# Behavior Analysis of Cold Expanded-Bolt Clamped AL2024-T3 Plate

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Abstract: For making rivet and bolt connections, making a hole is necessary. While basic S-N graph can be extracted from design documents, analysis of stress can be used for two different approaches. The first approach is the theoretical derivation of analytical relations with simplified assumptions like planar stress or uniform bolt load distribution. The other one is the numerical simulation using robust codes like Abaqus software. By using these two approaches, residual stress distribution around the hole can be extracted in various conditions. In this experiment, an aluminium 2024-T3 plate with 3.2 mm thickness is considered. The stress analysis results and basic S-N graph were combined and discursive S-N graphs were obtained for different cold-expanded bolted holes. These graphs were compared with experimental data in several steps. First, the bolt fastening was considered. In the second step, cold expansion was considered and in the final step, the effects of fastening bolts and nuts and cold expansion was considered simultaneously. At last, a comparison between various steps was drawn. The results of this study showed that this new analytical method on distribution of residual stresses around cold expansion holes is as effectiveness as old methods.

Keywords: Al2024-T3, Bolt clamping, Cold expansion, Finite element software, Fatigue life

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Technologies

plane stress conditions.

is one of the best known and most widely approaches

of improving fatigue life that during the last decades was broadly used for improving fatigue life of drilled

connections in aerospace structures. This method in

1965 was developed by the Boeing Company engineers

and later in the seventies and eighties by Fatigue

standardized [2]. Since the laboratory methods for

measuring residual stress can be expensive, researchers

have turned to analytical or numerical methods [3]. For

instance, of Naday's [4] analytical solutions gained

residual stress field around the hole for the model of

elastic-perfectly plastic material in von Misses and

Hsu and Forman [5] developed the accomplished work

of Naday in order to consider the ability of materials

work hardening. Rich and his co-workers [6] offered

residual stress distribution based on the classic solution

for thick-walled pipes by model of elastic perfectly

plastic material in plane strain conditions and Von

Misses criterion. Guo [7] introduces an exact solution

to find residual stress and strain field of this process for

plane stress conditions and Von Misses criterion and

modified Romberg-Osgood model. Chakherluo [2]

numerical analysis indicates that the environmental

residual stress is different over thickness, so that in

mandrel input screen this unit will be the minimum

amount. His fatigue test proved this subject as in cold

expansion sample under dynamic load; crack grew

close to login plate. Also Chakherlou [8] by laboratory

testing on cold expansion sheets that have open and

closed hole, obtained life stress curve of these sheets.

became

commercially

Company

# 1 INTRODUCTION

Fatigue life was recognized as the head of destruction reason in aerospace industries. This destruction moreover excessive recompense, causes decimation so that engineers try to produce numerous methods and plans for deducting fatigue life and increasing the strength of working tools in aerospace industries. Drilling is counting usual operations in essential equipment manufacture through the aerospace industry. These drills are usually required in the segment of attachment with using bolts or nuts and rivet, these attachments not only make easy assembling of components but also cause easier transfer of loading among them. In addition to these positive effects, we must consider the negative effects too. Their main weakness point is creating stress concentration around the hole, so dynamic load makes fatigue life structure decrease. Creating compressive residual stress around drill in one of the major ways which supports that, and reduces stress concentration, so in conclusion, it increases fatigue life. Compressive residual stress influence causes result in the delay of creating cracks and reducing the growth rate of creating cracks of fatigue too, so it increases component life [1]. Cold expansion method means passing conic pin or ball pin through a hole with smaller diameter than pins that is shown in Fig. 1. During these operations the hole expands and surrounding area to certain radius turned out to be plastic strain, whereas in farther areas from the hole, strains remains in the elastic limit, after leaving pins or balls of holes, elastic area tries to return to its previous position even though plastic zone resists against this recurrence, plastic zone is placed under the compressive residual stress (negative) and elastic zone is placed under tensile residual stress (positive) and since the creation of residual stresses occurs without heat treatment, we call that cold expansion operation, and also it is called cold working. Existence of tangential compressive residual stresses delays the creation and growth of cracks around the hole that improvement of life in drilled connection is observed as a result of this delay [2].



For gaining this goal, cold expansion process that is used more in the manufacture of aerospace structures since 1970 till now, has been considered. This method

astic zone resists placed under the and elastic zone ss (positive) and s occurs without g. Existence of g. Existence of

coefficient both causes the rise of upper bound pressure [9]. The most studies of cold expansion were carried out into open hole sheets. Cold expansion was used for bolt and riveting couplings that have different action in contrast with open hole sheets. Separable mechanical attachments are used for components construction. Generally, in aircraft through recent years by usage increase of pneumatic industry in military and civil problems more concentration on flight safety is conserved to the first preference of designing engineers.

In this article aluminum sheet property T3-2024 with a thickness of 3.2 mm has been considered. These alloys due to the unique features have many usages in aerospace industry, through recent researches that have been done for measuring the residual stresses around the hole, have been required cold expansion in that plastic zone radius and as a result of pressure from

mandrel, into the hole through the test, is obtained through testing. Whatever has been studied here is an analytical solution for plane strain by using the theory of plasticity and using the theory of strain gradient plasticity to obtain the residual stresses around the cold expansion hole, since in reality model sheet is with closing bolts or rivets, also bolt and nuts relations has been considered, now the hole with cold expansion and it is closed by bolts and nuts placed under tension. By considering the stress concentration factor around the hole and obtained combine relations structure, and also obtained quite analytical solution from drilled sheet, cold expansion that is closed by bolts will be obtained according to accomplished fatigue tests. On these sheets modeling was done in Abaqus software and numerical and analytical solution's results has been compared with laboratory results.

# 2 ANALYTICAL SOLUTION OF RESIDUAL STRESS FIELD

Analytical methods based on mathematics are always attended by mechanical engineers since they are cheaper and simple in the meantime progress of plasticity science, thanks to analytical and mathematical solutions that were done in Second World War. Cold expansion analytical methods are as same as analytical solution of elastic-plastic of thick tanks. This is one of the most fundamental solved problems in plasticity [7].

Various analytical models that have been introduced till now to predict the residual stress distribution in cold expansion method according to assumptions, and simplifications applied, differ from each other, these assumptions and simplifications contains yield criteria (Von Misses or Tresca), state of stress (plane stress or plane strain), material model (elastic-completely plastic or elastic with nonlinear hardening), plasticity theory and unloading (elastic or elastic by reverse yielding). The complexity and analytical model precision depends on these assumptions and simplifications specially unloading material model [3].

All offered models have something in common models to predict the distribution of residual stress in cold expansion component that is their two dimensional nature. The complexity of three dimensional states nearly makes impossible the analytical solution offers. Therefore, the analytical also, like experimental methods of residual stress measurement cannot predict the distribution of residual stress in work pieces' thickness.

# 2.1. Theory of Strain Gradient Plasticity

Classic plastic theories are based on Cauchy's stress law in which basic hypothesis is that; the state of stress in a point of unit environment, substance is only affected by neighbor point. Classic theories, because of lack of dimension and related parameter cannot explain the dependence to effects of parts size in several experimental tests that have been done in small and micro dimension. The simplest edit of this gradient theory defines yielding function as follows [10]:

$$\sigma_{\rm e} = \sigma_{\rm e}^{\rm H} - c \nabla^2 \varepsilon_{\rm e} \tag{1}$$

In which  $\sigma_e$  and  $\sigma_e^H$ , respectively are total and homogenous amount of effective stress,  $\mathcal{E}_e$  is effective plastic strain, c is measure coefficient of strain gradient, and  $\nabla^2$  is Laplacian operator. In this case yielding function  $\sigma_e^H \leq \sigma_y$  can be described as  $\sigma_e \leq \sigma_y$ . The boundary conditions of considering the strain gradient are defined as below [10]:

$$\varepsilon_{e} = \overline{\varepsilon_{e}}$$
 or  $\frac{\partial \varepsilon_{e}}{\partial n} = 0$  above  $\partial^{p} B$  (2)

Including boundary of plastic area and normal vector on bar sign above the quantity means; that is obvious. Another equation in classic plasticity stays changing. Existing equations (1) and (2) with Hencky's deformation theory are consumed for gaining the equations in plastic area [11].

# 2.2. Analytical Equation of Cold Expansion

Analysis based on classic elasticity theory (Hook's relation) for elastic area and classic plasticity theory and theory of Hencky's total strain was done in plane strain condition in plastic area [12]. Strain gradient of plasticity theory has been considered for gaining effective stress in plastic area and Von Misses for yielding. Bauschinger effect and yielding effect have not been considered in unloading direction, and only elastic unloading has been considered. Axis and hole contact condition like entrance speed or even exit speed and coefficient of friction have affected created stresses. In this article, final conditions have been considered.

The hole outer radius  $r_0$ , inner radius  $r_1$ , plastic areas

radius  $r_c$ , internal pressure  $p_i$ , yield stress  $\sigma_y$ , elasticity coefficient hardening strain m, strength coefficient k, Poisson coefficient  $\upsilon$ , Lame coefficient  $\lambda$ , strain of inner surface of the hole D, radially direction with index r, environmental direction with index  $\theta$ , axial direction with index z, radial displacement u, stress  $\sigma$ , strain  $\varepsilon$ , deflection stress

 $\sigma_{ii}$ , and hydrostatic stress  $\sigma_m$  are considered [13].

4

# **2.3. Solution in plastic area** $(r_c \le r)$

Stress–strain equation in this area is linear and is  $\sigma_i = E\epsilon_i$ . Since two century ago numerous articles among Lame solution about elastic solution in hole has offered results as follow [14]:

$$\sigma_{\rm rr}^{\rm e} = \frac{P_{\rm c} r_{\rm c}^2}{r_{\rm o}^2 - r_{\rm c}^2} \left( 1 - \frac{r_{\rm o}^2}{r^2} \right)$$
(3)

$$\sigma_{\theta\theta}^{e} = \frac{P_{c}r_{c}^{2}}{r_{o}^{2} - r_{c}^{2}} \left(1 + \frac{r_{o}^{2}}{r^{2}}\right)$$
(4)

$$\sigma_{zz}^{e} = \frac{2\nu P_{c} r_{c}^{2}}{r_{o}^{2} - r_{c}^{2}}$$
(5)

Considering equation between stress and strain:

$$\varepsilon_{ij} = \frac{1+\nu}{E} \sigma_{ij} - \frac{\nu}{E} \left( \sigma_{xx} + \sigma_{yy} + \sigma_{zz} \right) \delta_{ij}$$
(6)

Strain in elastic area can be obtained as follow:

$$\varepsilon_{\rm rr} = \frac{1 + \nu}{E} \sigma_{\rm rr} - \frac{\nu}{E} \left( \sigma_{\rm rr} + \sigma_{\theta\theta} + \sigma_{\rm zz} \right) \tag{7}$$

$$\varepsilon_{\theta\theta} = \frac{1+\nu}{E} \sigma_{\theta\theta} - \frac{\nu}{E} \left( \sigma_{rr} + \sigma_{\theta\theta} + \sigma_{zz} \right)$$
(8)

$$\varepsilon_{zz} = 0$$
 (9)

Elastic displacement can be expressed as follow:

$$\mathbf{u} = \frac{\mathbf{r}}{\mathbf{E}} \left\{ \sigma \begin{bmatrix} 1 - \nu (1 + 2\nu) \\ + (1 + \nu) (\sigma - \mathbf{P}_{c}) \frac{\mathbf{r}_{c}^{2}}{\mathbf{r}^{2}} \end{bmatrix} \right\}$$
(10)

Elastic-plastic boundary condition is:

$$\sigma_{e}(r=r_{c})=\sigma_{y}$$
(11)

# **2.4.** Solution in plastic area $(r_i \le r \le r_c)$

Equations of this section by considering assumptions are obtained; small deformations, isotropic hardening, incompressible and units loading by Hanky's total strain theory and plasticity theory of strain gradient. Stress–strain equation in this area is considered nonlinear as follow:

$$\sigma_i = E \epsilon_i^n \tag{12}$$

Considering balance equation in cylindrical coordinates:

$$\frac{\partial \sigma_{\rm r}}{\partial r} + \frac{1}{r} \frac{\partial \tau_{\rm r\theta}}{\partial \theta} + \frac{\sigma_{\rm r} - \sigma_{\theta}}{r} + F_{\rm r} = 0$$
(13)

By considered boundary condition:

$$\sigma_{\theta\theta} - \sigma_{rr} = r \frac{d\sigma_{rr}}{dr}$$
(14)

Compatibility relation and also structural equation is as follow:

$$\varepsilon_{\rm rr} - \varepsilon_{\theta\theta} = r \frac{d\varepsilon_{\theta\theta}}{dr}$$
(15)

$$\varepsilon_{\theta\theta} = \frac{\varepsilon_{e}}{\sigma_{e}} \left( \sigma_{\theta\theta} - \frac{1}{2} \sigma_{rr} \right)$$
(16)

$$\varepsilon_{\rm rr} = \frac{\varepsilon_{\rm e}}{\sigma_{\rm e}} \left( \sigma_{\rm rr} - \frac{1}{2} \sigma_{\rm \theta\theta} \right) \tag{17}$$

The Von Misses relation of two dimensional stresses is:

$$\sigma_{\rm e} = \sqrt{\sigma_{\rm rr}^2 - \sigma_{\rm rr}\sigma_{\theta\theta} + \sigma_{\theta\theta}^2}$$
(18)

Hencky's total strain theory in plastic area that expresses the plastic strain based on deflection stress and hydrostatic strain can be written as follow:

$$\varepsilon_{ij}^{p} = \lambda \sigma_{ij} = \frac{3}{2} \frac{\varepsilon_{e}}{\sigma_{e}} \left( \sigma_{ij} - \sigma_{m} \right)$$
<sup>(19)</sup>

$$\varepsilon_{\rm rr} = \frac{3}{4} \frac{\varepsilon_{\rm e}}{\sigma_{\rm e}} (\sigma_{\rm rr} - \sigma_{\theta\theta}) = -\varepsilon_{\theta\theta}$$
(20)

Plasticity theory of strain gradient is:

$$\sigma_{\rm e} = K \varepsilon_{\rm e}^{\rm m} - c \nabla^2 \varepsilon_{\rm e} \tag{21}$$

Von Misses stress by plane strain assumption can be simplified as:

$$\sigma_{\rm e} = \frac{\sqrt{3}}{2} \left( \sigma_{\rm \theta\theta} - \sigma_{\rm rr} \right) \tag{22}$$

Boundary conditions in this problem are:

# Int J Advanced Design and Manufacturing Technology, Vol. 10/ No. 2/ June – 2017

$$\sigma_{\rm rr}(r=r_{\rm i})=-p_{\rm i} \tag{23}$$

$$\sigma_{rr}(r=r_c) = -p_c \tag{24}$$

$$\varepsilon_{e}(r=r_{i})=D$$
 (25)

$$\varepsilon_{e} \left( r = r_{c} \right) = \frac{\sigma_{y}}{E}$$
(26)

By substituting equation (22) in equation (20), it can be written as:

$$\varepsilon_{\rm rr} = -\frac{\sqrt{3}}{2}\varepsilon_{\rm e} \tag{27}$$

$$\varepsilon_{\theta\theta} = \frac{\sqrt{3}}{2}\varepsilon_{\rm e} \tag{28}$$

By substituting equations (27) and (28) in equation (15), it can be written as:

$$\frac{\mathrm{d}\varepsilon_{\mathrm{e}}}{\varepsilon_{\mathrm{e}}} = -2\frac{\mathrm{d}r}{\mathrm{r}} \tag{29}$$

By integration equation (29) from  $r_i$  till r and by using boundary condition (25) it can be written as:

$$\varepsilon_{\rm e} = D \frac{r_{\rm i}^2}{r^2} \tag{30}$$

By substituting equation (30) in boundary condition (26), it can be written as:

$$D = \frac{\sigma_y}{E} \frac{r_c^2}{r_i^2}$$
(31)

By substituting equation (30) in equation (21), it can be written as:

$$\sigma_{\rm e} = KD^{\rm m} \left(\frac{r_{\rm i}}{r}\right)^{2\rm m} - 6cD\frac{r_{\rm i}^2}{r^4}$$
(32)

By substituting equations (22) and (32) in equation (14), it can be written as:

$$d\sigma_{\rm rr} = \frac{2}{\sqrt{3}} \left( {\rm KD}^{\rm m} \frac{{\rm r_i}^{2\rm m}_i}{{\rm r}^{2\rm m+1}} - 6{\rm cD} \frac{{\rm r_i}^2}{{\rm r}^5} \right) dr$$
(33)

By integration equation (33) from  $r_i$  till r and using boundary condition (23), it can be written as:

$$\sigma_{rr}^{p} = -p_{i} + \frac{2}{\sqrt{3}} \left( \frac{KD^{m}}{2m} \left( 1 - \frac{r_{i}^{2m}}{r^{2m}} \right) \right) - cDr_{i}^{2} \left( \frac{1}{r_{i}^{4}} - \frac{1}{r^{4}} \right)$$
(34)

By substituting equation (34) in (24) it can be written as:

$$p_{i} - p_{c} = \frac{2}{\sqrt{3}} \left( \frac{KD^{m}}{2m} \left( 1 - \frac{r_{i}^{2m}}{r_{c}^{2m}} \right) \right) - cDr_{i}^{2} \left( \frac{1}{r_{i}^{4}} - \frac{1}{r_{c}^{4}} \right)$$
(35)

By substituting equations (3), (4) and (22) in equation (11) it can be written as:

$$p_{c} = \frac{\sigma_{y} \left( r_{o}^{2} - r_{c}^{2} \right)}{\sqrt{3} r_{o}^{2}}$$
(36)

Now we can gain pressure of mandrel entrance into hole by substituting equation (36) in equation (35):

$$p_{i} = \frac{\sigma_{y}}{\sqrt{3}} \frac{\left(r_{o}^{2} - r_{c}^{2}\right)}{r_{o}^{2}} + \frac{2}{\sqrt{3}} \left(\frac{KD^{m}}{2m} \left(1 - \frac{r_{i}^{2m}}{r_{c}^{2m}}\right)\right) - cDr_{i}^{2} \left(\frac{1}{r_{i}^{4}} - \frac{1}{r_{c}^{4}}\right)$$
(37)

Residual stresses are:

$$\sigma^{p}_{\theta\theta} = -p_{i} + \frac{2}{\sqrt{3}} \begin{pmatrix} \frac{KD^{m}}{2m} + \\ \frac{KD^{m}r_{i}^{2m}}{r^{2m}} \begin{pmatrix} 1 - \frac{1}{2m} \end{pmatrix} - \\ cD\left(\frac{1}{r_{i}^{2}}\right) - 3cD\left(\frac{r_{i}^{2}}{r^{4}}\right) \end{pmatrix}$$
(38)

By substituting equations (30) and (31) in equations (27) and (28) it can be written as:

$$\varepsilon_{\rm rr} = -\frac{\sqrt{3}}{2} \frac{\sigma_y}{E} \frac{r_{\rm c}^2}{r^2}$$
(40)

$$\varepsilon_{\theta\theta} = \frac{\sqrt{3}}{2} \frac{\sigma_y}{E} \frac{r_c}{r^2}$$
(41)

# Int J Advanced Design and Manufacturing Technology, Vol. 10/ No. 2/ June - 2017

6

$$\varepsilon_{\rm rr} = \frac{{\rm d}u}{{\rm d}r} \tag{42}$$

$$\varepsilon_{\theta\theta} = \frac{\mathbf{u}}{\mathbf{r}} \tag{43}$$

By substituting equation (41) in (43), it can be written as:

$$u = \frac{\sqrt{3}}{2} \frac{\sigma_y}{E} \frac{r_c^2}{r}$$
(44)

#### 2.5. Unloading solution

In this part, by elastic unloading assumption, unloading solution is exactly as same as elastic solution with different boundary conditions. So in this way, whole area around the hole gets to elastic unloading and it would not be just outer ring. Based on these equations can be written as:

$$\sigma_{\rm rr}^{\rm un} = \left(\frac{p_{\rm i} r_{\rm i}^2}{r_{\rm o}^2 - r_{\rm i}^2} \left(1 - \frac{r_{\rm o}^2}{r^2}\right)\right) \tag{45}$$

$$\sigma_{\theta\theta}^{\mathrm{un}} = \left(\frac{p_{\mathrm{i}}r_{\mathrm{i}}^{2}}{r_{\mathrm{o}}^{2} - r_{\mathrm{i}}^{2}} \left(1 + \frac{r_{\mathrm{o}}^{2}}{r^{2}}\right)\right) \tag{46}$$

$$\sigma_{zz}^{un} = \left(\frac{2\nu p_i r_i^2}{r_o^2 - r_i^2}\right)$$
(47)

# 2.6. Residual stresses field solution

Field of residual stresses is the result of stresses during loading minus stresses in unloading. So in this radius area  $r_i \le r \le r_c$  which is under the plastic behavior and can be written as:

$$\sigma_{rr}^{res} = \begin{pmatrix} -p_{i} + \frac{2}{\sqrt{3}} \left( \frac{KD^{m}}{2m} \left( 1 - \frac{r_{i}^{2m}}{r^{2m}} \right) \right) \\ -cDr_{i}^{2} \left( \frac{1}{r_{i}^{4}} - \frac{1}{r^{4}} \right) \\ -\left( \frac{p_{i}r_{i}^{2}}{r_{o}^{2} - r_{i}^{2}} \left( 1 - \frac{r_{o}^{2}}{r^{2}} \right) \right) \end{cases}$$
(48)

$$\sigma_{\theta\theta}^{res} = \begin{pmatrix} -p_{i} \\ \frac{2}{\sqrt{3}} \left( \frac{KD^{m}}{2m} + \frac{KD^{m}r_{i}^{2m}}{r^{2m}} \left( 1 - \frac{1}{2m} \right) \\ -cD\left( \frac{1}{r_{i}^{2}} \right) - 3cD\left( \frac{r_{i}^{2}}{r^{4}} \right) \end{pmatrix} \end{pmatrix}$$
$$- \left( \frac{p_{i}r_{i}^{2}}{r_{o}^{2} - r_{i}^{2}} \left( 1 + \frac{r_{o}^{2}}{r^{2}} \right) \right)$$
(49)

$$\sigma_{zz}^{\text{res}} = \begin{pmatrix} -\mathbf{p}_{i} \\ +\frac{1}{\sqrt{3}} \left( \frac{\mathbf{K}\mathbf{D}^{m}}{m} + \frac{\mathbf{K}\mathbf{D}^{m}\mathbf{r}_{i}^{2m}}{r^{2m}} \left(1 - \frac{1}{m}\right) \\ -2c\mathbf{D}\mathbf{r}_{i}^{2} \left(\frac{1}{\mathbf{r}_{i}^{4}} + \frac{1}{\mathbf{r}^{4}}\right) \end{pmatrix} \right) \\ - \left( \frac{2\nu\mathbf{p}_{i}\mathbf{r}_{i}^{2}}{\mathbf{r}_{o}^{2} - \mathbf{r}_{i}^{2}} \right)$$
(50)

And in radial period  $r_c \le r \le r_o$ , that elastic area is in loading path and can be written as:

$$\sigma_{\rm rr}^{\rm res} = \frac{P_{\rm c} r_{\rm c}^2}{r_{\rm o}^2 - r_{\rm c}^2} \left( 1 - \frac{r_{\rm o}^2}{r^2} \right) - \left( \frac{p_{\rm i} r_{\rm i}^2}{r_{\rm o}^2 - r_{\rm i}^2} \left( 1 - \frac{r_{\rm o}^2}{r^2} \right) \right)$$
(51)

$$\sigma_{\theta\theta}^{\text{res}} = \frac{P_c r_c^2}{r_o^2 - r_c^2} \left( 1 + \frac{r_o^2}{r^2} \right)^{-} \left( \frac{p_i r_i^2}{r_o^2 - r_i^2} \left( 1 + \frac{r_o^2}{r^2} \right) \right)$$
(52)

$$\sigma_{zz}^{\text{res}} = \frac{2\nu P_{c} r_{c}^{2}}{r_{o}^{2} - r_{c}^{2}} - \left(\frac{2\nu p_{i} r_{i}^{2}}{r_{o}^{2} - r_{i}^{2}}\right)$$
(53)

# 2.7. Analytical equation of tension and fasten screw

In main equations of stress for tension statue, we cannot assume that there are not any unpredictable changes in surveyed members section. Machine components are parts with variations in their own sections. Many of the components have hole, oil path, and grooves. Any kinds of discontinuity in machine components cause changes of stress distribution around the discontinuity. So that the simple equation of stress is no longer able to express the stress statue in a section, these discontinuities are called stress raiser and the regions that they happened there, are called areas of stress concentration.

It is obvious from Fig. 2 that the stress lines path is uniform everywhere except around the hole. Stress concentration is completely a positional effect. Material does not have any effects on stress concentration factor and it is only depended on segment figure.

$$\sigma_{\rm rr}^{\rm t} = k_{\rm t} \sigma_0 \tag{54}$$

$$\sigma_{\theta\theta}^{t} = 0 \tag{55}$$

$$\sigma_{zz}^{t}=0$$
 (56)

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Int J Advanced Design and Manufacturing Technology, Vol. 10/ No. 2/ June - 2017



Fig. 2 Stress distribution around the hole [15]

In this article through studied geometry, the stress concentration factor has been considered as 2.45 and tension is 138.8 MPa till 312.5 MPa. Used assumptions in stress solution around the hole in tension are:

- In all directions stress is ignored.
- Stress in closest point of hole has maximum amount and in farthest point of hole has minimum amount that is as same as stress in distant.
- Stress along the width of the sheet are uniformly reduced to gain the stress in distant.

In this article half thread screw specifications M5\*0.8 with hexagon and indentation strength 8.8 with nut are used, and torque is 4 N-m. In the following equation of preloads  $F_i$ , external tensile load P, parts of P that screw tolerated it  $P_b$ , Parts of P that members tolerated it  $P_m$ , bar resultant on the bolt  $F_b$ , bar resultant on members  $F_m$ , cross-tensile area  $A_t$ , confidence resistance  $S_p$ , tighten torque T, largest diameter of screw gear d, most gage pressure on main page  $P_1$ , by considering the problem conditions and also no external load, equations between torque and force on members are [15] :

$$F_{cl} = E_{bush} A_{bush} \varepsilon_m$$
(57)

$$P=P_{b}+P_{m}=0$$
  
 $P_{b}=P_{m}=0$  (58)

$$F_{\rm b} = F_{\rm i}$$

$$F_{\rm m} = -F_{\rm i}$$
(59)

7

For unlock able connections can be written as:

$$F_i = 0.75 F_p$$
 (60)

$$F_{p} = AS_{p}$$
(61)

$$S_{p} = 0.85S_{y}$$
 (62)

According to washer's dimensions, cross section under pressure is:

$$A = \frac{\pi (d_o^2 - d_i^2)}{4}$$
(63)

$$T=0.2F_{i}d$$
 (64)

$$\sigma = \frac{F_i}{A} \tag{65}$$

By considering a linear pressure distribution from contact plates to main pages, we can gain the stress distribution under washer's area for main plate:

$$P_1 = \sigma_{zz}^b + m(r-2.5)$$
 (66)

# **3 FOUNDATIONS OF FINITE ELEMENT METHOD**

Finite element is one of the most important engineering survey tools. Simulations that are done with this tool are good alternatives for difficult and expensive experimental tests [16]. Finite element modeling of this research is done in Abaqus software environment, version 6-10. High performance of this software in nonlinear analysis field has made it to one of the most important software solutions in mechanical engineering even in other engineering professions.



Fig. 3 The final dimension of geometric model of perforated Aluminum sheet in mm

8

Figure 3 and figure 4 are dimensions of the main model and pin of cold expansion by 1.5% degrees of expansion. Figure 5 is a perfect view of model in software. In the meantime, the size of elements is important. For checking that the answer depends on size of the mesh, simple tension analysis was done on the model and stress around the hole was gained by changing the grain size (grading).



Fig. 4 Final dimension of mandrel with 1.5% degree of cold expansion according to mm



Fig. 5 Three-dimensional view of geometric model in finite element software

Figure 6 shows the stress distribution according to grain size. As can be seen by decreasing grading, stress amount will increase to converge to a fixed value. It should be noted that the smallest size of elements substantially increases the time resolution, so in this article 0.8 has been considered for elements. Pin surface friction coefficient of cold expansion and the hole of wall have been considered 0.16. This amount was gained by comparing finite element methods and experimental tests. Friction coefficient among steel

washer and contact plates and also among contact plates and main plates is 0.288.



Fig. 6 Sensitive to mesh by changing the grain size

This amount was gained by experimental test on ramp. In table 1 Aluminum alloy properties has been shown [8]. In Fig. 7 and Fig. 8 comparison of tangential and radial residual stresses in entrance plate of mandrel line movement has been shown between numerical and analytical solution. As can be seen stresses distribution is acceptable.

Table 1 Mechanical pro	operties of Al 2024-T3
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Alloy	0.2% Proof stress (MPa)	Tensile strength (MPa)	Modulus of elasticity (GPa)	Poisson's ratio
2024-T3	375	570	71.5	0.3



**Fig. 7** Comparison of radial residual stress on entrance plate of mandrel movement by 1.5% degree of expansion



Fig. 8 Comparison of tangential residual stress on entrance plate of mandrel in movement by 1.5% degree of expansion

It should be noted that radial residual stress at the edge of the hole must be zero. Although these points are on free surface and they do not need to enter any external force, the finite element's conclusion is gained in integral points and these points are not exactly on free surface, so the radial residual stress that obtained in these points is not necessarily the exact amount of residual stress at the edge of the hole, so it is not zero.



Fig. 9 Tangential residual stress field distribution in cross section and changes after closing bolts by three different pressure forces in mandrel entrance plate

# 4 SIMULATION OF RESIDUAL STRESS FIELD OF BELT IN EFFECT OF BOLT CLOSURE

Conclusion of finite element simulation for radial and tangential distribution of residual stress field and it's changing after close bolts is drown by three different pressing forces in Fig. 9 and Fig. 10. For normalized radial, tangential residual stresses were used from yield strength of Aluminum alloy 2024-T3. The distance from the edge of the hole along cross-section (r) is normalized by using hole radius (a).

According to these figures, residual stress in the further area of hole, has rather equal distribution before and after closing bolts. But, it's distribution in close points of hole, means where washer and contact plates have contacts. It is strongly changed by increasing the pressure force of bolts and nuts, these changes increase. Tangential residual stress changes are more than radial residual stress changes. Away from the edge of the hole, radial and tangential residual stresses decrease.





# 5 MULTIAXIAL STRESSES AND CALCULATION OF LIFE

States of multiaxial stresses are so common and avoidance of multiaxial strains is difficult. For instance, in a rod under tension, strains are usually triaxial. With longitudinal strain  $\mathcal{E}$ , there are two lateral strains too -V $\mathcal{E}$ . That is V, Poisson's ratio. The state of stress in tracks are usually triaxial and they are not like stress state in rest of the piece. For example, at the root of a screw tooth, state of stress is biaxial, while in the main body, stress state is maybe uniaxial. In addition, stress state and stress and strain concentration coefficient are not equal [17].

For multiaxial fatigue analysis, understanding the state of stress and strain in one piece or structure is necessary. State of stress and strain in a point of a body can be described by 6 stress components and 6 strain

9

components which are applied on octahedral plates. Octahedral plates in yield prediction and fatigue analysis are important. There are eight octahedral plates that make same angels with three directions of main stress. Shear stress on these plates is obtained via following equation [17]:

$$\tau_{oct} = \frac{1}{3} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$
(67)

Normal stress on octahedral plates is hydrostatic stress (that named average normal stress), that is obtained by following equation [17]:

$$\sigma_{\text{oct}} = \sigma_{\text{h}} = \sigma_{\text{ave}} = \frac{1}{3} (\sigma_1 + \sigma_2 + \sigma_3)$$
(68)

In equation (68),  $\sigma_1$ ,  $\sigma_2$ ,  $\sigma_3$  are main stresses.

In this article for calculation of fatigue strength stress or equivalent stress that makes possible determining the life, Sine's method was used. Sine's method uses octahedral shear stress for alternative stresses (equation 67 based on main alternative stresses) and hydrostatic stress for average stresses (equation (68) based on residual nominal stresses or main stresses) which can be shown by following equation [17]:

$$\sqrt{\left(S_{a1} - S_{a2}\right)^{2} + \left(S_{a2} - S_{a3}\right)^{2} + \left(S_{a3} - S_{a1}\right)^{2} } + m\left(S_{mx} + S_{my} + S_{mz}\right) = \sqrt{2}S_{Nf} }$$
(68)

That m, is impact factor of average stress and  $S_{\rm Nf}$  is fatigue strength. Impact factor, can be determined by obtaining fatigue strength with a non-zero average stress for laboratory.  $S_{a1}$ ,  $S_{a2}$ ,  $S_{a3}$  are main alternative stresses.  $S_{mx}$ ,  $S_{my}$ ,  $S_{mz}$  are main average nominal stresses.

#### 6 EXPERIMENTAL RESULTS

Fatigue test specimens were classified into six batches each undergoing different combinations of cold expansion and torque clamping (Fig. 11). The fatigue tests were carried out using sinusoidal cycles at the frequency of 12 Hz and load ratio of R=0 by means of Zwick Roell Amsler HA250 servo-hydraulic pull-push fatigue test machine. These tests were performed at eight load levels from 8 to 18 kN (equivalent remote stress ranges of 139-312 MPa) for every load level. Three specimens were used and the resulting average life was displayed in a semi-log S-N diagram in Fig. 12. As the results indicate, clamping leads to an appreciable improvement in fatigue life as expected. However, a salient point according to these results, is that the specimens clamped and cold expanded with 1.5% degree (batch of 1.5% & 4 N-m) exhibits much longer fatigue life compared to specimens clamped and cold expanded with 4.7% degree (batch of 4.7% & 4 N-m). As seen, increasing the cold expansion degree to 4.7% for the clamped specimens decreases the fatigue life, such that their fatigue life becomes even lower than that of the only-clamped specimens (batch of 0% & 4 N-m) [8].



Fig. 11 The fixture used in the tests and the part used for finite element modeling [8]



Fig. 12 S-N data obtained from fatigue tests [8]

# 7 EFFECTS OF COLD EXPANSION AND CLOSE BOLTS ON SEGMENT LIFE

In this part, for checking effects of cold expansion and close bolts on segment life, modeling was performed in finite element software then, the stress is obtained in critical point or in other words at the edge of hole. By using the stress-life diagram of alloy Al2024-T3 in Fig. 11, we can obtain the equation of this curve, and by this equation we can obtain segment life in effect of various processes including cold expansion and close bolts [18]. In Figs. 12 and 13 the effects of this process has been shown on segment life. As can be seen in figures, cold expansion process is creasing the life of segment by increasing the expansion degree. Also close bolts by increasing torque and as a result of the load increased on the plate, is increasing the life of parts

simultaneous, combining of closing bolts and cold expansion process improves the segment life, but it does not follow a specific pattern based on increasing the degree of expansion and screw load. These simultaneous rise cause changes in material properties and creates plastic flow. That could shorten the life of the segment in compare with using any of those processes separately. The optimal use for expansion degree 1.5% and increase of bolts load can be seen in the increase of regular segment's life.



Fig. 13 The S-N diagram for Al 2024-T3



Fig. 14 The S-N diagram for Al 2024-T3 with bolt load 67 MPa



Fig. 15 The S-N diagram for Al 2024-T3 with bolt load 305 MPa

# 8 CONCLUSION

Based on the passed in this article, we have results in below:

- Analysis based on classical elasticity theory (Hooke's law) for elastic area and Hencky's total strain theory in plate strain condition in plastic area with Von Misses criteria for yielding and strain gradient theory for graining effective stresses in plastic area that have stress concentration predicts residual stress distribution field around the piercing cold work.
- Using analytical solution with considering assumptions that has been considered in every solution separately can be a good alternative or numerical and experimental solution.
- Close bolts on cold expansion hole in addition to increase measure stresses distribution uniform around the hole.
- Using cold expansion process or close bolts on the hole, improved fatigue life separately, Expansion degree increment or pressure of close bolts increases fatigue life.
- Cold expansion sheets that were bolted with different pressure forces in compared with cold expansion sheets with open hole, shows more fatigue life, Cold expansion degree's increment in equal screw load cause fatigue life decrement that is because of plastic flows of material.

• Cold expansion sheets that were bolted with different pressure forces in compared with sheets that just were bolted, according to the cold expansion degree and pressure forces can shows or decrease fatigue life. For example, in cold expansion with expansion degree 1.5%, increase of load of bolt causes increase of fatigue life than just use bolt without cold expansion but, in cold expansion degree 4.7%, increasing load of bolt causes decreasing fatigue life than just use bolt without cold expansion.

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# Int J Advanced Design and Manufacturing Technology, Vol. 10/ No. 2/ June – 2017

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