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QCD FACTORIZATION IN HADRONIC $B \to J/\psi(\pi, K)$ DECAYS*

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Abstract – Using QCD factorization for the hadronic matrix elements, we show that existing data, in particular the branching ratios BR ($\overline{B} \to J/\psi K$) and BR ($\overline{B} \to J/\psi\pi$), can be accounted for in this approach. We analyze the decay $B \to J/\psi K(\pi)$ within the framework of QCD factorization. The calculation of the relevant hard-scattering kernels for twist-2 and twist-3 is completed. We calculate this decay in a special scale ($\mu = m_b$) and in two schemes for Wilson coefficients in NLO. We consider three functions for J/ψ. The twist-3 contribution involves the logarithmically divergent integral, we consider $\rho_H = 0$ the canceling divergent. The obtained results are in agreement with available experimental data.

Keywords – B Meson, hard scattering, QCD factorization, 2 and 3-twist

1. INTRODUCTION

There are many ways that the quarks produced in a nonleptonic weak decay can arrange themselves into hadrons. There are many complicated trees of gluon and quark interactions, pair production, and loops that link the final state into the intial state. These make the theoretical description of nonleptonic decays difficult [1]. The idea of factorization in hadronic decays of heavy mesons is already quite old. Factorization is a property of the heavy quark limit, in which we assume that the *b*-quark mass is parametrically large. The *b* quark then decays into a set of very energetic partons. How these partons and what is left of the *B* meson hadronize into two mesons depends on the identity of these mesons [2]. Color transparency is the basis for the *factorization hypothesis*, in which amplitudes factorize into products of two current matrix elements. This ansatz is widely used in heavy-quark physics, as it is almost the only way to treat hadronic decays. However its validity is not demonstrated by any quantitative theoretical argument, and there are some instances in which this approach is not applicable. The most obvious cases are those in which the final state is chosen in such a way that the quark pair of one of the currents does not correspond to a final state particle. Whether factorization "works" or not depends on the particular decay considered. Surprisingly, it seems to be applicable in many cases. It has been used mainly in hadronic two-body decays [3, 4], but it may also be applicable to certain multibody decays [1, 5]. For a long time, exclusive two-body *B*-decay amplitudes have been estimated in the "naive" factorization approach or modifications thereof. In many cases, this approach provides the correct order of magnitude for branching fractions, but it cannot predict direct *CP* asymmetries due to the assumption of no strong rescattering. It is, therefore, no longer adequate for a detailed phenomenological analysis of *B*-factory data. Naive factorization has now been superseded by *QCD* factorization [6, 7]. Although this scheme has not proved rigorous yet, it provides the means to compute two-body decay amplitudes from first principles. Its

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accuracy is limited only by power corrections to the heavy quark limit and the uncertainties of theoretical inputs such as quark masses, form factors and light cone distribution amplitudes [8].

Weak decays of heavy mesons involve three fundamental scales, the weak interaction scale M_W, the *b*-quark mass m_b, and the *QCD* scale Λ_{QCD}, which are strongly ordered: $M_W \rangle\rangle m_b \rangle\rangle \Lambda_{QCD}$. The underlying weak decay being computable, all theoretical work concerns strong-interaction corrections [7]. The strong interaction effects which involve virtualities above the scale m_b are well understood. They renormalize the coefficients of local operators O_i in the weak effective Hamiltonian. Assuming the standard model of flavour violation, the amplitude for the decay $B \to M_1 M_2$ is given by,

$$A(B \to M_1 M_2) = \frac{G_F}{\sqrt{2}} \sum_i \lambda_i C_i(\mu) \langle M_1 M_2 | O_i | B \rangle (\mu) \tag{1}$$

in which, G_F is the Fermi constant. Each term in the sum is the product of a *CKM* factor λ_i, a coefficient function $C_i(\mu)$, which incorporates strong-interaction effects above the scale $\mu = m_b$, and a matrix element of an operator O_i. In extension of the standard model, there may be further operators and different flavour violating couplings, but the strong interaction effects below the scale μ are still encoded by the matrix elements of local operators. Therefore, the theoretical problem is to compute these matrix elements. Since they depend on m_b and Λ_{QCD}, one should take advantage of the fact that $m_b \rangle\rangle \Lambda_{QCD}$ and compute the short distance part of the matrix element. Then, the remainder depends only on Λ_{QCD}, and it-to leading order in Λ_{QCD} / m_b -turns out to be much simpler than the original matrix element [2].

Beneke et al. [6] considered general two body nonleptonic decays of *B* mesons extensively including a light-light meson system as well as a heavy-light system in the final state. The general idea is that in the limit $m_b \rangle\rangle \Lambda_{QCD}$, the hadronic matrix elements can be schematically represented as

$$\langle M_1 M_2 | O_i | B \rangle (\mu) = \langle M_1 | j_1 | B \rangle \langle M_2 | j_2 | 0 \rangle [1 + \sum_n r_n \alpha_s^n + O(\Lambda_{QCD} / m_b)] \tag{2}$$

where M_1, M_2 are final-state mesons and O_i is a local current-current operator in the weak effective Hamiltonian. If we neglect radiative corrections in α_s and power corrections in Λ_{QCD}, we get the factorized result with a form factor times decay constant. At higher order in α_s, this simple factorization is broken, but the corrections can be calculated systematically in terms of short distance Wilson coefficients and meson light-cone distribution amplitudes. We call this, the *QCD*-improved factorization [9].

2. EFFECTIVE WEAK HAMILTONIAN

In any phenomenological treatment of the weak decays of hadrons, the starting point is the weak effective Hamiltonian at low energy. It is obtained by integrating out the heavy fields (e.g., the top quark, $W^{\pm}$ and *Z* bosons) from the standard model Lagrangian [10]. The effective weak Hamiltonian for hadronic *B* decays consists of a sum of local operators O_i multiplied by short distance coefficients C_i and products of elements of the quark mixing matrix, $\lambda_p = V_{pb} V_{ps}^*$ or $\lambda_p = V_{pb} V_{pd}^*$ [8]. It can be written as,

$$H_{eff}^{\Delta B=1} = \frac{G_F}{\sqrt{2}} \sum_i V_{CKM} C_i(\mu) O_i(\mu) \tag{3}$$

where, G_F is the Fermi constant, V_{CKM} is the CKM matrix element, $C_i(\mu)$ are the Wilson coefficients, $O_i(\mu)$ are the operators entering the operator product expansion (OPE) and μ represents the renormalization scale. In the present case, since we take into account tree and penguin operators, the matrix elements of the effective weak Hamiltonian reads

$$\left\langle M_1 M_2 \left| H_{eff}^{\Delta B=1} \right| B \right\rangle = \frac{G_F}{\sqrt{2}} [V_{pb} V_{pq}^* \sum_{i=1}^{2} C_i(\mu) \left\langle M_1 M_2 \left| O_i^p \right| B \right\rangle (\mu)$$

$$- V_{tb} V_{tq}^* \sum_{i=3}^{10} C_i(\mu) \left\langle M_1 M_2 \left| O_i \right| B \right\rangle (\mu)] + h.c \tag{4}$$

where $q = d$ or s according to the transition $b \to d$ or $b \to s$ ($p = u, c$). $\langle M_1 M_2 | O_i | B \rangle (\mu)$ are the hadronic matrix elements, and $M_i M_j$ indicates either a pseudo-scalar and a vector in the final state or two pseudo-scalar mesons in the final state. The matrix elements describe the transition between the initial and final states at scales lower than μ and include, up to now, the main uncertainties in the calculation because they involve non perturbative physics. The operator product expansion is used to separate the calculation of the amplitude, $A(M \to F) \propto C_i(\mu) \langle M_1 M_2 | O_i | B \rangle (\mu)$, into two distinct physical regimes. One is called *hard* or short distance physics, represented by $C_i(\mu)$ and calculated by a perturbative approach, the other is called *soft* or long distance physics. This part is described by $O_i(\mu)$, and is derived by using a non perturbative approach such as the $1/N_C$ expansion, *QCD* sum rules or hadronic sum rules. The operators $O_i(\mu)$ can be understood as local operators which govern effectively a given decay, reproducing the weak interaction of quarks in a point like approximation. The definitions of the operators O_i are recalled for completeness:

Current-current operators:

$$O_1^p = (\bar{p}_\alpha b_\beta)_{V-A} (\bar{q}_\beta p_\alpha)_{V-A}, \qquad O_2^p = (\bar{p}_\alpha b_\alpha)_{V-A} (\bar{q}_\beta p_\beta)_{V-A} \tag{5}$$

QCD penguin operators:

$$O_3 = (\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\beta q_\beta)_{V-A}, \qquad O_4 = (\bar{q}_\alpha b_\beta)_{V-A} \sum_{q'} (\bar{q}_\beta q_\alpha)_{V-A}$$

$$O_5 = (\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\beta q_\beta)_{V+A}, \qquad O_6 = (\bar{q}_\alpha b_\beta)_{V-A} \sum_{q'} (q'_\beta q_\alpha)_{V+A} \tag{6}$$

Electroweak penguin operators:

$$O_7 = (\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} \frac{3}{2} e'_q (\bar{q}'_\beta q_\beta)_{V+A}, \qquad O_8 = (\bar{q}_\alpha b_\beta)_{V-A} \sum_{q'} \frac{3}{2} e'_q (\bar{q}'_\beta q_\alpha)_{V+A}$$

$$O_9 = (\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} \frac{3}{2} e'_q (\bar{q}'_\beta q_\beta)_{V-A}, \qquad O_{10} = (\bar{q}_\alpha b_\beta)_{V-A} \sum_{q'} \frac{3}{2} e'_q (\bar{q}'_\beta q_\alpha)_{V-A} \tag{7}$$

where $(\bar{q}_1 q_2)_{V \pm A} = \bar{q}_1 \gamma_\mu (1 \pm \gamma_5) q_2$; *α, β* are colour indices, e'_q are the electric charges of the quarks in units of |*e*|, and a summation over all the active quarks ($q' = u, d, s, c$) is implied. In equation (5) *p* denotes the quark *u* or *c* and *q* denotes the quark *d* or *s*, according to the given transition $b \to d$ or $b \to s$ [10]. The effective Hamiltonian relevant for $B \to J/\psi K$ $(b \to s)$ has the form:

$$H_{eff} = \frac{G_F}{\sqrt{2}} (V_{cb} V_{cs}^* (C_1 O_1 + C_2 O_2) - V_{tb} V_{ts}^* \sum_{i=3}^{10} C_i O_i) \tag{8}$$

in this case we have $\langle O^u{}_{1,2} \rangle = 0$. Where

$$O_1 = \bar{s}_\alpha \gamma^\mu (1 - \gamma_5) c_\beta \cdot \bar{c}_\beta \gamma_\mu (1 - \gamma_5) b_\alpha \quad , \qquad O_2 = \bar{s}_\alpha \gamma^\mu (1 - \gamma_5) c_\alpha \cdot \bar{c}_\beta \gamma_\mu (1 - \gamma_5) b_\beta$$

$$O_3 = \bar{s}_\alpha \gamma^\mu (1 - \gamma_5) b_\alpha \cdot \sum_q \bar{q}_\beta \gamma^\mu (1 - \gamma_5) q_\beta \, , \qquad O_4 = \bar{s}_\alpha \gamma^\mu (1 - \gamma_5) b_\beta \cdot \sum_q \bar{q}_\beta \gamma^\mu (1 - \gamma_5) q_\alpha$$

$$O_5 = \bar{s}_\alpha \gamma^\mu (1 - \gamma_5) b_\alpha \cdot \sum_q \bar{q}_\beta \gamma^\mu (1 + \gamma_5) q_\beta \quad , \qquad O_6 = \bar{s}_\alpha \gamma^\mu (1 - \gamma_5) b_\beta \cdot \sum_q \bar{q}_\beta \gamma^\mu (1 + \gamma_5) q_\alpha$$

$$O_7=\bar{s}_\alpha\gamma^\mu(1-\gamma_5)b_\alpha\cdot\sum_q\frac{3}{2}e'_q\bar{q}_\beta\gamma^\mu(1+\gamma_5)q_\beta\ ,O_8=\bar{s}_\alpha\gamma^\mu(1-\gamma_5)b_\beta\cdot\sum_q\frac{3}{2}e'_q\bar{q}_\beta\gamma^\mu(1+\gamma_5)q_\alpha$$

$$O_9=\bar{s}_\alpha\gamma^\mu(1-\gamma_5)b_\alpha\cdot\sum_q\frac{3}{2}e'_q\bar{q}_\beta\gamma^\mu(1-\gamma_5)q_\beta\ ,\ O_{10}=\bar{s}_\alpha\gamma^\mu(1-\gamma_5)b_\beta\cdot\sum_q\frac{3}{2}e'_q\bar{q}_\beta\gamma^\mu(1-\gamma_5)q_\alpha \qquad (9)$$

O_3-O_6 are the QCD penguin operators and O_7-O_{10} are the electroweak penguin operators [11]. For $B\to J/\psi\pi\ \ (q=d)$, we have $b\to d$ transition and then $\langle O^u{}_{1,2}\rangle=0$ (hence we will find that $a_1^u=a_2^u=0$ in section 4 and we have Wilson coefficients in a_1^c or a_2^c , which is dependent on the kind of decays), so we only replace $V_{cb}V_{cd}^*$ and $V_{tb}V_{td}^*$ instead of the *CKM* matrix elements in (8).

3. THE FACTORIZATION FURMULA

We consider weak decays $B\to M_1M_2$ in the heavy-quark limit. The formal expression of the previous discussion is given by the following result for the matrix element of an operator O_i in the weak effective Hamiltonian, which is valid up to corrections of the order of Λ_{QCD}/m_b [2]:

$$\begin{aligned}\langle M_1M_2|O_i|B\rangle=&\sum_j F_j^{B\to M_1}(m_2^2)\int_0^1 duT_{ij}^I(u)\phi_{M_2}+(M_1\leftrightarrow M_2)\\&+\int_0^1 d\xi dudvT_i^{II}(\xi,u,v)\phi_B(\xi)\phi_{M_1}(v)\phi_{M_2}(u)\end{aligned} \qquad (10)$$

if M_1 and M_2 are both light, and

$$\langle M_1M_2|O_i|B\rangle=\sum_j F_j^{B\to M_1}(m_2^2)\int_0^1 duT_{ij}^I(u)\phi_{M_2}(u) \qquad (11)$$

if M_1 is heavy and M_2 is light.

Here, $F_j^{B\to M}(m_{2,1}^2)$ denotes a $B\to M_{1,2}$ form factor, and ϕ_M is the lightcone distribution amplitude for the quark–antiquark Fock state of meson *M*. $T_{ij}^I(u)$ and $T_i^{II}(\xi,u,v)$ are hard scattering functions, which are perturbatively calculable. Finally, $M_{1,2}$ denote the light meson masses. The second line of (10) is somewhat simplified and may require including an integration over transverse momentum in the *B* meson starting from order α_s^2. Equation (10) applies to decays into two light mesons, for which the spectator quark in the *B* meson can go to either of the final-state mesons. An example is the decay $B^-\to\pi^0K^-$. If the spectator quark can go only to one of the final-state mesons, such as, for example, in $\bar{B}_d\to\pi^+K^-$, we call this meson M_1 and the second form factor term on the right-hand side of (10) is absent. The factorization formula simplifies when the spectator quark goes to a heavy meson (see (11)), such as in $\bar{B}_d\to D^+\pi^-$. In this case the hard interactions with the spectator quark can be dropped because they are power suppressed in the heavy-quark limit. In the opposite situation that the spectator quark goes to a light meson but the other meson is heavy, factorization does not hold as discussed above [2].

This method works well for the case with two light mesons like $\pi\pi$ or πK [6, 12], in which the final state mesons carry large momenta. Interestingly enough, when there is a heavy quark in the final state such as $B\to D^+\pi^-$, this method still works when a spectator quark of the *B* meson is absorbed by, say, a *D* meson [6, 13]. However, when the spectator quark is absorbed by a light quark, say, in $B\to D\pi^0$, nonfactorizable contributions are infrared divergent and the factorization breaks down.

4. $B\to J/\psi(K,\pi)$ DECAYS

When we consider the decay $B\to J/\psi K$, it looks ambiguous at first sight as to whether we can apply the same method used in $B\to\pi\pi$ or $B\to D^+\pi^-$, since the spectator quark in the *B* meson goes into a light *K* meson. However, what is special about J/ψ is that the size of the charmonium is so small ($\approx 1/\alpha_s m_c$) that the charmonium has a negligible overlap with the (B,K) system, hence enabling the same improved factorization method in the decay $B \to J/\psi K$.

When the mass of the J/ψ meson is not negligible, the light-cone wave function of the J/ψ meson should include higher twist contributions. The light cone wave functions are obtained in powers of $m_{J/\psi}/E$ or Λ_{QCD}/E where $E(\approx m_b)$ is the energy of the J/ψ meson. For B decays into two light mesons, the higher twist contributions are negligible since they are of order Λ_{QCD}/E . However, for $B \to J/\psi K$, higher twist contributions are important. Therefore, we expect that the decay rate using only the leading, asymptotic wave function of J/ψ will be smaller than the experimental result. When we use light cone meson wave functions for exclusive decays, $B \to J/\psi K$ the transition amplitude of an operator O_i in the weak effective Hamiltonian is given by

$$\left\langle J/\psi K(\pi) \middle| O_i \middle| \overline{B} \right\rangle =$$

$$\sum_i F_i^{B\to K(\pi)}(m_\psi^2)\int_0^1 dx T_{ij}^I(x)\phi_{J/\psi}(x) + \int_0^1 d\xi dx du T_i^{II}(\xi,x,u)\phi_B(\xi)\phi_{J/\psi}(x)\phi_{K(\pi)}(u) \qquad (12)$$

where $F_j^{B\to K(\pi)}(m_{J/\psi}^2)$ is the form factors for $B \to K(\pi)$, and $\phi_M(x)$ is the lightcone wave function for the meson *M*. $T_{ij}^I(u)$ and $T_i^{II}(\xi,u,v)$ are hard scattering amplitudes, which are perturbatively calculable. The second term in (12) represents spectator contributions. Under naive factorization, the decay amplitude of $B \to J/\psi K(\pi)$ reads

$$A(B \to J/\psi K(\pi)) = \frac{G_F}{\sqrt{2}}[V_{cb}V_{cs(d)}^* a_2 - V_{tb}V_{ts(d)}^*(a_3 + a_5 + a_7 + a_9)]X^{(BK(\pi),J/\psi)} \qquad (13)$$

where ($a_2 = a_2^c$),

$$X^{(BK,J/\psi)} \equiv f_{J/\psi} m_{J/\psi} F_1^{BK}(m_{J/\psi}^2)(2\varepsilon^* \cdot p_B)$$

$$X^{(B\pi,J/\psi)} \equiv \frac{1}{\sqrt{2}} f_{J/\psi} m_{J/\psi} F_1^{B\pi}(m_{J/\psi}^2)(2\varepsilon^* \cdot p_B)$$

and in naive factorization [9, 14], $a_{2i} = C_{2i} + (1/N_c)C_{2i-1}$ and $a_{2i-1} = C_{2i-1} + (1/N_c)C_{2i}$. Wilson coefficients are presented in Table 1. In Table 2, we computed these parameters. And ε^* is the polarization vector of J/ψ . There is only one non-vanishing helicity amplitude. In the rest frame of the decaying B meson only longitudinally polarized J/ψ is produced. $\varepsilon^*.p_B$ is then given by

Table 1. Wilson coefficient for Leading Order (*LO*) and Next Leading Order (*NLO*) in NDR and HV scheme ($\mu = m_b$) , $\alpha = 1/129$

	LO	NLO(NDR)	NLO(HV)
C_1	1.144	1.082	1.105
C_2	-0.308	-0.185	-0.228
C_3	0.014	0.014	0.013
C_4	-0.030	-0.035	-0.029
C_5	0.009	0.009	0.009
C_6	-0.038	-0.041	-0.033
C_7/α	0.045	-0.002	0.005

Table 1. (Continued)

C_8/α	0.048	0.054	0.060
C_9/α	-1.280	-1.292	-1.283
C_{10}/α	0.328	0.263	0.266

Table 2. Numerical values of a_i in Naive Factorization

Naïve	NLO(NDR)	NLO(HV)	LO
a_1	1.02	1.029	1.041
a_2	0.175	0.140	0.073
a_3	0.002	0.0033	0.004
a_4	-0.030	-0.024	-0.025
a_5	-0.004	-0.002	-0.0036
a_6	-0.038	-0.030	-0.035
a_7	0.0001	0.0001	0.00047
a_8	0.0004	0.0004	0.00048
a_9	-0.009	-0.009	-0.009

$$p_B.\varepsilon^* = \frac{m_B}{m_{J/\psi}}|P| \tag{14}$$

where $|P|$ is the absolute value of the 3-momentum of the J/ψ (or the K^-) in the B rest frame [15]. There are two serious problems with the naive factorization approximation. First, the Wilson coefficients $C_i(\mu)$, and hence, a_i are renormalization scale and γ^5-scheme dependent, whereas the decay constants and form factors are not. Hence, the amplitude (13) is not physical. However, if we include the α_s correction to the amplitudes, it turns out that the μ dependence of the Wilson coefficients is cancelled and the overall amplitude is insensitive to the renormalization scale. Second, nonfactorizable effects, which play an essential role in colour suppressed modes, are not taken into account. Nonfactorizable contributions at order α_s come from the radiative corrections of the operators O_1, O_4, O_6, O_8 and O_{10} and the relevant Feynman diagrams are shown in Fig. 1. The radiative corrections with a fermion loop do not contribute due to the color structure. For each operator O_1, O_4, O_6, O_8 and O_{10} if we add all the diagrams in Fig. 1 and symmetrize the result with respect to $\xi \leftrightarrow 1-\xi$, the infrared divergence of each diagram cancels and the remaining amplitude is infrared finite. One thing to note is that there appear imaginary parts in the nonfactorizable contribution, which are due to the final state interaction. The strong phase can be calculated in the QCD-improved factorization and it is important in exploring the CP violation in nonleptonic decays.

The aforementioned two difficulties for naive factorization are resolved in the *QCD* factorization approach, in which the inclusion of vertex corrections and hard spectator interactions (see Fig. 1) yields [11, 16]:

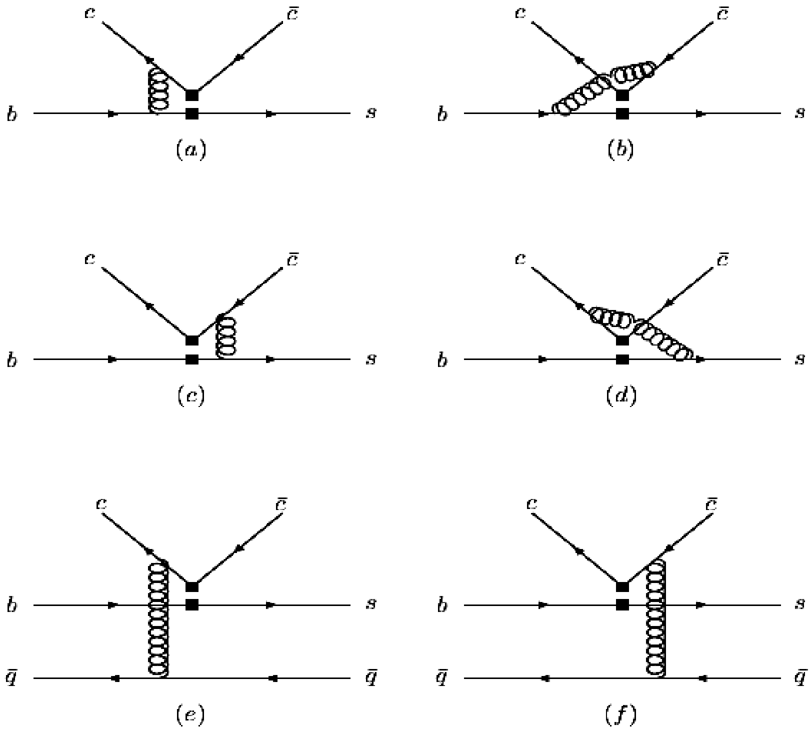

Fig. 1. Vertex and spectator corrections to $B \to J/\psi K$ decay

$$a_2 = C_2 + \frac{C_1}{N_c} + \frac{\alpha_s}{4\pi}\frac{C_F}{N_c}C_1[-\begin{pmatrix}18\\14\end{pmatrix} - 12\ln\frac{\mu}{m_b} + f_I + f_{II}]$$

$$a_3 = C_3 + \frac{C_4}{N_c} + \frac{\alpha_s}{4\pi}\frac{C_F}{N_c}C_4[-\begin{pmatrix}18\\14\end{pmatrix} - 12\ln\frac{\mu}{m_b} + f_I + f_{II}]$$

$$a_5 = C_5 + \frac{C_6}{N_c} - \frac{\alpha_s}{4\pi}\frac{C_F}{N_c}C_6[-\begin{pmatrix}6\\18\end{pmatrix} - 12\ln\frac{\mu}{m_b} + f_I + f_{II}]$$

$$a_7 = C_7 + \frac{C_8}{N_c} - \frac{\alpha_s}{4\pi}\frac{C_F}{N_c}C_8[-\begin{pmatrix}6\\18\end{pmatrix} - 12\ln\frac{\mu}{m_b} + f_I + f_{II}]$$

$$a_9 = C_9 + \frac{C_{10}}{N_c} + \frac{\alpha_s}{4\pi}\frac{C_F}{N_c}C_{10}[-\begin{pmatrix}18\\14\end{pmatrix} - 12\ln\frac{\mu}{m_b} + f_I + f_{II}] \quad (15)$$

where the upper entry of the matrix is evaluated in the naive dimension regularization (*NDR*) scheme and the lower entry in the Hooft-Veltman (*HV*) renormalization scheme, $C_F = (N_c^2 - 1)/(2N_c)$, and N_c is the number of colors. Wilson coefficients are presented for leading order (*LO*) and the next leading order (in *NDR* and *HV* scheme) in Table 1. The hard scattering functions f_I arise from the vertex corrections, Figs. 1(a, c)-1(b, d), while f_{II} arises from the hard spectator interactions Figs. 1(e)-1(f). Formally, the coefficients a_i are scale and γ^5-scheme independent. The results for the hard scattering functions f_I are (Tables 3, 4) [11]:

$$f_I = f_I' + \frac{F_0^{BK(\pi)}(m_{J/\psi}^2)}{F_1^{BK(\pi)}(m_{J/\psi}^2)} g_I \quad (16)$$

where the contributions of f_I' are from Fig. 1(a, c)

$$f_I' = \int_0^1 d\xi \phi^{J/\psi}(\xi)\{\frac{2z\xi}{1-z(1-\xi)} + (3-2\xi-8\xi^2)\frac{\ln\xi}{1-\xi} + (-\frac{3}{1-z\xi} + \frac{1+8\xi}{1-z(1-\xi)} - \frac{2z\xi}{[(1-z(1-\xi)]^2})$$
$$z\xi\ln z\xi + (3(1-z)+2z\xi-8z\xi^2 + \frac{2z^2\xi^2}{1-z(1-\xi)})\frac{\ln(1-z)-i\pi}{1-z(1-\xi)}$$

and the contributions of g_I are from Fig. 1(b, d)

$$g_I = \int_0^1 d\xi\phi^T_{J/\psi}(\xi)\{\frac{4\xi(2\xi-1)}{(1-z)(1-\xi)}\ln\xi + \frac{z\xi}{[1-z(1-\xi)]^2}\ln(1-z) + (\frac{1}{(1-z\xi)^2} - \frac{1}{[1-z(1-\xi)]^2}$$
$$-\frac{8\xi}{(1-z)(1-z\xi)} + \frac{2(1+z-2z\xi)}{(1-z)(1-z\xi)^2})z\xi\ln z\xi - i\pi\frac{z\xi}{[1-z(1-\xi)]^2}\}$$

where $z = m^2_{J/\psi}/m^2_B$ and ξ is the momentum fraction of a c quark inside the J/ψ meson, and the asymptotic wave functions ($\phi(\xi)$, $\phi^T(\xi)$) for the J/ψ meson are symmetric functions under $\xi \leftrightarrow 1-\xi$. The asymptotic form of the distribution amplitudes $\phi(\xi)$ and $\phi^T(\xi)$ is the same.

Table 3. f_I, f^2_{II}, f^3_{II} for $B \to J/\psi K$ ($q = s$), $\mu = m_b$

	f_I	f^2_{II}	$f^3_{II}(\rho_H = 0)$
$\phi_{J/\psi}(\xi) = 6\xi(1-\xi)$	-0.813-6.61i	4.95	4.89
$\phi_{J/\psi}(\xi) = \delta(\xi - 1/2)$	-0.517-6.94i	3.30	3.26
$\phi_{J/\psi} = 9.58x(1-x)[\frac{x(1-x)}{1-2.8x(1-x)}]^{0.7}$	-0.672-6.74i	4.02	3.98

Table 4. f_I, f^2_{II}, f^3_{II} for $B \to J/\psi\pi$ ($q = d$), $\mu = m_b$

	f_I	f^2_{II}	f^3_{II}
$\phi_{J/\psi}(\xi) = 6\xi(1-\xi)$	-0.813-6.61i	4.91	4.85
$\phi_{J/\psi}(\xi) = \delta(\xi - 1/2)$	-0.517-6.94i	3.27	3.24
$\phi_{J/\psi} = 9.58x(1-x)[\frac{x(1-x)}{1-2.8x(1-x)}]^{0.7}$	-0.672-6.74i	3.99	3.94

And [9]:

$$\frac{F_0^{BK(\pi)}(m^2_{J/\psi})}{F_1^{BK(\pi)}(m^2_{J/\psi})} = \frac{m^2_B - m^2_{J/\psi}}{m^2_B}$$

as for the hard scattering function f_{II} that is orginated from spectator diagrams, we write [11]:

$$f_{II} = f^2_{II} + f^3_{II} + \ldots$$

where, the superscript denotes the twist dimension of LCDA. To the leading-twist order, we obtain (Tables 3, 4),

$$f^2_{II} = \frac{4\pi^2}{N}\frac{f_K f_B}{F_1^{BK}(m^2_{J/\psi})m^2_B}\frac{1}{1-z}\int_0^1 d\bar\rho\frac{\phi_1^B(\bar\rho)}{\bar\rho}\int_0^1 d\xi\frac{\phi^{J/\psi}(\xi)}{\xi}\int_0^1 d\bar\eta\frac{\phi^{K(\pi)}(\bar\eta)}{\bar\eta} \quad (17)$$

however, we shall see that numerically the twist-2 nonfactorizable effects are small; the predicted decay rate of $B \to J/\psi K$ is too small by a factor of 7 ~ 10. Therefore, it is inevitable that higher-twist effects which are seemingly power suppressed should play an essential role. Chirally enhanced corrections arise from twist-3 two-particle light cone distribution amplitudes, whose normalization involves the quark condensate (Table 3, 4). Consequently [9, 11],

$$f_{II}^{3} = (\frac{2\mu_{\chi}}{m_B})\frac{4\pi^2}{N}\frac{f_{K(\pi)}f_B}{F_1^{BK(\pi)}(m_{J/\psi}^2)m_B^2}\int_0^1\frac{d\overline{\rho}}{\overline{\rho}}\phi_1^B(\overline{\rho})\int_0^1\frac{d\xi}{\xi}\phi^{J/\psi}(\xi)\int_0^1\frac{d\overline{\eta}}{\overline{\eta}^2}\frac{\phi_{\sigma}^{K(\pi)}(\overline{\eta})}{6(1-z)^3} \tag{18}$$

the contributions to, for example, $B \to K\pi$ from the $(S-P)(S+P)$ penguin operators are enhanced by the factor

$$\frac{2\mu_{\chi}}{m_b} = \frac{2m_K^2}{(m_s+m_u)m_b} \approx 12\frac{\Lambda_{QCD}}{m_b} \approx O(1) \tag{19}$$

because the current masses of light quarks are difficult to fix, we would like to take $r_K = r_\pi$, which is proportional to the quark condensate. The logarithmic divergence of the $\overline{\eta}$ integral in (17) implies that the spectator interaction is dominated by soft gluon exchanges between the spectator quark and the charmed or anti-charmed quark of J/ψ. The twist-3 contribution involves the logarithmically divergent integral ($M = K$ or π)

$$X_H^M \equiv \int_0^1\frac{d\overline{\eta}}{\overline{\eta}} = Ln\frac{m_B}{\Lambda_{QCD}}(1+\rho_H e^{i\varphi_H}),\quad 0 \le \rho_H \le 1,\ -180^\circ \langle \varphi_H \langle 180^\circ \tag{20}$$

because this divergence is associated with a soft interaction of the ejected meson with the spectator quark, the divergence arises specifically from the region $\overline{\eta} \approx \Lambda_{QCD}/m_B$, and therefore one expects that $X_H^M \approx Ln(m_B/\Lambda_{QCD})$. The choice for the values of X_H^M introduces unavoidable model dependence in the predictions [16, 17]. Here, we considered that $\rho_H = 0, \varphi_H = 0$ and computed a_i coefficients in *QCD* factorization (Tables 5, 6, 7, 8) by different 3- twist contributions.

Table 5. Numerical value of a_i for $B \to J/\psi K$ ($q = s$), $\mu = m_b$, by using f_{II}^2

$\phi_{J/\psi}(\xi)$		$6\xi(1-\xi)$	$\delta(\xi-1/2)$	$9.58\xi(1-\xi)[\frac{\xi(1-\xi)}{1-2.8\xi(1-\xi)}]^{0.7}$
Twist 3		f_{II}^2		
NDR	a_2	0.0689-0.0505i	0.062-0.053i	0.067-0.051i
	a_3	0.0054+0.0016i	0.0056+0.0017i	0.0054+0.0016i
	a_5	-0.0045-0.0019i	-0.0047-0.0020i	-0.0046-0.00195i
	a_7	0.000105+0.0000196i	0.000108+0.0000205i	0.000105+0.0000199i
	a_9	-0.00919-0.0000953i	-0.0092-0.0001i	-0.0092-0.0000971i
HV	a_2	0.0629-0.0516i	0.056-0.054i	0.062-0.053i
	a_3	0.00502+0.00135i	0.0052+0.0014i	0.0050+0.0014i
	a_5	-0.00523-0.00154i	-0.0054-0.0016i	-0.0053-0.00157i
	a_7	0.000145+0.0000215i	0.000148+0.0000228i	0.000146+0.0000221i
	a_9	-0.00914-0.0000963i	-0.0092-0.000101i	-0.0091-0.0000983i

Table 6. Numerical value of a_i for $B \to J/\psi K$ ($q = s$), $\mu = m_b$, by using f_{II}^3

$\phi_{J/\psi}(\xi)$		$6\xi(1-\xi)$	$\delta(\xi - 1/2)$	$9.58\xi(1-\xi)[\frac{\xi(1-\xi)}{1-2.8\xi(1-\xi)}]^{0.7}$
Twist 3		f_{II}^3		
NDR	a_2	0.106-0.050i	0.083-0.053i	0.093-0.051i
	a_3	0.0042+0.0016i	0.0049+0.0017i	0.0046+0.0016i
	a_5	-0.0031-0.0019i	-0.0039-0.0020i	-0.0036-0.00195i
	a_7	0.000091+0.0000195i	0.00099+0.00002i	0.000096+0.0000199i
	a_9	-0.0091-0.0000953i	-0.00917-0.0001i	-0.0091-0.0000971i
HV	a_2	0.101-0.051i	0.077-0.054i	0.087-0.052i
	a_3	0.0040+0.0013i	0.0046+0.0014i	0.0043+0.0013i
	a_5	-0.0041-0.0015i	-0.0047-0.0016i	-0.0045-0.00157i
	a_7	0.00012+0.0000217i	0.000139+0.00002i	0.000135+0.0000221i
	a_9	-0.0090-0.0000964i	-0.0091+0.0001i	-0.0090-0.0000983i

Table 7. Numerical value of a_i for $B \to J/\psi \pi$ ($q = d$), $\mu = m_b$, by using f_{II}^2

$\phi_{J/\psi}(\xi)$		$6\xi(1-\xi)$	$\delta(\xi - 1/2)$	$9.58\xi(1-\xi)[\frac{\xi(1-\xi)}{1-2.8\xi(1-\xi)}]^{0.7}$
Twist 3		f_{II}^2		
NDR	a_2	0.068-0.050i	0.058-0.053i	0.063-0.051i
	a_3	0.0054+0.0016i	0.0057+0.0017i	0.0056+0.0016i
	a_5	-0.0045-0.0019i	-0.0049-0.0020i	-0.0047-0.0019i
	a_7	0.000105+0.0000195i	0.000109+0.0000205i	0.000107+0.0000199i
	a_9	-0.0092-0.0000953i	-0.0092-0.000100i	-0.0092-0.0000971i
HV	a_2	0.062-0.051i	0.052-0.054i	0.056-0.052i
	a_3	0.0050+0.0013i	0.0053+0.0014i	0.0051+0.0014i
	a_5	-0.0052-0.0015i	-0.0055-0.0016i	-0.0054-0.0016i
	a_7	0.000145+0.0000217i	0.000150+0.0000228i	0.000148+0.0000221i
	a_9	-0.0091-0.0000964i	-0.0091-0.000101i	-0.0091-0.0000983i

Table 8. Numerical value of a_i for $B \to J/\psi \pi$ ($q = d$), $\mu = m_b$, by using f_{II}^3

$\phi_{J/\psi}(\xi)$		$6\xi(1-\xi)$	$\delta(\xi - 1/2)$	$9.58\xi(1-\xi)[\frac{\xi(1-\xi)}{1-2.8\xi(1-\xi)}]^{0.7}$
Twist 3		f_{II}^3		
NDR	a_2	0.105-0.050i	0.083-0.053i	0.092-0.051i
	a_3	0.0042+0.0016i	0.0049+0.0017i	0.0046+0.0016i
	a_5	-0.0031-0.0019i	-0.0040-0.0020i	-0.0036-0.00195i
	a_7	0.00009+0.0000195i	0.0001+0.0000205i	0.000096+0.0000199i
	a_9	-0.0091-0.0000953i	-0.00917-0.0001i	-0.0091-0.0000971i

Table 8. (Continued)

HV	a_2	0.10-0.051i	0.077-0.054i	0.087-0.052i
	a_3	0.0040+0.0013i	0.0046+0.0014i	0.0043+0.0013i
	a_5	-0.0041-0.0015i	-0.0048-0.0016i	-0.0045-0.00157i
	a_7	0.00012+0.0000217i	0.0001+0.0000228i	0.00013+0.0000221i
	a_9	-0.0090-0.0000964i	-0.0091-0.0001i	-0.0090-0.0000983i

5. WAVE FUNCTIONS OF J/ψ, *K, Π*, B

The leading-twist (twist-2) LCDAs of J/ψ can be expanded as,

$$\phi_{J/\psi}(\xi) = 6\xi(1-\xi)(1+\frac{3}{2}a_2[5(2\xi-1)^2-1])$$
$$\phi^T_{J/\psi}(\xi) = 6\xi(1-\xi)(1+\frac{3}{2}a^T_2[5(2\xi-1)^2-1]) \quad (21)$$

where the parameters a_2 and a_2^T are defined by the matrix element of the twist-2 conformal operator with conformal spin 3 [18], while twist-2 DAs ϕ_K and ϕ_π can be expanded in terms of Gegenbauer polynomials $C_{3/2}$:

$$\phi^{K(\pi)}(\overline{\eta},\mu^2) = 6\overline{\eta}(1-\overline{\eta})(1+\sum_{n=1}^{\infty} a_{2n}^{K(\pi)}(\mu^2)C_{2n}^{3/2}(2\overline{\eta}-1) \quad (22)$$

as before, $\overline{\eta}$ is the light-cone momentum fraction of the anti-quark in K and π. Here, we consider the asymptotic function with the values of the Gegenbauer moments a_{2n}^K to be an available from [19]. In the far ultraviolet $\mu \to \infty$, we have $\alpha_i^M \to 0$ so at the scale $\mu \approx m_b$, which is still large compared to the nonperturbative scale of *QCD*, the Gegenbauer moments α_i^M are expected to be small. The asymptotic form of the distribution amplitudes $\phi(\xi)$ and $\phi^T(\xi)$ is the same, which is given as $\phi(\xi) = \phi^T(\xi) = 6\xi(1-\xi)$. In the numerical analysis, we also consider the wave function of the form: $\phi(\xi) = \phi^T(\xi) = \delta(\xi - 1/2)$, $\phi(\xi) = \phi^T(\xi) = 9.58\xi(1-\xi)[\frac{\xi(1-\xi)}{1-2.8\xi(1-\xi)}]^{0.7}$ [20].

Twist-3 LCDAs, $\phi_p^{K(\pi)}$ and $\phi_\sigma^{K(\pi)}$ of the kaon and pion are defined in the pseudoscalar and tensor matrix elements. They can be expanded in terms of Gegenbauer polynominals:

$$\phi_p^{K(\pi)}(\overline{\eta}) = 1 + aC_2^{1/2}(\overline{\eta}) + bC_4^{1/2}(\overline{\eta}) + \ldots$$
$$\phi_\sigma^{K(\pi)}(\overline{\eta}) = 6\overline{\eta}(1-\overline{\eta})(1 + dC_2^{3/2}(\overline{\eta}) + \ldots) \quad (23)$$

in which we can find the coefficients *a, b, d* in [19]. Twist-3 DAs of pseudoscalar mesons are associated with a chiral enhancement factor μ_χ. We take its asymptotic form, then we apply $\phi_\sigma^{K(\pi)}(\overline{\eta}) = 6\overline{\eta}(1-\overline{\eta})$ and $\phi_p^{K(\pi)}(\overline{\eta}) = 1$. We find that the twist-3 kaon (pion) LCDA $\phi_\sigma^{K(\pi)}$ contributes to spectator diagrams in $B \to J/\psi K$ ($B \to J/\psi\pi$) decay. For the *B* meson, we use [21]

$$\phi_1^B(\overline{\rho}) = N_B\overline{\rho}^2(1-\overline{\rho})^2 \exp[-\frac{1}{2}(\frac{\overline{\rho}m_B}{\omega_B})^2] \quad (24)$$

where, $\omega_B = 0.25GeV$ and N_B are a normalization constant, $\int_0^1 \phi_B(\xi)d\xi = 1$. This B meson wave function corresponds to $\lambda_B = 300MeV$, which is defined by $\int_0^1 d\xi \frac{\phi_B(\xi)}{\xi} \equiv \frac{m_B}{\lambda_B}$. This can be understood since the B meson wave function is peaked at small ξ: It is of order m_B / Λ_{QCD} at $\overline{\rho} \approx \Lambda_{QCD} / m_B$. Hence, the integral over $\phi_B(\overline{\rho}) / \overline{\rho}$ produces a m_B / Λ_{QCD} term [9, 11].

6. BRANCHING RATIO

The decay rate is simply given by

$$\Gamma = \frac{S}{16\pi m_B} \left| \left\langle M_1 M_2 \left| H_{eff} \right| \overline{B} \right\rangle \right|^2$$

where $S = 1/2$, if M_1 and M_2 are identical, and $S = 1$ otherwise [8]. Also, the decay rates for $B \to M_1 M_2$ are given by

$$\Gamma\left(B_s \to M_1 M_2\right) = \frac{p_c}{8\pi m_{B_s}^2} \left| M\left(B \to M_1 M_2\right)\right|^2 \tag{25}$$

where

$$p_c = \frac{\sqrt{(m_B^2 - (m_{M_1} + m_{M_2})^2)(m_B^2 - (m_{M_1} - m_{M_2})^2)}}{2 m_B} \tag{26}$$

is the *C.M.* momentum of the decay particles [22]. We assume that in the limit, in which m_b goes to infinity, $m_{J/\psi}$ is heavy enough to regard the size of the J/ψ meson as small, but light enough to employ the leading-twist light cone wave function for J/ψ. Then, $P_c = m_B / 2$ and $p_B.\varepsilon^* = m_B^2 / 2m_{J/\psi}$. The branching ratio is given by

$$BR\left(B \to M_1 M_2\right) = \frac{\Gamma_i}{\Gamma_{tot}} = \tau_B \frac{1}{8\pi} |M|^2 \frac{|p_c|}{m_B^2} \tag{27}$$

where $\tau_B = \hbar / \Gamma_{tot}$ =1.638 ps, Γ_{tot} = (4.2±0.3) ×10^{-13} [23].

7. FORM FACTORS

B-to-light form factors are parametrized in the LCSR as

$$F(q^2) = \frac{F(0)}{1 - a_F \frac{q^2}{m_B^2} + b_F (\frac{q^2}{m_B^2})^2} \tag{28}$$

where the relevant fitted parameters a_F and b_F are in Table 9. (Here, we want to find form factors in $q^2 = m_{J/\psi}^2$). The momentum dependence of the form factors is given by [24],

$$F_0^{B\pi}(q^2) = -0.28(\frac{5.4^2}{5.4^2 - q^2}) \frac{q^2}{m_B^2 - m_\pi^2} + F_1^{B\pi}(q^2)$$

$$F_0^{BK}(q^2) = -0.32(\frac{5.8^2}{5.8^2 - q^2}) \frac{q^2}{m_B^2 - m_K^2} + F_1^{BK}(q^2) \tag{29}$$

in this section, form factors are calculated by (25).

Table 9. Values of a_F and b_F

	$F(0)$	a_F	b_F
F_+^π	0.3±0.04	1.35	0.27
F_0^π	0.3±0.04	0.39	0.62
F_T^π	0.3±0.04	1.34	0.26
F_+^K	0.35±0.05	1.37	0.35
F_0^K	0.035±0.05	0.40	0.41
F_T^K	0.36±0.05	1.37	0.37

8. CP ASYMMETRY

Since there are no mixing effects present in the charged B meson system, non-vanishing CP asymmetries of the kind

$$A_{CP}(B^+ \to \bar{f}) \equiv \frac{\Gamma(B^+ \to \bar{f}) - \Gamma(B^- \to f)}{\Gamma(B^+ \to \bar{f}) + \Gamma(B^- \to f)} \tag{30}$$

would give us unambiguous evidence for "direct" CP violation in the B system, the CP asymmetries (30) arise from the interference between decay amplitudes with both different CP-violating weak and different CP-conserving strong phases. In the SM, the weak phases are related to the phases of the CKM matrix elements, whereas the strong phases are induced by final-state interaction processes. In general, the strong phases introduce severe theoretical uncertainties into the calculation of $A_{CP}(B^+ \to f)$, thereby destroying the clean relation to the CP-violating weak phases. However, there is an important tool to overcome these problems, which is provided by amplitude relations between certain nonleptonic B decays. For the charged B meson decays, the direct CP-violating asymmetries A_{CP}^{dir} can be defined as usual. For $B^+ \to J/\psi K^+$ decay, there is no direct CP violation, since there is no weak phase appearing in their decay amplitude [25].

$$A_{CP}^{dir}(B^+ \to J/\psi K^+) = 0.017 \pm 0.016$$

$$A_{CP}^{dir}(B^+ \to J/\psi \pi^+) = 0.09 \pm 0.08$$

In this scheme in the standard model, there is no contribution to CP asymmetry in the decay amplitude since the CKM matrix elements involved here are all real. The CP asymmetry totally comes from $B^0 - \overline{B}^0$ mixing.

For the $B^0 \to M_1 M_2$ decays, because these decays are neutral B meson decays, we should consider the effects of $B^0 - \overline{B}^0$ mixing. In the case of $B \to J/\psi K$, we have to deal with both current–current, i.e. tree-diagram-like, and with penguin contributions. For the B^0 decay, the CP asymmetry is time dependent.

$$A_{CP}(t) = A_{CP}^{dir}\cos(\Delta m t) + A_{CP}^{mix}\sin(\Delta m t) \tag{31}$$

the direct and mixing induced CP-violating asymmetries A_{CP}^{dir} and A_{CP}^{mix} can be written as

$$A_{CP}^{dir} = \frac{|\lambda_{CP}|^2 - 1}{1 + |\lambda_{CP}|^2} = \frac{2r\sin\delta\sin\gamma}{1 + 2r\cos\delta\cos\gamma + r^2}$$

$$A_{CP}^{mix} = \frac{2\,\mathrm{Im}(\lambda_{CP})}{1+|\lambda_{CP}|^2} = -\frac{\sin 2\beta + 2r\cos\delta\sin(2\beta+\gamma) + r^2\sin 2(\beta+\gamma)}{1+2r\cos\delta\cos\gamma + r^2} \tag{32}$$

where γ, β, δ are used in the Wolfenstein approximation. The CP-violating parameter λ_{CP} is

$$\lambda_{CP} = \eta_f e^{-2i\beta} \frac{\langle f|H_{eff}|\overline{B}^0\rangle}{\langle f|H_{eff}|B^0\rangle} \tag{33}$$

η_f is the CP-eigenvalue of the final states and $r = P_{(Penguin)}/T_{(Tree)}$ [26-28]. Keeping only linear terms in r,

$$C \equiv A_{CP}^{dir} \approx 2r\sin\delta\sin\gamma \quad , \quad S \equiv A_{CP}^{mix} \approx -\sin 2\beta - \overbrace{2r\cos 2\beta\cos\delta\sin\gamma}^{\Delta S}$$

in $b \to c\bar{c}s$ quark-level decays, the time-dependent CP violation parameters measured from the interference between decays with and without mixing are $S_{c\bar{c}s} = -\eta_{CP}\sin 2\beta$ and $C_{c\bar{c}s} = 0$, to a very good approximation. The theoretically cleanest case if $B \to J/\psi K(\pi)$, where

$$\lambda_{J/\psi K} = -e^{-2i\beta}\,,\ \lambda_{J/\psi\pi} = e^{-2i\beta} \tag{34}$$

and so

$$\mathrm{Im}\,\lambda_{J/\psi K} = \sin 2\beta\,,\ \mathrm{Im}\,\lambda_{J/\psi\pi} = -\sin 2\beta$$

then

$$A_{CP}^{mix}(B \to J/\psi K) = \sin(2\beta)$$

$$A_{CP}^{mix}(B \to J/\psi\pi) = \sin(-2\beta)$$

$$A_{CP}(t) = \pm\sin(2\beta)\cos(\Delta m_{d,s}t) \tag{35}$$

one more important implication of the SM is [26, 29-31],

$$A_{CP}^{dir}(B_d \to J/\psi K) \approx 0 \approx A_{CP}(B^+ \to J/\psi K^+)$$

This theoretical expectation agrees well with the data [25],

$$A_{CP}^{dir}(B^0 \to J/\psi K^0) = -0.018 \pm 0.025$$

$$A_{CP}^{dir}(B^0 \to J/\psi\pi^0) = -0.11 \pm 0.25$$

in [20], the values of $\sin(2\beta)$ are expressed for $B^0 \to J/\psi\pi^0$ and compared with experimental data ($\beta = 22.2$), and we have redescribed these values in Table 10 and compared them with the experimental data which are for $B^0 \to J/\psi K^0$ in $\beta = 20.1$.

Table 10. Determination of weak phase β through mixing-induced CP asymmetry

β(deg)	$S_{J/\psi K}$	$S_{J/\psi\pi}$
18.0	0.585	-0.585
18.3	0.593	-0.593
18.6	0.601	-0.601
18.9	0.610	-0.610

Table 10. (Continued)

19.2	0.618	-0.618
19.5	0.626	-0.626
19.8	0.634	-0.634
20.1	0.642	-0.642
20.4	0.650	-0.650
20.7	0.658	-0.658
21.0	0.666	-0.666
21.3	0.674	-0.674
21.6	0.681	-0.681
21.9	0.689	-0.689
22.2	0.696	-0.696
22.5	0.704	-0.704
22.8	0.711	-0.711
23.1	0.718	-0.718
23.4	0.726	-0.726
23.7	0.733	-0.733
24.0	0.740	-0.740
24.3	0.747	-0.747
24.6	0.754	-0.754
24.9	0.760	-0.760
Exp. [25]	0.642±0.035	-0.69±0.25

8. INPUT

For numerical analysis, we use the following input parameters [9, 11]:

$m_b = 4.4GeV$, $m_c = 1.5GeV$, $m_B = 5.28GeV$, $m_{J/\psi} = 3.1GeV$

$f_{J/\psi} = 405MeV$, $f_B = 190MeV$, $f_K = 160MeV$, $f_\pi = 133MeV$

$\Lambda_{QCD} = 300MeV$, $\lambda_B \approx 300MeV$, $\alpha_s(\mu = m_b) \approx 0.2$

$F_1^{B\to K}(m_{J/\psi}^2) = 0.7$, $F_0^{B\to K}(m_{J/\psi}^2) = 0.418$

$F_1^{B\to \pi}(m_{J/\psi}^2) = 0.587$, $F_0^{B\to \pi}(m_{J/\psi}^2) = 0.351$ [22, 32 and 33]

$C_F = \frac{N^2-1}{2N} = \frac{4}{3}$, $\qquad G_F = 1.166\times 10^{-5}$

$$V_{CKM} = \begin{pmatrix} 0.9603 & 0.223 & 0.0037e^{-i(1.2\pm 0.08)} \\ -0.225 - 0.0001e^{i(1.2\pm 0.08)} & 0.969 - 0.00003e^{i(1.2\pm 0.08)} & 0.041 \\ 0.009 - 0.0035e^{i(1.2\pm 0.08)} & -0.040 - 0.0008e^{i(1.2\pm 0.08)} & 0.989 \end{pmatrix}$$

$$|V_{cb}V_{cs}^*| = 0.039, \; |V_{tb}V_{ts}^*| = 0.041$$

$$|V_{cb}V_{cd}^*| = 0.009, \; |V_{tb}V_{td}^*| = 0.0319 - 0.014i$$

9. DISCUSSION AND RESULTS

The hadronic decays $B \to J/\psi K(\pi)$ are interesting because, experimentally, they are the only color suppressed modes which have been measured, and theoretically they are calculable by *QCD* factorization, even the emitted meson J/ψ is heavy. We computed a_i coefficients in Naïve factorization (Table 2) and in QCD factorization (Tables 5-8) by two 3-twist contributions, and then we obtained decay rates (Tables

11, 12) and branching ratios for two decays (Tables 13, 14). We compare branching ratios in Tables 13, 14 which, for the $\phi^{J/\psi} = 6\xi(1-\xi)$ function, is in agreement with the experiments.

Table 11. Decay rates in Naïve Factorization and QCD Factorization for $B \to J/\psi K$ (GeV),($\mu = m_b$)

	$\Gamma(LO)$	$\Gamma(NLO)$ (NDR)	$\Gamma(NLO)$ (HV)
NF	1.61×10^{-16}	8.46×10^{-16}	5.32×10^{-16}
QCDF			
$\phi_{J/\psi}(\xi) = 6\xi(1-\xi)$	f_{II}^2	2.07×10^{-16}	1.92×10^{-16}
$\phi_{J/\psi}(\xi) = \delta(\xi - 1/2)$		1.37×10^{-16}	1.31×10^{-16}
$\phi_{J/\psi} = 9.58x(1-x)[\frac{x(1-x)}{1-2.8x(1-x)}]^{0.7}$		2.01×10^{-16}	1.92×10^{-16}
$\phi_{J/\psi}(\xi) = 6\xi(1-\xi)$	f_{II}^3	3.77×10^{-16}	3.59×10^{-16}
$\phi_{J/\psi}(\xi) = \delta(\xi - 1/2)$		2.71×10^{-16}	2.52×10^{-16}
$\phi_{J/\psi} = 9.58x(1-x)[\frac{x(1-x)}{1-2.8x(1-x)}]^{0.7}$		3.12×10^{-16}	2.91×10^{-16}

Table 12. Decay rates in Naïve Factorization and QCD Factorization for $B \to J/\psi\pi$ (GeV), ($\mu = m_b$)

	$\Gamma(LO)$	$\Gamma(NLO)$ (NDR)	$\Gamma(NLO)$ (HV)
NF	1.02×10^{-17}	4.44×10^{-17}	2.71×10^{-17}
QCDF			
$\phi_{J/\psi}(\xi) = 6\xi(1-\xi)$	f_{II}^2	1.21×10^{-17}	1.19×10^{-17}
$\phi_{J/\psi}(\xi) = \delta(\xi - 1/2)$		1.08×10^{-17}	1.06×10^{-17}
$\phi_{J/\psi} = 9.58x(1-x)[\frac{x(1-x)}{1-2.8x(1-x)}]^{0.7}$		1.13×10^{-17}	1.11×10^{-17}
$\phi_{J/\psi}(\xi) = 6\xi(1-\xi)$	f_{II}^3	1.96×10^{-17}	1.96×10^{-17}
$\phi_{J/\psi}(\xi) = \delta(\xi - 1/2)$		1.53×10^{-17}	1.51×10^{-17}
$\phi_{J/\psi} = 9.58x(1-x)[\frac{x(1-x)}{1-2.8x(1-x)}]^{0.7}$		1.68×10^{-17}	1.69×10^{-17}

Table 13. Branching ratios in Naïve Factorization and QCD Factorization for $B \to J/\psi K$ ($\mu = m_b$)

	$BR(LO)$	$BR(NLO)$ (NDR)	$BR(NLO)$ (HV)
NF	$3.8^{+0.3}_{-0.3}\times10^{-4}$	$20.1^{+1.6}_{-1.3}\times10^{-4}$	$12.6^{+1.0}_{-0.8}\times10^{-4}$
QCDF			
$\phi_{J/\psi}(\xi) = 6\xi(1-\xi)$	f_{II}^2	$4.9^{+0.4}_{-0.3}\times10^{-4}$	$4.5^{+0.4}_{-0.3}\times10^{-4}$
$\phi_{J/\psi}(\xi) = \delta(\xi - 1/2)$		$3.2^{+0.3}_{-0.2}\times10^{-4}$	$3.1^{+0.2}_{-0.2}\times10^{-4}$

Table 13. (Continued)

$\phi_{J/\psi} = 9.58x(1-x)[\frac{x(1-x)}{1-2.8x(1-x)}]^{0.7}$		$4.7^{+0.4}_{-0.3}\times10^{-4}$	$4.5^{+0.4}_{-0.3}\times10^{-4}$
$\phi_{J/\psi}(\xi) = 6\xi(1-\xi)$	f_{II}^3	$8.9^{+0.7}_{-0.6}\times10^{-4}$	$8.5^{+0.7}_{-0.6}\times10^{-4}$
$\phi_{J/\psi}(\xi) = \delta(\xi-1/2)$		$6.4^{+0.5}_{-0.4}\times10^{-4}$	$6.0^{+0.4}_{-0.4}\times10^{-4}$
$\phi_{J/\psi} = 9.58x(1-x)[\frac{x(1-x)}{1-2.8x(1-x)}]^{0.7}$		$7.4^{+0.6}_{-0.5}\times10^{-4}$	$6.9^{+0.5}_{-0.5}\times10^{-4}$
Exp. [25]	$(10.07\pm 0.35)\times10^{-4}$		

Table 14. Branching ratios in Naïve Factorization and QCD Factorization for $B \to J/\psi\pi$ ($\mu = m_b$)

	$BR(LO)$	$BR(NLO)$ (NDR)	$BR(NLO)$ (HV)
NF	$0.24^{+0.02}_{-0.02}\times10^{-4}$	$0.10^{+0.01}_{-0.01}\times10^{-4}$	$0.64^{+0.05}_{-0.04}\times10^{-4}$
QCDF			
$\phi_{J/\psi}(\xi) = 6\xi(1-\xi)$	f_{II}^2	$0.28^{+0.03}_{-0.02}\times10^{-4}$	$0.28^{+0.02}_{-0.02}\times10^{-4}$
$\phi_{J/\psi}(\xi) = \delta(\xi-1/2)$		$0.25^{+0.02}_{-0.01}\times10^{-4}$	$0.25^{+0.02}_{-0.02}\times10^{-4}$
$\phi_{J/\psi} = 9.58x(1-x)[\frac{x(1-x)}{1-2.8x(1-x)}]^{0.7}$		$0.26^{+0.02}_{-0.01}\times10^{-4}$	$0.26^{+0.02}_{-0.02}\times10^{-4}$
$\phi_{J/\psi}(\xi) = 6\xi(1-\xi)$	f_{II}^3	$0.46^{+0.04}_{-0.03}\times10^{-4}$	$0.46^{+0.04}_{-0.03}\times10^{-4}$
$\phi_{J/\psi}(\xi) = \delta(\xi-1/2)$		$0.36^{+0.03}_{-0.02}\times10^{-4}$	$0.35^{+0.03}_{-0.02}\times10^{-4}$
$\phi_{J/\psi} = 9.58x(1-x)[\frac{x(1-x)}{1-2.8x(1-x)}]^{0.7}$		$0.40^{+0.03}_{-0.03}\times10^{-4}$	$0.40^{+0.03}_{-0.03}\times10^{-4}$
Exp. [25]	$(0.49\pm0.06)\times10^{-4}$		

In the colour suppressed $B \to J/\psi K$ and $J/\psi\pi$ decays, non factorizable contribution is more important. Our result on the color suppressed $B \to J/\psi K$ and $B \to J/\psi\pi$ decays is still sensitive to the values of both $F_1^{B\to\pi}(m^2_{J/\psi})$ [or $F_1^{B\to K}(m^2_{J/\psi})$].

We considered coefficients in $\mu = m_b$ and three functions for J/ψ, in which the numerical results are better for $\phi^{J/\psi} = 6\xi(1-\xi)$. Also, we assumed $\rho_H = 0$.

To leading-twist contributions from the light-cone distribution amplitudes (LCDAs) of the mesons, vertex corrections and hard spectator interactions, which include m_c effects, imply results in Tables 13, 14. Hence, the predicted branching ratio is too small by a factor of 5; the nonfactorizable corrections to naive factorization to leading-twist order are small.

We study the twist-3 effects due to the kaon. The prediction $BR(B \to J/\psi K(\pi))$ is in agreement with the experiments [25]:

$$BR(B \to J/\psi K) = (10.07 \pm 0.35)\times 10^{-4} \qquad BR(B \to J/\psi\pi) = (0.49 \pm 0.06)\times 10^{-4}$$

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