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"Research Note"

A BOUNDARY INTEGRAL MODEL FOR SIMULATING SPILLING BREAKERS*

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Abstract – A boundary integral method is used to simulate spilling breakers. The bottom is also included in the introduced closed boundary and the problem is directly solved in the physical plane. The method has been shown to be remarkably stable, and no numerical instability has occurred in any of the calculations. The results reveal that both the momentum and total energy are almost constant in time during the simulation period. As a result, the breaking process of a spilling breaker is fairly well simulated.

Keywords – Breaking waves, numerical simulation, spilling, Cauchy's integral theorem

1. INTRODUCTION

Over the last two decades, many studies have been carried out to achieve a better understanding of the breaking of surface water waves [1-4]. Knowledge of the dynamics of breaking waves is necessary to explain the mechanisms of air sea interactions such as energy and momentum transfer from wind to water and from waves to currents, and the generation of turbulence in the surface layer of oceans. In the surf zone, consideration of the breaking wave's effects is an essential requirement in most engineering practices. Breaking waves are not only responsible for impact loading on the vertical breakwaters and sea-walls [5-7], but also have significant influence on the cross-shore hydrodynamics, suspension and transport of sediments, and beach profile change [8]. The study of breaking waves is also important in applications to naval hydrodynamics due to their damaging effects on ships and offshore structures in heavy seas.

In this study, a mathematical model was developed to simulate the developing process of spilling breakers from non breaking conditions. The model is based on a Boundary Integral Method (BIM) and the problem is stated as an initial-value problem. The wave is stepped forward in time by solving the exact free-surface boundary conditions. Besides, a bottom is included in the closed boundary to solve the problem directly in the physical plane.

2. FORMULATION OF THE MODEL

The problem is formulated as a mixed Eulerian-Lagrangian description. This means that the free surface elevation will be described by the positions of fluid particles at the free surface with reference to the fixed coordinate system (x, y) with its origin in the mean water level (see Fig. 1). $\gamma(t)$ is assumed to be the closed contour consisting of the bottom, the free surface and two vertical boundaries a distance L apart. The fluid is assumed to be homogeneous, incompressible and irrotational. Therefore, the potential theory

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is applicable. Further, the fluid motion is assumed to be 2-dimensional. Thus, the velocity potential, φ, and the stream function, ψ, can describe the fluid motions. The complex potential is given as:

$$\beta(z;t) = \varphi(z;t) + i\psi(z;t) \tag{1}$$

where Z = x + iy. The positions of the fluid particles at the free surface are integrated in time, using the kinetic boundary condition stating that a fluid particle at the free surface will remain at the free surface,

$$\frac{Dz}{Dt} = u + iv = \overline{q} \tag{2}$$

where a bar sign denotes complex conjugation.

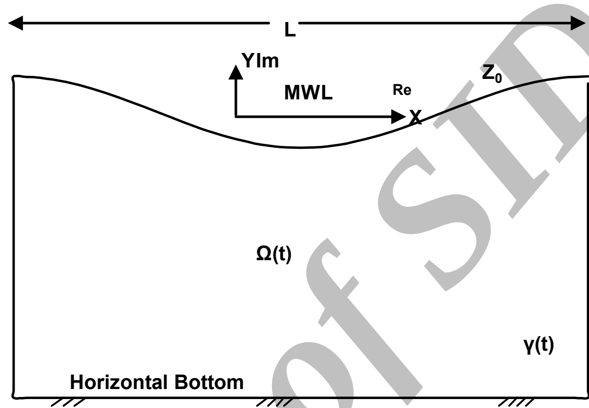

Fig. 1. Fluid domain $\Omega(t)$ and closed boundary $\gamma(t)$

The dynamic free surface boundary condition is given from Bernoulli's equation:

$$\frac{\partial\varphi}{\partial t} = -\frac{1}{2}q\overline{q} - gy - \frac{P_s}{\rho} \tag{3}$$

where Ps is an arbitrary pressure distribution applied at the free surface. In the fluid domain, Laplace's equation is satisfied for both the velocity potential, φ, and the stream function, ψ, and therefore, $\beta(z;t) = \varphi(z;t) + i\psi(z;t)$ is an analytic function of z. In this case Cauchy's integral theorem is valid;

$$\oint_{\gamma(t)} \frac{\beta(z;t)}{z - z_0} dz = 0 \tag{4}$$

where $\gamma(t)$ is the closed boundary defined before, and Z_0 is situated outside $\gamma(t)$. When β(z; t) and $\partial\beta(z;t)/\partial t$ are know along the boundary $\gamma(t)$, the complex potential and its time derivative at any point Z_0 inside the boundary can be found.

3. NUMERICAL SOLUTION

Control points were used along the free surface of the water, the left and right vertical boundaries, and the seabed (Fig. 1). At the free surface, the desired non-breaking waves were considered according to the suitable characteristics satisfying deep water conditions. Initial values of the surface elevation and the velocity potential along the free surface were calculated from stokes wave theory. The collocation method is applied when solving equations. A linear variation of the complex potential and its derivative between the nodal points is assumed in z. In order to integrate equations (1) and (2) in time, Hamming's fourth-order predictor/corrector method is applied with a Runge Kutta starting procedure. The Runge Kutta algorithm was used for the time stepping with a constant of about T/400. This method has been shown to be remarkably stable, and no numerical instability has occurred in any of the calculations.

A Stokes wave with a slight steepness has run for one wave period to check the solution for stationary conditions. To check the solution for non-stationary breaking waves, the validity of kinematic and dynamic boundary conditions is investigated. As the kinematic boundary condition, the flux of fluid through the free surface is checked to be zero at different time steps. As the dynamic boundary conditions, the following are controlled; a) The change in time of the total energy is checked to be equal to the work done by the pressure at the free surface, and b) The change in time of the momentum in the x-direction of the fluid flow is checked to be equal to the total horizontal force acting on the fluid at the free surface.

4. RESULTS

Time variations of the kinetic, potential and total energy of the wave described below are shown in Fig. 2. In this Figure, the energy is divided by ¼ $\rho g a^2 L$ and the horizontal momentum by $1/2\ \rho\ (g\ k)^{1/2} a^2\ L$, where a is the amplitude of the initial wave. As can be seen, both the momentum and total energy are almost constant in time, except at the very end of the calculations, as should be expected.

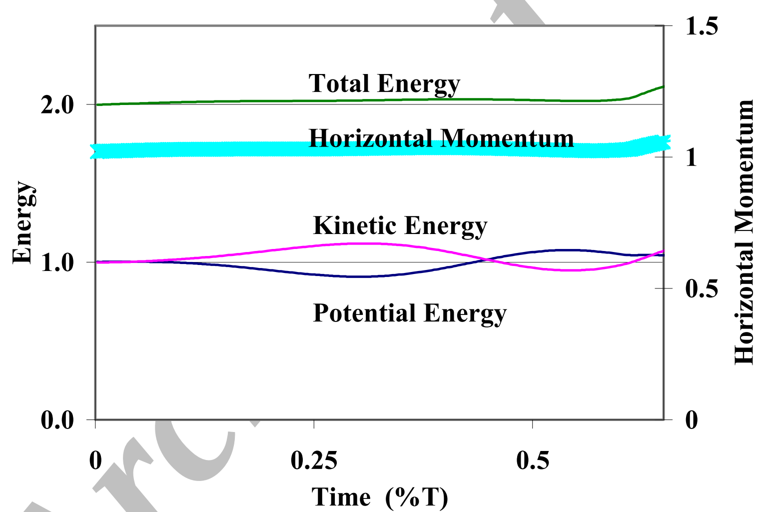

Fig. 2. Variation of the energy and horizontal momentum in time

In order to get a spilling breaker, we have given a Stokes wave with a large steepness and let it run freely from deep to shallow water. Stokes theory is used to calculate the starting values of the potential at the undisturbed free surface. The characteristics of the wave presented in this paper as a representative case are described in Table 1.

Table 1. Initial wave characteristics of representative case

Water Depth	Wave Height	Wave Period	Wave Length	Rational Depth	Wave Steepness
D[m]	H[m]	T[s]	L[m]	D/L[-]	H/L[-]
60.0	15.0	8.4	110.0	0.545	0.136

Figure 3 shows deformation and progress of this wave in its breaking zone. The number of nodal points used on the free water surface, bottom and each vertical side of the closed boundary are also indicated.

Figure 3 obviously represents the development of a spilling breaker. As the wave progresses, it becomes asymmetric and the wave front becomes steeper. Then, a smooth jet of fluid is ejected from the wave crest. In a further stage, the ejected jet plunges into the wave front face and shortly after this the calculations breakdown. The breaking process, including the overturning of the wave front face near the crest, develops in a very short period of time. As seen in Fig. 3, this period is about 0.66 T for a spilling breaker, where T denotes the wave period. It is also notable that the time necessary for the wave front to become vertical is 0.61 T.

5. CONCLUSIONS

The boundary integral model can, quite well, simulate the breaking process of a spilling breaker. Cauchy's integral theorem was successfully used to solve a boundary value problem on a closed boundary consisting of the bottom, the free surface and two vertical boundaries a distance L apart. The collocation method is applied when solving the equations. A linear variation was assumed in variable z of the complex potential function and its derivative between the nodal points. The Runge Kutta algorithm was used for the time stepping with a constant of about T/400. This method has been shown to be remarkably stable, and no numerical instability has occurred in any of the calculations. The results reveal that the breaking process of a spilling breaker, including the overturning of the wave front face near the crest, develops in a very short period of time of about 0.66 T, where T denotes the wave period.

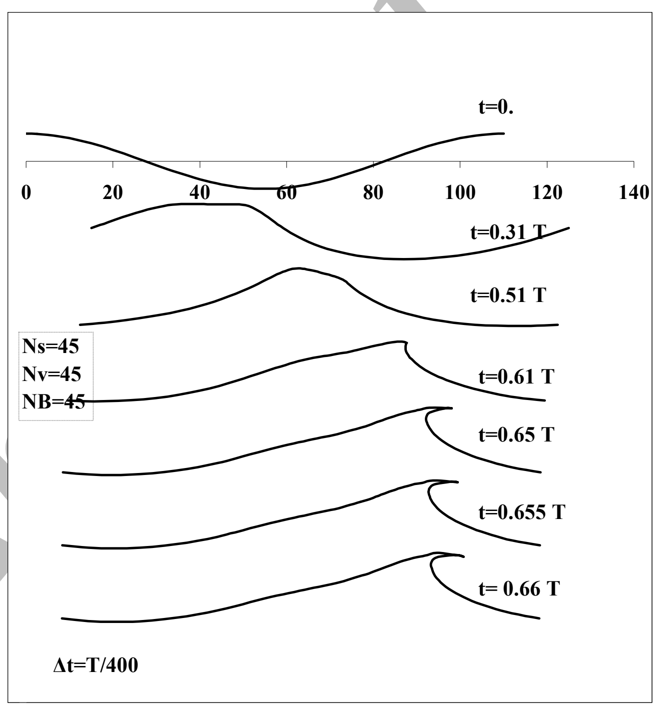

Fig. 3. Deformation of a steep stokes wave in deep water results in a spilling breaker

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