Effects of Slip Boundaries on Mixed Convection of Al_2O_3-water Nanofluid in Microcavity

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Abstract: Due to the importance of the slip effect on modeling of microchannel and microcavity, numerical investigations have been introduced in this work for studying the mixed convection of Al_2O_3-water nanofluid in a square microcavity. Governing equations are discretized and solved using the Finite Volume Method and SIMPLER algorithm. The Knudsen number is selected between 0.001 and 0.1 to consider slip velocity and the temperature jump boundary conditions in slip flow regime. In this study we investigate the influence of the Knudsen number on the average Nusselt number and heat transfer rate of Al_2O_3-water nanofluid. Results shows that the average Nusselt number is the function of Richardson number, Knudsen number and volume fraction of nanoparticles. Increasing the Richardson number, makes the forced convection less effective and leads in reduction of the Nusselt number. Hence, increasing the Knudsen number, leads to the temperature gradient reduction and reducing the average Nusselt number. As a result, the average Nusselt number could be enhanced up to 10.93% by using nanoparticles in the base fluid.

Keywords: Knudsen Number, Mixed Convection, Microcavity, Nano fluid, Slip Flow

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1 INTRODUCTION

Flows in micro-devices constitute an emerging application field of fluid dynamics [1]. Under certain conditions such as very low pressure, hydrophobic surfaces, and small-size channels with characteristic lengths between 1 μm and 1 mm, the continuum assumption may not be accurate, particularly in micro-devices, which find applications in medicine, fuel cells, biomedical reaction chambers, Lab-On-a-Chip technology and heat exchangers for electronics cooling. Therefore, it is important to investigate slip flows in order to provide useful prediction tools for convective heat transfer in micro-devices. Also lid-driven cavity flow is considered as an internal flow that is driven by a moving lid of a cavity which is rectangular in most studies. It is widely used as a benchmark problem for various fluid dynamics methods.

Ismael et al., [2] studied laminar mixed convection inside a lid-driven square cavity filled with water. The lid is due to the movement of the isothermal top and bottom walls and a partial slip condition was imposed in these two moving walls. They investigated that there are critical values of the partial slip parameter at which the convection is declined. These critical slips (>0) where found to be sensitive to the Richardson number and the direction of the moving walls where they exist at $0.01 < Ri < 100$ for counter-direction and at $0.01 < Ri < 10$ for indirection moving walls. They also found for nonzero partial slip parameter's values, the average Nusselt number is an increasing function of the Richardson number.

Recent advances in nanotechnology have led to the development of a new, innovative class of heat transfer fluids (nanofluids) created by dispersing nanoparticles (10–50 nm) in traditional heat transfer fluids. Nanofluids appear to have the potential to significantly increase heat transfer rates in a variety of areas such as industrial cooling applications, nuclear reactors, transportation industry, micro-electromechanical systems (MEMS), electronics and instrumentation, and biomedical. Possible improved thermal conductivity translates into higher energy efficiency, better performance, and lower operating costs. Many studies related to the heat transfer enhancement using nanofluids both experimentally and theoretically was conducted by a number of investigators and also many review articles involving the progress of nanofluid investigation were published in the past several years [3–8]. Talebi et al., [9] studied laminar mixed convection flows through a copper–water nanofluid in a square lid-driven cavity. They found that at a fixed Reynolds number, the solid concentration affects the flow pattern and thermal behavior particularly for a higher Rayleigh number. In addition, they observed that the effect of solid concentration decreases by the increase of Reynolds number.

Based on the Knudsen number (Kn), the flow in micro-devices have been classified. For $Kn < 0.001$, the flow is continuum and it is actually modeled by the Navier–Stokes equations with classical no-slip boundary condition. For $0.001 < Kn \leq 0.1$, the flow is a slip flow and the Navier–Stokes equations remain applicable, provided a velocity slip and a temperature jump are taken into account at the walls. These new boundary conditions point out that rarefaction effects become sensitive at the wall first [10]. The flow in most application of these systems, such as micro gyroscope, accelerometer, flow sensors, micro nozzles, and micro valves, is in slip flow regime, which is characterized by slip flow at wall.

Kuddusi et al., [11] studied slip flow in rectangular micro-channels heated at constant and uniform heat flux. Their study is extended to eight possible thermal versions that are formed of different combinations of heated and adiabatic walls. They found that rarefaction has a decreasing effect on heat transfer in the micro-channels exposed to any of the eight thermal versions. Renksizbulut et al., [12] studied a rarefied gas flow in the entrance region of rectangular micro-channel in the slip-flow regime. They used a control-volume based numerical method to solve the Navier–Stokes and energy equations with velocity-slip and temperature-jump conditions at the walls. They observed significant reductions in the friction factor and Nusselt number due to rarefaction effects, which also extend to the fully developed region. They investigated that the friction and heat transfer coefficients are less sensitive to rarefaction effects in corner-dominated flows as in square channels when compared to flows between parallel plates.

Mizzi et al., [13] highlighted differences between Navier-Stokes-Fourier (NSF) slip/jump solutions and direct simulation Monte-Carlo (DSMC) computations for a micro lid-driven cavity problem. They have shown that for complex flows, such as the driven cavity, non-equilibrium effects are more appreciable and their onset occurs at lower Knudsen numbers than expected. Hettiarachchi et al., [14] studied three-dimensional laminar slip flow in rectangular micro-channel. They used finite-volume method to solve Navier–Stokes and energy equation with velocity slip and temperature jump at the walls. They showed that the effect of velocity slip is to increase the Nusselt number, while the temperature-jump tends to decrease it, and the combined effect could result in an increase or a decrease in the Nusselt number. Perumal et al., [15] numerically simulated gaseous micro flows by Lattice Boltzmann Method (LBM). They applied LBM to simulate the pressure driven micro-channel flows and micro lid-driven cavity flows. After comparing the pressure distribution and other parameters with available experimental and analytical data, they found good agreement between the results. Kuo et al., [16] studied the thermal convection under various slip boundary conditions in a 2D box with aspect ratio equal to two. Their results have shown that the slip boundary conditions of vertical side walls and horizontal plates will affect the pattern selections of the flow and temperature fields.

Liu et al., [17] studied the pressure-driven flow in a long micro-channel via lattice Boltzmann equation method. They presented a method for implementing slip boundary conditions with local effective Knudsen number in a LBGK model for microscale gas flows. They studied the flows in a long micro-channel driven by a pressure gradient under different conditions. They've also investigated the effects of rarefaction and compressibility on the deviation of the pressure distribution from the linear one.

Babaie et al., [18] studied the Thermal transport characteristics of electroosmotic flow of power-law fluids in the presence of pressure gradient through a slit micro-channel. They investigated that the non-Newtonian characteristic of the fluid can influence the thermal behaviors of the flow by affecting the rate of heat convection and viscous dissipation; however, its influence diminishes at higher values of the dimensionless Debye-Hückel parameter. They also found that the zeta potential, whose value is dependent on the channel wall and the electrolyte solution characteristics, can highly affect the thermal behaviors of the flow, especially at smaller values of the dimensionless Debye-Hückel parameter.

Shojaeian et al., [19] studied numerically three-dimensional slip flow through in a triangular micro-channel. They solved Navier–Stokes equations in conjunction with slip/jump boundary conditions. Their result showed that the rarefaction decreases the Poiseuille number, while its effect on the Nusselt number completely depends on the interaction between velocity slip and temperature jump. They also found that the aspect ratio has an important role in the analysis, but the variation of Reynolds number is less remarkable.

Shojaeian et al., [20] also studied analytically the convective heat transfer and entropy generation in Newtonian and non-Newtonian fluid flows between parallel-plates with velocity slip boundary for both isoflux and isothermal thermal boundary conditions. Their results indicated that an increase in the slip coefficient leads to an increase in both Nusselt number and Bejan number, whereas it gives rise to a decrease in global entropy generation rate. Brinkman number and power-law index have opposite effects on the Nusselt number, Bejan number, and entropy generation rate compared to slip coefficient.

Shetab Boushehri et al., [21] studied the effect of temperature jump boundary condition on conjugate heat transfer in parallel plate micro-channel heat sink. They implemented new method for coupling equations between fluid and solid domains with temperature jump boundary condition in Open FOAM. They observed that temperature jump has a considerable effect in temperature field and disregarding that leads to overestimation of heat transfer.

In the present study, mixed convection of Al_2O_3-water nanofluid inside a square lid-driven microcavity is investigated numerically. Mostly, previous studies focused on micro-channels and to the best of our knowledge, in most researches done on nanofluids in a lid-driven microcavity, thermal behavior with temperature jump boundary condition have not been studied yet. Due to the lack of comprehensive study in thermal behavior of lid-driven microcavity, the main purpose of this study is to investigate the effects of the Knudsen number on the average Nusselt number. The method that is used in this study helped us to solve the energy equation using temperature jump boundary condition for the first time in this case.

2 PROBLEM DESCRIPTION

Geometry of the problem is shown in Figure 1. The left and the right walls of the cavity as well as its top wall are maintained at a constant temperature T_c, here referred as "cold" temperature. The enclosure's bottom wall, which moves in its own plane from left to right with a constant speed u_b, is kept at a constant temperature T_h, here referred as "hot" temperature. The cavity is filled with Al_2O_3-water nanofluid. The nanofluid is incompressible and the flow is laminar. Also it is assumed that the liquid and solid are in thermal equilibrium. The thermophysical properties of the nanofluid are assumed to be constant. The physical and thermal properties of base fluid and nanoparticles at the base temperature, i.e. at 20◦C have been summarized in Table 1.

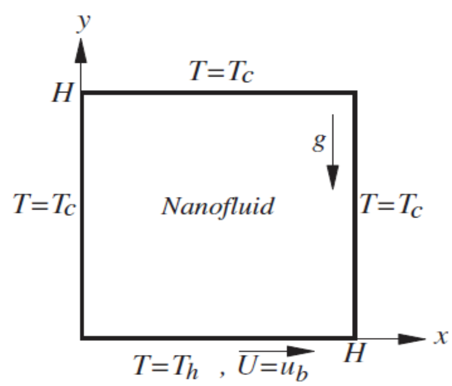

Fig. 1 Geometry of the problem

3 MATHEMATHICAL FORMULATION

The problem is considered to be steady-state and the Knudsen number is in this range $0.001 < Kn \leq 0.1$. In this case, Navier-Stokes and energy equations are applicable with slip velocity and temperature jump boundary conditions. The following dimensionless parameters are used to derive non-dimensional equations:

$$X = \frac{x}{H}, Y = \frac{y}{H}, U = \frac{u}{u_b}, V = \frac{v}{u_b}, \quad \theta = \frac{T - T_c}{T_h - T_c}, \; P = \frac{p}{\rho_{nf} u_b^2} \tag{1}$$

Using these dimensionless variables, the governing equations could be written in the following form:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{2}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{\text{Re}}\frac{\mu_{nf}}{\vartheta_f \rho_{nf}}\nabla^2 U \tag{3}$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{\text{Re}}\frac{\mu_{nf}}{\vartheta_f \rho_{nf}}\nabla^2 V + \frac{(\rho\beta)_{nf}}{\rho_{nf}\beta_f} Ri\,\theta \tag{4}$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{\alpha_{nf}}{\alpha_f}\frac{1}{\text{Pr.Re}}\nabla^2 \theta \tag{5}$$

Table 1 Thermophysical properties of base fluid and nanoparticles [22]

Property	Fluid (water)	Solid (Al_2O_3)
c_p (J/kgK)	4179	3970
ρ (kg/m^3)	997.1	765
k (W/mK)	0.613	25
μ (Ns/m^2)	10.03×10^{-4}	–

Reynolds number, Richardson number and Prandtl number are defined as:

$$\text{Re} = \frac{u_b H}{\vartheta_f}, \; Ri = \frac{g\beta_f (T_h - T_c) H}{u_b^2}, \; \text{Pr} = \frac{\vartheta_f}{\alpha_f} \tag{6}$$

Moreover, $Ri = \frac{Gr}{\text{Re}^2}$ where the Grashof number, is defined as:

$$Gr = \frac{g\beta_f (T_h - T_c) H^3}{\vartheta_f^2} \tag{7}$$

Density, heat capacity and thermal expansion coefficient of nanofluid are calculated as follows [23], [24]:

$$\rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_s \tag{8}$$

$$(\rho c_p)_{nf} = (1-\varphi)(\rho c_p)_f + \varphi(\rho c_p)_s \tag{9}$$

$$(\rho\beta)_{nf} = (1-\varphi)(\rho\beta)_f + \varphi(\rho\beta)_s \tag{10}$$

The effective thermal conductivity of the nanofluid is evaluated from the Maxwell model [25]:

$$\frac{k_{nf}}{k_f} = \frac{(k_s + 2k_f) - 2\varphi(k_f - k_s)}{(k_s + 2k_f) + \varphi(k_f - k_s)} \tag{11}$$

The effective dynamic viscosity of the nanofluid is obtained from Brinkman model [26]:

$$\mu_{nf} = \frac{\mu_f}{(1-\varphi)^{2.5}} \tag{12}$$

In the present study, the first order Maxwell non-dimensionalized velocity slip and temperature jump are considered as boundary conditions [27]:

$$U_{slip} - U_w = \frac{2-\sigma_v}{\sigma_v} Kn \frac{\partial U_s}{\partial n} + \frac{3}{2\pi}\left(\frac{\gamma - 1}{\gamma}\right)\frac{Kn^2 \text{Re}}{Ec}\frac{\partial T}{\partial s} \tag{13}$$

$$T_{slip} - T_w = \frac{2-\sigma_T}{\sigma_T}\left(\frac{2\gamma}{\gamma+1}\right)\frac{Kn}{\text{Pr}}\frac{\partial T}{\partial n} \tag{14}$$

Here σ_v, and σ_T are the tangential momentum and energy accommodation coefficients, respectively and are assumed to be 1.0 in this work. Since, cavity walls are isothermal, $\frac{\partial T}{\partial s} = 0$ and the second term of the right side of equation (13) will be vanished.

4 NUMERICAL IMPLEMENTATION

The governing mass, momentum, and energy equations are discretized and solved using a FORTRAN code utilizing control volume method and the SIMPLER algorithm. The diffusion terms are discretized using a second-order central difference scheme; while, a hybrid scheme is employed to discretize the convective terms. An under-relaxation scheme is employed to obtain converged solutions.

5 BENCHMARKING OF THE CODE

In order to validate the numerical procedure, a simulation of a mixed convection heat transfer with no-slip boundary conditions in a square cavity is performed using the code. The results are compared with Abu-Nada and Chamkha as shown in Table 2 [28]. As it can be observed from the table, the agreements between the results are very pleasing.

6 GRID INDEPENDENCE STUDY

To determine an optimum grid for the numerical simulations, a grid independence study is undertaken for mixed convection in a cavity filled with pure water with Pr =6.84 and *Ri* =0.01. Six different uniform grids, namely, 21 ×21, 41×41, 61×61, 81×81, 101×101, and 121×121 are employed for the numerical simulations. The results for the X-component of the velocity along the vertical centerline of the cavity for these grids are shown in Figure 2. Based on the results of this figure, an 81×81 uniform grid is used for all of the subsequent numerical calculations.

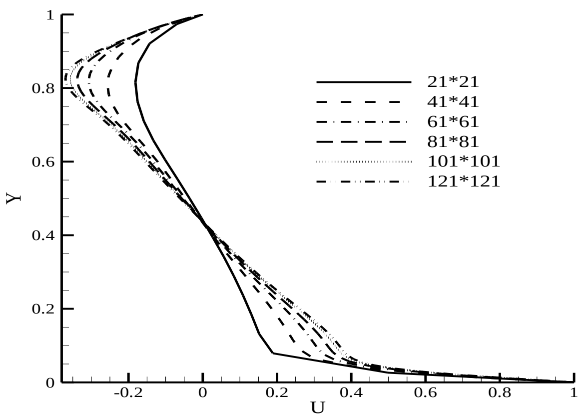

Fig. 2 The X-component of velocity along the cavity vertical centerline for different uniform grids

Table 2 Comparisons of the present results for the average Nusselt number of the hot wall with the results of Abu-Nada and Chamkha [28]

Ri	*φ*	Nu_{avg} (Our study)	Nu_{avg} (Abu-Nada)	% Error
0.5	0	2.128211	2.183122	2.52
0.5	2	2.207584	2.254814	2.09
0.5	5	2.328127	2.365185	1.57
5	0	1.325834	1.325823	0.00
5	2	1.379991	1.375144	0.35
5	5	1.463651	1.453406	0.70

7 RESULTS AND DISCUSSION

The results for $Gr = 10^4$ and the Richardson number is in the range of 0.01 to 100, the Knudsen number varies from 0.001 to 0.1 and the volume fraction of nanoparticles is considered from 0 to 5%. The streamlines and isotherms for $Gr = 10^4$, Ri =100 and *φ* =3% at different Knudsen numbers are shown in Figure 3. As it could be seen, using velocity slip and temperature jump boundary conditions strongly affect the whole simulation. At low Knudsen numbers such as 0.001, results are the same as the case of using no-slip boundary conditions, while, by increasing the Knudsen number to 0.1, the results are not similar anymore. So, in the case of slip flow regime, no-slip boundary conditions is not admissible.

a

b

c

d

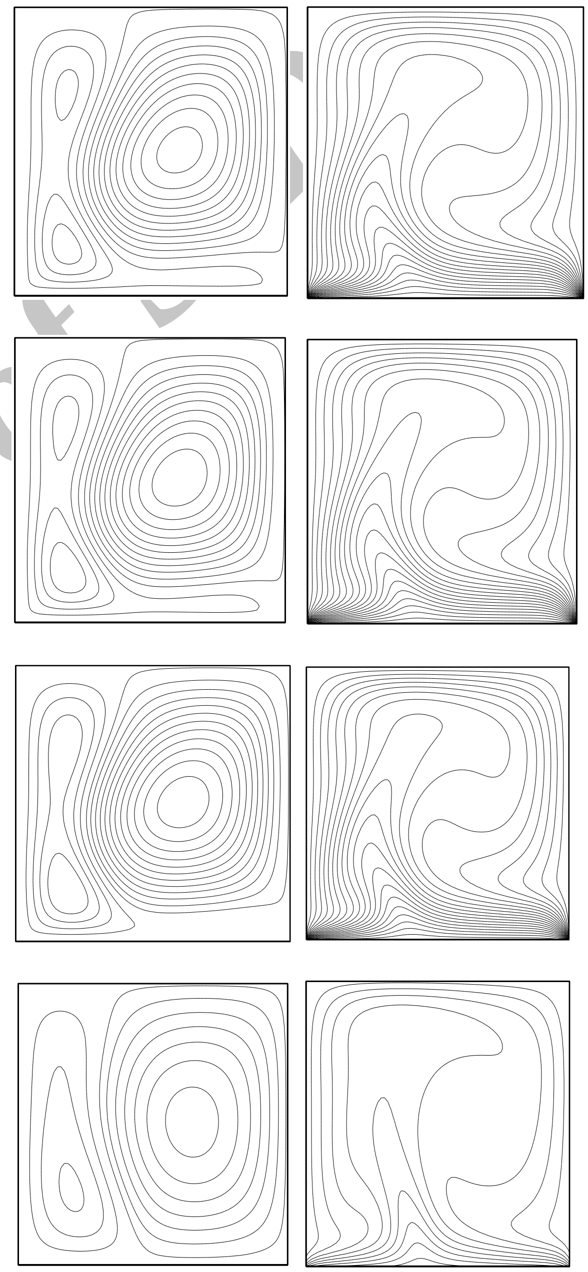

Fig. 3 Streamlines and isotherms for $Gr = 10^4$, Ri =100 and *φ* =3%. (*a*) No-slip (*b*) *Kn* =0.001, (*c*) *Kn* =0.01, (*d*) *Kn* =0.1

The average Nusselt number of the hot wall for $Gr = 10^4$ and Ri =100 at different volume fractions of nanoparticles is displayed in Figure 4. As it is shown in this figure, by keeping the Grashof and Richardson numbers constant and increasing the volume fraction of nanoparticles, the average Nusselt number will increase. Also, the average Nusselt number decreases with the increase of the Knudsen number. Figure 5 shows the average Nusselt number of the hot wall for $Gr = 10^4$ and $Kn = 0.01$ at different volume fractions of nanoparticles. As it is shown, average Nusselt number decreases with the increase of the Richardson number. Increasing the Richardson number, or in other words decrease of Reynolds number, makes the forced convection less effective with respect to the heat transfer inside the cavity and leads in reduction of Nusselt number. Table 3 indicates the average Nusselt number of hot wall for different Richardson and Knudsen numbers for two cases of nanofluid with volume fraction of 5% and the base fluid. Results show that the maximum increase in average Nusselt number due to using nanoparticles, occurs in the case of $Ri = 1.0$ and $Kn = 0.1$ with the value of 10.93%.

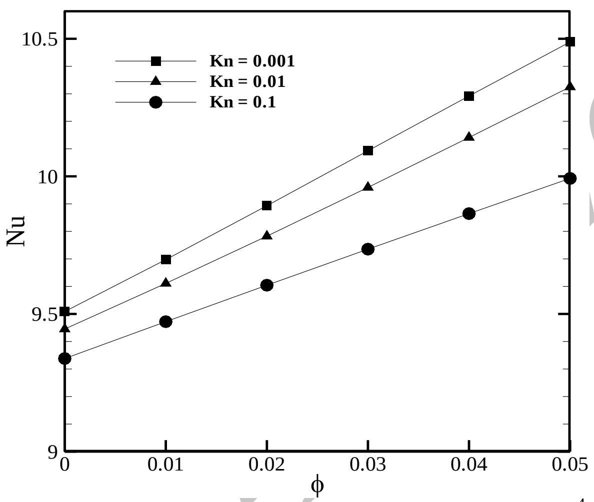

Fig. 4 Average Nusselt number of the hot wall for $Gr = 10^4$ and $Ri = 100$

Table 3 Percentage of increase in average Nusselt number of nanofluid ($\varphi = 5\%$) to the base fluid

Ri	*Kn*	Nu_{ave} ($\varphi = 0$)	Nu_{ave} ($\varphi = 5\%$)	% Increase
0.01	0.001	46.60	50.40	8.15
	0.01	42.85	46.43	8.35
	0.1	23.31	25.75	10.47
1.0	0.001	15.42	16.82	9.08
	0.01	14.87	16.24	9.21
	0.1	10.70	11.87	10.93
100	0.001	9.51	10.49	10.30
	0.01	9.44	10.32	9.32
	0.1	9.34	9.99	6.96

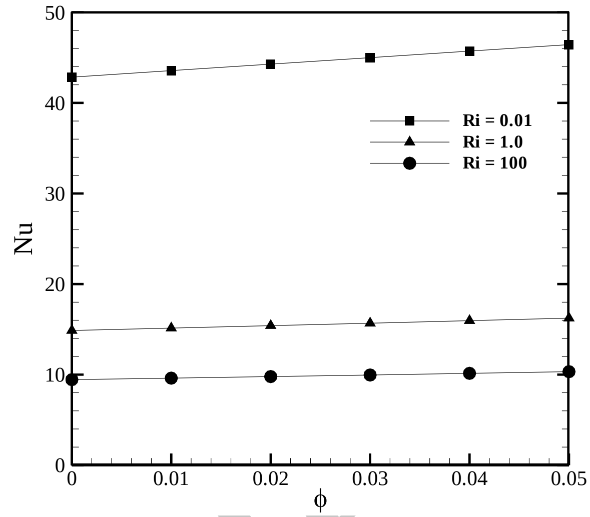

Fig. 5 Average Nusselt number of the hot wall for $Gr = 10^4$ and $Kn = 0.01$

8 CONCLUSION

Mixed convection of Al_2O_3-water nanofluid inside a square microcavity was studied numerically. The forced convective flow within the cavity is attained by moving the bottom wall of the cavity, while the natural convective effect is obtained by subjecting the bottom wall to a higher temperature than those of the other walls which are kept at the same temperature. Navier-Stokes equations along with velocity slip and temperature jump boundary condition were applied to simulate the slip flow regime. Results showed that using velocity slip and temperature jump boundary conditions strongly affects the whole simulation and in this case (slip flow regime), using no-slip boundary conditions is not admissible.

It was also shown that, increasing the volume fraction of nanoparticles, leads to increase of the average Nusselt number and it could be seen that the presence of Al_2O_3 nanoparticles in pure water can significantly enhance heat transfer coefficient. Furthermore, the average Nusselt number decreases with increase of the Knudsen number. Besides, the average Nusselt number decreases with increase of the Richardson number. Results indicate that the maximum increase in average Nusselt number due to using nanoparticles, occurs in the case of $Ri = 1.0$ and $Kn = 0.1$ with the value of 10.93%.

9 LIST OF SYMBOLS AND GREEK SYMBOLS

c_p	Specific heat, J/kg K
Gr	Grashof number
g	gravitational acceleration, m/s^2

h	heat transfer coefficient, W/m^2 K
H	enclosure length, m
k	thermal conductivity, W/mK
Kn	Knudsen number
Nu	Nusselt number
p	pressure, N/m^2
P	dimensionless pressure
Pr	Prandtl number
q	heat flux per unit area, W/m^2
Re	Reynolds number
Ri	Richardson number
T	temperature, K
u,v	velocity components, m/s
U,V	dimensionless velocity components
x,y	Cartesian coordinates, m
X,Y	dimensionless Cartesian coordinates
α	thermal diffusivity, m^2/s
β	thermal expansion coefficient, K^{-1}
γ	specific heat ratio
θ	dimensionless temperature
μ	dynamic viscosity, kg/m s
ϑ	kinematic viscosity, m^2/s
ρ	density, kg/m^3
φ	volume fraction of the nanoparticles

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