Multi Layered Finite Element Analysis of Graded Coatings in Frictional Rolling Contact

R. Jahedi
Department of Mechanical and Aerospace Engineering,
Science and Research Branch,
Islamic Azad University, Tehran, Iran
E-mail: re.jahedi@gmail.com

S. Adibnazari*
Department of Mechanical and Aerospace Engineering,
Science and Research Branch,
Islamic Azad University, Tehran, Iran
E-mail: s.adibnazar@gmail.com
*Corresponding author

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Abstract: A plain strain analysis of frictional rolling contact on an elastic graded coating is presented in this paper. Finite element method is applied to gain an understanding of the stresses and contact zone properties caused during rolling contact. The effects of friction, material stiffness ratio and coating thickness on stresses in contact zone and coating/substrate interface are studied. Shear modulus of softening and stiffening graded coatings change with exponential, power law and linear functions. The substrate is homogenous and the rigid cylindrical roller moves in a steady state condition with constant velocity. The coating is modelled in multi layers and a 2-D hard contact of rolling surfaces is considered. The analytical results verify the present method and show a good agreement. It is shown that thinner thicknesses have more effects on stresses and energy density, but these effects are not seen for thicknesses larger than a specific limit.

Keywords: Frictional Rolling Contact, Finite Element Method, Graded Coating, Geometrical Effects

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Biographical notes: R. Jahedi received his PhD in mechanical engineering and currently is a faculty member of Islamic Azad University. He has co-authored one book and many papers on stress analysis of structures and composites, contact mechanics of coatings, finite element method and mechanical behaviour of materials. **S. Adibnazari** is a professor in the Department of Aerospace Engineering at the Sharif University of Technology and adjunct professor at Islamic Azad University, Tehran Science and Research Branch. His research interests and activities are in fretting fatigue and contact mechanics, fracture mechanics, and fatigue of composites. He has several publications in the area of contact mechanics of graded coatings in recent years.

1 INTRODUCTION

Theoretical modelling and finite element analysis of interactions between a rolling component and a supporting bed of material is the interest of contact mechanics. Functional coatings may be applied to change the surface properties of the substrate, such as adhesion, wet ability, corrosion resistance, wear resistance or thermal and electrical properties. Using functionally graded materials (FGMs) as coatings may be more beneficial than common coatings. They tend to reduce stresses resulting from material property mismatch, increase the bonding strength [1], improve the surface properties and provide protection against adverse thermal and chemical environments. There are also important potential applications of FGMs in contact situations. They are mostly load transfer problems to study the stress distribution and singularities in deformable and semi-rigid solids contact, generally in the presence of friction like bearings, gears, cams and machine tools [2], [6].

Recent attentions in contact problems are focused on stress and crack initiation between the coating and substrate. The positive mechanical and thermal effects of graded coatings on interface stresses are studied [3-5] but our analysis on coating thickness leads to growth of graded coatings science. Furthermore, other studies on the rolling contact problems indicate that the peak of the contact pressure, normal stresses and the creepage significantly affect the wear rate of rolling components [7], [8]. Movement of two bodies over each other forms the rolling contact in a wide variety of tribo-components. Contact mechanics approach which deals with the singular stress field at the free edge that causes crack formation (assuming perfect bonding between the film and the substrate and no pre-existing cracks) is taken in to account in this paper. Sliding contact on graded substrates is studied by Suresh et al., in normal and tangential loadings [9]. They proved that the contact stress can be calculated by punch deformation. Also non-frictional sliding contact with linear variation of material constants [10], [11] and normal contact for various coating material variations are modelled [12]. They used Fourier transforms to show that the critical tensile stresses are in trailing edge of sliding triangular punches. The FGM components in these works are simulated in few layers. Different stiffness ratios are considered for contact of stamps with graded coatings [3], [13]. The results indicate that larger contact surfaces increases the contact normal force. Frictional sliding contact on a graded coating is studied analytically to find critical stresses by Guler et al., [3]. They considered constant Poisson ratio and friction coefficient to investigate the positive effect of inhomogeneity factor, γ, on contact stresses. Integral equations of contact problem have been solved by Guler and Erdogan to examine the influence of constants on stress decrease in contact of two negative curvature solids [14]. In a series of articles, many models of finite element based on non-linear behavior of materials and methods of simulation were proposed. These researches with 2D plane stress or plane strain results show an appropriate compatibility with analytical ones [15-17]. Models are designed for sliding and normal contact of stamps on composite and graded materials. The FEM codes discretization approaches to the numerical analysis of functionally graded materials and some homogeneous parts with variations in mechanical properties [18-20]. The quasi static contact is applied in their models and meshless methods would not be applicable in common normal contact problems. FEM modelling of thin coated members and sliding contact on laterally graded substrates are investigated by Guler and Dag and continued by Adibnazari et al., on finite element models for frictional sliding of stamps which shows the importance of medium properties [21], [22], [23]. Guler et al., recently solved two coupled Cauchy singular integral equations for an analytical rolling contact problem [24-28]. In these works, the effects of stiffness and creep ratios on stress as well as slip and stick zones are studied. As stated in previous paragraphs, sliding of frictional and non-frictional contacts of FGM components and coatings are studied by numerous researchers; however, some analytical studies have dealt with the FGM rolling contact problem of graded coatings. This paper introduces a complementary finite element analysis and parameters effects on rolling contact of graded coatings beside these few analytical and mathematical studies. A number of works have been carried out on the knowledge of the FGM properties and their influence on stress variations, but more detailed study would be needed in design of components. Investigating the effects of coating thickness, *h*, as well as inhomogeneity constant, *γ*, on contact zone and stresses is the aim. Also the other novelties of the present study would be stated as analysis of the strain energy and material variation functions by FE modelling. The FE stress analysis of coating/substrate interface under frictional rolling contact is missing in literature. Two linked FE subroutines modify the material properties and coating modelling. Then the models material property variation in exponential, power law and linear trends are applied as a new study and the results show stress differences in contact zone. The verification of results shows a good accuracy in method and results.

2 INTRODUCING THE PROBLEM AND FUNDAMENTAL FORMULATION

A two-dimensional elastic contact is shown in Fig. 1; two concentrated forces P and Q act at the center of a cylindrical rigid roller of radius R which rolls on an elastic coated substrate. The third dimension of both roller and substrate are as long as the simplified plain strain problem is considered. The substrate is a homogeneous half-space with shear modulus μ_s and Poisson's ratio υ. Contact surface between roller and substrate is coated with a thin FG medium of thickness h. The cylinder rolls with constant velocity, V, in the negative x direction (see Fig. 1).

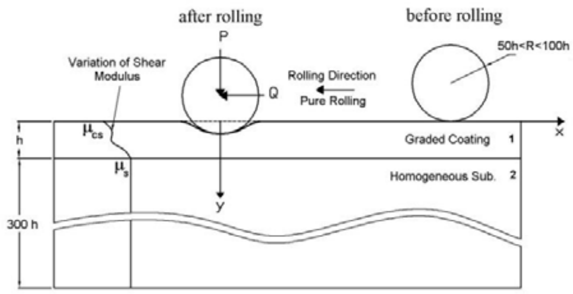


Fig. 1 Graded coated substrate in frictional rolling contact with rigid cylindrical roller; shear modulus varies in coating thickness with different functions (h, thickness of coating).

The modelling of a contact problem by Lagrange approach is a frequent method which deals with unilateral contact conditions. An additional set of finite element contact constraints are needed to be imposed on the degrees of freedom of the nodes. Some constraints are adopted to preclude penetration of components and satisfy certain friction law whereas the normal contact tractions are ensured. Generally the shear modulus and Poisson's ratio of the functionally graded coating may be described in an exponential format as Eq. (1). In the graded medium ($0<y< h_c$) the spatial variation of Poisson's ratio is assumed to be negligible. The shear module of the coating surface, μ_{cs}, and the shear modulus of substrate, μ_s , are constant. The shear modulus of the graded coating $\mu(y)$ is approximated as Eq. (1) [24],

$$\mu(y) = \mu_{cs} e^{\gamma y} \qquad 0 < y < h_{fgm} \qquad (1)$$
$$\mu(y) = \mu_s \qquad h_{fgm} < y$$

γ, the material inhomogeneity parameter can be calculated by inserting coating thickness in exponential form of shear modulus variation,

$$\gamma = -\frac{1}{h_{fgm}} \ln(\Gamma) \qquad (2)$$

Where Γ is the stiffness ratio and is defined as:

$$\Gamma = \frac{\mu_s}{\mu_{cs}} \qquad (3)$$

The equilibrium equations are satisfied, if p(x) and q(x) are the continuous functions of normal and tangential loads in contact area; the following equations are considered.

$$P = -\int_{contact} p(\xi)d\xi \qquad (4)$$
$$Q = \int_{contact} q(\xi)d\xi$$

Fundamental equations for 2-D contact of homogeneous bodies would be extracted in elasticity theorems as coupled forms [29]. These equations can be decoupled in an analytical approach if both contact parts are the same material. In present research, the equations cannot be decoupled unless using numerical methods in finite element codes. Goodman approximation would be used in some numerical methods with acceptable approximated results [24]. The equations of elasticity for graded coatings in the absence of body forces were written by Guler et al. [1], ($0<y< h_c$)

$$(k_c+1)\frac{\partial^2 v_c}{\partial y^2} + (k_c-1)\frac{\partial^2 v_c}{\partial x^2} + 2\frac{\partial^2 u_c}{\partial x\,\partial y} + \gamma(3-k_c)\frac{\partial u_c}{\partial x} + \gamma(k_c+1)\frac{\partial v_c}{\partial y} = 0 \qquad (5)$$

$$(k_c+1)\frac{\partial^2 u_c}{\partial x^2} + (k_c-1)\frac{\partial^2 u_c}{\partial y^2} + 2\frac{\partial^2 v_c}{\partial x\,\partial y} + \gamma(k_c-1)\frac{\partial u_c}{\partial y} + \gamma(k_c-1)\frac{\partial v_c}{\partial x} = 0 \qquad (6)$$

Where u_c and v_c are the displacement components of the graded coating in x and y directions. The Kolosov`s constant for present 2-D problem would be [24],

$$k_c = 3 - 4\upsilon \qquad (7)$$

In other work, related to mixed boundary-value contact problems, Guler and Erdogan found another form of governing stress equations for contact problem of a graded coating/substrate system [3], [14]. The model describes that a cylinder and a half-plane are in constrained contact without penetrating phenomenon by hard contact method. Substrate basement is fixed in all directions and roller moves in x-y coordinates as degrees of freedom. Other displacement and traction boundary conditions of the model are in continuity form as follows:

$$u_{fgm}(x,h) = u_{sub}(x,h) \qquad (8)$$

$$v_{fgm}(x, h) = v_{sub}(x, h) \quad (9)$$

$$\sigma_{yy_{fgm}}(x, h) = \sigma_{yy_{sub}}(x, h) \quad (10)$$

$$\sigma_{xy_{fgm}}(x, h) = \sigma_{xy_{sub}}(x, h) \quad (11)$$

Usually the dimensions of half-planes are too much larger than other parts in models, so the elasticity theorem defines the approximate zero stresses far from the contact applied loads

$$\lim_{|x^2+y^2|\to} \sigma_{xy_{sub}} = 0 \quad (12)$$

$$\lim_{|x^2+y^2|\to} \sigma_{yy_{sub}} = 0 \quad (13)$$

And stresses on surface of contact can be written in a simple form like:

$$\sigma_{yy_{fgm}}(x, 0) = \sigma(x) \quad (14)$$

$$\sigma_{xy_{fgm}}(x, 0) = \tau(x) \quad (15)$$

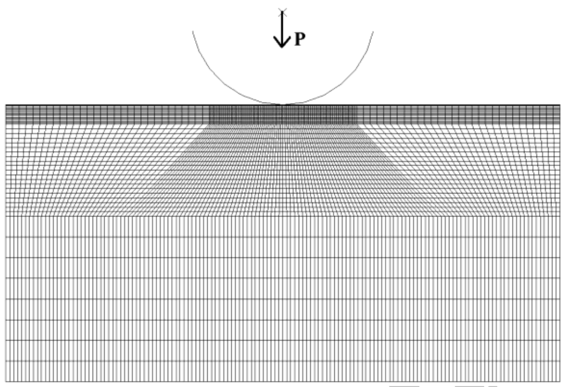


Fig. 2 Meshing and loading of coating model; verification of graded coating finite element model by simulation of coating in several layers.

2.1. Assumptions

Some of the assumptions used to simplify the problem are as follow:

1. No thermal effect is considered and mechanical properties such as Poisson`s ratio and coefficient of friction are constant.
2. The roller and half-plane are considered too much long in the direction perpendicular to rolling plane. This lets the true consideration of 2-D behavior for present problem.
3. Contact problem is in steady state condition while the linear velocity of roller is constant in pure rolling and the problem would be solved in an implicit approach.
4. Rolling is simulated as a quasi-static process, i.e., time dependent phenomena are not analyzed. Hence, dynamic effects are ignored and material properties do not depend on the strain rate.
5. No penetration in contact area and no delamination in coating/substrate interface occur due to FE modelling and node positioning.
6. Small deformations, controlled time periods and ideal material behavior are the other assumptions which are common considerations in FE analysis of coatings and films.

3 FINITE ELEMENT MODELING

Contact mechanics approach which deals with stress field is the study method in the present paper. Pure rolling of two bodies with respect to their initial geometries and external loading makes the stresses variation and surface deformation. FE codes are developed to analyze this rolling contact on a graded coating. Verification of FE modelling and assumptions for graded coating is the first step and the contact analysis of effects of this geometry and material property are the main objectives.

Two special modelling of different meshing and elements have been developed. The first is a 2-D model which concentrates on FGM coating. Fig. 2 shows the loading and meshing of the first simulation for the purpose of coating model verification. The contact force, P, is applied to make a vertical deformation about 20% of coating thickness in elastic range of coating and substrate. Meshing in the first model is symmetric and rigid cylinder pushes the surface normal to contact zone. This model simulates a graded material in several layers by different properties. Also these material constants are applied at the integration point of each finite element. The number of layers was increased to achieve the convergence of stress results in graded layers. Γ, stiffness ratio, has an important effect on stress variation in contact area, so this parameter can make FGM coating different from normal ceramic and metallic coatings. The shear modulus would change in coating thickness with several functional patterns; some examples are as exponential, linear, power law or both exponential-linear compound functions.

The thickness of substrate is sufficiently large compared to the coating (300:1); this confirms the coatings assumption. Hence, the coating/substrate system is modelled as a semi-infinite continuum which can be checked when the stresses tend to zero far away from contact zone. A FE subroutine is written to apply the material properties and graded coating constants. In fact, the variation of shear modulus and other mechanical properties are defined in first code, and then number of layers and distribution trend of these properties in coating thickness are calculated by subroutine for convergence error of less than 0.1%. Our dual FE codes methodology helps us to model more loading and material conditions. Also let the trial and error in some situations to improve the material variation function through the coating thickness. Coating thickness (*h*) is divided in several layers in each step. Properties of each layer are defined through the thickness and all the layers are at the same thickness as mentioned in Fig. 3. The non-linear 8-node elements are used in this model which have compatible displacement shapes and are well suited to model curved boundaries in elastic contact.

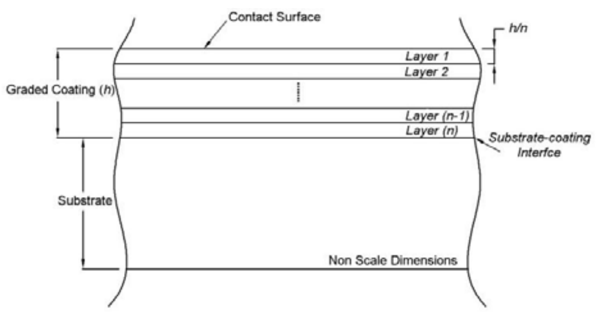


Fig. 3 Dividing the graded coating to several layers

Simulation of frictional rolling contact is the objective of the second model. A global coordinate exists in the initial directions of *x* and *y* (see Fig. 1) and local coordinates are defined for deformation of elements.

Planar non-linear 8 nodal quadratic elements are used to build the finite element mesh. Various mesh schemes are tried to achieve convergence. The optimized model has totally 148004 elements which 800 elements are in contact region of interest. Less than 1% of them are triangular related to nonlinear geometry. These elements provide acceptable accurate results for mixed (quadrilateral-triangular) meshes and can tolerate irregular and non-linear shapes (especially in deformable contact problems) without as much loss of accuracy.

In addition, typical element formulation is based on the use of second order polynomial interpolation functions of the dependent variables, e.g. displacements and stresses. The values of the dependent variables at the element nodes uniquely determine the coefficients of their interpolating polynomials. In this formulation, it is assumed that the contact area is small compared to radii of roller, so the standard Hertzian assumption can be used. This assumption is true whereas the rigid roller is in contact with half-plane substrate, but may be different in some applications of contact problems like clutches, brakes and couplings. This model uses R/h=100 and L/R>10, (*L*, length of roller and half-plane) which are adequate to simulate a plane strain 2-D modelling [20]. The roller/coating contact surfaces were modelled using surface-to-surface approach with respect to node place contact discretization. In this approach the roller is the master and the coating plays the slave role. The Lagrange multiplier method was used for contact simulation. The Lagrange multiplier formulation adds more degrees of freedom to the model in order to guarantee no penetration of contact bodies occurs [30]. The penalty friction law and linear elasticity are applied in contact zone.

Present FE code solutions are based on infinitesimal strain theory which is due to the constant loads in steady conditions of contact. For this non-linear problem, small load steps are used toward incremental quasi-static contact. Values of the contact force, stress tensor, deformations, and other outputs are recorded at each load step; modulus of time scale in elastic analysis is fixed in a long term condition.

Boundary conditions are applied in first steps and continued to last step. Then the external loads are applied in second steps. This method helps the model to simulate the exact contact conditions and solving of equations is simpler, so the infinitesimal movement of roller would happen in an appropriate time.

In order to capture the accurate sharp variations of the stress components especially near the ends of contact zone, FE mesh density is increased significantly in the vicinity of the contact region. The aspect ratio is tried to be controlled (see Fig. 4). Fig. 4 shows the roller before movement; the time period of solution is set that the analysis of contact is done in left half of substrate. Therefore, the elements in right half of roller are coarser and too much fine meshing with more accuracy is used in left half to decrease the solving time.

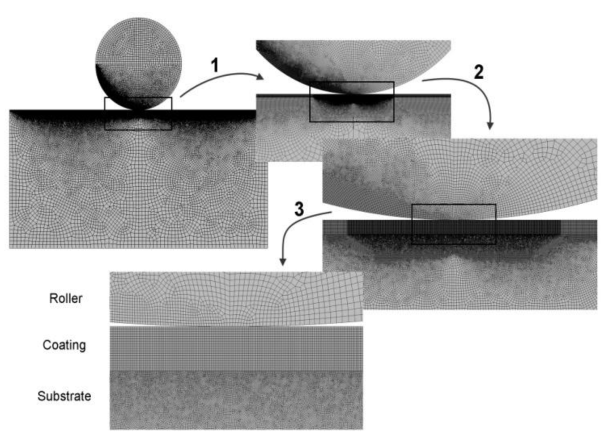


Fig. 4 Main modeling of rolling on FGM coating before loading and movement; Non symmetric element shapes and meshing are used because of ccw rotational direction.

In the present simulation, Abaqus 6.12-1 as a commercial finite element software linked with a developed manual FE code in Matlab is used. The subroutine code changes the coating thickness, material and mechanical properties in layers and determines the variation of graded coating constants for second FE model which does the main contact analysis.

4 RESULTS AND DISCUSSION

The mechanical contact of a cylindrical roller on a graded coating is affected by several material and geometrical parameters. Some of them are studied and discussed here via FE analysis which is verified by analytical results.

4.1. Results verification

The contact stress and force as well as contact zone have been considered here for the purpose of model verification. The accurate capability of the FE methodology for contact problems in comparison with the analytical results found in the literature is proved. The results can be verified by Guler et al., through many published articles [24].

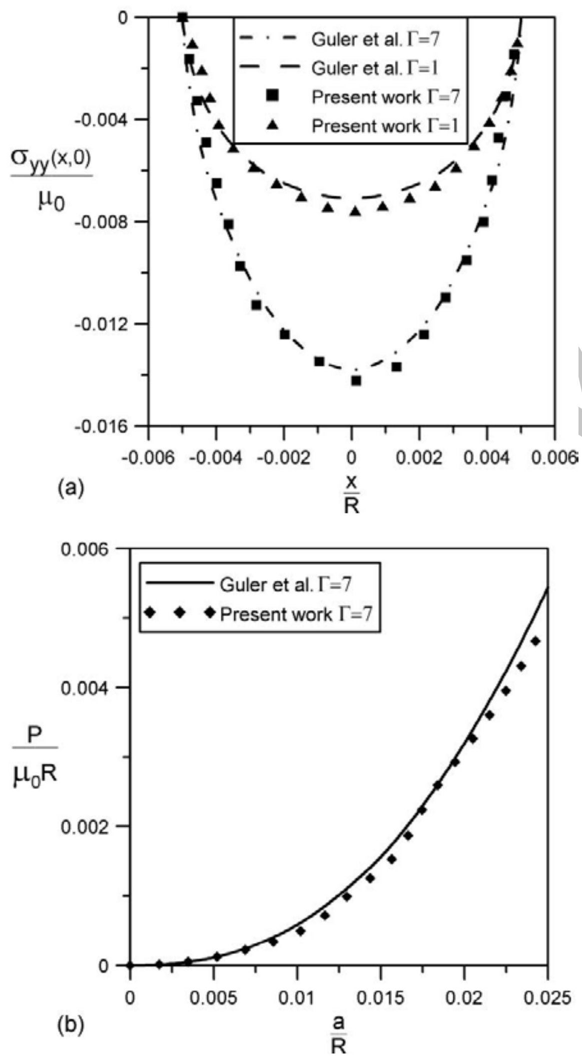


Fig. 5 Verification of FEM results with analytical solution Guler et al. [24]; solid curves present Guler results and symbols present proposed modeling (υ=0.3, Q/ ηP=-0.75, β/η=-1, a/h=0.5).

The results are compared in two sections which are shown in Fig. 5. First the normal stresses on contact surface are verified for sample loading and constant contact area (*a/h*=cte.) in two stiffness ratios. Also the relationship between vertical force and contact zone length is verified for a stiffening graded coating (Γ>1). Comparing both sections simultaneously helps us to verify the reliability of FEM results such as stress analysis and deformation of contact area. These verifications show a good agreement between the achieved FEM results and analytical solution. The maximum relative difference between these two results was about 10% at stress peak point and larger contact area. Also the FEM code is tested on a simple rolling contact of 2-D long roller on a homogenous non-frictional half-plane; this was for initial checking of models.

4.2. Graded coating simulation

Mesh convergence rate and number of elements improvement are discussed in Fig. 6. The number of elements increase and more fine mesh leads to much less error and convergence in the results. Basis of comparison is contact length ratio (*a/h*=0.5) for two different roller radiuses *R*. Thereafter, more increasing the number of elements after convergence makes a negligible inappropriate error. Verification of contact stresses $\sigma_{yy}(x,0)$ and $\sigma_{xx}(x,0)$ show a good agreement with reference curves (such as previous section), whereas the contact length (2*a*) in FE modelling is the same with Guler results [24].

The capability of the present finite element code in graded coating is tested by simulating in several layers. The dimensionless results which are shown in Fig. 7 compare the trend of stress variation by increasing the number of layers. Both normal contact and Von Misses stresses are decreased and converged to a constant value in contact zone. The error curve shows about 0.003% difference in stresses in comparison with 5 and 6 layers of graded coatings. Although the coating thickness is too much less than half-plane thickness, but this result represents a very exact simulation of FGM coating with appropriate 6 layers. One innovation of this paper is modelling of FGM coating by two linked finite element codes which can converge the equations in implicit solution.

4.3. Effects of coating thickness variation (*h*)

In addition to mechanical property variation through the coating thickness, the thickness of this graded coating has a positive effect on a contact analysis. Although the coatings are thin related to substrates and punches in contact problems, but Fig. 8 shows a very small and negligible interface stress variation by the effect of thickness. The stress components in coating/substrate interface at the center of contact zone, (0,*h*), is affected by graded coating geometry (See Fig. 8). This position analysis is important in order to design against coating/substrate delamination and cracking failure. The stresses σ_{yy} and σ_{xx} are invariably compressive; they decrease as the coating thickness increases. Thinner coatings have more effect on variation of stress components and these variation decreases strongly when the thickness-radius ratio (*h/R*) tends to 0.025.

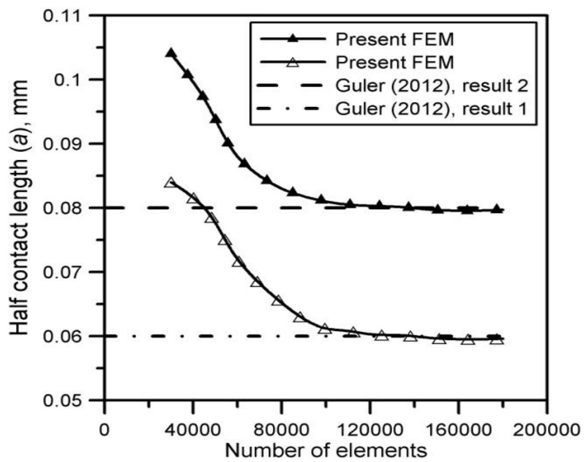
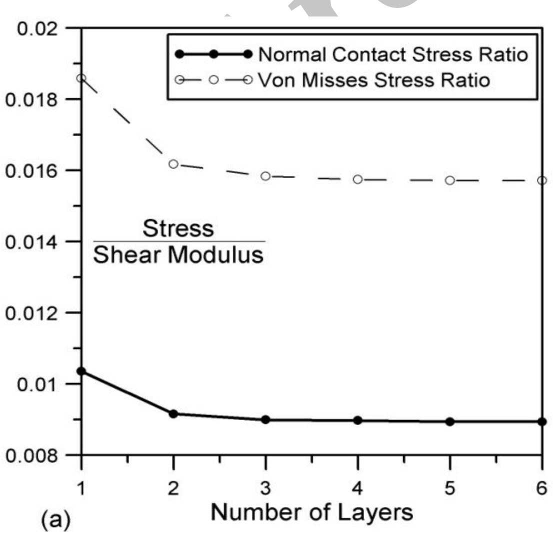


Fig. 6 Mesh improvement in FE analysis with respect to Guler et al. [24], (a/h=0.5, h/R=0.01, υ=0.3, Q/ ηP=-0.75, β/η=-1)

Tensile shear stress, σ_{xy}, decreases by thickness and assures the safer bonding of coating/substrate in thicker coatings. This phenomenon can be explained by decrease in subsurface stresses originated from the surface loads (*p(x)* and *q(x)*) when the distances from the contact surface increases.



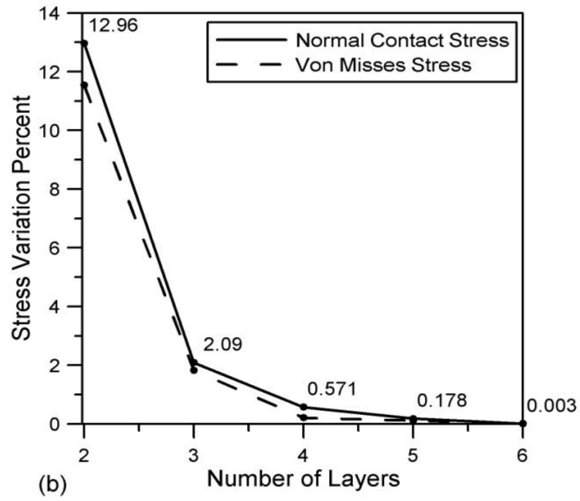


Fig. 7 Comparison of the number of graded coating layers, (a) variation of normal and Von Misses stresses, (b) percent of difference in variation of stresses by increase in layers quantity

The interface stress components, (0,*h*), are shown in Fig. 8 for various values of coefficient of friction. Unlike the in-plane stress (σ_{xx}), the shear and normal stresses (σ_{xy}, σ_{yy}) are not too affected by contact surface friction variation. Comparison of stress curves with respect to friction effects show the less effects of larger coefficients of friction, e.g. η=0.56 and 0.28. In fact, the frictions of more than a limit have a negligible effect on stress component variation in coating/substrate interface. On the other hand increasing this contact parameter increases the contact stresses which would be destructive in fatigue crack initiation.

Elastic strain energy is generated by surface disturbances and maximum strain energy density occurs in contact surface where the maximum stress and tractions are available. The elastic strain density is defined as:

$$u_e = \frac{\sigma^2}{2E} \tag{16}$$

The thermal effects are not supposed, so the elastic modulus, E, and other mechanical properties are constant on contact surface. The maximum total strain energy density on the contact surface of roller and stiffening graded coating is shown in Fig. 9. Energy density decreases by increase in coating thickness, it tends to a minimum variation when the thicker coatings are used. Thinner coatings have more effects on elastic strain energy density. Contacts of coatings with more friction coefficient increase the contact stresses which may lead to increase in strain energy.

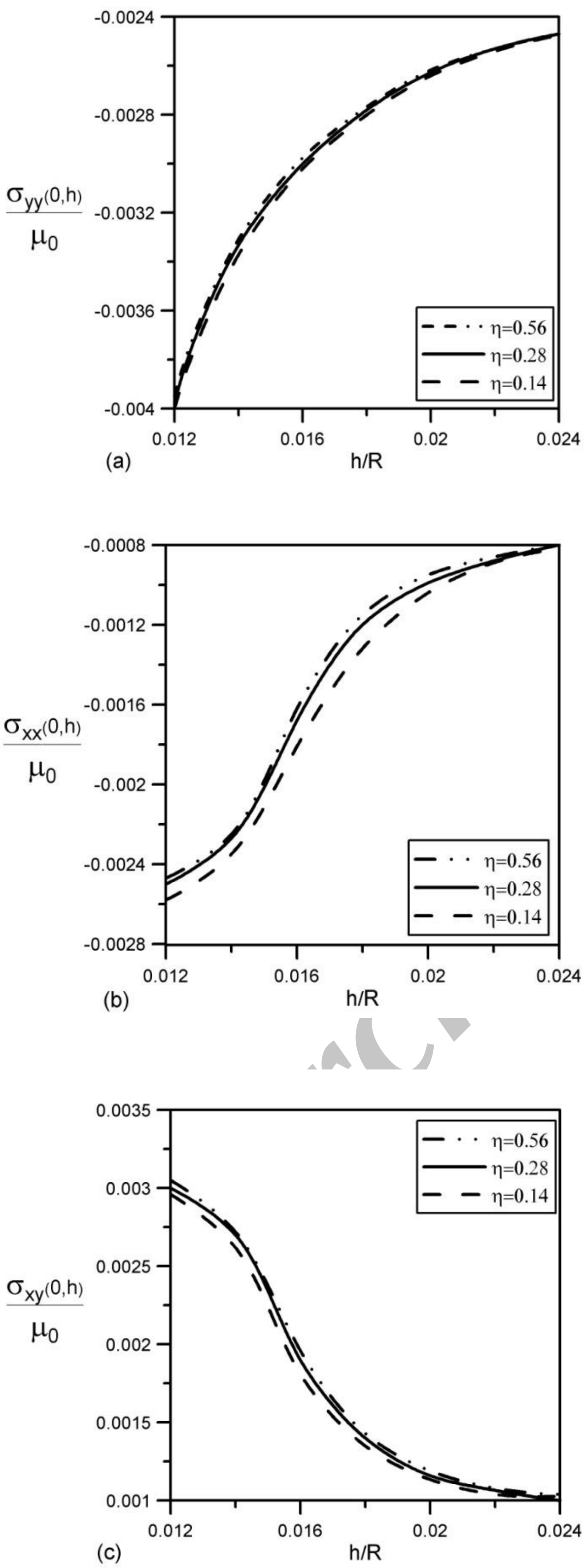


Fig. 8 Variation of stress components of coating/substrate interface, (0,h), by coating thickness in various coefficients of friction, (υ=0.3, Q/ηP= -0.75, β/η=not cte., a/h=not cte., Γ=7)

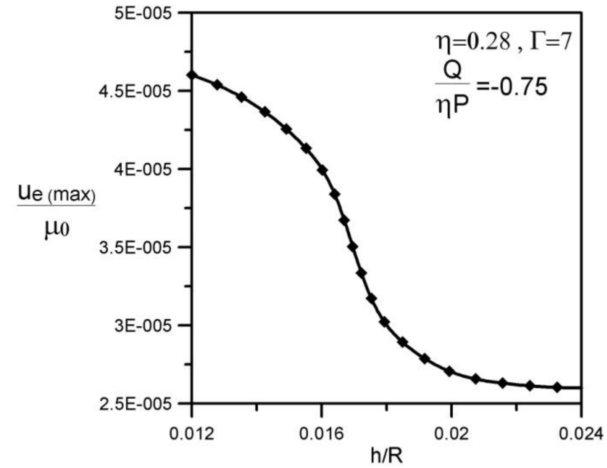


Fig. 9 Effect of coating thickness on maximum total elastic strain energy ($u_{e\ (max)}$) of contact surface (x, 0)

Coating thickness has different effects on surface stresses. Analysis of contact stresses of frictional rolling contact plays a significant role in design of graded coatings. Normal contact stress experiences a peak in center of contact zone. This maximum compressive stress decreases as the dimensionless ratio of coating thickness (*h/R*) increases (See Fig. 10a). Minor thickness ratios have more effects on stress distribution; the reason would be explained as the shear modulus for substrate (μ_s) and stiffness ratio (Γ) for all models are the same. Contact area length, 2*a*, may be studied in this modelling. Larger contact area would be occurred by increase in thickness of graded coating. Increase in maximum stress when the surface loadings (P, Q and friction coefficient) are constant will lead to increase in contact area length.

Fig. 10b shows the in-plane stress distribution on surface of coating, $\sigma_{xx}(x,0)$. It is apparent that the surface maximum tensile stress in trailing edge of cylinder decreases by increase in coating thickness. In fact, the position of this critical stress moves by changes in the contact area length. A remarkable point is that the coating thickness effects on peak of stress are more than other parts of contact area. The FEM modelling of stress distribution for *h/R*=0.012 has been verified by Guler results (see verification section) and then the other thickness effects have been developed.

As stated earlier, shear modulus μ vary exponentially through the coating thickness (Eq. 1).

The inhomogeneity constant γ affects the contact results like area length as shown in Fig. 11 for the interval described before (See Fig. 8). This constant, γ, is in reverse relationship with coating thickness which let the true comparison of results in Figs. 11 and 10a. The ratio of contact area to coating thickness *(a/h)* decreases by increase of *h*, but the larger contact zones are generated. This phenomenon is due to note that the increase of contact length is not as much as increase in coating thickness. Effect of friction coefficient η is also studied. The increase in contact area by increase in friction is obvious but the rate of changes in results decreases for thinner coatings or larger inhomogeneity constants.

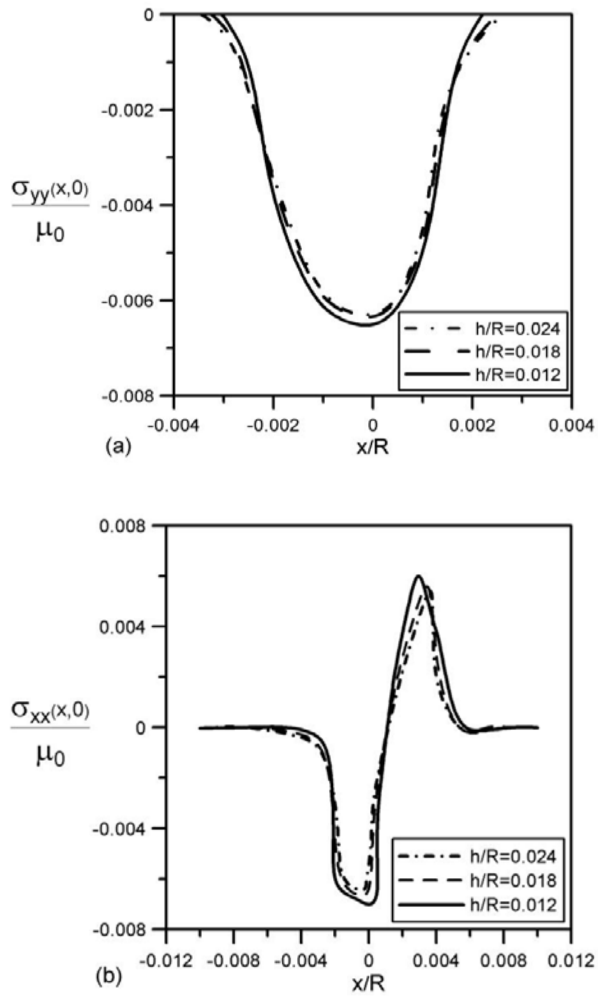


Fig. 10 Stress variation on contact surface due to coating thickness (υ=0.3, Q/ηP= -0.75, β/η=-1, a/h=not cte., Γ=7)

4.4. Effects of material variation in graded coating

This section presents FEM results for the effect of material variation in layers of graded coating on stress distributions of the coating/substrate system due to the frictional contact of a roller. The shear modulus of coating changes continuously through the thickness according to exponential function [24],

$$\mu(y) = \mu_{cs} \exp(\gamma \frac{y}{h}) \tag{17}$$

A power law function can be used as studied in literature for a normal contact frictionless punch [31],

$$\mu(y) = \mu_{cs} + (\mu_s - \mu_{cs})(\frac{y}{h})^n \tag{18}$$

Also linear variation of shear modulus would be considered when the gradient index n in power law equals to 1.

$$\mu(y) = \mu_{cs} + \left(\frac{\mu_s - \mu_{cs}}{h}\right) y \tag{19}$$

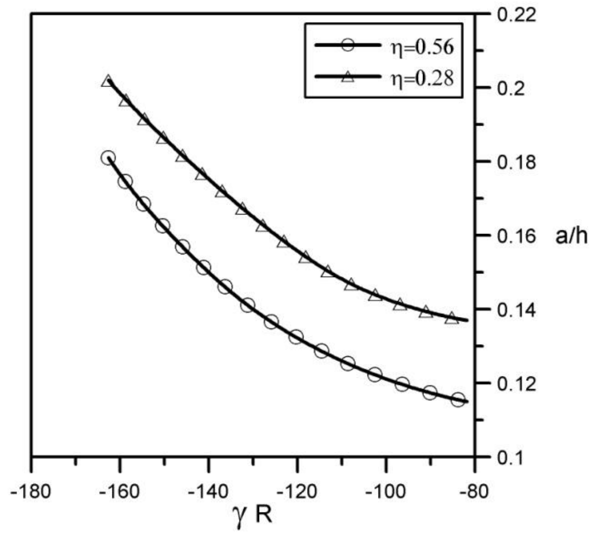


Fig. 11 Effect of friction coefficient on the relationship between contact area a/h and the inhomogeneity constant γ.

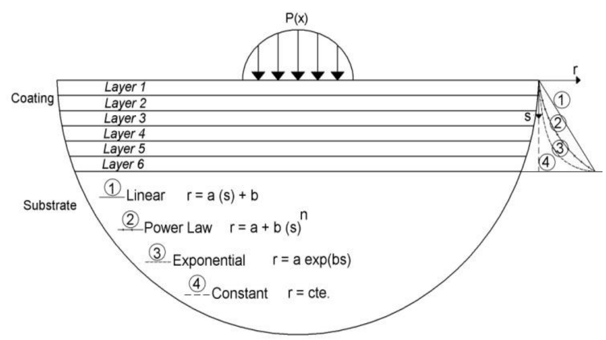


Fig. 12 Configuration of material property variation through the thickness of coating in linear, power law and exponential approaches (*a, b* and *n* are the constants, *s* and *r* are the local coordinate).

It can be shown that shear modulus in contact surface (y=0), μ_{cs} and at substrate interface ($y=h$), μ_c are the same in all three above approaches. Poisson's ratio is taken constant within the structure for simplicity. The power law for unit value of n (n=1) forms a linear variation of material. The purpose of selecting these three different shear modulus variations in FE analysis is to compare and identify the gradient type that is more effective in suppressing the rolling contact stresses. Fig. 12 shows the configuration of mechanical property variation through the coating thickness in a local coordinate (*s, r*).

In the first step, a normal contact of a punch is considered with a vertical force, *P*, pushes the punch to surface of graded coating. This normal force is in a direct relation by contact zone. As the contact force increases the contact length, 2*a*, increases (See Fig. 13) and this note may be so important in design of components. More contact length in frictional rolling components may play a significant role in wear of contact surfaces especially in higher contact stresses. The modellings are considered in following conditions:

$$\frac{\mu_{cs}}{\mu_s} = \Gamma = 0.33$$

$$\upsilon = 0.3 \;, \quad P = 15 \text{ KN}$$

$$\frac{R}{h} = 250$$

The results show that the effect of material distribution in graded layers by exponential or power law forms would be more distinctive in higher contact loads (0.1<*a/h*<0.2). For small loadings there is not too much difference in the results affected by trends of material variation, exponential or power law. In other words, the same contact region of power law and exponential functions are created by contact loads which have about 8% difference. This point is verified by Yang and Ke [31] who studied a normal contact of a punch on two homogenous and graded coatings.

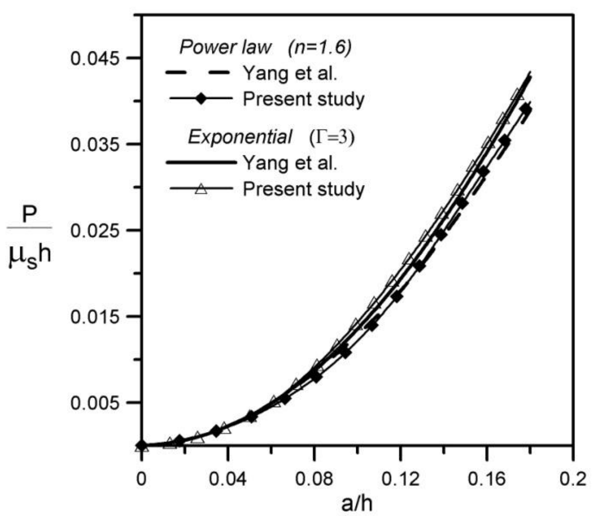


Fig. 13 Variation of contact zone affected by contact force verified by Yang and Ke [31]

The distribution of surface stresses would be effective in contact failure analysis like fatigue and wear [32]. The normal contact stress in frictional rolling contact of a cylinder is investigated in Fig. 14. The maximum normal stress in contact surface ($\sigma_{yy}(x,0)$) occurs in center of contact zone (*x/a*=0). This stress decreases by approaching to the leading and trailing edges of roller. The exponential, power law and linear variations of material in the graded coating present similar trends in stress distribution on contact surface, but the results of power law and exponential forms have less difference in comparison to the linear one. The reason would be explained by higher rate of material property change in linear function, but the exponential and power law functions make a lower rate of material variation near to surface (*y*=0).

Fig. 14 shows the close effects of three material variation forms near to the trailing edge, but the differences in leading edge of roller are more. The in-plane stress in contact has a tensile peak in trailing edge and this stress in linear function is more than other forms of material variation. This note may lead to a negative point in selection of linear trend of material variation. Also the stresses in inner layers of graded coatings (specially second and third layers) for linear form are more critical than exponential one.

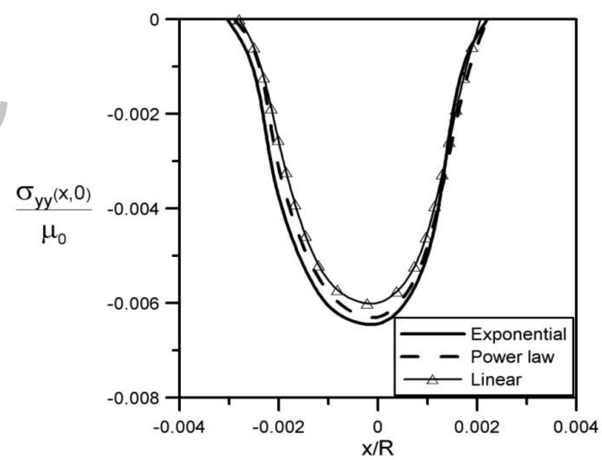


Fig. 14 Normal stress variation on contact surface ($\sigma_{yy}(x,0)$) in three trends of exponential, power law and linear variation of material through the graded coating thickness.

The comparison of material variation in power law and exponential forms for various stiffness ratios is shown in Fig. 15. Investigation of roller contact length (2*a*) according to Γ variation is another required result which is so important in design of rolling contact parts like gears. Stiffening coating (Γ>1), softening coating (Γ<1) and homogenous one (Γ=1) are studied in Fig. 15. Some parameters such as friction and loading are controlled and only the effect of changes in shear modulus is investigated. Decreasing the shear modulus of graded coating in free surface increases the contact length and this curve rate has a maximum slope around

1<Γ<2. Generally speaking, the contact zone expands over the free surface of graded coating as the stiffness ratio, Γ, increases. Contact length for present problem is approximately 20% of coating thickness which is useful for coating design of parts against abrasion and fatigue. As shown, the contact area for the softening coating is less than that of the homogeneous material while it is opposite for the stiffening one. The effect of power law and exponential coating material variations on contact zone of stiffening coatings is much more than which can be analyzed for softening ones.

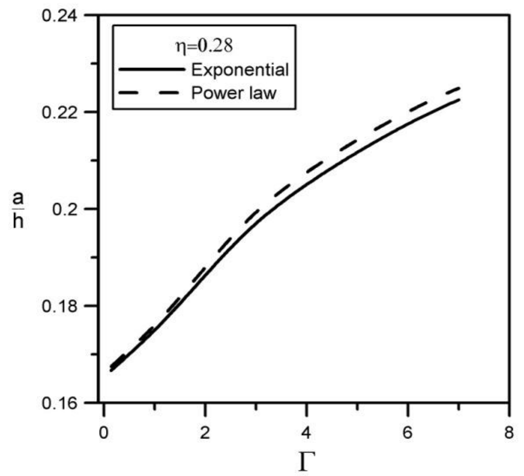


Fig. 15 Variation of contact zone length with stiffness ratio of graded coating in two approaches of power law and exponential material variation in coating (friction and loading condition are constant.)

5 CONCLUSION

The FGM coatings permit a smooth transition in the material properties at the interface and overcome some of the shortcomings in homogeneous substrate and coating. FE modelling was applied to simulate the frictional rolling contact of a cylindrical component on a graded coating. The verification of method with analytical results shows a good agreement. The effects of geometry and coating material variation on performance of coating were studied. The results of this study may be used as a guide line for designing thin films and graded coatings bonded to homogeneous materials under rolling contact loads. Some of the main conclusions would be as follows:

- FE analysis of a graded coating in minimum six layers indicates an exact modelling; this method can simulate the coating performance in rolling contact with less than 0.1% convergence error.
- Coating thickness affects on the stresses in coating/substrate interface (0, *h*). Variation in thickness of thinner coatings has more effect on stress distribution. Generally for each value of stiffness ratio and roller diameter there is a specific value of coating thickness that the stress state remains at constant level.
- The interface stresses variation decreases significantly for the coating thickness ratios of more than 2.5%, (*h/R*>0.025). Tensile shear stress, σ_{xy}, decreases by increase of thickness and assures the safer bonding of coating/substrate in thicker coatings.
- Coefficient of friction affects the in-plane stress (σ_{xx}) in coating/substrate interface more than shear and normal stresses. Also this phenomenon decreases by increase in coefficient of friction.
- Energy density decreases by increase in coating thickness. If the thicker coatings are applied, the energy density tends to a minimum variation. Also larger contact zone would be occurred by increase in graded coating thickness.
- The effect of material distribution in graded layers by exponential or power law forms would be more distinctive in higher contact loads (0.1<*a/h*<0.2). The comparison of material variation in power law, exponential and linear functions show the almost similar results near the trailing edge, but different results are seen in leading edge of roller.
- The effect of power law and exponential material variations in coatings show more different results for contact zones of stiffening coatings (Γ>1), but in softening ones (Γ<1) the results are more the same.
- Larger contact lengths are generated by increase in thickness *h*, but the ratio of contact area to coating thickness *a/h* decreases. The rate of change in contact zone and stresses decreases by increase in inhomogeneity constant, γ.

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