzy hyperideal* of R if and only if the conditions (1), (2), (3) and (4) in Proposition 4.9 hold.

Definition 4.11. A fuzzy subset μ of a ternary semihyperring $(R, +, f)$ is called an $(\overline{\in}, \overline{\in} \vee \overline{q})$-*fuzzy hyperideal* of R if

(i) $z_{t \wedge s} \overline{\in} \mu$ implies $x_t \overline{\in} \vee \overline{q} \mu$ and $y_s \overline{\in} \vee \overline{q} \mu$ for all $z \in x + y$, $t, s \in (0, 1]$ and $x, y \in R$,
(ii) $f(x, y, z)_t \overline{\in} \mu$ implies $x_t \overline{\in} \vee \overline{q} \mu$ for all $t \in (0, 1]$ and $x, , y, z \in R$,
(iii) $f(x, y, z)_t \overline{\in} \mu$ implies $y_t \overline{\in} \vee \overline{q} \mu$ for all $t \in (0, 1]$ and $x, , y, z \in R$,
(iv) $f(x, y, z)_t \overline{\in} \mu$ implies $z_t \overline{\in} \vee \overline{q} \mu$ for all $t \in (0, 1]$ and $x, , y, z \in R$.

Proposition 4.12. *A fuzzy subset μ of a ternary semihyperring $(R, +, f)$ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy hyperideal of R if and only if it satisfies:*

(1) $\mu(x) \wedge \mu(y)) \leq \bigwedge_{z \in x+y} (\mu(z) \vee 0.5)$ *for all* $x, y \in R$,
(2) $\mu(x) \leq \mu(f(x, y, z)) \vee 0.5$ *for all* $x, y, z \in R$,
(3) $\mu(y) \leq \mu(f(x, y, z)) \vee 0.5$ *for all* $x, y, z \in R$,
(4) $\mu(z) \leq \mu(f(x, y, z)) \vee 0.5$ *for all* $x, y, z \in R$.

Proof. The proof is similar to the proof of Proposition 4.9. □

Definition 4.13. Let $\alpha, \beta \in [0, 1]$ and $\alpha < \beta$. Let μ be a fuzzy subset of a ternary semihyperring $(R, +, f)$. Then μ is called a *fuzzy hyperideal with thresholds of* R, if

(1) $\mu(x) \wedge \mu(y) \wedge \beta \leq \bigwedge_{z \in x+t} (\mu(z) \vee \alpha)$ for all $x, y \in R$,
(2) $\mu(x) \wedge \beta \leq \mu(f(x, y, z)) \vee \alpha$ for all $x, y, z \in R$,
(3) $\mu(y) \wedge \beta \leq \mu(f(x, y, z)) \vee \alpha$ for all $x, y, z \in R$,
(4) $\mu(z) \wedge \beta \leq \mu(f(x, y, z)) \vee \alpha$ for all $x, y, z \in R$.

Now, we give a characterization of fuzzy hyperideals with thresholds by using their level subsets.

Theorem 4.14. *A fuzzy subset μ of a ternary semihyperring $(R, +, f)$ is a fuzzy hyperideal with thresholds (α, β) of R if and only if $\mu_t (\neq \emptyset)$ is a hyperideal of R for all $t \in (\alpha, \beta]$.*

Proof. Suppose that μ is a fuzzy hyperideal with thresholds of R and $t \in (\alpha, \beta]$. Let $x, y \in \mu_t$. Then $\mu(x) \geq t$ and $\mu(y) \geq t$. Now,

$$\alpha < t = t \wedge \beta \leq \mu(x) \wedge \mu(y)) \wedge \beta \leq \bigwedge_{z \in x+y} (\mu(z) \vee \alpha).$$

So for every $z \in x + y$ we have $\mu(z) \vee \alpha \geq t > \alpha$ which implies that $\mu(z) \geq t$ and $z \in \mu_t$. Hence $x + y \subseteq \mu_t$.

Now, suppose that $x, y, z \in R$, $x \in \mu_t$ and $t \in (\alpha, \beta]$. Then $\mu(x) \geq t$. Thus, we have

$$\alpha < t = t \wedge \beta \leq \mu(x) \wedge \beta \leq \mu(f(x, y, z)) \vee \alpha.$$

Then, we obtain $\mu(f(x, y, z)) \geq t$ or $f(x, y, z) \in \mu_t$. Hence, $f(\mu_t, y, z) \subseteq \mu_t$. Similarly, we obtain $f(x, \mu_t, z) \subseteq \mu_t$ and $f(x, y, \mu_t) \subseteq \mu_t$. Therefore, μ_t is a hyperideal of R.

Conversely, let μ be a fuzzy subset of R such that $\mu_t (\neq \emptyset)$ is a hyperideal of R for all $\alpha < t \leq \beta$. If there exist $x, y, z \in R$ with $z \in x + y$ such that $\mu(z) \vee \alpha < \mu(x) \wedge \mu(y) \wedge \beta = t$, then $t \in (\alpha, \beta]$, $\mu(z) < t$, $x, y \in \mu_t$. Since μ_t is a hyperideal of R so $x + y \subseteq \mu_t$. Hence, $\mu(z) \geq t$ for all $z \in x + y$. This is a contradiction with $\mu(z) < t$. Therefore, $\mu(x) \wedge \mu(y)) \wedge \beta \leq \mu(z) \vee \alpha$ for all $x, y, z \in R$ which implies that

$$\mu(x) \wedge \mu(y)) \wedge \beta \leq \bigwedge_{z \in x+y} (\mu(z) \vee \alpha)$$

for all $x, y \in R$. Hence, the first condition of definition holds. Now, if there exist $x, y, z \in R$ such that $\mu(f(x,y,z)) \vee \alpha < (\mu(x) \vee \mu(y)) \wedge \beta = \mu(x) \wedge \beta = t_0$, then $t_0 \in (\alpha, \beta]$, $\mu(f(x,y,z)) < t_0$ and $x \in \mu_{t_0}$. Since μ_{t_0} is a hyperideal of R, so $f(x,y,z) \in \mu_{t_0}$. Hence, $\mu(f(x,y,z)) \geq t_0$. This is a contradiction. □

By the above theorem, we have the following corollary.

Corollary 4.15. Let μ be a fuzzy subset of a ternary semihyperring R. Then

(i) μ is an $(\in, \in \vee q)$-fuzzy hyperideal of R if and only if the set $\mu_t (\neq \emptyset)$ is a hyperideal of R for all $t \in (0, 0.5]$.
(ii) μ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy hyperideal of R if and only if the set $\mu_t (\neq \emptyset)$ is a hyperideal of R for all $t \in (0.5, 1]$.

Fuzzy logic is an extension of set theoretic variables in terms of the linguistic variable truth. Some operators, like $\wedge, \vee, \neg, \rightarrow$ in fuzzy logic are also defined by using truth tables, the extension principle can be applied to derive definitions of the operators.

In the fuzzy logic, truth value of fuzzy proposition P is denoted by $[P]$. In the following, we display the fuzzy logical and corresponding set-theoretical notions used in this paper:
$[x \in A] = A(x)$,
$[x \notin A] = 1 - A(x)$,
$[P \wedge Q] = \min\{[P], [Q]\}$,
$[P \rightarrow Q] = \min\{1, 1 - [P] + [Q]\}$,
$[\forall x P(x)] = \inf[P(x)]$,
$\models P$ if and only if $[P] = 1$.

A function $I : [0,1] \times [0,1] \longrightarrow [0,1]$ is called fuzzy implication if it is monotonic with respect to both variables (separately) and fulfils the binary implication truth table: $I(0,0) = I(0,1) = I(1,1) = 1$, $I(1,0) = 0$. By monotonicity $I(0,x) = I(x,1) = 1$ for all $x \in [0,1]$, where I is decreasing with respect to the first variable $(I(1,0) < I(0,0))$ and I is increasing with respect to the second variable $(I(1,0) < I(1,1))$.

Of course, various implication operators have been defined. The following are the most important multi-valued implications:

$$I_g(\alpha, \beta) = \begin{cases} 1 & \text{if } \alpha \leq \beta \\ \beta & \text{if } \alpha > \beta, \end{cases} \qquad I_{cg}(\alpha, \beta) = \begin{cases} 1 & \text{if } \alpha \leq \beta \\ 1 - \alpha & \text{if } \alpha > \beta, \end{cases}$$
$$I_{gr}(\alpha, \beta) = \begin{cases} 1 & \text{if } \alpha \leq \beta \\ 0 & \text{if } \alpha > \beta. \end{cases}$$

In the following definition we consider the definition of implication operator in the Łukasiewicz system of continuous-valued logic.

Definition 4.16. A fuzzy subset μ of a ternary semihyperring $(R, +, f)$ is called a *fuzzifying hyperideal* of R if and only if it satisfies:

(i) for any $x, y \in R$,
$$\models [[x \in \mu] \wedge [y \in \mu] \longrightarrow [\forall z \in x + y, z \in \mu]],$$

(ii) for any $x, y, z \in R$,
$\models [[x \in \mu] \longrightarrow [f(x, y, z) \in \mu]]$,
(iii) for any $x, y, z \in R$,
$\models [[y \in \mu] \longrightarrow [f(x, y, z) \in \mu]]$,
(iv) for any $x, y, z \in R$,
$\models [[z \in \mu] \longrightarrow [f(x, y, z) \in \mu]]$.

Clearly, a fuzzyfying hyperideal is an ordinary fuzzy hyperideal. In [49], the concept of t-tautology is used, i.e., $\models_t P$ if and only if $[P] \geq t$ for all valuations. Now, we consider the following definition.

Definition 4.17. A fuzzy subset μ of a ternary semihyperring $(R, +, f)$ is called a *t-implication-based fuzzy hyperideal* of R with respect to implication $\longrightarrow$ if and only if satisfies:

(1) for any $x, y \in R$
$\models_t [[x \in \mu] \wedge [y \in \mu] \longrightarrow [\forall z \in x + y, z \in \mu]]$,
(2) for any $x, y, z \in R$
$\models_t [[x \in \mu] \longrightarrow [f(x, y, z) \in \mu]]$,
(3) for any $x, y, z \in R$
$\models_t [[y \in \mu] \longrightarrow [f(x, y, z) \in \mu]]$,
(4) for any $x, y, z \in R$
$\models_t [[z \in \mu] \longrightarrow [f(x, y, z) \in \mu]]$.

Corollary 4.18. A fuzzy subset μ of a ternary semihyperring $(R, +, f)$ is a t-implication-based fuzzy hyperideal of R with respect to implication I if and only if

(i) $I\left(\mu(x) \wedge \mu(y), \bigwedge\limits_{z \in x+y} \mu(z)\right) \geq t$ for all $x, y \in R$,
(ii) $I(\mu(x), \mu(f(x, y, z))) \geq t$ for all $x, y, z \in R$,
(iii) $I(\mu(y), \mu(f(x, y, z))) \geq t$ for all $x, y, z \in R$,
(iv) $I(\mu(z), \mu(f(x, y, z))) \geq t$ for all $x, y, z \in R$.

Theorem 4.19. *Let μ be a fuzzy subset of a ternary semihyperring $(R, +, f)$.*

(i) *Let $I = I_{gr}$. Then μ is a 0.5-implication-based fuzzy hyperideal of R if and only if μ is a fuzzy hyperideal with thresholds $\alpha = 0$ and $\beta = 1$ of R.*
(ii) *Let $I = I_g$. Then μ is a 0.5-implication-based fuzzy hyperideal of R if and only if μ is a fuzzy hyperideal with thresholds $\alpha = 0$ and $\beta = 0.5$ of R.*
(iii) *Let $I = I_{cg}$. Then μ is a 0.5-implication-based fuzzy hyperideal with thresholds if and only if μ is a fuzzy hyperideal with thresholds $\alpha = 0.5$ and $\beta = 1$ of R.*

Proof. It is straightforward. □

5. **Bi-hyperideals**

Definition 5.1. A sub-semihyperring A of a ternary semihyperring $(R, +, f)$ is called a *bi-hyperideal* of R if $f(A, f(R, A, R), A) \subseteq A$.

In general, if A is a bi-hyperideal of a ternary semihyperring R and B is a bi-hyperideal of A, then B is not a bi-hyperideal of R. But, in particular, we have the following proposition.

Proposition 5.2. *Let A be a bi-hyperideal of a ternary semihyperring R and B be a bi-hyperideal of R such that $f(B,B,B)=B$. Then B is a bi-hyperideal of R.*

Proof. It is straightforward. □

Proposition 5.3. *Let A, B and C be ternary sub-semihyperrings of a ternary semihyperring R and $D=f(A,B,C)$. Then D is a bi-hyperideal of R if at least one of A, B, C is a right, a lateral, or a left hyperideal of R.*

Proof. We consider only one case. The proofs of other cases are similar. Suppose that A is a right hyperideal of R. Then we have

$$\begin{aligned} f(D,f(R,D,R),D) &= f(f(A,B,C),f(R,f(A,B,C),R),f(A,B,C)) \\ &\subseteq f(A,f(f(R,R,R),f(R,R,R),f(R,R,B),C)) \\ &\subseteq f(A,f(R,R,B),C) \subseteq f(f(A,R,R),B,C) \\ &\subseteq f(A,B,C)=D. \end{aligned}$$

So D is a bi-hyperideal of R. □

Definition 5.4. A fuzzy sub-semihyperring μ of R is called a *fuzzy bi-hyperideal* of R if

$$\min\{\mu(x),\mu(y),\mu(z)\} \leq \mu(f(x,f(a,y,b),z))$$

for all $x,y,z,a,b \in R$.

Example 11. Consider $\mathbb{Z}^-$, the set of all negative integers. Then $\mathbb{Z}^-$ is a ternary semihyperring (indeed, a ternary semiring) under the usual addition and multiplication. Let $S=2\mathbb{Z}^-$. We define a fuzzy subset $\mu:\mathbb{Z}^- \longrightarrow [0,1]$ by

$$\mu(x)=\begin{cases} 0.7 & \text{if } x \in S \\ 0.1 & \text{otherwise.} \end{cases}$$

Then μ is a fuzzy bi-hyperideal of $\mathbb{Z}^-$.

The following proposition gives a characterization of fuzzy bi-hyperideals in terms of level subsets.

Proposition 5.5. *Let μ be a fuzzy subset of R. Then μ is a fuzzy bi-hyperideal of R if and only if μ_t $(\neq \emptyset)$ is a bi-hyperideal of R.*

Proof. Suppose that μ is a fuzzy hyperideal of R. Then μ is a fuzzy sub-semihyperring of R and so according to the proof of Theorem 4.4, μ_t is a sub-semihyperring of R. In order to show that $f(\mu_t,f(R,\mu_t,R),\mu_t) \subseteq \mu_t$, suppose that $\omega \in f(\mu_t,f(R,\mu_t,R),\mu_t)$ is an arbitrary element. Then there exist $x,y,z \in \mu_t$ and $a,b \in R$ such that $\omega = f(x,f(a,y,b),z)$. Since μ is a fuzzy bi-hyperideal, we have $\min\{\mu(x),\mu(y),\mu(z)\} \leq \mu(\omega)$. Since $x,y,z \in \mu_t$, $t \leq \min\{\mu(x),\mu(y),\mu(z)\}$. Thus $t \leq \mu(\omega)$ or $\omega \in \mu_t$. Therefore, μ is a bi-hyperideal of R.

Conversely, suppose that μ_t $(t \in Im\mu)$ is a bi-hyperideal of R. Then μ_t is a sub-semihyperring of R and so according to the proof of Theorem 4.4, μ is a fuzzy sub-semihyperring of R. Now, let $x, y, z, a, b \in R$ such that $\mu(x) \leq \mu(y) \leq \mu(z)$. If we set $\mu(x) = t$, $\mu(y) = r$ and $\mu(z) = s$, then $x, y, z \in \mu_t$. So $f(x, f(a, y, b), z) \in \mu_t$ which implies that $\mu(f(x, f(a, y, b), z)) \geq t$. Hence, $\min\{\mu(x), \mu(y), \mu(z)\} \leq \mu, (f(x, f(a, y, b), z))$. Therefore, μ is a fuzzy bi-hyperideal of R. □

Proposition 5.6. *Let μ be a fuzzy bi-hyperideal of a ternary semihyperring R and λ be a fuzzy sub-semihyperring of R. Then $\mu \cap \lambda$ is a fuzzy bi-hyperideal of R.*

Proof. It is straightforward. □

References

[1] R. Ameri and H. Hedayati, *On k-hyperideals of semihyperrings*, J. Discrete Math. Sci. Cryptogr., **10** (2007), 41-54.

[2] S. M. Anvariyeh and B. Davvaz, *Strongly transitive geometric spaces associated to hypermodules*, Journal of Algebra, **322** (2009), 1340-1359.

[3] S. M. Anvariyeh, S. Mirvakili and B. Davvaz, θ^**- Relation on hypermodules and fundamental modules over commutative fundamental rings*, Communications in Algebra, **36(2)** (2008), 622-631.

[4] S. K. Bhakat and P. Das, *Fuzzy subrings and ideals redefined*, Fuzzy Sets and Systems, **81** (1996), 383-393.

[5] A. Chronowski, *On ternary semigroups of lattice homomorphisms*, Quasigroups Related Systems, **3** (1996), 55-72.

[6] A. Chronowski, *Congruences on ternary semigroups*, Ukrainian Math. J., **56** (2004), 662-681.

[7] P. Corsini, *Prolegomena of hypergroup theory*, Second edition, Aviani editor, 1993.

[8] P. Corsini and V. Leoreanu, *Applications of hyperstructure theory*, Advances in Mathematics, Kluwer Academic Publishers, Dordrecht, 2003.

[9] G. Crombez, *On* (n, m)*-rings*, Abh. Math. Semin. Univ. Hamburg, **37** (1972), 180-199.

[10] G. Crombez and J. Timm, *On* (n, m)*-quotient rings,* Abh. Math. Semin. Univ. Hamburg, **37** (1972), 200-203.

[11] B. Davvaz, *Fuzzy* H_v*-groups*, Fuzzy Sets and Systems, **101** (1999), 191-195.

[12] B. Davvaz, *Fuzzy* H_v*-submodules*, Fuzzy Sets and Systems, **117** (2001), 477-484.

[13] B. Davvaz, *Hyperideals in ternary semihyperrings*, submitted.

[14] B. Davvaz, P. Corsini and V. Leoreanu-Fotea, *Fuzzy n-ary subpolygroups*, Computers & Mathematics with Applications, **57** (2009), 141-152.

[15] B. Davvaz, P. Corsini and V. Leoreanu-Fotea, *Atanassov's intuitionistic* (S, T)*-fuzzy n-ary subhypergroups and their properties*, Information Sciences, **179** (2009), 654-666.

[16] B. Davvaz and V. Leoreanu, *Applications of interval valued fuzzy n-ary polygroups with respect to t-norms (t-conorms)*, Computers & Mathematics with Applications, **57** (2009), 1413-1424.

[17] B. Davvaz, J. Zhan and K. P. Shum, *Generalized fuzzy* H_v*-ideals of* H_v*-rings*, Int. J. General Systems, **37(3)** (2008), 329-346.

[18] B. Davvaz, J. Zhan and K. P. Shum, *Generalized fuzzy* H_v*-submodules endowed with interval valued membership functions*, Information Sciences, **178(15)** (2008), 3147-3159.

[19] B. Davvaz, *Approximations in n-ary algebraic systems*, Soft Computing, **12** (2008), 409-418.

[20] B. Davvaz, *Rings derived from semihyper-rings*, Algebras Groups Geom., **20** (2003), 245-252.

[21] B. Davvaz, *Some results on congruences in semihypergroups*, Bull. Malays. Math. Soc., **23(2)** (2000), 53-58.

[22] B. Davvaz and V. Leoreanu-Fotea, *Hyperring theory and applications*, International Academic Press, USA, 2007.

[23] B. Davvaz and V. Leoreanu-Fotea, *Binary relations on ternary semihypergroups*, Communications in Algebra, to appear.

[24] B. Davvaz and A. Salasi, *A realization of hyperrings*, Communications in Algebra, **34(12)** (2006), 4389-4400.

[25] B. Davvaz and T. Vougiouklis, *Commutative rings obtained from hyperrings (H_v-rings) with α^*-relations*, Communications in Algebra, **35(11)** (2007), 3307-3320.

[26] B. Davvaz and T. Vougiouklis, *n-Ary hypergroups*, Iranian Journal of Science and Technology, Transaction A, **30(A2)** (2006), 165-174.

[27] B. Davvaz, W. A. Dudek and S. Mirvakili, *Neutral elements, fundamental relations and n-ary hypersemigroups*, International Journal of Algebra and Computation, **19(4)** (2009), 567-583.

[28] B. Davvaz, W. A. Dudek and T. Vougiouklis, *A generalization of n-ary algebraic systems*, Communications in Algebra, **37** (2009), 1248-1263.

[29] V. N. Dixit and S. Dewan, *A note on quasi and bi-ideals in ternary semigroups*, Internat. J. Math. & Math. Sci., **18** (1995), 501-508.

[30] W. Dörnte, *Untersuchungen über einen verallgemeinerten Gruppenbegriff*, Math. Z., **29** (1928), 1-19.

[31] W. A. Dudek, *Idempotents in n-ary semigroups*, Southeast Asian Bull. Math., **25** (2001), 97-104.

[32] W. A. Dudek *On the divisibility theory in (m, n)-rings*, Demonstratio Math., **14** (1981), 19-32.

[33] T. K. Dutta and S. Kar, *On regular ternary semirings*, Advances in Algebra, Proceedings of the ICM Satellite Conference in Algebra and Related Topics, World Scientific, New Jersey, (2003), 343-355.

[34] K. Iséki, *Ideals in semirings*, Proc. Japan Acad., **34** (1958), 29-31.

[35] E. Kasner, *An extension of the group concept*, Bull. Amer. Math. Soc., **10** (1904), 290-291.

[36] O. Kazancı, S. Yamak and B. Davvaz, *The lower and upper approximations in a quotient hypermodule with respect to fuzzy sets*, Information Sciences, **178(10)** (2008), 2349-2359.

[37] D. H. Lehmer, *A ternary analogue of abelian groups*, American J. Math., **59** (1932), 329-338.

[38] D. H. Lehmer, *A ternary analogue of abelian groups*, Amer. J. Math., **59** (1932), 329-338.

[39] V. Leoreanu-Fotea and B. Davvaz, *Roughness in n-ary hypergroups*, Information Sciences, **178** (2008), 4114-4124.

[40] V. Leoreanu-Fotea and B. Davvaz, *n-hypergroups and binary relations*, European Journal of Combinatorics, **29** (2008), 1027-1218.

[41] W. G. Lister, *Ternary rings*, Trans. Amer. Math. Soc., **154** (1971), 37-55.

[42] W. J. Liu, *Fuzzy Invariant subgroups and fuzzy ideals*, Fuzzy Sets and Systems, **8** (1982), 133-139.

[43] F. Marty, *Sur une generalization de la notion de group*, 8th Congress Math. Scandenaves, Stockholm, (1934), 45-49.

[44] V. Maslov, *New superposition principle for optimization problems*, Sem. sur les Equations avec Dérivées Partielles 1985/6, Centre Mathématique de l'Ecole Polytechnique Palaiseau, esp. 24, 1986.

[45] S. Mirvakili and B. Davvaz, *Relations on Krasner (m, n)-hyperrings*, European Journal of Combinatorics, to appear.

[46] S. Mirvakili, S. M. Anvariyeh and B. Davvaz, *On α-relation and transitivity conditions of α*, Communications in Algebra, **36** (2008), 1695-1703.

[47] D. Monk and F. M. Sioson, *m-semigroups, semigroups, function representations*, Fund. Math., **59** (1966), 233-241.

[48] A. Nakassis, *Expository and survey article recent results in hyperring and hyperfield theory*, Internat. J. Math. Math. Sci., **11** (1988), 209-220.

[49] P. M. Pu and Y. M. Liu, *Fuzzy topology I, neighborhood structure of a fuzzy point and Moors-Smith convergence*, J. Math. Anal., **76** (1980), 571-599.

[50] E. L. Post, *Polyadic groups*, Trans. Amer. Math. Soc., **48** (1940), 208-350.

[51] F. Poyatos, *The Jordan-Hölder theorem for A-semimodules (Spanish)*, Rev. Mat. Hisp.-Amer., **32(4)** (1972), 251-260; ibid. **33(4)** (1973) 36-48; ibid. **33(4)** (1973), 122-132.

[52] A. Rosenfeld, *Fuzzy groups*, J. Math. Anal. Appl., **35** (1971), 512-517.

[53] S. A. Rusakov, *Some applications of n-ary group theory*, Belaruskaya Navuka, Minsk, 1998.

[54] M. K. Sen and M. R. Adhikari, *On k-ideals of semirings*, Internat. J. Math. & Math. Sci., **15** (1992), 347-350.
[55] F. M. Sioson, *Ideal theory in ternary semigroups*, Math. Jpn., **10** (1965), 63-84.
[56] T. Vougiouklis, *Hyperstructures and their representations*, Hadronic Press, USA, 1994.
[57] T. Vougiouklis, *On some representations of hypergroups*, Ann. Sci. Univ. Clermont-Ferrand II Math., **26** (1990), 21-29.
[58] T. Vougiouklis, *The fundamental relation in hyperrings, the general hyperfield*, Proc. Fourth Int. Congress on Algebraic Hyperstructures and Applications (AHA 1990), World Scientific, (1991), 203-211.
[59] X. H. Yuan, C. Zhang and Y. H. Ren, *Generalized fuzzy groups and many valued applications*, Fuzzy Sets and Systems, **138** (2003), 205-211.
[60] M. M. Zahedi, M. Bolurian and A. Hasankhani, *On polygroups and fuzzy subpolygroups*, J. Fuzzy Math., **3** (1995), 1-15.

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