Thermo-Mechanical Nonlinear Vibration in Nano-Composites Polyethylene Shell Reinforced by CNT's Embedded Elastic

M. Raminnia
Department of Mechanical Engineering,
Islamic Azad University-Khomeinishahr Branch, Iran
E-mail: Morteza.Raminnia@iaukhsh.ac.ir
*Corresponding author

A. Ghorbanpour Arani
Department of Mechanical Engineering,
University of Kashan, Country Iran
E-mail: Aghorban@kashan.ac.ir

A. Manouchehrifar
Department of Mechanical Engineering,
Islamic Azad University-Khomeinishahr Branch, Iran
E-mail: Manouchehri@iaukhsh.ac.ir

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Abstract: In this study, thermo-mechanical nonlinear vibration of a polyethylene (PE) cylindrical shell embedded in an elastic foundation was investigated. The shell is reinforced by armchair Carbon nanotubes (CNTs) where characteristics of the equivalent composite being determined using Mori-Tanaka model. The elastic medium is simulated using the spring constant of the Winkler-type. The governing equations were obtained by employing nonlinear terms of strains-displacements based on Donell's theory, stress-strain relation, first order shear deformation theory and Hamilton's principal. Differential quadrature method (DQM) is used to calculate the nonlinear frequency of the shell. The influences of geometrical parameters, orientation angle of CNTs and elastic foundation constants on the nonlinear vibration of the shell were investigated. Results showed that the nonlinear effect represented by nonlinear frequency ratio is considerable at lower K_w.

Keywords: Cylindrical Shell, DQM, Elastic Foundation, Nonlinear Vibration, Nano-Composites

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Biographical notes: M. Raminnia received his MSc from Islamic Azad University of Khomeinishahr in Isfahan, Iran, in 2010. His current research interests are nanotubes, and FGMs. **A. Ghorbanpour Arani** received his PhD from Isfahan University of Technology in Isfahan, Iran, in 2001. His current research interests are stress analyses, stability and vibration of nanotubes, and FGMs. **A. Manouchehrifar** is Assistant Professor of Mechanical engineering at Islamic Azad University of Khomeinishahr, Iran. He received his PhD and MSc in Aviation engineering from Moscow Aviation Institute-MAI, Russia and his BSc in Mechanical engineering from Isfahan University of Technology-IUT, Iran. His current research focuses on Helicopter Design, stability and vibration.

1 INTRODUCTION

The use of composite materials is rapidly increasing because of their advantageous properties such as high specific strength/modulus, corrosion resistance and fatigue life. For example, the Boeing 787 uses composite materials accounting for about 50% of structural weight which leads to significantly increased fuel efficiency and reduced part count compared to the similar sized airplanes. Composite materials are also being used in other industries such as automotive and various sporting goods. Composite materials result from the integration of two or more distinct components (fiber and matrix) such that superior physical and mechanical properties are realized. In addition, some composite materials have other advantages, like electrical conductivity and thermal properties, which make them suitable as multifunctional materials.

The development of multifunctional composite materials/structures is aimed at providing innovative functionality to structures in addition to their load carrying capability [1]. NASA's Marshall Space Flight Center has established fabrication techniques for the manufacture of composite material pipes/shells and storage vessels of conventional and unique shapes which possess characteristics like lightweight, corrosion resistance, capable of handling diverse fluids, large diameter pipes for high pressure applications, capable of withstanding high temperatures [1].

There is available abundant literature on isotropic shells dealing with the thermal buckling subjected to constant temperature, temperature varying in circumferential direction and temperature varying along the generator of the shell. As pointed by Earl Thornton in his recent review paper on thermal buckling of plates and shells, the availability of studies on thermal buckling in composite shells is scarce. Thangaratnam et al. have performed linear thermal buckling analysis of laminated composite cylindrical and conical shells using finite element method and examined the nature of buckling under thermal load and mechanical load with respect to different fiber orientation [2], [3]. Radhamohan and Venkataramana have made a complete study of thermal buckling of composite cylindrical shell made of fiberglass reinforced plastics [4].

A uniform temperature rise throughout the shell was considered for buckling studies. Very recently the authors have also contributed to the research literature on the thermal buckling studies and the influence of axi-symmetric temperature variation on the natural frequencies of the composite cylindrical shells [5]. Birman and Bert considered studies on the buckling and post buckling response of composite shells subjected to high temperature using the equilibrium equations for shells under the simultaneous action of thermal and axial load [6].

Similar to thermal buckling studies of isotropic shells, studies on pipes/shells conveying fluids are abundant in literature. Paidoussis and Li have reviewed and compiled the exhaustive literature available on the studies related with the dynamics of pipes conveying fluids [7]. Contributions by Paidoussis in the area of dynamics of pipes conveying fluid is enormous and one can find a host of problems and solutions associated with fluid flowing through slender structures in the book written by Professor M.P. Paidoussis [8].

This paper makes the first attempt to study thermo mechanical nonlinear vibration in Nano-composites cylindrical shell reinforced by CNT's for embedded gas pipes using Hamilton's principle and DQM. The effects of geometrical parameters, orientation angle of CNTs and elastic foundation constants on the nonlinear vibration of the shell were investigated.

2 MORI-TANAKA MODEL

In this section, the effective modulus of the composite shell reinforced by CNTs is developed. Different methods are available to estimate the overall properties of a composite [9]. Due to its simplicity and accuracy even at high volume fractions of the inclusions, the Mori-Tanaka method is employed in this section [10]. To begin with, the CNTs are assumed to be aligned and straight with the dispersion of uniform in the polymer. The matrix is assumed to be elastic and isotropic, with the Young's modulus and the Poisson's ratio. The constitutive relations for a layer of the composite with the principal axes parallel to the r,θ and z directions are as follows [11]:

$$\begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{23}\\ \sigma_{13}\\ \sigma_{12}\end{Bmatrix}=\begin{bmatrix}k+m & l & k-m & 0 & 0 & 0\\ l & n & l & 0 & 0 & 0\\ k-m & l & k+m & 0 & 0 & 0\\ 0 & 0 & 0 & P & 0 & 0\\ 0 & 0 & 0 & 0 & m & 0\\ 0 & 0 & 0 & 0 & 0 & P\end{bmatrix}\begin{Bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{33}\\ \gamma_{23}\\ \gamma_{13}\\ \gamma_{12}\end{Bmatrix} \tag{1}$$

Where σ_{ij} , ε_{ij} , γ_{ij} , k, m, n, l, p are the stress components, the strain components and the stiffness coefficients respectively? According to the Mori-Tanaka method the stiffness coefficients are given by the following relations [10]:

$$k = \frac{E_m\{E_m c_m + 2k_r(1+\nu_m)[1+c_r(1-2\nu_m)]\}}{2(1+\nu_m)[E_m(1+c_r-2\nu_m)+2c_m k_r(1-\nu_m-2\nu_m^2)]}$$

$$l = \frac{E_m\{c_m\nu_m[E_m+2k_r(1+\nu_m)]+2c_r l_r(1-\nu_m^2)]\}}{(1+\nu_m)[E_m(1+c_r-2\nu_m)+2c_m k_r(1-\nu_m-2\nu_m^2)]}$$

$$n = \frac{E_m^2 c_m(1+c_r-c_m\nu_m)+2c_m c_r(k_r n_r-l_r^2)(1+\nu_m)^2(1-2\nu_m)}{(1+\nu_m)[E_m(1+c_r-2\nu_m)+2c_m k_r(1-\nu_m-2\nu_m^2)]} + \frac{E_m[2c_m^2 k_r(1-\nu_m)+c_r n_r(1+c_r-2\nu_m)-4c_m l_r\nu_m]}{E_m(1+c_r-2\nu_m)+2c_m k_r(1-\nu_m-2\nu_m^2)}$$

$$p = \frac{E_m[E_m c_m+2p_r(1+\nu_m)(1+c_r)]}{2(1+\nu_m)[E_m(1+c_r)+2c_m p_r(1+\nu_m)]}$$

$$m = \frac{E_m[E_m c_m+2m_r(1+\nu_m)(3+c_r-4\nu_m)]}{2(1+\nu_m)\{E_m[c_m+4c_r(1-\nu_m)]+2c_m m_r(3-\nu_m-4\nu_m^2)\}} \quad (2)$$

Where C_m and C_γ are the volume fractions of the matrix and the CNTs respectively and k_r, l_r, n_r, p_r, m_r are the Hills elastic modulus for the CNTs.

3 STRAIN-DISPLACEMENT RELATIONSHIPS

In order to calculate the middle-surface strain and curvatures, using Kirchhoff-Law assumptions, the displacement components of an arbitrary point may be written as follows [12]:

$$\begin{aligned}
\mathrm{u}(x,\theta,z) &= u_0(x,\theta) - z\,\frac{\partial w(x,\theta)}{\partial x},\\
\mathrm{v}(x,\theta,z) &= v_0(x,\theta) - z\,\frac{\partial w(x,\theta)}{\partial \theta},\\
w(x,\theta,z) &= w(x,\theta).
\end{aligned} \quad (3)$$

Using Donnell's theory, strains may be obtained by a combination of linear, nonlinear and curvature change terms as

$$\begin{aligned}
\varepsilon_{xx} &= u,_x + \frac{1}{2}w,_x{}^2 - zw,_{xx},\\
\varepsilon_{\theta\theta} &= \frac{v,_\theta}{R} + \frac{w}{R} + \frac{1}{2R^2}w,_\theta{}^2 - \frac{z}{R^2}w,_{\theta\theta},\\
2\varepsilon_{x\theta} &= \frac{u,_\theta}{R} + v,_x + \frac{w,_x w,_\theta}{R} - \frac{2z}{R}w,_{xy},
\end{aligned} \quad (4)$$

where x and θ denote axial and circumferential direction of coordinate system, respectively, z is the distance from an arbitrary point to the middle surface and R is the radius of the shell.

4 ENERGY FORMULATION

The total energy, U, consists of the potential energy K, the virtual kinetic energy W as well as the virtual work due to external forces including, elastic medium is modeled using spring Winkler and shear Pasternak constants. Considering the governing Eq. (3) and strain displacement Eq. (4), U, K and W may be expressed as follows [13]:

$$\begin{aligned}
U = \int_{-\frac{h}{2}}^{\frac{h}{2}}\int_A \Bigg(&\sigma_{xx}\left(\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - z\,\frac{\partial^2 w}{\partial x^2}\right) + \\
&\sigma_{\theta\theta}\left(\frac{\partial v}{R\,\partial\theta} + \frac{w}{R} + \frac{1}{2}\left(\frac{\partial w}{R\,\partial\theta}\right)^2 - z\,\frac{\partial^2 w}{R^2\partial\theta^2}\right) + \\
&\sigma_{x\theta}\left(\frac{\partial u}{R\,\partial\theta} + \frac{\partial v}{\partial x} + \frac{\partial w}{R\,\partial\theta}\frac{\partial w}{\partial x} - 2z\,\frac{\partial^2 w}{R\,\partial\theta\,\partial x}\right)\Bigg).dA\,dz \\
K = \int \Bigg(\frac{\rho}{2}\Bigg(&\frac{h^3}{12}\left(\left(\frac{\partial^2 u}{\partial t\,\partial x}\right)^2 + \left(\frac{\partial^2 w}{\partial t\,\partial\theta}\right)^2\right) + \\
&h\left(\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial v}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2\right)\Bigg)\Bigg)dA
\end{aligned} \quad (5)$$

and

$$W = -\int (F_e) w\,dA = -\int \left(K_w\,w - K_g\nabla^2 w\right) w\,dA, \quad (6)$$

Where dA is the surface element. Dimensionless parameter are then defined as:

$$\begin{aligned}
&\gamma = \frac{h}{l}, \quad \xi = \frac{x}{l}, \quad \beta = \frac{h}{R},\\
&\{\bar{u},\bar{v},\bar{w}\} = \frac{\{u_1,v_1,w_1\}}{h}\bar{C}_{ij} = \frac{C_{ij}}{C_{11}}, i, j = 1,2,6\\
&\bar{t} = \frac{t}{h\sqrt{\frac{\rho}{C_{11}}}}, \ \bar{C} = \frac{hc_v}{\sqrt{C_{11}\rho}}, \ \bar{K}_w = \frac{hk_w}{C_{11}},\\
&\bar{K}_g = \frac{k_g}{hC_{11}}, \quad N^*_{x0} = \frac{N^*_{x0}}{lC_{11}},\\
&N^*_{x\theta 0} = \frac{N^*_{x\theta 0}}{lC_{11}}, N^*_{\theta 0} = \frac{N^*_{\theta 0}}{lC_{11}}
\end{aligned} \quad (7)$$

Applying Hamilton principle

$$\int_0^t (\delta W - \delta U)dt = 0$$

and rearranging the governing equation in mechanical displacement directions(U, V and W) yield the following coupled thermo-mechanical equations.

$$
\begin{aligned}
&(\gamma^2)\left(\frac{\partial^2\bar{u}}{\partial\xi^2}+\frac{\partial\bar{w}}{\partial\xi}\frac{\partial^2\bar{w}}{\partial\xi^2}\right)+\\
&\gamma\beta\bar{C}_{12}\left(\frac{\partial^2\bar{v}}{\partial\xi\partial\theta}+\frac{\partial\bar{w}}{\partial\xi}+\beta\frac{\partial\bar{w}}{\partial\theta}\frac{\partial^2\bar{w}}{\partial\xi\partial\theta}\right)+\\
&\beta^2\bar{C}_{66}\left(\frac{\partial^2\bar{u}}{\partial\theta^2}+\frac{1}{\beta}\frac{\partial^2\bar{v}}{\partial\xi\partial\theta}+\frac{\partial\bar{w}}{\partial\xi}\frac{\partial^2\bar{w}}{\partial\theta^2}+\right.\\
&\left.\frac{\partial\bar{w}}{\partial\xi}+\frac{\partial\bar{w}}{\partial\theta}\frac{\partial^2\bar{w}}{\partial\xi\partial\theta}\right)=\bar{C}\frac{\partial\bar{u}}{\partial\bar{t}}+\frac{\partial^2\bar{u}}{\partial\bar{t}^2}
\end{aligned}
\tag{8}
$$

$$
\begin{aligned}
&\beta\bar{C}_{12}\left(\frac{\partial^2\bar{u}}{\partial\xi\partial\theta}+\frac{\partial\bar{w}}{\partial\xi}\frac{\partial^2\bar{w}}{\partial\xi\partial\theta}\right)+\\
&\beta^2\bar{C}_{22}\left(\frac{\partial^2\bar{v}}{\partial\theta^2}+\frac{\partial\bar{w}}{\partial\theta}+\beta\frac{\partial\bar{w}}{\partial\theta}\frac{\partial^2\bar{w}}{\partial\theta^2}\right)\\
&+\gamma\bar{C}_{66}\left(\frac{\beta\,\partial^2\bar{u}}{\partial\xi\partial\theta}+\frac{\partial^2\bar{v}}{\partial\xi^2}+\frac{\beta\,\partial^2\bar{w}}{\partial\theta\,\partial\xi}\frac{\partial\bar{w}}{\partial\xi}+\right.\\
&\left.\frac{\beta\,\partial\bar{w}}{\partial\theta}\frac{\partial^2\bar{w}}{\partial\xi^2}\right)=\bar{C}\frac{\partial v}{\partial\bar{t}}+\frac{\partial^2\bar{v}}{\partial\bar{t}^2}
\end{aligned}
\tag{9}
$$

$$
\begin{aligned}
&\frac{\gamma^2}{12}\left(-\gamma^2\frac{\partial^4\bar{w}}{\partial\xi^4}-\bar{C}_{12}\beta^2\frac{\partial^4\bar{w}}{\partial\xi^2\partial\theta^2}\right)+\\
&\frac{1}{12}\left(-\gamma^2\beta^2\bar{C}_{12}\frac{\partial^4\bar{w}}{\partial\xi^2\partial\theta^2}-\beta^4\bar{C}_{22}\frac{\partial^4\bar{w}}{\partial\theta^4}\right)\\
&-\frac{\gamma\beta\bar{C}_{12}}{3}\left(\frac{\partial\bar{u}}{\partial\xi}+\frac{\gamma}{2}\left(\frac{\partial\bar{w}}{\partial\xi}\right)^2\right)-\\
&\beta\bar{C}_{22}\left(\frac{\partial\bar{v}}{\partial\theta}+\beta\bar{w}+\frac{\beta^2}{2}\left(\frac{\partial\bar{w}}{\partial\theta}\right)^2\right)-\\
&\left(\left(\beta^2\alpha_x++\beta^2\bar{C}_{12}\alpha_\theta\right)\frac{\partial^2\bar{w}}{\partial\theta^2}\Delta T-\right.\\
&\left.\left(\gamma^2\alpha_x++\gamma^2\bar{C}_{12}\alpha_\theta+\right)\frac{\partial^2\bar{w}}{\partial\xi^2}\right)\Delta T+\\
&K_w\bar{w}-K_g\left(\gamma^2\frac{\partial^2\bar{w}}{\partial\xi^2}+\beta^2\frac{\partial^2\bar{w}}{\partial\theta^2}\right)+\\
&\gamma N^*_{x0}\frac{\partial^2\bar{w}}{\partial\xi^2}+\beta N^*_{x\theta0}\frac{\partial^2\bar{w}}{\partial\xi\partial\theta}+\\
&\beta^2N^*_{\theta0}\frac{\partial^2\bar{w}}{\partial\theta^2}=\bar{C}\frac{\partial\bar{w}}{\partial\bar{t}}+\frac{\partial^2\bar{w}}{\partial\bar{t}^2}.
\end{aligned}
\tag{10}
$$

As can be seen, these are nonlinear equations which could not be solved analytically. Hence, DQM is employed which in essence approximates the partial derivative of a function, with respect to a spatial variable at a given discrete point, as a weighted linear sum of the function values at all discrete points chosen in the solution domain of the spatial variable [14]. Let F be a function representing $\bar{u}$, $\bar{v}$ and $\bar{w}$ with respect to variables ξ and θ in the following domain of ($0<\xi<L, 0<\theta<2\pi$) having $N_\xi \times N_\theta$ grid points along these variables. The n[th]-order partial derivative of $F(\xi,\theta)$ with respect to ξ, the m[th]-order partial derivative of $F(\xi,\theta)$ with respect to θ and the (n + m)[th]-order partial derivative of $F(\xi,\theta)$ with respect to both ξ and θ may be expressed discretely [13] at the point (ξ_i,θ_i) as :

$$
\frac{d^nF(\xi_i,\theta_j)}{d\xi^n}=\sum_{k=1}^{N_\xi}A_{ik}^{(n)}F(\xi_k,\theta_j) \qquad n=1,\ldots,N_\xi-1, \tag{11}
$$

$$
\frac{d^mF(\xi_i,\theta_j)}{d\theta^m}=\sum_{l=1}^{N_\theta}B_{jl}^{(m)}F(\xi_i,\theta_l) \qquad m=1,\ldots,N_\theta-1, \tag{12}
$$

$$
\frac{d^{n+m}F(\xi_i,\theta_j)}{d\xi^nd\theta^m}=\sum_{k=1}^{N_\xi}\sum_{l=1}^{N_\theta}A_{ik}^{(n)}B_{jl}^{(m)}F(\xi_k,\theta_l), \tag{13}
$$

where $A_{ik}^{(n)}$ and $B_{jl}^{(m)}$ are the weighting coefficients associated with n[th]-order partial derivative of $F(\xi,\theta)$ with respect to ξ at the discrete point ξ_i and m[th]-order derivative with respect to θ at θ_i, respectively, whose recursive formulae can be found in reference [15]. A more superior choice for the positions of the grid points is Chebyshev polynomials as expressed in reference [15]. According to HDQM, mechanical clamped and free electrical boundary conditions at both ends of the shell may be written as :

$$
\begin{cases}
w_{i1}=v_{i1}=u_{i1}=0 & \sum_{j=1}^{N_\theta}A_{2j}w_{ji}=0\\
w_{N_xi}=v_{N_xi}=u_{N_xi}=0 & \sum_{j=1}^{N_\theta}A_{(N_x-1)}w_{ji}=0
\end{cases}
\tag{14}
$$

For $i=1\cdots N_\theta$

Applying these boundary conditions into the governing Eqs. (8-10) yields the following coupled assembled matrix equations.

$$\left(\left[\underbrace{K_L+K_{NL}}_{K}\right]+\Omega[C]+\Omega^2[M]\right)\begin{Bmatrix}\{d_b\}\\ \{d_d\}\end{Bmatrix}=0, \tag{15}$$

Where d^b and d^d represent boundary and domain points expressed as:

$$\begin{aligned}\{d^b\}&=\{\bar{u}_{i1},\bar{v}_{i1},\bar{w}_{i1},\bar{w}_{i2},\Phi_{i1},\bar{u}_{iN_\theta},\\ &\qquad\bar{v}_{iN_\theta},\bar{w}_{iN\theta},\bar{w}_{i(N_\theta-1)},\Phi_{iN_\theta}\}\\ &\qquad i=1,\cdots,N_x\\ \{d^d\}&=\{\bar{u}_{ij},\bar{v}_{ij},\bar{w}_{i(j+1)},\Phi_{ij}\}\\ &\qquad j=2,\cdots,N_x-1\end{aligned} \tag{16}$$

As well as M is the mass matrix, $\{C\}$ is the damping matrix and $\{K_{NL}+K_L\}$ are the nonlinear and linear stiffness matrixes.

5 NUMERICAL RESULTS AND DISCUSSION

In order to obtain the nonlinear frequency ratio (Ω_{NL}/Ω_L) for a polyethylene (PE) cylindrical shell reinforced with CNTs, embedded in the Pasternak foundation, DQM was used in conjunction with a program being written in MATLAB, where the effect of dimensionless parameters such as aspect ratios of length to radius of the shell, (L/R), aspect ratios of thickness to length (h/L),Winkler, K_W module, damp constant C and orientation angel θ were investigated.

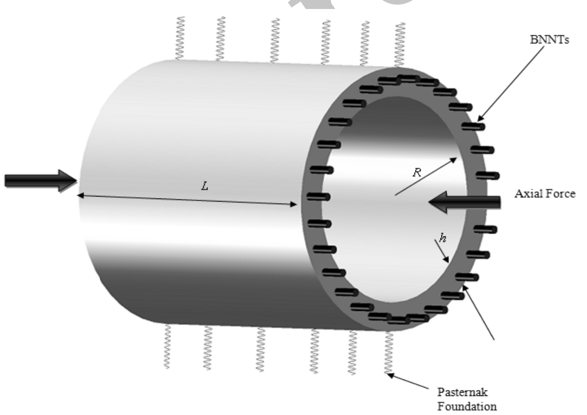

Fig. 1 Polymeric cylindrical composite shell reinforced with CNTs embedded in an elastic medium

The materials tested include (PE) as matrix and carbon nano-tube (CNT) as matrix reinforce, whose properties are taken from reference [9].

Fig. 2 illustrates the effects of aspect ratio (L/R) on the nonlinear frequency ratio versus maximum transverse amplitude $(w_{\max})$. Ω_{NL}/Ω_L is reduced substantially, when L/R is increased; such that nonlinear frequency tends to be linear (i.e. Ω_{NL}/Ω_L tends to unity). This is perhaps because increasing aspect ratio of length to radius decreases the shell stiffness.

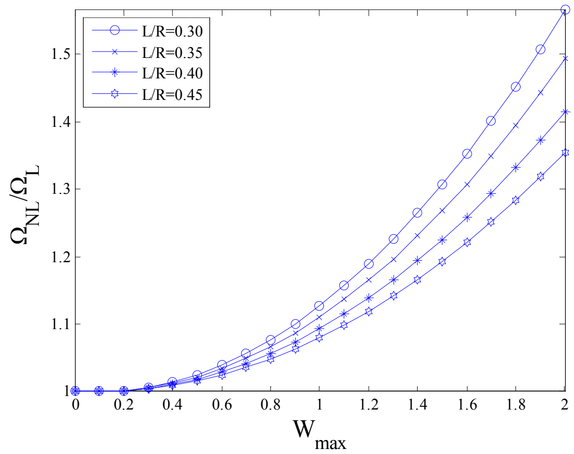

Fig. 2 Effect of aspect ratio L/R on the nonlinear frequency ratio

Fig.3 shows the effects of aspect ratio (h/L)on the nonlinear frequency ratio versus maximum transverse amplitude $(w_{\max})$. Ω_{NL}/Ω_L is increased substantially when aspect ratio of thickness to length is increased. This is perhaps because increasing h/L increases the shell stiffness.

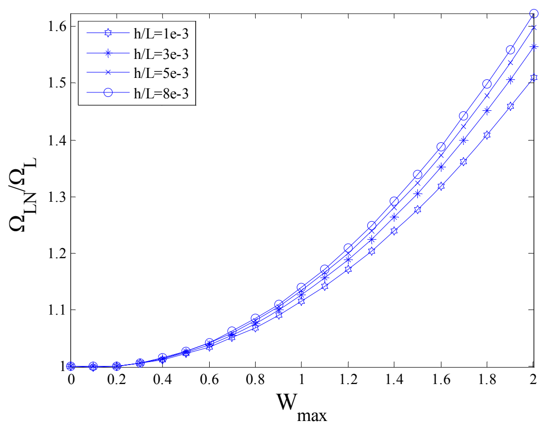

Fig. 3 Effect of aspect ratio h/L on the nonlinear frequency ratio

Effect of elastic medium on nonlinear vibration studied in Fig. 4. This figure illustrates that nonlinear frequency is increased substantially when K_w is increased. Such that nonlinear frequency tends to be linear (i.e. NFR tends to unity). This suggests that at higher K_w 's, the nonlinear effect is considerable, while this might be neglected for lower K_w 's.

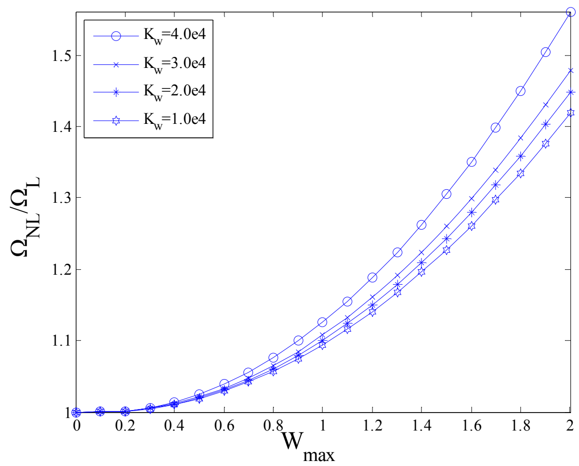

Fig. 4 Effect of Winkler constant K_W on the nonlinear frequency ratio

To illustrate the effect of damper constant on Ω_{NL}/Ω_L, Fig. 5 shows that nonlinear frequency ratio is reduced when damper constant is increased.

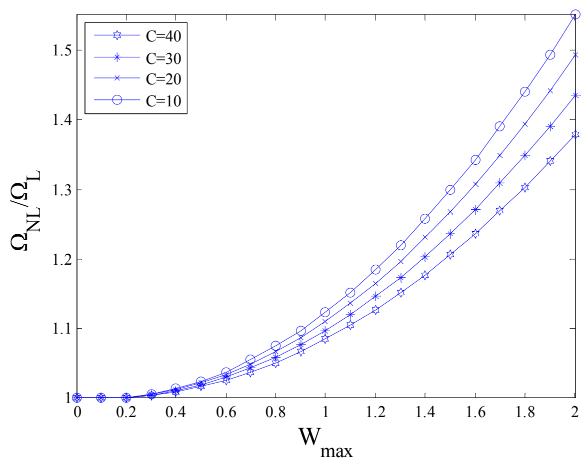

Fig. 5 Effect of damper constant C on the nonlinear frequency ratio

Fig. 6 shows the dimensionless frequency ratio Ω_{NL}/Ω_L versus maximum transverse amplitude $(w_{\max})$ at various orientation angle θ of CNTs in the polymer. This is a periodic function with a period of π, indicating that Ω_{NL}/Ω_L is same when θ values are zero and π. For orientation angle between two above values, Ω_{NL}/Ω_L is lower.

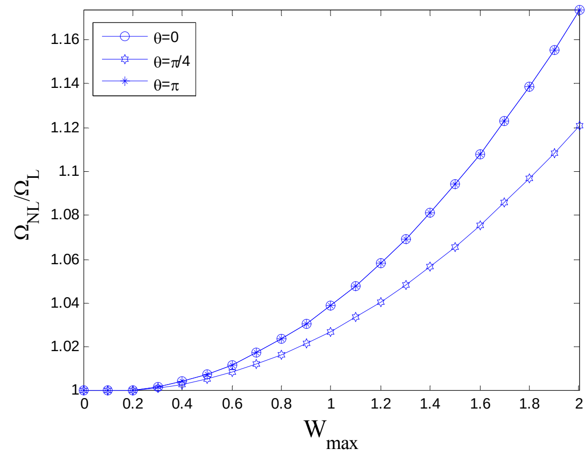

Fig. 6 Effect of orientation angel θ on the nonlinear frequency ratio

6 CONCLUSION

Thermo-mechanical nonlinear vibration of a poly ethylene (PE) cylindrical shell on the elastic foundation was investigated. Using DQM the derived governing equations were discretized, and solved to obtain the nonlinear frequency with clamped boundary conditions. The results have indicated that Ω_{NL}/Ω_L is increased substantially when aspect ratio of thickness to length is increased. This is perhaps because of increasing h/L ratio, increases the shell stiffness. Nonlinear frequency ratio is reduced substantially when L/R is increased; such that nonlinear frequency tends to be linear (i.e. Ω_{NL}/Ω_L tends to unity). This is perhaps because of increasing aspect ratio of length to radius, decreases the shell stiffness.

REFERENCES

[1] Nabors, S. A., "Technology opportunity composite technology: strong, lightweight composite tanks and pipes for demanding applications", NASA_s Marshall Space Flight Center.
[2] Thornton, E., "Thermal buckling of plates and shells", Appl. Mech. Rev.Vol. 46, No. 10, 1993, pp. 485-506.
[3] Thangartnam, R. K., Palaninathan, R., and Ramachandran J., "Thermal buckling of laminated composite shells", Journal AIAA, Vol. 28, No. 5, 1990, pp. 859-60.
[4] Radhamohan, S. K., and Venkatramana, J., "Thermal buckling of orthotropic cylindrical shells", Journal AIAA, Vol. 13, No. 3, 1975, pp. 397-9.
[5] Ganesan, N., and Kadoli, R., "Buckling and dynamic analysis of piezothermoelastic composite cylindrical

shell", Compos. Struct., Vol. 59, No. 1, 2003, pp. 45-60.

[6] Birman, V., and Bert, C. W., "Buckling and post-buckling of composite plates and shells subjected to elevated temperature", ASME Journal Appl. Mech., Vol. 60, 1993, pp. 514-9.

[7] Paidoussis, M. P., and Li, G. X., "Pipes conveying fluid: a model dynamic problem", Journal Fluids Struct., Vol. 7, 1993, pp. 137-204.

[8] Paidoussis, M. P., "Fluid-structure interactions: slender structures and axial flow", London Academic Press; Vol. I, 1998.

[9] Wan, H., Delale, F., and Shen, L., "Effect of CNT length and CNT-matrix interphase in carbon nanotube (CNT) reinforced composites", Mechanics Research Communications, Vol. 3, 2005, pp. 481-489.

[10] Shi, D. L., Feng, X. Q., Huang, Y. Y., and Hwang, K. C., Gao, H., "The Effect of nanotube waviness and agglomeration on the elastic property of carbon nanotube-reinforced composites", Department of Engineering Mechanics, Vol. 126, 2004, pp. 250-257.

[11] Kollar, L. P.; and Springer, G. S., "Mechanics of Composite Structures", Cambridge University Press NY, USA, 2003.

[12] Amabili, M., "Nonlinear Vibrations and Stability of Shells and Plates", New York: Cambridge University Press, 2008.

[13] Mosallaie Barzoki, A. A., Ghorbanpour Arani, A., Kolahchi, R, and Mozdianfard, M. R., "Electro-thermo-mechanical torsional buckling of a piezoelectric polymeric cylindrical shell reinforced by DWBNNTs with an elastic core", Appl. Math. Model In Press.

[14] Liew, K. M., Han, J. B., and Xiao, Z. M., "Differential quadrature method for thick symmetric cross-ply laminates with first-order shear flexibility", Int Journal Solids Struct., Vol. 33, 1996, pp. 2647-2658.

[15] Civalek, Ö., "Application of differential quadrature (DQ) and harmonic differential quadrature (HDQ) for buckling analysis of thin isotropic plates and elastic columns", Eng. Struct., Vol. 26, 2004, pp. 171-186.