Maximizing First Natural Frequency of 2D Structures Using Bee Colony Optimization (BCO) and Fuzzy Logic

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Abstract: Nowadays, different methods are used to solve the optimization problems. One of the newest methods is BCO, which is inspired of the nature and derived from bee's interacting with each other. It can be noted that BCO is derived from the bees' social life. In this paper, the fuzzy logic is used for bee's decision stage. In a nutshell, we solve two different problems that have already been solved by ESO (Evolution Structural Optimization) and BESO (Bi direction Evolution Structural Optimization) methods. In this study we demonstrate that the BCO is able to achieve better solutions and it is quite appropriate even for constrained problems.

Keywords: Fuzzy Logic, Finite Element, Meta-heuristics, Natural Frequency

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1 INTRODUCTION

Nature-Inspired Algorithms are motivated by a variety of biological and natural processes. The popularity of the Nature-Inspired Algorithms is primarily caused by the ability of biological systems to effectively adjust to frequently changeable environment. Evolutionary computation, neural networks, ant colony optimization, particle swarm optimization, artificial immune systems, and bacteria foraging algorithm are the algorithms and concepts that were motivated by nature. Swarm behavior is one of the main characteristics of different colonies of social insects (bees, wasps, ants, etc.). This type of behavior is first and foremost characterized by autonomy, distributed functioning and self-organizing. Swarm Intelligence is the area of Artificial Intelligence that is based on study of actions of individuals in various decentralized systems [1]. When creating Swarm Intelligence models and techniques, researchers apply some principles of the natural swarm intelligence.

The BCO meta-heuristic has recently been used as a toll for solving large and complex real-world problems. It has been shown that the BCO poses an ability to find high quality solutions of difficult combinatorial problems within a reasonable amount of computer time. The BCO is a stochastic, random-search technique. This technique uses an analogy between the way in which bees in nature search for a food, and the way in which optimization algorithms search for an optimum of (given) combinatorial optimization problems. The basic idea behind the BCO is to build the multi agent system (colony of artificial bees) able to effectively solve difficult combinatorial optimization problems. Artificial bees investigate through the search space looking for the feasible solutions. In order to find better and better solutions, autonomous artificial bees collaborate and exchange information. Using collective knowledge and sharing information among themselves, artificial bees concentrate on more promising areas, and slowly abandon solutions from the less promising areas. Step by step, artificial bees collectively generate and/or improve their solutions. The BCO search is running in iterations until some predefined stopping criteria is satisfied.

Reference [2] shows the new methods of bee colony optimization that fuzzy logic is used for decision stage. The Fuzzy Bee System (FBS) capable of solving combinatorial optimization problems is characterized by uncertainty. In this paper, for solving optimization problems, during decision making stage (bees), BCO method and fuzzy logic is utilized. The work shows the proposed BCO can be effectively used to solve optimization problems for 2D structures considering Natural Frequency.

This paper is organized as follows: in section 2, the bee colony optimization is described. In Section 3, topology optimization problems and material interpolation scheme is derived. In Section 4, various examples are studied and advantages of the BCO are discussed. At the end in Section 5, the conclusions are presented.

2 THE BEECOLONY OPTIMIZATION: THENEW COMPUTATIONALPARADIGM

Social insects (bees, wasps, ants, etc.) have lived on Earth for millions of years, building nests and more complex dwellings, organizing production and procuring food. The colonies of social insects are very flexible and can adapt well to the changing environment. This flexibility allows the colony of social insects to be robust and maintain its life in spite of considerable disturbances.

The dynamics of the social insect population is a result of different actions and interactions of individual insects with each other, as well as with their environment. The interactions are executed via multitude of various chemical and/or physical signals. The final product of different actions and interactions represents social insect colony behavior. Interaction between individual insects in the colony of social insects has been well documented. These communication systems between individual insects contribute to the formation of the "collective intelligence" of the social insect colonies.

2.1. BEES IN THE NATURE

Self-organization of bees is based on a few relatively simple rules of individual insect's behavior. The best example is the collection and processing of nectar, the practice of which is highly organized. Each bee decides to reach the nectar source by following a nest mate who has already discovered a patch of flowers. Each hive has a so-called dance floor area in which the bees that have discovered nectar sources dance, in that way trying to convince their nest mates to follow them. If a bee decides to leave the hive to get nectar, she follows one of the bee dancers to one of the nectar areas. Upon arrival, the foraging bee takes a load of nectar and returns to the hive relinquishing the nectar to a food store bee. After she relinquishes the food, the bee can (a) abandon the food source and become again uncommitted follower, (b) continue to forage at the food source without recruiting the nest mates, or (c) dance and thus recruit the nest mates before the return to the food source. The bee opts for one of the above alternatives with a certain probability. Within the dance area, the bee dancers "advertise" different food areas.

The mechanisms by which the bee decides to follow a specific dancer are not well understood, but it is considered that "the recruitment among bees is always a function of the quality of the food source" [3]. It is also noted that not all bees start foraging simultaneously. The experiments confirmed, "new bees begin foraging at a rate proportional to the difference between the eventual total and the number presently foraging". In ref [4, 5], it is shown that who was the first in using the basic principles of collective bee intelligence in solving combinational optimization problems. They introduced the Bee System (BS) and tested it in the case of Traveling Salesman Problem. The Bee Colony Optimization (BCO) Meta heuristic that has been proposed in this paper represents further improvement and generalization of the Bee System, and the basic characteristics of the BCO Meta heuristic are described.

2.2. THE BEE COLONY OPTIMIZATION META HEURISTIC

Within the Bee Colony Optimization Meta heuristic (BCO), agents collaborate in order to solve difficult combinatorial optimization problem. All bees are located in the hive at the beginning of the search process. During the search process, bees communicate directly. Each bee makes a series of local moves, and in this way incrementally constructs a solution for the problem. Bees are adding solution components to the current partial solution until they create one or more feasible solutions. These arch process is composed of iterations. The first iteration is finished when bees create one or more feasible solutions for the first time. The best discovered solution during the first iteration is saved, and then the second iteration begins. Within the second iteration, bees again incrementally construct solutions of the problem, etc. There are one or more partial solutions at the end of each iteration. The analyst-decision maker prescribes the total number of iterations.

When flying through the space our bees perform forward pass or backward pass. During forward pass, bees create various partial solutions. They do this via a combination of individual exploration and collective experience from the past. After that, they perform backward pass, i.e. they return to the hive. In the hive, all bees participate in a decision-making process. We assume that every bee can obtain the information about solutions'quality generated by all other bees. In this way, bees exchange information about quality of the partial solutions created. Bees compare all generated partial solutions. Based on the quality of the partial solutions generated, every bee decides whether to abandon the created partial solution and become again uncommitted follower, continue to expand the same partial solution without recruiting the nest mates, or dance and thus recruit the nest mates before returning to the created partial solution. Depending on the quality of the partial solutions generated, every bee possesses certain level of loyalty to the path leading to the previously discovered partial solution. During the second forward pass, bees expand previously created partial solutions, and after that perform again the backward pass and return to the hive. In the hive bees again participate in a decision-making process, perform third forward pass, etc. The iteration ends when one or more feasible solutions are created.

Like Dynamic Programming, the BCO also solves combinatorial optimization problems in stages. Each of the defined stages in volves one optimizing variable. Let us denote by $ST=\{st_1,st_2,\ldots,st_m\}$ a finite set of pre-selected stages, where 'm' is the number of stages. By 'B' we denote the number of bees to participate in the search process and by 'I' the total number of iterations. These to f partial solutions at stages t_j is denoted by S_j (j=1, 2, ..., m). The following is pseudo-code of the Bee Colony Optimization [2]:

Bee Colony Optimization

1. Initialization Determine the number of bees 'B', and the number of iterations 'I' Select the set of stages ST= $\{st_1, st_2, \ldots, st_m\}$. Find any feasible solution 'x' of the problem. This solution is the initial best solution.
2. Set i=1Untili=I, repeat the following steps.
3. Set j = 1 Until j = m, repeat the following steps.

Forward pass: Allow bees to fly from the hive and to choose 'B' partial solutions from the set of partial solutions S_j at stages t_j.

Backward pass: Send all bees back to the hive. Allow bees to exchange information about quality of the partial solutions created and to decide whether to abandon the created partial solution and become again uncommitted follower, continue to expand the same partial solution without recruiting the nest mates, or dance and thus recruit the nest mates before returning to the created partial solution.

Set, j:=j+1.

4. If the best solution x_i obtained during the i^{th} iteration is better than the best known solution, update the best known solution ($x = x_i$).
5. Set, i:=i+1.

Alternatively, forward and backward passes could be performed until some other stopping condition is satisfied. The possible stopping conditions could be for example, the maximum total number of forward/backward passes, or the maximum total number of forward/backward passes between two objective function value improvements.

Within the proposed BCO Meta heuristic, various sub-models describing bees' behavior and/or "reasoning" could be developed and tested. In other words, various BCO algorithms could be developed. These models should describe the ways in which bees decide to abandon the created partial solution, to continue to expand the same partial solution without recruiting the nest mates, or to dance and thus recruit the nest mates before returning to the created partial solution [2].

2.3. THE FUZZY BEE SYSTEM

Bees face many decision-making problems while searching for the best solution. The following are bees' choice dilemmas:

1. What is the next solution component to be added to the partial solution?
2. Should the partial solution be abandon or not?
3. Should the same partial solution be expanded without recruiting the nest mates?

The majority of the choice models are based on random utility modeling concepts. These approaches are highly rational. They are based on assumptions that decision-makers possess perfect information processing capabilities and always behave in a rational way (trying to maximize utilities). In order to offer alternative modeling approach, researchers started to use less normative theories. The basic concepts of Fuzzy Sets Theory, linguistic variables, approximate reasoning, and computing with words introduced by Zadeh, have more understanding for uncertainty, imprecision, and linguistically expressed observations [6], [7]. According to this idea in the chosen model, this assumption is considered that bees are using fuzzy logic.

Artificial bees use approximate reasoning and rules of fuzzy logic in their communication and acting [6-8]. During the j^{th} stage bees fly from the hive and choose 'B' partial solutions from the set of partial solutions S_i at stage st_j (forward pass).

When adding the solution component to the current partial solution during the forward pass, specific bee perceives specific solution component as "less attractive", "attractive", or "very attractive". We also assume that an artificial bee can perceive a specific attributes as "short", "medium" or "long", "cheap", "medium", or "expensive", etc. (Figure 1).

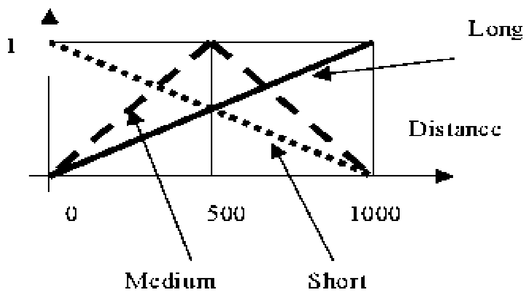


Fig. 1 Fuzzy sets describing distance [2]

2.3.1. Calculating the solution component attractiveness and choice of the next solution component to be added to the partial solution

The approximate reasoning algorithm for calculating the solution component attractiveness consists of the rules of the following type:

If the attributes of the solution component are VERY GOOD
Then the considered solution component is VERYATTRACTIVE

The main advantage of using the approximate reasoning algorithm for calculating the solution component attractiveness is that it is possible to calculate solution component attractiveness even if some of the input data were only approximately known. Let us denote by f_i the attractiveness value of solution component 'i'. The probability p_i for solution component 'i' to be added to the partial solution is equal to the ratio of f_i to the sum of all considered solution component attractiveness values, Eq. (1) [2].

$$P_i = \frac{f_i}{\sum_j^m f_i} \tag{1}$$

2.3.2. Bee's partial solutions comparison mechanism

In order to describe bee's partial solutions comparison mechanism, we introduce the concept of partial solution badness. We define partial solution badness in the Eq. (2).

$$L_K = \frac{L^{(K)} - L_{MIN}}{L_{MAX} - L_{MIN}} \tag{2}$$

Where:
L_K: badness of the partial solution discovered by the k^{th} bee
$L^{(k)}$: the objective function values of the partial solution discovered by the k^{th} bee

L_{min}: the objective function value of the best discovered partial solution from the beginning of the search process
L_{max}: the objective function value of the worst discovered partial solution from the beginning of the search process

The approximate reasoning algorithm to determine the partial solution badness consists of the rules of the following type:

If the discovered partial solution is BAD

Then loyalty is LOW

Bees use approximate reasoning, and compare their discovered partial solutions with the best, and the worst discovered partial solution from the beginning of the search process. In this way, "historical facts" discovered by all members of the bee colony have significant influence on the future search directions [2].

2.3.3. Calculating the number of bees changing the path

Every partial solution (partial path) that is being advertised in the dance area has two main attributes:
(a) the objective function value, and (b) the number of bees that are advertising the partial solution (partial path). The latter number is a good indicator of bees' collective knowledge. It shows how bee colony perceives specific partial solutions. The approximate reasoning algorithm to determine the advertised partial solution attractiveness consists of the rules of the following type:

If the length of the advertised path is SHORT and the number of bees advertising the path is SMALL

Then the advertised partial solution attractiveness is MEDIUM

Path attractiveness calculated in this way can take values from the interval [0,1]. The higher the calculated value, the more attractive is advertised path. Bees are less or more loyal to "old" paths. At the same time, advertised paths are less, or more attractive to bees. Let us note paths p_i and p_j. We denote by n_{ij} the number of bees that will abandon path p_i, and join nest mates who will fly along path p_j. The approximate reasoning algorithm to calculate the number of shifting bees consists of the rules of the following type:

If bees' loyalty to path p_i is LOW and path p_j's attractiveness is HIGH
Then the number of shifting bees from path p_i to path p_j is HIGH

In this way, the number of bees flying along specific path is changed before beginning of the new forward pass. Using collective knowledge and sharing information, bees concentrate on more promising search paths, and slowly abandon less promising ones [2].

3 TOPOLOGY OPTIMIZATION PROBLEMS AND MATERIAL INTERPOLATION SCHEME

3.1. TOPOLOGY OPTIMIZATION PROBLEMS

In the finite element analysis, the dynamic behavior of a continuum structure can be represented by the following general eigen value problem by Eq. (3).

$$(K-\omega^2 M)\, u_i = 0 \tag{3}$$

Where 'K' is the global stiffness matrix and 'M' is the global mass matrix, 'i' is the i^{th} natural frequency and u_i is the eigenvector corresponding to ω_i. The natural frequency ω_i and the corresponding eigenvector u_i are related to each other by Rayleigh quotient, Eq. (4).

$$\omega_i^2 = \frac{u_i^T K u_i}{u_i^T M u_i} \tag{4}$$

Here, we consider the topology optimization problems for maximization of the i^{th} natural frequency of vibrating continuum structures. For a solid-void design, the optimization problem can be stated as Eq. (5).

$$\begin{aligned} &\text{Maximize: } \omega_i \\ &\text{Subject to: } V^* - \sum_{i=1}^{N} (V_i x_i) = 0 \\ &x_i \geq x_{min} \text{ or } 1 \end{aligned} \tag{5}$$

Where V_i is the volume of an individual element and V^* is the prescribed structural volume. 'N' is the total number of elements in the structure. The binary design variable x_i denotes the density of the i^{th} element and small value x_{min} (e.g. 10^{-3}) is used to denote a void element [9].

3.2. Material interpolation scheme

To obtain the gradient information of the design variable, it is necessary to interpolate the material between x_{min} and 1. A popular material interpolation scheme is the so-called power-law penalization model (the SIMP model (solid isotropic material with penalization)). For a solid-void design, the material density and Young's modulus are assumed to be functions of the design variable x_i as Eq. (6).

$$\begin{aligned} &\rho(x_i) = (x_i)\, \rho^{1} \\ &E(x_i) = (x_i)^P E^{1} \end{aligned} \tag{6}$$

($0<x_{min}\leq x_i\leq 1$)

(Normally $p\geq 3$is used in the SIMPmodel for topology optimization problems [9]. Therefore, $p=3$ is used throughout this paper), where ρ^{1} and E^{1} are the density and Young's modulus of solid material.

4 NUMERICAL EXAMPLES

Example 1:

The example presented in this section is two-dimensional plane stress problem; with only in-plane vibration considered .The elements used here are of the four-nodded linear quadrilateral type. The two driving criteria are the minimization of the mean compliance and the maximization of the first mode of natural frequency, as shown in Fig. 2 (Reference [9] shows this example based on a rectangular plate model that was used by Xie and Steven in their study of ESO for dynamic problems).

A rectangular aluminum plate of dimension 0.15m×0.1m is fixed at two diagonal corners, with two horizontal loads (each 100 N) applied on the other two diagonal corners as shown in Fig. 2. These are included for the linear static stress analysis, but are removed for the frequency analysis. The physical data are as follows:

Young's modulus E =70 G Pa, Poisson's ratio ν=0.3, thickness t =0.01m, and density ρ =2700 kg/m^3. The domain is divided into 45×30 square elements (The first mode natural frequency, 2498.9 Hz and mean compliance,0.0001751 N.m).

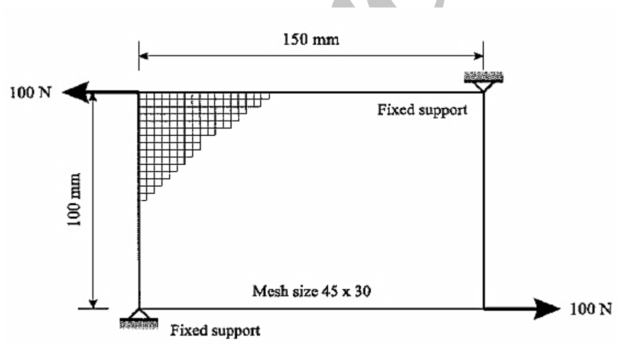


Fig. 2 Initial design domain of rectangular plate under loading with fixed supports [10]

This example is multi objective (i.e.: minimization of the mean compliance and the maximization of the first mode of natural frequency). For this purpose, we use weighted sum method (WSM). Table 1 and figure 3 shows the comparison between the first mode natural frequency and the mean compliance of the structure for a range of different weightings of the criteria and for a 30% volume reduction (w_s is weightings factor of stiffness and w_f is weightings factor of frequency). As presented in Table 1, in addition to the fact that the initial frequency in the BCO method is more than the ECO method, nonetheless, the main compliance 'c' (dependent on the structure stiffness) of BCO method is also less than the ESO method, where this is an indication of ability of this method in solving such problems.

Table 1 Results of solved problems, using ESO and BCO methods

Weighting factor	ESO		BCO	
	C(N.m)	First Mode Natural frequency (Hz)	C(N.m)	First Mode Natural frequency (Hz)
ws=0.2, wf=0.8	0.000205	2950	0.00017721	3041.9
ws=0.1, wf=0.9	0.00025	2990	0.00017679	3031
ws=0.3, wf=0.7	0.0001977	2940	0.00017881	3024

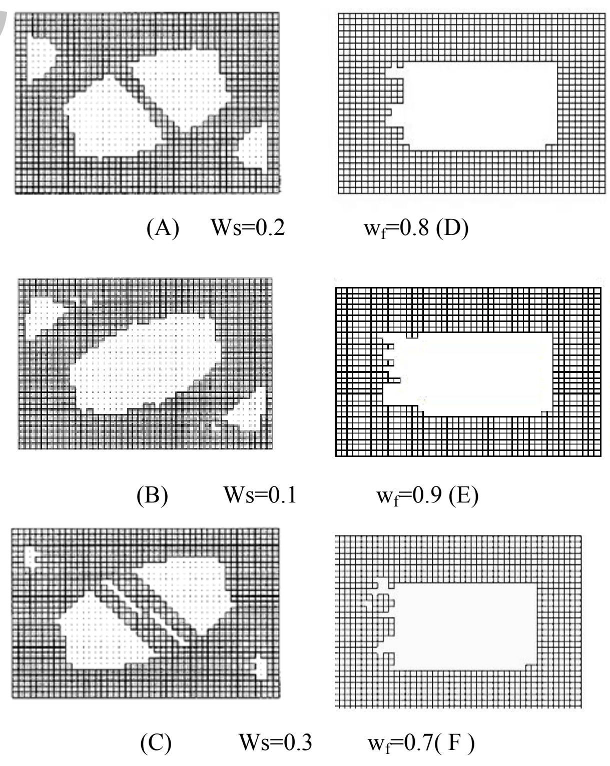


Fig. 3 Optimal designs of rectangular plate for different weighting criteria of stiffness and natural frequency; material removed, 30%. (A)-(C) [10], (D) – (F) BCO Method

Example 2

Fig. 4 shows a plate supported at its two diagonal corners with a full design domain of dimensions0.15 × 0.1 m. The Young's modulus E = 70 G Pa, Poisson's ratio v= 0.3, thickness t = 0.01 m, and density r = 2,700 kg/m^3 are assumed. The design domain is divided into 50 × 50 rectangular elements. The prescribed weight is 50% of the full design domain [11].

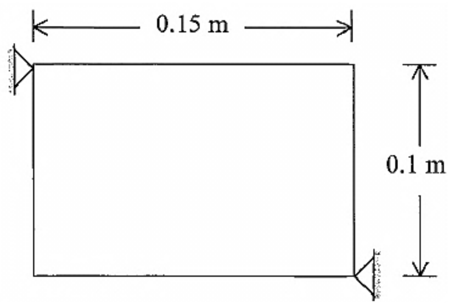
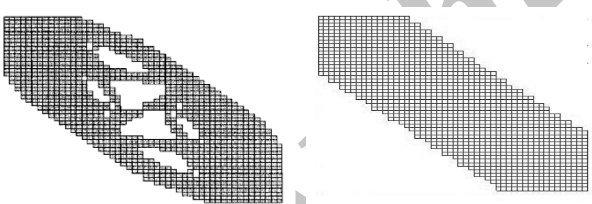


Fig. 4 Diagonally Supported Rectangular Plate [11]

Table 2 Results of solving example 2 by ESO,BESO [11] and BCO method

Method	First Mode Natural Frequency (Hz)
ESO	2845
BESO	2667.6
BCO	3395.98



(A) (B)

Fig. 5 Optimal designs of rectangular plate for First natural frequency; material removed, 50% (A) [11], (B) BCO Method

This example is not similar to the first example in solving multi-objective problems; the reason in choosing such a problem is to show the ability of BCO method in solving single objective problems as well. There are already examples of ways ESO and BESO has been solved in reference [11]. The results shown in Table 2 and Fig. 5, reveals the superiority of the BCO method over two other techniques.

5 CONCLUSION

The vibration of mechanical systems has been a major concern for scientists and engineers for several centuries. During this time, almost all new mechanical designs required some kind of vibration study. A multi criterion structural optimization design methodology has been developed which eliminates most of the costly trial-and-error testing currently required.

In this study, we have demonstrated that a multiple criterion optimization algorithm based on a weighting method can be introduced into the BCO method so that can solve a wide range of stiffness and frequency optimization problems.

In this paper, we use this method by using finite element method to solve two examples which had different objective functions. In the first multi-objective problem, the objective is to increase the initial frequency and increasing the structure stiffness, while in the second problem, the objective is merely increasing the initial frequency. As already mentioned, the BCO method could be successful in both examples and achieve better results than ESO and BESO methods. Therefore, we conclude that optimization method of BCO for solving such problems could be more reliable and efficient.

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