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Free Vibration of Simply Supported Rectangular Composite Plates with Patch Mass

R. Akbari Alashti[1], R. Alibakhshi[2*]

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Abstract: The effect of a distributed patch mass on the natural frequency of vibration of a laminated rectangular plate with simply supported boundaries is investigated. The third order displacement field of a composite laminated rectangular plate is defined using the two- variable refined plate theory. Equations of motion of the plate are obtained with the help of the calculus of variation. Parametric study of non-dimensional natural frequencies of vibration is carried out and the effects of geometrical parameters such as the aspect ratio of the plate, size and location of the patch mass on these frequencies are studied. The results are then compared with those reported using the third order shear deformation theory. The findings are found to be in a very good agreement.

Keywords: Free Vibration, Patch Mass, Laminated Composite Plate, Refined Plate Theory

1. Introduction

Composite materials have found various engineering applications due to their high strength to weight ratio, high degrees of anisotropy and low rigidity in transverse shear compared with other materials, and are widely used in the automotive, aerospace and other application in structures. These materials are practically subjected to various loading conditions such as distributed patch mass, transverse and in-plane loadings, hence it is necessary to study their response to these loading conditions. Withney and Pagano [1] investigated free vibration response of a composite plate using the first order shear deformation theory (FSDT). This theory needs shear correction factor to rectify the unrealistic variation of the shear strain-stress through the thickness. To overcome the limitations of FSDT, higher order shear deformation theory (HSDT) were developed by Levinson [2], Reddy [3], Ren [4], Kant and Pandta [5] and Mohan et al. [6].

Singh et al. [7] studied natural frequencies of composite plates with random material properties using higher order shear deformation theory which rotary inertia effect was also considered. Noor [8] presented the exact three dimensional elasticity solutions of the vibration of isotropic, orthotropic and anisotropic composite plates. Reddy [9] carried out free vibration analysis of anti-symmetric angle-ply laminated plates considering the effect of transverse shear deformation using finite element method. Khdeir and Reddy [10] obtained the free vibration response of laminated composite plate with the help of the second order shear deformation theory. Wong [11] studied the effect of distributed patch mass on the plate vibration response. A two-variable refined plate theory (RPT2) which has two unknown functions was developed by Shimpi [12] for isotropic plates. Shimpi and Patel [13,14] could extend the two- variable refined plate theory for free vibration analysis of simply supported orthotropic plates. Alibeigloo et al. [15] studied the vibration response of anti-symmetric rectangular plates with distributed patch mass using the third order shear deformation theory (TSDT).

Seung-Eock et al. [16,17] employed the two- variable refined plate theory for vibration and buckling analysis of cross-ply and angle-ply laminated composite plates under the action of the transverse and in-plane force. While there are many papers on

1. Assistant Professor, Department of Mechanical Engineering, Babol University of Technology, Babol, Iran (raalashti@nit.ac.ir)
2*. Corresponding Author: M. Sc., Department of Mechanical Engineering, Babol University of Technology, Babol, Iran (sh_a_bakhshi@yahoo.com)

plate vibrations with added point masses, very few reports on plate vibration with patch mass can be found in this case such as Kompaz and Telli [18], and Wong [11]. The aim of this paper is to develop RPT2 for laminated composite plates with mass. The most important feature of this theory is that it does not require shear correction factor which is essential for making an equilibrium condition at the top and bottom faces of the plates, and has considerable similarities with the classical plate theory in boundary conditions, moment expressions and constitutive equations.

2. Basic formulation

A rectangular plate with length, width and thickness equal to *a*, *b* and *h* respectively is considered. The plate supports an arbitrary patch mass, M_{mass}, with dimensions equal to *c* and *d* in the *x* and *y*-direction respectively, with its centre positioned at (x', y') on the upper surface of the plate, as shown in Fig. 1. The global Cartesian coordinate system is chosen with the origin at the corner and on the middle plane of the plate, $z=0$. Therefore, the domain is defined as $0 \le x \le a,\quad 0 \le y \le b$ and $-h/2 \le z \le h/2$. In order to proceed with the formulation of the problem using the two- variable refined plate theory (RPT2), it is assumed that the displacement components (u,v,w) of the plate are small in comparison with the thickness of the plate.

Also, the transverse normal stress in the z-direction, σ_z, is very small in comparison with the in-plane stresses, σ_x and σ_y. In view of the above assumptions, the stress– strain relations can be reduced from a 6×6 matrix to a 5×5 matrix that reduces the complexity of the problem. The total displacements of the plate in the x and y-direction (U,V) are inclusive of three components, u, u_b, u_S and the total displacement in z-direction (W) is assumed to be consisting of three components, w_a, w_b and w_s which are the functions of x, y and time [16].

$$U = u - z(\partial w_b/\partial x) + z\left(\frac{1}{4} - \frac{5}{3}(z/h)^2\right)\frac{\partial w_s}{\partial x} \tag{1}$$

$$V = v - z(\partial w_b/\partial y) + z\left(\frac{1}{4} - \frac{5}{3}(z/h)^2\right)\frac{\partial w_s}{\partial y} \tag{1}$$

$$W = w_a + w_b + w_s$$

Where subscripts, *a*, *b* and *s* denote extension, bending and shearing components, respectively. In order to define the stress-strain relations in the geometrical coordinate system of the plate, that is the global Cartesian coordinate system, the components of the reduced stiffness tensor should be transformed according to the transformation law of the fourth order tensors. The strain energy and the kinetic energy of the plate are defined as:

$$U_{Strain} = \frac{1}{2}\int_V \sigma_{ij}\varepsilon_{ij}dV$$

$$T = \frac{1}{2}\int_V \rho\left(\dot{U}^2 + \dot{V}^2 + \dot{W}^2\right)dxdydz \tag{2}$$

As a matter of fact, the stress resultants of the total N layers of the plate are found in terms of the mid plane strain and curvatures of the plate.

3. Equations of motion

The total kinetic energy of the plate with the patch mass is the summation of the kinetic energies of the plate and the uniformly localized patch mass acting on the top surface of the plate:

$$T_{total} = T_{plate} + T_{mass} \tag{3}$$

$$\begin{aligned} T_{total} = {} & \frac{1}{2}\int_0^b\!\!\int_0^a I_0\left[\dot{u}^2 + \dot{v}^2 + (\dot{w}_a + \dot{w}_b + \dot{w}_s)^2\right]dxdy \\ & + \frac{1}{2}\int_0^b\!\!\int_0^a I_2\left[\left(\frac{\partial \dot{w}_b}{\partial x}\right)^2 + \left(\frac{\partial \dot{w}_b}{\partial y}\right)^2\right]dxdy \\ & + \frac{1}{2}\int_0^b\!\!\int_0^a \frac{I_2}{84}\left[\left(\frac{\partial \dot{w}_s}{\partial x}\right)^2 + \left(\frac{\partial \dot{w}_s}{\partial y}\right)^2\right]dxdy + \end{aligned} \tag{4}$$

$$\frac{1}{2}\int_{x'-c/2}^{x'+c/2}\int_{y'-d/2}^{y'+d/2} \rho_m h_m \left(\begin{aligned} & \left[\dot{u} - \frac{h}{2}\left(\frac{\partial \dot{w}_b}{\partial x}\right) - \frac{h}{12}\left(\frac{\partial \dot{w}_s}{\partial x}\right)\right]^2 \\ & + \left[\dot{v} - \frac{h}{2}\left(\frac{\partial \dot{w}_b}{\partial y}\right) - \frac{h}{12}\left(\frac{\partial \dot{w}_s}{\partial y}\right)\right]^2 \\ & + \left[\dot{w}_a + \dot{w}_b + \dot{w}_s\right]^2 \end{aligned}\right) dydx \Bigg|_{z=h/2}$$

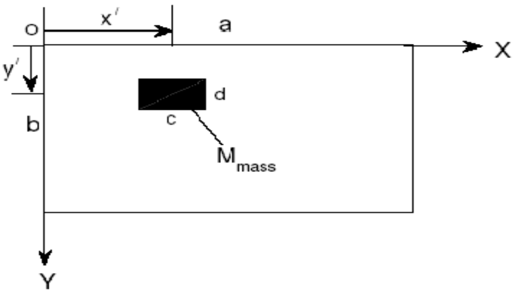

Fig. 1. A rectangular plate with a distributed patch mass

Where $I_0, I_2 = \int_{-h/2}^{h/2} \rho(1, z^2)dz$ are the inertia terms, ρ_m and h_m are the density of the patch mass and its thickness in the z-direction respectively. The first variation of the Lagrangian, (i.e., Hamilton's principle [19]) is employed to obtain the coefficients of mass and stiffness matrix.

$$\int_{t_1}^{t_2} \delta(U_{Strain} - T_{total}) = 0 \tag{5}$$

Where δ presents a variation with respect to x and y. Here U_{Strain} denotes the strain energy. Since this study here is in the free vibration analysis, the potential energy due to the applied force leads to zero. Substituting the displacements field in the relevant strain energy and kinetic energy terms, integrating the equations by parts and collecting the coefficients of $\delta u, \delta v, \delta w_b, \delta w_s$ and δw_a, the equations of motion for homogeneous laminates are derived as follows:

$$1.\ \delta u \rightarrow \frac{d}{dt}\Big(\int_0^a\int_0^b\left(\frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}\right)dydx+\int_0^a\int_0^b I_0\dot{u}^2dydx+\int_{x'-c/2}^{x'+c/2}\int_{y'-d/2}^{y'+d/2}3.125I_0\dot{u}^2dydx\Big)=0, \tag{6}$$

$$2.\ \delta v \rightarrow \frac{d}{dt}\Big(\int_0^a\int_0^b\left(\frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}\right)dydx+\int_0^a\int_0^b I_0\dot{v}^2dydx+\int_{x'-c/2}^{x'+c/2}\int_{y'-d/2}^{y'+d/2}3.125I_0\dot{v}^2dydx\Big)=0,$$

$$3.\ \delta w_b \rightarrow \frac{d}{dt}\Big(\int_0^a\int_0^b\left[\frac{\partial^2 M_x^b}{\partial x^2}+2\frac{\partial^2 M_{xy}^b}{\partial x\partial y}+\frac{\partial^2 M_y^b}{\partial y^2}\right]dydx+\int_0^a\int_0^b I_0(\dot{w}_a+\dot{w}_b+\dot{w}_s)^2dydx+\int_{x'-c/2}^{x'+c/2}\int_{y'-d/2}^{y'+d/2}3.125I_0(\dot{w}_a+\dot{w}_b+\dot{w}_s)^2dydx-\int_0^a\int_0^b I_2\left[\frac{\partial^2\dot{w}_b}{\partial x^2}+\frac{\partial^2\dot{w}_b}{\partial y^2}\right]dydx-\int_{x'-c/2}^{x'+c/2}\int_{y'-d/2}^{y'+d/2}9.375I_2\left[\frac{\partial^2\dot{w}_b}{\partial x^2}+\frac{\partial^2\dot{w}_b}{\partial y^2}\right]dydx\Big)=0,$$

$$4.\ \delta w_s \rightarrow \frac{d}{dt}\Big(\int_0^a\int_0^b\Big(\frac{\partial^2 M_x^s}{\partial x^2}+2\frac{\partial^2 M_{xy}^s}{\partial y^2}+\frac{\partial^2 M_y^s}{\partial y^2}+\frac{\partial Q_{xz}^s}{\partial x}+\frac{\partial Q_{yz}^s}{\partial y}\Big)dydx+\int_0^a\int_0^b I_0(\dot{w}_a+\dot{w}_b+\dot{w}_s)^2dydx-\frac{I_2}{84}\left[\left(\frac{\partial\dot{w}_s}{\partial x}\right)^2+\left(\frac{\partial\dot{w}_s}{\partial y}\right)^2\right]dydx+\int_{x'-c/2}^{x'+c/2}\int_{y'-d/2}^{y'+d/2}3.125I_0(\dot{w}_a+\dot{w}_b+\dot{w}_s)^2dydx-\int_{x'-c/2}^{x'+c/2}\int_{y'-d/2}^{y'+d/2}0.2604I_0\left[\left(\frac{\partial\dot{w}_s}{\partial x}\right)^2+\left(\frac{\partial\dot{w}_s}{\partial y}\right)^2\right]dydx\Big)=0,$$

$$5.\ \delta w_a \rightarrow \frac{d}{dt}\Big(\int_0^a\int_0^b\left(\frac{\partial Q_{xz}^a}{\partial x}+\frac{\partial Q_{yz}^a}{\partial y}\right)dydx+\int_0^a\int_0^b I_0(\dot{w}_a+\dot{w}_b+\dot{w}_s)^2dydx+\int_{x'-c/2}^{x'+c/2}\int_{y'-d/2}^{y'+d/2}3.125I_0(\dot{w}_a+\dot{w}_b+\dot{w}_s)^2dydx\Big)=0.$$

Finally, the sets of governing equations of plate vibration are obtained in the form of $([K]-\omega^2[S])\{\lambda\}=\{0\}$, where $[K]$ and $[S]$ are the stiffness and mass matrices respectively, ω is the natural frequency of vibration of the plate and λ is a vector of unknown coefficients. For convenience, the non-dimensional natural frequency of plate is defined as: $\overline{\omega}=\omega\sqrt{\rho a^4/E_2h^2}$.

Here, two sets of boundary conditions are considered. The first set of boundary condition is called the SS-1 boundary condition that is applied for an anti-symmetric cross-ply laminate and the second one is called SS-2 boundary condition for an anti-symmetric angle-ply laminate.

SS-1 boundary condition:

For $i = a, b, s$, $m = 0, a$ and $n = 0, b$

$$u(x,n)=v(m,y)=0,\ w_i(m,y)=w_i(x,n)=0,$$
$$=0, \partial w_i/\partial y(m,y)=\partial w_i/\partial x(x,n)$$
$$N_x(m,y)=N_y(x,n)=M_x^b(m,y)$$
$$=M_x^s(m,y)=M_y^b(x,n)=M_x^s(x,n)=0$$

SS-2 boundary condition:

For $i = a, b, s$, $m = 0, a$ and $n = 0, b$

$$u(m,y) = v(x,n) = 0,\ w_i(m,y) = w_i(x,n) = 0,$$
$$\partial w_i/\partial y\,(m,y) = \partial w_i/\partial x\,(x,n)$$
$$N_{xy}(0,y) = N_{xy}(x,0) = M_x^b(0,y) = M_y^b(x,0) = M_x^s(0,y) = M_x^s(x,0) = 0$$
$$N_x(a,y) = N_y(x,b) = M_x^b(a,y) = M_y^b(x,b) = M_x^s(a,y) = M_x^s(x,b)$$

In order to satisfy the boundary conditions, the following displacement fields are assumed:

$$\begin{Bmatrix} u \\ v \\ w_b \\ w_s \\ w_a \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn}\cos\alpha_m x \sin\beta_n y \\ V_{mn}\sin\alpha_m x \cos\beta_n y \\ W_{bmn}\sin\alpha_m x \sin\beta_n y \\ W_{smn}\sin\alpha_m x \sin\beta_n y \\ W_{amn}\sin\alpha_m x \sin\beta_n y \end{Bmatrix} \quad (7)$$

Where $\alpha = m\pi/a$, $\beta = n\pi/b$, and $(U_{mn}, V_{mn}, W_{bmn}, W_{smn}, W_{amn})$ are coefficients.

4. Numerical results and discussion

Two sets of dimensionless material properties are considered:

MAT1:
$E_1/E_2 = open, G_{12}/E_2 = 0.6, G_{13}/E_2 = G_{23}/E_2 = 0.5, \upsilon_{12} = 0.25$

MAT2:
$E_1/E_2 = 25, G_{23}/E_2 = 0.4, G_{13}/E_2 = G_{12}/E_2 = 0.6, \upsilon_{12} = 0.3$

At first, the vibration response of a plate without the patch mass is studied. The first non-dimensional natural frequencies of vibration for a four-layer anti-symmetric cross-ply laminate made of MAT1 with $G_{13} = G_{23} = 0.6E_2$ and $a/h = 5$ are found with different values of E_1/E_2 as a parameter. The results are compared with those obtained using the first order shear deformation theory [1], the 3D-Elasticity solution [8] and higher order shear deformation theory [9], as shown in Table 1. As it is expected, the results obtained by the present theory are in a very good agreement with the results of the higher shear deformation theory.

Now, a distributed patch mass at the centre of the plate with the following properties is considered.

$$M_{mass}/M_{plate} = 0.5,\ c/a = d/b = 0.4 \rightarrow$$

$$\frac{\rho_m h_m cd}{\rho hab} = 0.5 \rightarrow \rho_m h_m = 3.125\rho h = 3.125 I_0$$

The effect of the distributed patch mass on the first non-dimensional natural frequency of angle ply laminated plates made of MAT1, $[45/-45]_2$ with different ratios of a/h and a/b are studied and the results obtained by the present method (RPT2) are compared with those obtained by (TSDT) method [15], as shown in Table 2. It is observed that, as the a/b ratio increases (the a/h ratio being constant), the first natural frequency increases. Comparison of the results of the first natural frequency between RPT2 and TSDT methods with distributed patch mass is shown in Fig. 2.

In the next step, the distributed patch mass is considered in three different positions on the plate.

These three positions of the distributed patch mass are:

1) $x'_1 = a/4,\ y'_1 = b/2$
2) $x'_2 = a/2,\ y'_2 = b/4$
3) $x'_3 = a/6,\ y'_3 = b/5$

and other parameters related to the distributed patch mass are:

$$M_{mass}/M_{plate} = 0.3,\ c/a = d/b = 0.2$$

The effect of position of the distributed patch mass on the first natural frequency of an angle-ply laminated plate made of MAT2 with $[30/-30]_2$ lamination is studied and the results are shown in Table 3. It is observed from Table 3, that the natural frequency of the plate with distributed patch mass at position 3 is higher than the other two positions under study, that is due to its larger stiffness induced comparing with the other two positions. The non-dimensional first natural frequency of a rectangular plate made of MAT2 for $[45/-45]_2$ lamination for four different positions of the patch mass is shown in Fig. 3. Due to the symmetry imposed by the boundary conditions of the plate, it is observed that the patch mass in position 1 and position 2 would have similar natural frequencies of vibration.

Table 1. The first non-dimensional natural frequency (ϖ), anti-symmetric cross-ply square plate, $(0/90)_2$, MAT1 with $G_{13}=G_{23}=0.6E_2$, $a/h=5$

	E_1/E_2				
Theory	**3**	**10**	**20**	**30**	**40**
FSDT[1]	6.4402	8.1963	9.6729	10.6095	11.2635
3D-E[8]	6.5455	8.1445	9.4055	10.165	10.6798
HSDT[9]	6.5008	8.1954	9.6265	10.5348	11.1716
Proposed	6.5008	8.1948	9.6251	10.5333	11.1705

Table 2. The non-dimensional first natural frequency (ϖ), MAT1, $[45/-45]_2$

a/b	Theory	a/h				
		10	**20**	**30**	**40**	**50**
0.2	TSDT [15]	5.3728	5.8127	5.9079	5.9423	5.9585
0.2	Proposed	5.4030	5.8227	5.8488	5.9122	5.9446
0.6	TSDT [15]	7.8768	9.0055	9.2748	9.3751	9.4227
0.6	Proposed	7.9366	9.0264	9.2845	9.3803	9.4258
0.8	TSDT [15]	9.5102	11.1744	11.5923	11.7503	11.8256
0.8	Proposed	9.5487	11.1861	11.5973	11.7527	11.8269
1.0	TSDT [15]	11.2448	13.5765	14.1934	14.4304	14.5442
1.0	Proposed	11.2868	13.5870	14.1974	14.4321	14.5448
1.2	TSDT [15]	13.0434	16.1777	17.0518	17.3932	17.5585
1.2	Proposed	13.1349	16.2067	17.0646	17.4000	17.5624
1.6	TSDT [15]	16.8026	21.9349	23.5225	24.1660	24.4826
1.6	Proposed	17.1292	22.0784	23.5961	24.2092	24.5103
2.0	TSDT [15]	20.7951	28.4084	31.0067	32.1021	32.6506
2.0	Proposed	21.4663	28.7637	31.2024	32.2214	32.7294

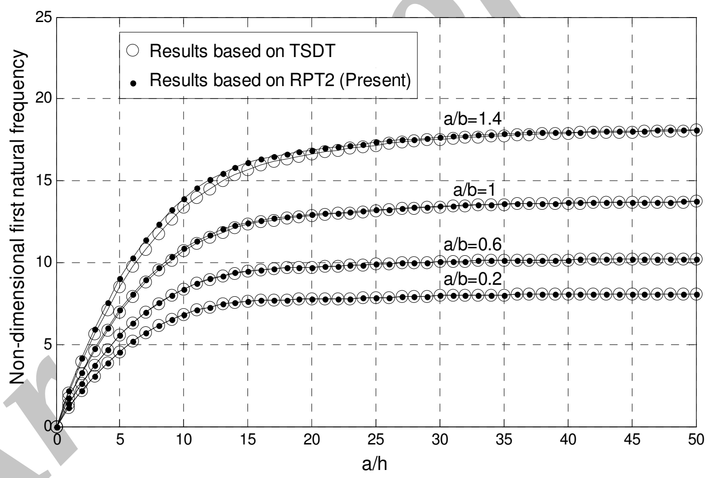

Fig. 2. Comparison of (ϖ) obtained by (RPT2) and (TSDT) methods, MAT1, $[45/-45]_2$.

Table 3. The effect of a/b and a/h on (ϖ) of a plate with patch mass, MAT2, $[30/-30]_2$

a/b	a/h				
	10	**20**	**30**	**40**	**50**
Pose 1					
0.2	7.4084	8.0523	8.1928	8.2440	8.2680
0.8	10.4399	11.6925	11.9822	12.0892	12.1397
1.4	15.3371	17.7550	18.3529	18.5779	18.6852
2	21.3574	25.5037	26.5983	27.0189	27.2210
Pose 2					
0.2	7.4322	8.0611	8.1971	8.2464	8.2696
0.8	10.4516	11.6974	11.9846	12.0906	12.1407
1.4	15.3048	17.7390	18.3445	18.5730	18.6819
2	21.2429	25.4380	26.5627	26.9973	27.2067
Pose 3					
0.2	8.7569	9.5884	9.7614	9.8235	9.8524
0.8	12.2495	13.9077	14.2712	14.4030	14.4648
1.4	17.8157	21.0767	21.8435	22.1259	22.2592
2	24.5518	30.1950	31.6253	32.1623	32.4175

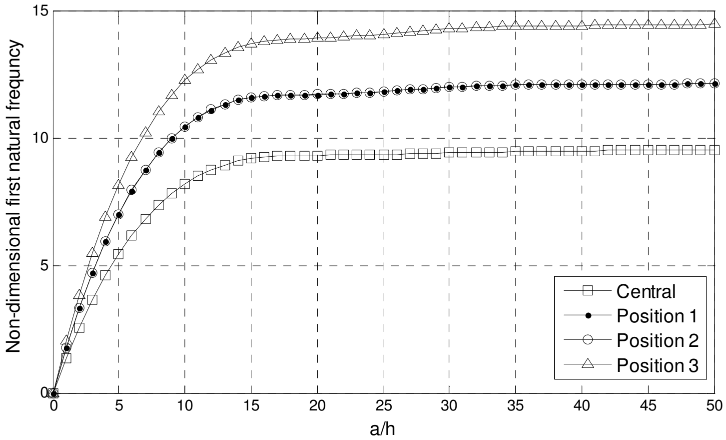

Fig. 3. Comparison of $(\overline{\omega})$ of a rectangular plate with patch mass, four positions, MAT2, $[45/-45]_2$.

5. Conclusions

In this paper, the free vibration of a rectangular composite plate carrying a distributed patch mass was presented. The governing equations have been obtained by the two-variable refined plate theory and the effect of aspect ratios, size and location of the patch mass on the free vibration of the plate studied. The main conclusions are as:

- The first natural frequency of vibration of a plate increases with an increase in the ratio of *a/b* (with *a/h* being constant).
- The results obtained by the two-variable refined plate theory is in a very good agreement with the results obtained by the higher order shear deformation theory.
- The lowest natural frequency of plates with symmetric boundary conditions occurs with the patch mass at the centre of the plate. The natural frequency increases with the patch mass moving towards the corner of the plate.

Nomenclature

w_a	displacement term in extension in the z-direction
w_b	displacement term in bending in the z-direction
w_s	displacement term in shear in the z-direction
σ_{ij}	Stress in the local coordinate
ε_{ij}	Strain in the local coordinate
G_{ij}	Modulus of elasticity in shear in the ij-direction
E_i	Modulus of elasticity in tension and compression in the i-direction
ν_{ij}	Poisson's ratio

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