# Analytical and Experimental Analysis of Axisymmetric Deep Drawing Process

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**Abstract:** There are mainly two methods of deep drawing analysis; experimental and analytical/numerical. Experimental analysis can be useful in analyzing the process to determine the process parameters in producing a defect free product. The analytical/numerical modeling can be used to model and analyze the process through all stages of deformation. This approach is less time consuming and more economical. Sheet metal forming often includes biaxial in-plane deformation with non-proportional strain paths. In deep drawing of cylindrical cup, the deformation in the flange is dominated by pure shear deformation, while it changes to plane strain when the material is drawn into the die. This paper deals with the analysis of deep drawing of circular blanks into axi-symmetric cylindrical cup using numerical modeling. The blank drawability has been related both theoretically and experimentally with the initial diameter of the blank and deep drawing parameters. The strains in the radial and circumferential directions have been measured. A correlation on the flange thickness variation by taking into account the work hardening with the analytical and experimental values also has been searched.

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## 1   INTRODUCTION

Production of high quality products in a short time and at low cost is an ultimate goal in manufacturing. To reach this goal, continuous progress made at the design and at the shop floor levels of forming process [1]. The stages of research and development in CAD/CAM/CAE in relation with the analysis and design of forming parts can be observed from several papers published in proceedings of international meetings [2-3]. To avoid trial and error tryout procedures, the sheet metal forming simulation is increasingly being used in the stamping industry to evaluate the deformation paths and the forming defects such as fracture and wrinkling. Many research groups are still developing and improving finite element codes for the analysis when an initial design of the blank and the tools is done and when the forming conditions are defined.

Several tool analyses are described in the proceedings [2-3]. They are based on membrane, shell or even solid elements, considering static or dynamic, implicit or explicit approaches. These tool analyses could be very precise if used by well-trained engineers, but they are time consuming and needs expensive computer resources. Computer simulation can also be very attractive and helpful for the process and tooling engineers to define the initial blank (thickness, contour and surface), some process parameters (boundary conditions, holding forces, lubrication conditions, drawbed types and positions, etc.) and the material properties (yield stress, hardening, anisotropy, etc.).

This has been recognized by some industrial and academic research and development groups. As a result, a number of methods have been developed in the last decade. They are mainly based on the fact that the shape of the desired part is known. A comparison is then made with the initial flat blank to estimate the deformation of the final product taking into account simple constitutive equations and assumptions regarding the tool actions. These simplified procedures have been called different names: 'geometrical mapping method' [4], 'single and multi-stage forming formulations' [5], 'one step solution' [6], 'ideal forming theory' [7], 'Inverse Approach' (IA) [8-12] and 'Simplified Approach' [13].

The inverse approach is based on a discretization of the final workpiece by simple triangular flat facet shell elements. For a large number of industrial applications, the membrane effects are dominant, but it has been necessary to consider bending effects using a simple discrete Kirchhoff shell element [14-16]. Assumptions were made regarding the action of the tools (punch and die) at the end of the forming process. Logarithmic strains and total deformation theory of plasticity were considered. The equilibrium of the workpiece leads to a set of nonlinear equations. These nonlinear equations could be solved by different techniques such as the Newton–Raphson static implicit approach, the dynamic relaxation method or the dynamic explicit algorithm [17-21].

The convergence difficulties can be encountered for practical situations involving deep drawing workpieces (with almost vertical walls) and a low plastic hardening law. A simplified scheme in order to estimate the trimming part of the workpiece for a given blank shape, flat or curved has been developed. The inverse approach has been continuously evaluated by comparing the numerical results with experimental and other numerical results obtained by incremental approaches. The procedure has been found very efficient and quite precise at the preliminary tool design stage [22-23]. Based on the idea of the simplified inverse approach two industrial codes have been developed [24-26]. The codes are routinely used at the preliminary forming design stage of car panels and thin walled structural members. Some backwards simulation codes have been developed and used in the industry [27-29].

In this paper, in order to increase the efficiency and accuracy of the modeling procedure, the yielding criteria and radial and circumference strains among with strain hardening and thickness variations during the forming process has been employed. Moreover experimental results are present to evaluate the theoretical method.

## 2   THE DIFFERENT DEFORMATION MODES

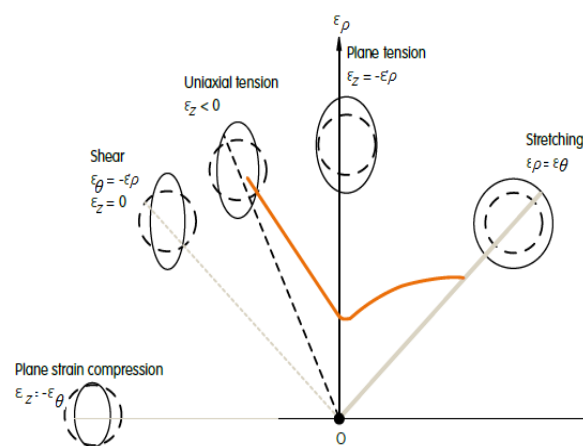Figure 1 shows different deformation modes that can occur in a cup drawing.



**Fig. 1**     Strain diagram for steel forming showing a typical forming limit curve for steel blanks [30]

By convention, any type of deformation in a sheet can be represented by a point in the ερ vs εθ diagram, where ερ is the principal strain in the plane of the sheet and εθ is the strain perpendicular to ερ. The thickness strain εz is obtained from the conservation law of volume: ερ + εθ + εz = 0.

This diagram illustrates different possible types of deformation during drawing. The deformation mode is visually represented in the chart by the deformation of the circle. In reality, the magnitude and mode of deformation of a component are measured by optical measurement of a full pattern of circles or dots printed on a blank before deformation. Characteristic cases or deformation modes are:

• Biaxial tension: this is accompanied by thinning and typically occurs in spherical shapes such as at the punch nose. The magnitude of deformation is largely defined by the shape and the restraining force on the flange, exerted by the blankholder.

• Plane strain tension: generally occurs in the vertical walls of the part, due to the tensile stress caused by retention under the blankholder, or just before the end of forming when sliding in the tool is restricted.

• Uniaxial tension: this is the situation encountered in a tensile test on a specimen, and it occurs in transition regions between zones of plane strain tension and plane strain compression.

• Shear deformation: represents the ideal forming mode; here, all tensile strains are being compensated by compressive strains, so that the thickness remains constant. This type of deformation can occur beneath the blankholder if the flow is uniform (axisymmetric drawing).

• Drawing: involves thickening due to a reduction in width (circumference). It is mostly unavoidable during deep drawing but can be controlled beneath the blankholder. However, its occurrence in unsupported areas will lead to wrinkling or folding [30].

**Sheet metal forming at axisymmetric conditions**

The basic analytical theory used in the solution of the problem of deep-drawing of a cylindrical cup is based mainly on the research works in references [31-40]. The analytical model is established on the following assumptions:

• Elastic strains are neglected, since they are small in comparison between plastic strains, and isotropic material.

• Von Misses material with non-linear strain-hardening is used.

• Radial, circumference and thickness directions are considered principal directions.

• Bending/unbending effects are neglected since their effect is negligible for a die profile radius to sheet thickness.

• Shear stress is neglected across the sheet thickness.

• A straight cup wall is assumed.

The principal directions in the problem of deep-drawing of a cylindrical cup are the radial, circumferential and thickness directions with ρ, θ and z designations, respectively. The governing plasticity equations of the analytical model are as follows: effective stress, for a material free from Bauschinger effects, Von Misses or effective stress can be defined as follows [41-42]:

$$\sigma_s = \{\frac{1}{2}[(\sigma_\rho - \sigma_\theta)^2 + (\sigma_\theta - \sigma_z)^2 + (\sigma_z - \sigma_\rho)^2]\}^{1/2} \qquad (1)$$

Plastic Strains for the three principal directions; circumferential, thickness, and radial directions can be expressed as:

$$\varepsilon_\theta = ln\ \rho/R \quad \varepsilon_z = ln\ S/S0 \quad \varepsilon_\rho = -(\ \varepsilon_\theta + \varepsilon_z\ ) \qquad (2)$$

S0 is primary thickness of blank and R is primary blank diameter. Effective Strain: The effective incremental strain can be stated as:

$$d\varepsilon_i = \{\frac{2}{9}[d\varepsilon_\rho - d\varepsilon_\theta)^2 + (d\varepsilon_\theta - d\varepsilon_z)^2 + (d\varepsilon_z - d\varepsilon_\rho)^2]\}^{1/2}$$

$$(3)$$

Stress-Strain Relationship: Based on Levy-Lode stress-strain relationship:

$$d\varepsilon_\rho = \frac{3}{2}\frac{d\varepsilon_i}{\sigma_s}(\sigma_\rho - \sigma_0), \qquad (4)$$

$$d\varepsilon_\theta = \frac{3}{2}\frac{d\varepsilon_i}{\sigma_s}(\sigma_\theta - \sigma_0), \qquad (5)$$

$$d\varepsilon_z = \frac{3}{2}\frac{d\varepsilon_i}{\sigma_s}(\sigma_z - \sigma_0), \qquad (6)$$

Flow Equation: The flow equation that describes the strain hardening of the material is the Ludwik-Hollomon power law which is given by [41-42]:

$$\sigma_s = K\varepsilon^n \qquad (7)$$

*K* and *n* are work hardening parameters depending on mechanical properties of material. The equilibrium equation for axi-symmetrically condition with variable thickness is:

$$\rho \frac{d\sigma}{d\rho} + \sigma_\rho (1 + \frac{\rho\, ds}{s\, d\rho}) - \sigma_\theta = 0 \qquad (8)$$

The major strains in circular and thickness directions are:

$$d\varepsilon_\theta = \frac{d\rho}{\rho}, \quad d\varepsilon_z = \frac{ds}{s} \qquad (9)$$

Substituting Eq. (9) in Eq. (8) and considering Eq. (6):

$$\rho \frac{d\sigma_\rho}{d\rho} + 2 \frac{\sigma_\rho^2 - \sigma_\rho\sigma_\theta + \sigma_\theta^2}{2\sigma_\theta - \sigma_\rho} = 0 \qquad (10)$$

Solving this equation requires simplifications. Nazaryan [43] represent the following equation:

$$d\sigma_\rho = \sigma_s d\varepsilon_i \qquad (11)$$

By assumption $d\varepsilon i = |d\varepsilon\theta|$ and using yield criterion $\sigma p - \sigma\theta = \sigma s$:

$$\rho \frac{d\sigma_\rho}{d\rho} + \sigma_\rho - \sigma_\theta = 0 \qquad (12)$$

This equilibrium equation is usually applied in researches without considering work hardening effects and thickness changes.

**Analysis of thin ring plates deformation**

Let's consider thin ring plate having the sizes R0, and S0 before drawing and r and s after drawing. Some of researchers represent the following equations to evaluate thin ring plate behaviour during deep drawing process by taking into account work hardening and thickness changes and strains during deep drawing of cylindrical cups [44-47].

$$\varepsilon_\rho = \frac{1+n}{2}(1 + \cos 2\phi - \frac{\sqrt{3}}{3}\sin 2\phi)$$

$$\varepsilon_\theta = -\frac{1+n}{2}(\cos 2\phi + \frac{\sqrt{3}}{3}\sin 2\phi) \qquad (13)$$

$$\varepsilon_z = -\frac{1+n}{2}(1 - \frac{2\sqrt{3}}{3}\sin 2\phi)$$

Where $\varphi$ is the angle between rolling and drawing directions.

**Experimental Section**

The sheet metal used in this research had 2 mm thickness with 148.5 mm diameter carbon steel sheet according to German Standard DIN–EN10130-FePO4-(St14). Chemical composition and mechanical properties of the material are shown in tables 1 and 2.

**Table 1**   Chemical Composition of Steel

| Steel Grade | % C | % P | % Mn | % S |
|---|---|---|---|---|
| St 14 | Max 0.08 | Max 0.03 | Max 0.4 | Max 0.03 |

**Table 2**   Mechanical Properties of Steel

| Steel Grade | Sheet Thickness $S_0$ (mm) | Mechanical Properties | | Parameters of power Function $\sigma = k\varepsilon^n$ | |
|---|---|---|---|---|---|
| | | $\sigma_{0,2}$ MPa | $\sigma_b$ MPa  %El | K | n |
| *St* 14 | 2.0 | 210 | 350    36 | 710 | 0.307 |

The die was made from CK 45 medium carbon steel where the punch surface and matrix was nitride hardening. Also the polyethylene papers (nylon) were used as lubricant. The primary sheet size diameter was 148.5 cm that was demarcation the rolling direction. After drawing, the stains were calculated in the rolling direction. The drawing process is shown in Fig. 2. The 50 tons hydraulic press was used to draw the cups.
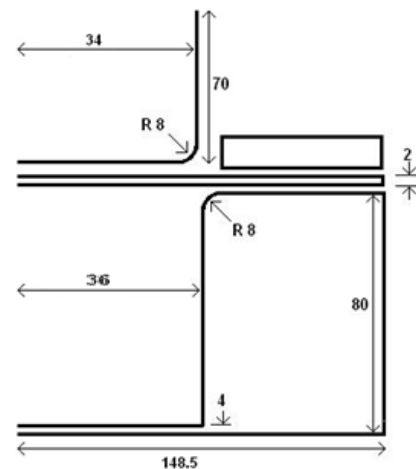


**Fig. 2**    Schematic of Cylindrical Cup Drawing Process

After the drawing process, the drawn parts were cut in rolling direction and the thickness was measured according to Fig. 3.
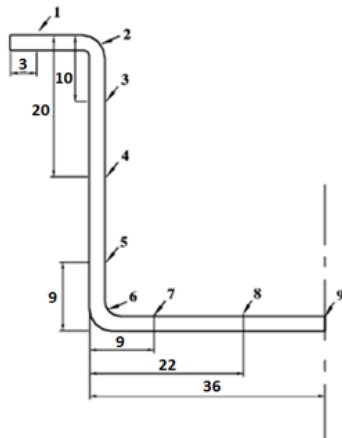


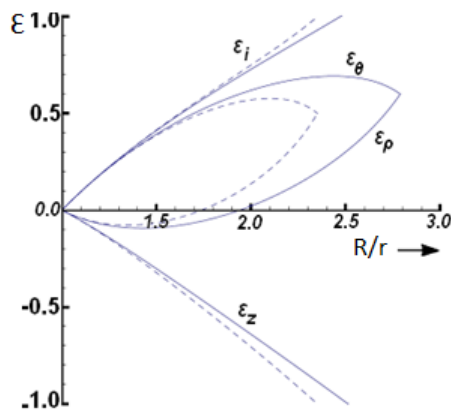**Fig. 3**   Measuring positions in drawing cups



**Fig. 4**      Distribution of non-dimensional equivalent and major strains for ring plates (r is punch diameter)

maximum thinning and thickness positions in drawn cups. According to the results of this research, for a blank with 2 mm thickness, the maximum thinning in drawn cups, occur in position 6 and it is next to 18% of blank thickness. Also the maximum thickness takes in position 1 and it is next to 1.11% of initial blank thickness. The schematic thickness changes in drawn cup during deep drawing process are shown in Fig. 6.
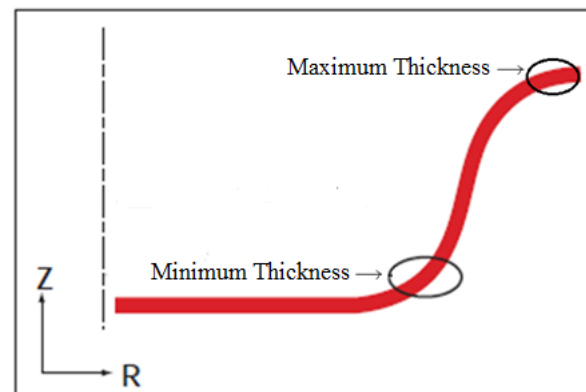


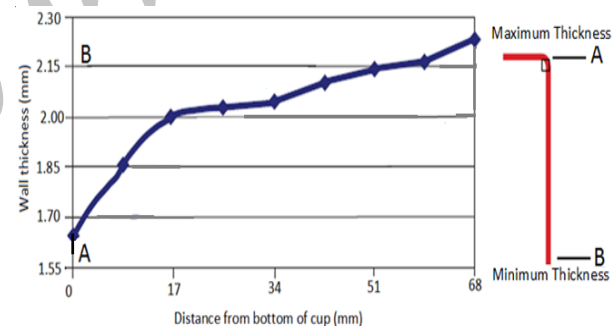**Fig. 5**      Maximum and minimum thickness positions in drawn cup



**Fig. 6**      Schematic illustration of thickness changes in cup drawing Process according to table 3 and Fig. 3

## 3    RESULTS AND DISSCISSION

**Analytical Modeling**
Taking into account Eqs. (3) and (13) and using computer MATLAB software to draw these equations, the relationship between the major strains in radial, circumference and thickness directions and equivalent strain  can be obtained while the effect of work hardening has been considered as shown in Fig. 4. Using Fig. 4, the major strains in radial, circumference and thickness directions and equivalent strain can be measured after deep drawing process.

**Experimental Results**
The thickness measuring results from 9 positions in 24 drawn parts are presented in table 3. Also, Fig. 5 shows

## 4    CONCLUSION

An analytical model to calculate the radial, circumference, thickness and equivalent strains of drawn cup has been developed by taking in to account work hardening effect. Experiments were conducted to evaluate theoretical analysis, where the experimental results are in agreement with theoretical results. The analysis method is useful for the prediction of thickness variation of the products prior to the carrying out of the actual drawing process. The analysis may be used to estimate the critical thickness that initiates the failure of the products and also to set the minimum clearness between the die and punch for the consequent ironing process.

Table 3    The thickness measurement results of experiments related to Fig. 3

| Experiment Position | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2.22 | 2.21 | 2.12 | 1.87 | 1.76 | 1.70 | 1.76 | 1.76 | 1.76 |
| 2 | 2.22 | 2.13 | 2.11 | 1.85 | 1.75 | 1.70 | 1.76 | 1.76 | 1.76 |
| 3 | 2.21 | 1.97 | 2.13 | 1.74 | 1.73 | 1.68 | 1.76 | 1.74 | 1.76 |
| 4 | 2.15 | 2.13 | 2.02 | 1.74 | 1.74 | 1.68 | 1.76 | 1.76 | 1.74 |
| 5 | 2.13 | 2.11 | 2.02 | 1.72 | 1.74 | 1.69 | 1.76 | 1.76 | 1.74 |
| 6 | 2.18 | 2.11 | 1.84 | 1.72 | 1.76 | 1.68 | 1.76 | 1.76 | 1.75 |
| 7 | 2.18 | 2.12 | 1.84 | 1.85 | 1.85 | 1.70 | 1.84 | 1.84 | 1.84 |
| 8 | 2.09 | 2.07 | 2.02 | 1.72 | 1.74 | 1.66 | 1.74 | 1.74 | 1.74 |
| 9 | 2.06 | 2.07 | 1.95 | 1.66 | 1.69 | 1.64 | 1.74 | 1.74 | 1.74 |
| 10 | 2.09 | 2.07 | 2.06 | 1.78 | 1.69 | 1.69 | 1.64 | 1.72 | 1.72 |
| 11 | 2.18 | 2.11 | 1.99 | 1.72 | 1.76 | 1.72 | 1.74 | 1.74 | 1.74 |
| 12 | 2.15 | 2.11 | 2.02 | 1.74 | 1.72 | 1.70 | 1.72 | 1.74 | 1.74 |
| 13 | 2.13 | 2.11 | 2.00 | 1.70 | 1.73 | 1.72 | 1.74 | 1.74 | 1.74 |
| 14 | 2.19 | 2.14 | 2.05 | 1.74 | 1.75 | 1.72 | 1.75 | 1.74 | 1.74 |
| 15 | 2.19 | 1.89 | 2.04 | 1.73 | 1.74 | 1.67 | 1.74 | 1.74 | 1.74 |
| 16 | 2.16 | 2.14 | 2.01 | 1.75 | 1.78 | 1.71 | 1.75 | 1.78 | 1.75 |
| 17 | 2.21 | 2.14 | 2.14 | 1.83 | 1.78 | 1.81 | 1.72 | 1.78 | 1.78 |
| 18 | 2.20 | 2.12 | 2.02 | 1.77 | 1.77 | 1.73 | 1.78 | 1.77 | 1.77 |
| 19 | 2.20 | 2.15 | 2.06 | 1.80 | 1.75 | 1.72 | 1.77 | 1.77 | 1.77 |
| 20 | 2.20 | 2.15 | 2.15 | 1.86 | 1.79 | 1.75 | 1.72 | 1.77 | 1.77 |
| 21 | 2.12 | 2.08 | 2.02 | 1.74 | 1.74 | 1.68 | 1.77 | 1.77 | 1.74 |
| 22 | 2.15 | 2.07 | 1.99 | 1.71 | 1.69 | 1.68 | 1.74 | 1.75 | 1.75 |
| 23 | 2.12 | 2.09 | 1.99 | 1.74 | 1.71 | 1.68 | 1.74 | 1.76 | 1.76 |
| 24 | 2.16 | 2.12 | 2.03 | 1.85 | 1.85 | 1.71 | 1.76 | 1.78 | 1.75 |
| 25 | 2.19 | 2.16 | 2.06 | 1.78 | 1.79 | 1.72 | 1.77 | 1.76 | 1.76 |

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