Optimal Motion Generating of Nonholonomic Manipulators with Elastic Revolute Joints in Generalized Point-to-Point Task

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Abstract: This paper presents optimal control method to path planning of mobile robots with flexible joints. Dynamic equations are derived and additional kinematic constraints are used to solve the extra Dofs arose from base mobility. Then with modeling the elasticity at each joint as a linear torsinal spring, the set of equations are formed. The Hamiltonian function is formed and the necessary conditions for optimality are derived from the Pontryagin's minimum principle. The obtained equations established a two point boundary value problem solved by numerical techniques. This problem is known to be complex in particular when combined motion of the base and manipulator, non-holonomic constraint of base and non-linear and complicated dynamic equations as a result of flexible nature of joints are taken into account. The study emphasizes on modeling of the complete optimal control problem by remaining all nonlinear state and costate variables as well as control constraints to establish accompanying boundary value problem. Another advantage of this method is obtaining various optimal trajectories with different characteristics by changing the penalty matrices values which enables the designer to choose the best trajectory. A mobile flexible joint manipulator is studied to verify the feasibility of the proposed approach.

Keywords: Mobile Manipulator- Flexible Joint- Trajectory Planning- Open Loop Optimal Control Problem

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1 INTRODUCTION

A mobile manipulator is basically a robotic arm mounted on a moving base and can be used to perform variety of tasks that are mostly related to materials handling application such as mining, construction, forestry, planetary exploration and the military. It has been determined that planning trajectory of such robots is a complex task that plays a crucial role in design and manufacturing of robots in task space. Hence, in many industrial applications such as high-speed assembly and heavy load carrying, the joint flexibility exists in most manipulators in the drive transmission systems (transmission belts, gears, shafts, etc.), that usually neglected to analysis of such systems.

Several different research works have been carried out for mobile robot arms, but in most analyses only Maximum Allowable Dynamic Load (MADL) are investigated [1] and limited research has been studied on path planning and finding optimal trajectory of mobile manipulators [2]. Open-loop optimal control method is proposed as an approach for trajectory optimization of flexible link mobile manipulator for a given two-end-point task in point-to-point motion [3]. A comprehensive literature survey on mobile manipulator systems can be found in refrence [4]. Korayem and Nikoobin formulated the flexible joint manipulators and found the maximum dynamic load carrying capacity of such robots with optimal control approach [5].

In above mentioned works only mobility of base or flexibility of joints has been considered, and the synthesis of mobile base with flexible joints robots has not been studied. In [6] a computational algorithm to MADL determination is presented on the basis of Iterative Linear Programming (ILP) approach for flexible mobile manipulators. But in these studies the end effector trajectories are given and path planning of such robots are not considered, Hence there is a need to re-investigate the modeling and analysis of the mobile robots with flexible joints to find the end effector trajectory and associated joints variables in the related movement.

Optimal control problems can be solved with direct and indirect techniques. In the direct method at, first the control and state variables are discretized and the optimal control problem is transcribed into a large, constrained and often sparse nonlinear programming problem; Then, the resulting nonlinear programming problem is treated by standard algorithm like interior point methods [7]. Famous realizations of direct methods are direct shooting methods [8] or direct collocation methods [9]. However, direct methods do not yield to exact results. They are exhaustively time consuming and quite inefficient due to the large number of parameters involved. Consequently, when the solution of highly complex problems such as the structural analysis of optimal control problems in robotics is required, the indirect method is a more suitable candidate. This method is widely used as an accurate and powerful tool in analyzing of the nonlinear systems.

The indirect method is characterized by a "first optimize, then discretize" strategy. Hence, the problem of optimal control is first transformed into a piecewise defined multipoint boundary value problem, which contains the full mathematical information about the respective optimal control problem. In the following step, this boundary value problem is discretized to achieve the numerical solution [10]. It is well known that this technique is conceptually fertile, and has given rise to far-reaching mathematical developments in the wide ranges of optimal dynamic motion planning problems. For example, it is employed in the path planning of flexible link manipulators [11], for the actuated kinematic chains [12] and for a large multibody systems [13]. A survey on this method is found in [14].

This paper focuses on analysis and path designing of wheeled mobile manipulators with joint flexibility. For the sake of this, open loop optimal control method is proposed. Dynamic equations of the link mobile manipulator are derived and the extra DOFs arose from base mobility are solved by using additional constraint functions and the augmented Jacobian matrix. After considering the equations that arises from joint flexibility, the complete form of the obtained equation in state-space form is used to forming Hamiltonian function for a proper objective function, and necessary conditions for optimality are obtained from the Pontryagin's minimum principle. The obtained equations establish a Two Point Boundary Value Problem (TPBVP) that was solved by numerical techniques. Finally, a two-link flexible joint manipulator with mobile base is simulated to illustrate the performance of the method.

2 MODELLING OF A MOBILE MANIPULATOR WITH MULTIPLE FLEXIBLE JOINTS

In mobile manipulators by defining $\vec{q} = \left(q_b{}^T, q_r{}^T\right)^T$ as generalized coordinate of the system that $\vec{q}_b$ is the generalized coordinates defining the mobile base motion, $\vec{q}_r$ is the generalized coordinates of the rigid body motion of links; When the link number of a flexible joint manipulator is m, position of the *i*th link is shown with $\theta_{2i-1} : i = 1,2,...,m$ and the position of the *i*th actuator with $\theta_{2i} : i = 1,2,...,m$; It is usual in the flexible joint manipulators literature to arrange these angles in a vector as [6]:

$$Q = \left[\theta_1, \theta_3, ...\theta_{2m-1} \mid \theta_2, \theta_4, ...\theta_{2m}\right]^T = [q_1^T, q_2^T]^T \qquad (1)$$

So by adding the joint flexibility with considering the elastic mechanical coupling between the *i*th joint the link is modeled as a linear torsional spring with constant stiffness coefficient k_i; The set of equations of motion comprising mobile base both link and joint flexibility can be rearranged into the following form:

$$M(q_1)\ddot{q}_1 + H(q_1,\dot{q}_1) + G(q_1) + K(q_1 - q_2) = 0$$
$$J\ddot{q}_2 + K(q_2 - q_1) = U \tag{2}$$

Where $K = diag\ [k_1, k_2, ..., k_m]$ is a diagonal stiffness matrix which models the joint elasticity, $J = diag\ [J_1, J_2, ..., J_m]$ is the diagonal matrix representing motor inertia, and U is the generalized force inserted into the actuator.

3 STATE-SPACE FORM OF DYNAMIC EQUATION

The system dynamics can be decomposed into two parts: one is corresponding to redundant set of variables, q_r, and another is corresponding to non-redundant set of them, q_{nr}. That is,

$$\begin{bmatrix} U_r \\ U_{nr} \end{bmatrix} = \begin{bmatrix} M_{r,r} & M_{r,nr} \\ M_{r,nr} & M_{nr,nr} \end{bmatrix} \begin{bmatrix} \ddot{q}_r \\ \ddot{q}_{nr} \end{bmatrix} + \begin{bmatrix} C_r + G_r \\ C_{nr} + G_{nr} \end{bmatrix}. \tag{3}$$

By considering the row associated with non-redundant part defining the state vector can be obtained as:

$$X_1 = q_1\ ;\ X_2 = \dot{q}_1\ ;\ X_3 = q_2\ ;\ X_4 = \dot{q}_2 \tag{4}$$

That q_1 is the non-redundant set of variables and $\dot{q}_1$ is its velocity. The state space form of Eq. (2) with remaining non-redundant part can be shown as:

$$\dot{X}_1 = X_2 \quad , \quad \dot{X}_2 = N(X_1, X_2) + D(X_1)K(X_3 - X_1)$$
$$\dot{X}_3 = X_4 \quad , \quad \dot{X}_4 = J^{-1}(U + K(X_1 - X_3)) \tag{5}$$

Where $N = -M_{nr}{}^{-1}(H_{nr}(X_1, X_2) + G_{nr}(X_1))$ and $D = M_{nr}{}^{-1}$. Then optimal control problem is to determine the joint and link trajectory $X_1(t)$ and $X_3(t)$, and their related velocity $X_2(t)$ and $X_4(t)$ and the joint torque $U(t)$ which optimize a well-defined performance measure when the model is given in [6].

4 FORMULATION OF THE OPTIMAL CONTROL PROBLEM

The basic idea to improve the formulation is to find the optimal path for a specified payload. For the sake of this, the following objective function is considered

$$\underset{U(t)}{Minimize}\, J_0 = \int_{t_0}^{t_f} L(X,U)dt\ , \tag{6}$$

where

$$L(X,U) = \frac{1}{2}\|X_1\|_{W_1}^2 + \frac{1}{2}\|X_2\|_{W_2}^2 + \frac{1}{2}\|U\|_R^2\ . \tag{7}$$

$\| X \|_K^2 = X^T K X$ is the generalized squared norm, W_1 and W_2 are symmetric, positive semi-definite ($m \times m$) weighting matrices and R is symmetric, positive definite ($m \times m$) matrices.

A remarkable emphasis is that the study is planned a trajectory in the joint space rather than in the operating space. It means the control system acts on the manipulator joints rather than on the end effector. Trajectory planning in the joint space would allow avoiding the problems arising with kinematic singularities and manipulator redundancy. Moreover, it would be easier to adjust the trajectory according to the design requirements if working in the joint space. By controlling manipulator joints can achieve the best dynamic coordination of joint motions, while minimizing the actuating inputs together with bounding the velocity magnitudes. It causes to ensure soft and efficient functioning while improving the manipulator working performances.

According to the Pontryagin's minimum principle, the following conditions must be satisfied

$$\dot{X} = \partial H / \partial \psi\ ; \qquad \dot{\psi} = -\partial H / \partial X \quad ;\ 0 = \partial H / \partial U \tag{8}$$

where by defining the nonzero costate vector $\psi = \begin{bmatrix} \psi_1^T & \psi_2^T \end{bmatrix}^T$ the Hamiltonian function can be obtained as:

$$\begin{aligned} \mathrm{H}(\mathrm{X},\mathrm{U},\psi) &= 0.5(\|X_1\|_{W_1}^2 + \|X_2\|_{W_2}^2 + \|U\|_R^2) \\ &\quad + \psi_1^T X_2 + \psi_2^T [N(X) + D(X)U] \end{aligned}. \tag{9}$$

The control values are limited with upper and lower bounds, so using Eq. (8) the optimal control is given by

$$U = \begin{cases} U^+ & -R^{-1}D^T\psi_2 > U^+ \\ -R^{-1}D^T\psi_2 & U^- < -R^{-1}D^T\psi_2 < U^+ \\ U^- & -R^{-1}D^T\psi_2 < U^- \end{cases} \tag{10}$$

The actuators which are used for medium and small size manipulators are the permanent magnet D.C. motor. The torque speed characteristic of such D.C. motors may be represented by the following linear equation:

$$U^+ = K_1 - K_2 X_2\ , U^- = -K_1 - K_2 X_2\ . \tag{11}$$

where $K_1 = [\tau_{s1}\ \tau_{s2} \cdots \tau_{sn}]^T$, $K_2 = dig[\tau_{s1}/\omega_{m1} \cdots \tau_{sn}/\omega_{mn}]$, $\dot{\theta} = [\dot{\theta}_1 \quad \dot{\theta}_2 \quad ... \quad \dot{\theta}_n]^T$, τ_s is the stall torque and ω_m is the maximum no-load speed of the motor, and the boundary values will be expressed as below:

$$X_1(0) = X_{10},\ X_2(0) = X_{20}\ ;\quad X_1(t_f) = X_{1f},\ X_2(t_f) = X_{2f} \tag{12}$$

In this formulation, for a specified payload value, 4m differential equations given in Eq. (8) are used to determine the 4m state and costate variables. The set of differential equations, control law and the boundary conditions Eq. (12) construct a standard form of TPBVP, which is solvable with available commands in different software such as MATLAB and MATHEMATICA.

5 SIMULATION FOR A FLEXIBLE PLANAR WHEELED MOBILE MANIPULATOR

A two-link planar flexible joint manipulator is mounted on a differentially driven mobile base at point F on the main axis of the base as shown in Fig.1. A concentrated payload of mass m_p is connected to the second link. All required parameters of the robot manipulator are given in Table 1.

Table 1 Simulation Parameters

Parameter	Value	Unit
Length of links	L_1=L_2=1	m
Mass of links	m_1=2, m_2=2	Kg
Max. no load speed	$w_{s1} = w_{s2}$ =4	Rad/s
Actuator stall torque	$\tau_{s1} = \tau_{s2} = 40$	N.m
Spring constant	$k_1 = k_2$ =1000	N/m
Moment of inertia	$J_1 = J_2 = 2$	Kg. m^2

In this simulation, the load will be carried from an initial to a final point during the overall time t_f = 1.5 seconds. The end effector degrees of freedom in the cartesian coordinate system will be m = 2. The system degree of freedom is equal to n = 5, hence the system has redundancy of order R = n-m = 3 and needs three additional kinematical constraints for proper coordination. Meanwhile, the mobile base has one non-holonomic constraint (c = 1) that it assures stability of the system from tipping over: $\dot{x}_f \sin(\theta_0) - \dot{y}_f \cos(\theta_0) + L_0\dot{\theta}_0 = 0$.

Hence, the number of kinematical constraints which must be applied to system for redundancy resolution is equal to r = R-c = 2. In this case, with the previously specified base trajectory during the motion, the user-specified additional constraints can be considered as the base position coordinates at point $F(x_f, y_f)$, which gives $x_f = X_{1z}$ and $y_f = X_{2z}$. X_{1z} and X_{2z} are functions in terms of. A fifth order polynomial function is considered for the base trajectory along a straight-line path from (0.5, 0.5) to (2, 1) during the overall time 1.5 seconds.

By defining the state vectors as:

$$X_1 = Q^T = [x_1 \quad x_3 \quad x_5 \quad x_7]^T,\qquad X_2 = \dot{Q}^T = [x_2 \quad x_4 \quad x_6 \quad x_8]^T. \tag{13}$$

In order to derive the equations associated with optimality conditions, penalty matrices can be selected as:

$$W_1 = diag(w_1, w_3, w_5, w_7);\quad W_2 = diag(w_2, w_4, w_6, w_8)\quad R = diag(r_1, r_2). \tag{14}$$

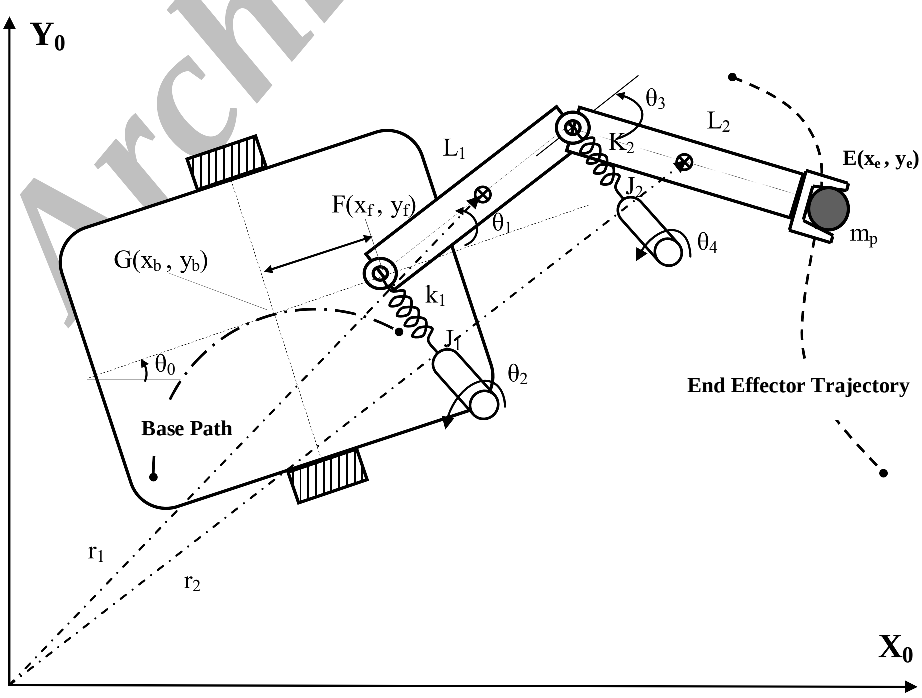

Fig. 1 Two-link mobile manipulator with flexible joints.

The boundary conditions can be expressed as:

$$
\begin{aligned}
&x_{10} = x_{50} = 60^{\circ}, x_{30} = x_{70} = -60^{\circ};\\
&x_{1f} = x_{5f} = -60^{\circ}, x_{30} = x_{7f} = 60^{\circ}\\
&x_{2i0} = x_{2if} = 0, \quad i = 1...4
\end{aligned}
\tag{15}
$$

In this case, the payload is considered to be 3 kg and the purpose is to find the optimal path between initial and final point of payload in such a way that the smallest control value can be applied and the angular velocity values of motors be bounded in ± 2 rad/s.

By considering the penalty matrices as: W_1=W_2= [0] and R=diag(0.01), the optimal path with minimum effort can be obtained, but the angular velocities are greater than ± 2 rad/s. Therefore for decreasing the velocities, W_2 must be increased. A range of values of W_2 which is used in simulation are given in TABLE II. W_1 and R remain without changes.

Table 2 Value of W_2 used in simulation

Case	1	2	3
K	Diag(0)	Diag(10)	Diag(100)

Figure 2 shows the angular position of joints with respect to time. This graph shows that by increasing the W_2, the angular position change to approach approximately to a straight line. Figure 3 shows the angular velocities of the first and second joints that shows reducing the velocities are the result of growing the W_2. The computed torque is plotted in Figure 4. As it can be seen, increasing the W_2 causes to raise the torques, so that for the last case the torque curves reach to their bounds at the beginning and end of the path.

This result is predictable, because increasing the W_2, decreases the proportion of R and the result of this is increasing the control values. Moreover it can be found from these Figures., for the trajectory with greater value of W_2, the oscillation amplitudes in velocity curves have been reduced considerably, but the magnitude of motor torques have been increased. Also, it is clearly observed that smaller speed path is smoother than the path with higher speed with the smaller amount of effort. The link positions in case 2 and end-effector trajectories in XY plane are shown in Figure 5 and Figure 6 respectively. Finally, the angular velocities of links and motors in case 2 are given in Figure 7.

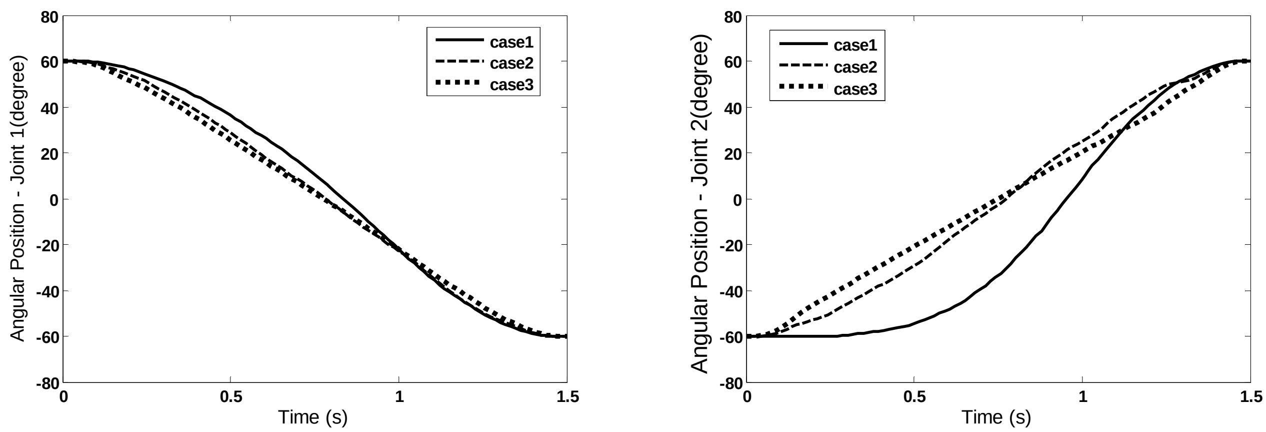

Fig. 2 Angular positions of joint 1 and 2.

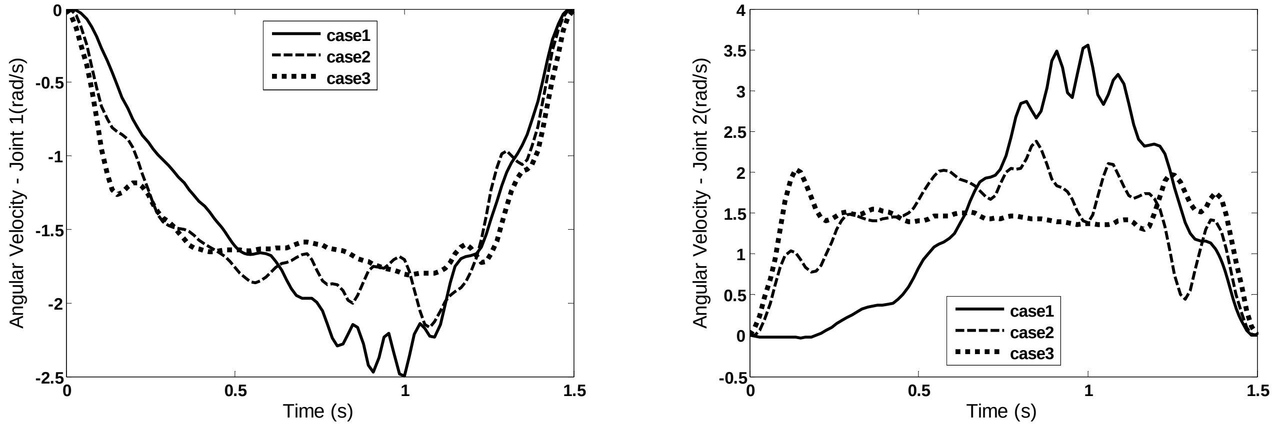

Fig. 3 Angular velocities of joints 1 and 2.

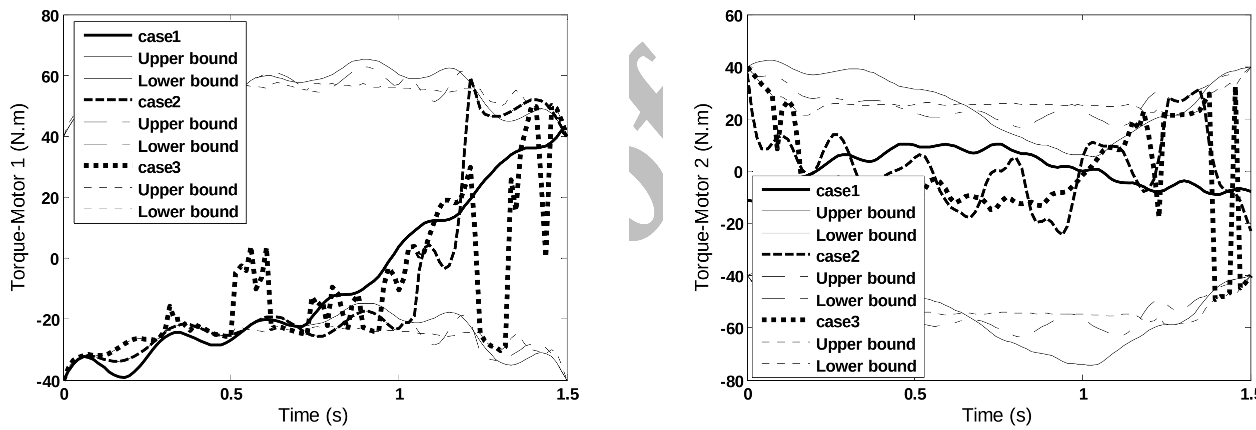

Fig. 4 Torques of motors 1 and 2.

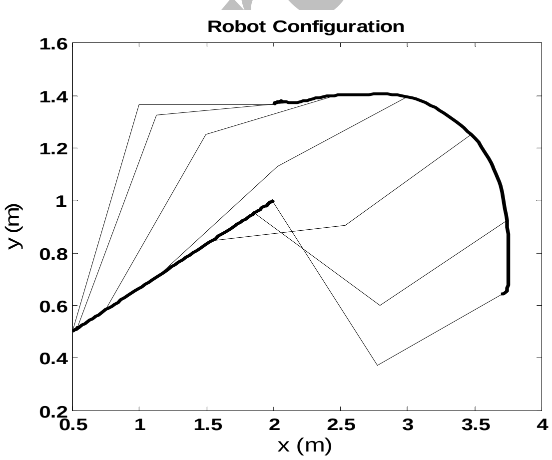

Fig. 5 Robot configuration in Cartesian plane.

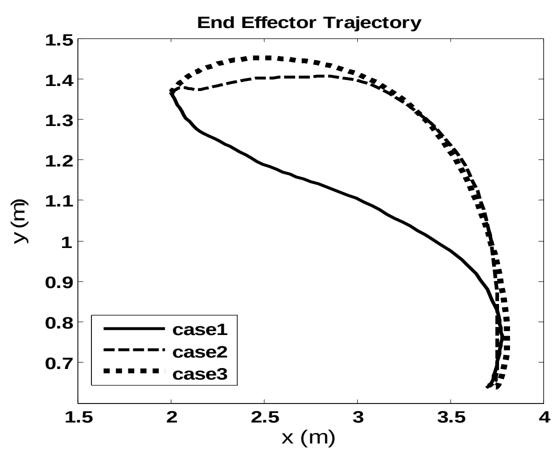

Fig. 6 End-effector trajectory in XY plane.

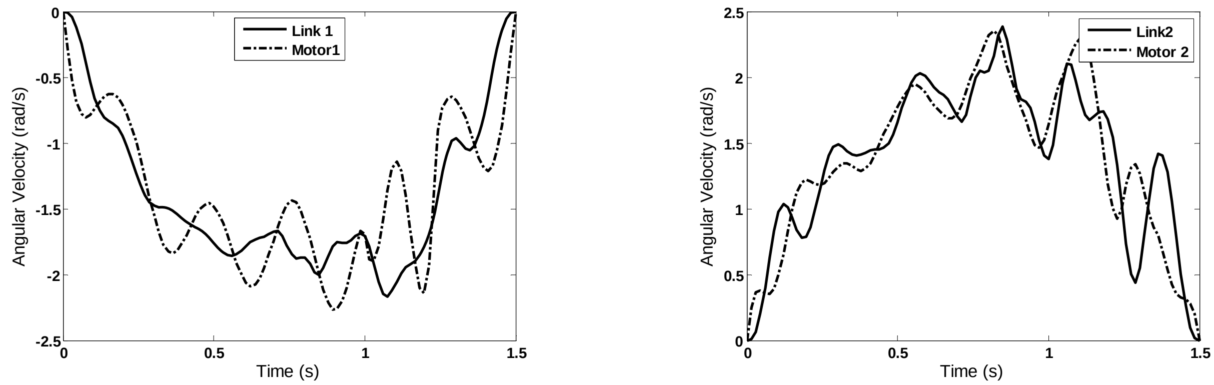

Fig. 7 Angular velocities of links and motors.

It can be seen on the basis of the objective contrast principle, the solution dose not satisfie all the desired objectives simultaneously. For example the optimal path with minimum effort has maximum velocity and the optimal path with minimum velocity has maximum effort. Consequently, in this method, designer compromises between different objectives by considering the proper penalty matrices.

6 DIFFERENT JOINT STIFFNESS TRAJECTORY

In this section, the effect of joint stiffness in performance characteristics of the robot is investigated. Penalty matrices are considered to be: W_1=W_2= [0] and R=diag (0.01).

Table 3 The values of K used in simulation

Case	1	2	3
K	Diag(500)	Diag(3000)	Rigid joints

The K values used in the simulation are given in Table 3. All simulation parameters are the same parameters as used in the previous section.

Angular velocities of joints are presented in figure 8. It is observed from figures that, increasing the joint stiffness caused the reducing oscillatory behavior of the system. Also, it is evident that increasing the elasticity in joints enlarges bounds of velocity. The computed torqueses are plotted in figure 9. It shows changing the joint stiffness changs the torque curves; in the way that decreasing the stiffness of joints caused to increase the torques of motors. The angular velocities of links and motors in case 1 are given in figure 10. It shows that both the link angular velocities have deviations from their respective rotor velocities. Thus, it is clear that joint flexibility significantly affects the link vibrations. Finally, end-effector trajectories in XY plane at case 1 is shown in figure 11.

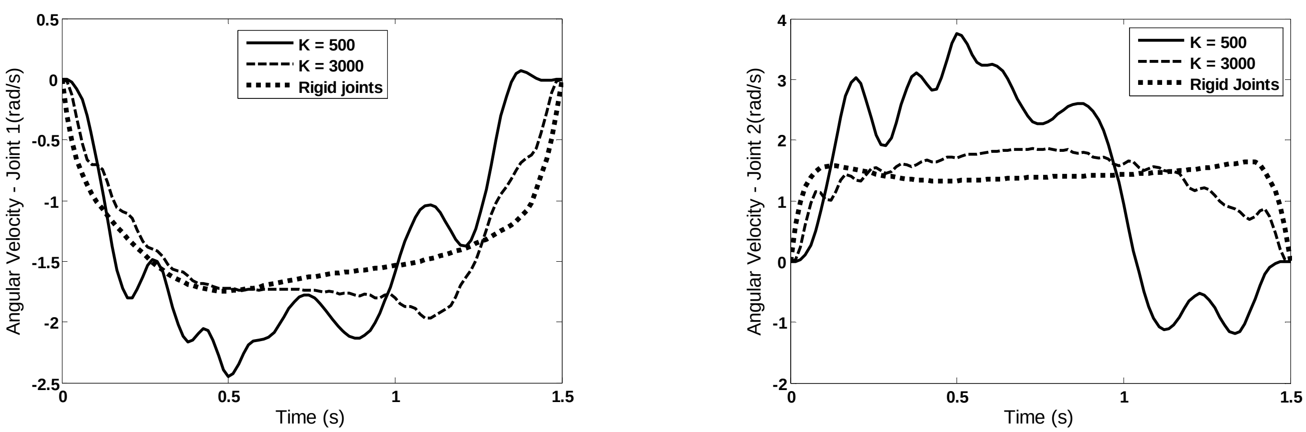

Fig. 8 Angular velocities of joints 1 and 2.

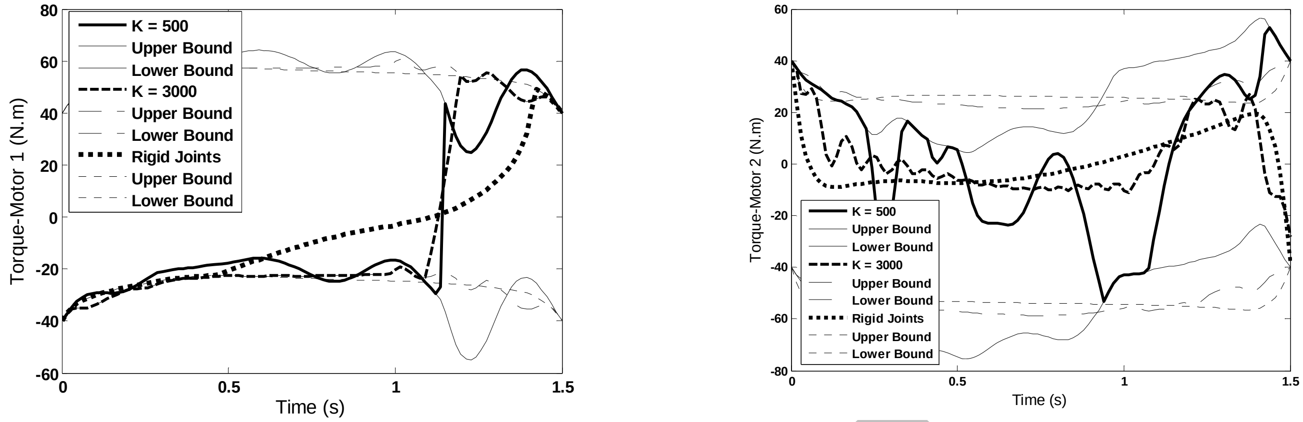

Fig. 9 Torques of motors 1 and 2.

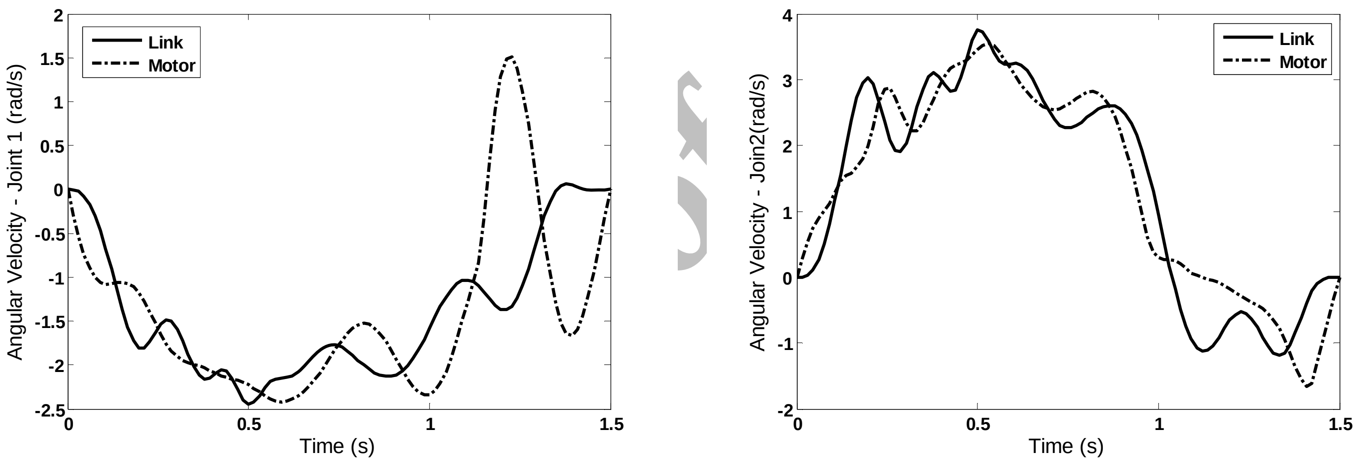

Fig. 10 Angular velocities of links and motors.

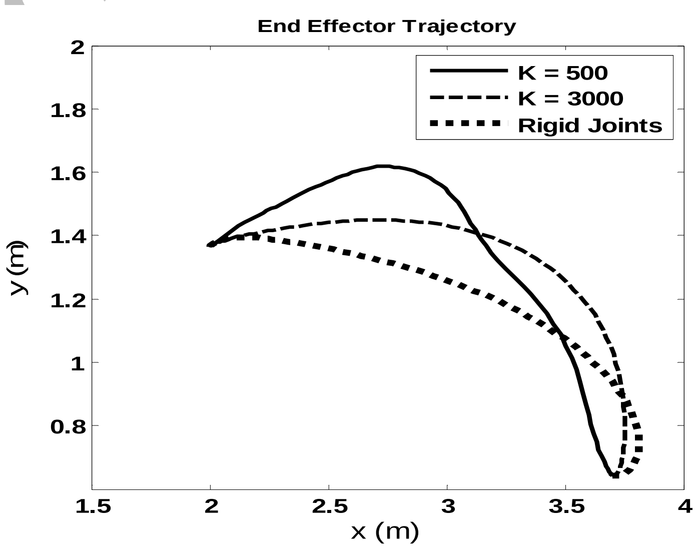

Fig. 11 End-effector trajectory in XY plane.

7 CONCLUSION

In this paper, formulation of the trajectory optimization for mobile flexible joint manipulator in point-to-point motion, based on the open-loop optimal control approach is presented. This formulation can be used for path planning of flexible mobile manipulators via defining the proper objective function and changing the penalty matrices to achieve the desired requirements. Therefore, an efficient solution on the basis of TPBVP is proposed to optimize the path in order to achieve the predefined objective. The main advantage of this method is that designer can compromise between different objectives by considering the proper penalty matrices and is able to choose the proper trajectory among the various paths. Highlighting the main contribution of the paper can be presented as:

- The proposed approach can be adapted to any general serial manipulator including both non-redundant and redundant systems with flexible joints and base mobility.
- In this approach the non-holonomic constraints do not appear in TPBVP directly, unlike the method given in [15] and [16].
- This approach allows completely nonlinear states and control constraints treated without any simplifications.
- The obtained results illustrate the power and efficiency of the method to overcome the high nonlinearity nature of the optimization problem, which with other methods, it may be very difficult or impossible.
- In this method, boundary conditions are satisfied exactly, while the results obtained by ILP method have a considerable error in final time.
- In this method, designer is able to choose the most appropriate path among various optimal paths by considering the proper penalty matrices.
- The optimal trajectory and corresponding input control obtained using this method can be used as a reference signal and feed forward command in control structure of such manipulators.

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