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ON A CHARACTERISTIC PROBLEM FOR A THIRD ORDER PSEUDOPARABOLIC EQUATION*

A. MAHER[1**] AND YE. A. UTKINA[2]

[1]Department of Mathematics, Faculty of Science, Assiut University, Egypt
Email: a_maher69@yahoo.com
[2]Department of Differential Equations, Kazan State University, Kazan, Russia

Abstract – In this paper, we investigate the Goursat problem in the class $C^{2+1}(D)\cap C^{n+0}(D\cup P)\cap C^{0+0}(D\cup Q)$ for a third order pseudoparabolic equation. Some results are given concerning the existence and uniqueness for the solution of the suggested problem.

Keywords – Third order pseudoparabolic equation, the Goursat problem

1. INTRODUCTION

In the domain $D=\{(x,y);\quad x_0<x<x_1,\quad y_0<y<y_1\}$ we consider the equation

$$L(u)\equiv u_{xxy}+au_{xx}+bu_{xy}+cu_x+du_y+eu=f, \tag{1}$$

where

$$a\ ,b,\ c,d,e,\ f\in C^{1+2}(D),$$

special cases of the equation (1) are encountered during the investigation of the processes of moisture absorption by plants [8], where the class C^{k+l} means the existence and continuity for all derivatives

$$\partial^{r+s}/\partial x^r\partial y^s \qquad (r=0,...k;s=0,...,l).$$

We will call the solution of the class as a regular.

For this equation, we investigate the following:

2. FORMULATION OF THE PROBLEM

To find the function

$$u\in C^{2+1}(D)\cap C^{\max(n_1,n_2)+0}(D\cup P)\cap C^{0+m}(D\cup Q),$$

which is the solution of Equation (1) in D and satisfies the following conditions:

$$D_x^{n_1}u(x_0,y)=\varphi_{n_1}(y),\ D_x^{n_2}u(x_0,y)=\varphi_{n_2}(y),\ \varphi_{n_1},\varphi_{n_2}\in C^1(P) \tag{2}$$

$$D_y^m u(x,y_0)=\psi_m(x),\ \psi_m\in C^2(Q), \tag{3}$$

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**Corresponding author

where

$$y \in P = [y_0, y_1], \qquad x \in Q = [x_0, x_1].$$

Here, we consider the conditions of function coincidence for the right parts (2) and (3) on the boundary of their definitions (co-ordination conditions) as satisfied:

$$\psi'(x_0) = \varphi_1(y_0),\ \psi(x_0) = \varphi(y_0),$$

where

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here, we consider the conditions of function coincidence for the right parts (2) and (3) on the boundary of their definitions (co-ordination conditions) as satisfied.

$$\psi'(x_0) = \varphi_1(y_0), \qquad \psi(x_0) = \varphi(y_0),$$

where

$$\varphi(y) = \varphi_0(y), \qquad \psi(x) = \psi_0(x).$$

When $n_1 = 0,\ n_2 = 1,\ m = 0$ in this statement, the Goursat problem considered in [9] is obtained. We should note that the Goursat problem for (1) is the most investigated. The cases $n_1 = 2, n_2 = 1, m = 0$; $n_1 = 0, n_2 = 2, m = 0$; $n_1 = 0, n_2 = 1, m = 1$; $n_1 = 2, n_2 = 1, m = 1$ and $n_1 = 0, n_2 = 2, m = 1$ are investigated in [3]. We will consider one of the variants through searching formulas for the definition of the boundary Goursat value. Our work is considered as a continuation of the results in [2-7]. More precisely, see in greater detail ([1] and [10]).

In order not to exceedingly increase the size of this paper, we will mainly be defining conditions of equality and types for the initial values that provide the chance to calculate the boundary Goursat value. Thus, in the final formulation of the obtained result we should assume that the initial values are sufficiently smooth. However, these smoothness conditions can be formulated, checking carefully what smoothness is required by all the stages of the conducted considerations.

Let us turn from the general statement to the one that is immediately studied in this paper.

Problem: To find the function

$$u \in C^{2+1}(D) \cap C^{n+0}(D \cup P) \cap C^{0+0}(D \cup Q)$$

which in D is the solution of Equation (1), which satisfies conditions (2) and (3) when $n_1 = n, n_2 = 1, m = 0$.

The proposed problem consists, probably, of finding $\varphi(y)$ in order to reduce to the Goursat problem. Let us first integrate Equation (1) with respect to y within the bounds from y_* to $y\,(y_*, y \in P)$ and then in the obtained relation, we direct y_* to y_0:

$$u_{xx}(x,y)-u_{xx}(x,y_0)+\int_{y_0}^{y}a(x,\eta)u_{xx}(x,\eta)d\eta+b(x,y)u_x(x,y)-b(x,y_0)u_x(x,y)+$$
$$+\int_{y_0}^{y}c(x,\eta)u_x(x,\eta)d\eta+d(x,y)u\ \ (x,y)-d(x,y_0)u\ \ (x,y)+$$
$$+\int_{y_0}^{y}\{-b_\eta(x,\eta)u_x(x,\eta)-d_\eta(x,\eta)u\ \ (x,\eta)+e(x,\eta)u\ \ (x,\eta)\}d\eta=0\,.$$

Differentiating this relation (*n*-2) times with respect to *x*, we obtain:

$$D_x^n u(x,y)-D_x^n u(x,y_0)+\int_{y_0}^{y}D_x^{n-2}[a(x,\eta)u_{xx}(x,\eta)]d\eta+$$
$$+D_x^{n-2}[b(x,y)u_x(x,y)]-D_x^{n-2}[b(x,y_0)u_x(x,y_0)]+$$
$$+\int_{y_0}^{y}D_x^{n-2}[c(x,\eta)u_x(x,\eta)]d\eta+D_x^{n-2}[d(x,y)u\ \ (x,y)]-$$
$$-D_x^{n-2}[d(x,y_0)u\ \ (x,y_0)]+\int_{y_0}^{y}\{-D_x^{n-2}[b_\eta(x,\eta)u_x(x,\eta)-$$
$$-D_x^{n-2}[d_\eta(x,\eta)u\ \ (x,\eta)]+D_x^{n-2}[e(x,\eta)u\ \ (x,\eta)]\}\,d\eta=0.$$

Directing x to x_0, we find:

$$\varphi_n(y)-\varphi_n(y_0)+\int_{y_0}^{y}\sum_{i\le n-2}C_{n-2}^{i}D_x^{i}[a(x_0,\eta)]\varphi_{n-i}(\eta)d\eta+$$
$$+\sum_{i\le n-2}C_{n-2}^{i}D_x^{i}[b(x_0,y)]\varphi_{n-(i+1)}(y)-$$
$$-\sum_{i\le n-2}C_{n-2}^{i}D_x^{i}[b(x_0,y_0)]\varphi_{n-(i+1)}(y_0)+$$
$$+\int_{y_0}^{y}\sum_{i\le n-2}C_{n-2}^{i}D_x^{i}[c(x_0,\eta)]\varphi_{n-(i+1)}(\eta)d\eta+$$
$$+\sum_{i\le n-2}C_{n-2}^{i}D_x^{i}[d(x_0,y\ \ )]\varphi_{n-(i+2)}(y\ \ )-$$
$$-\sum_{i\le n-2}C_{n-2}^{i}D_x^{i}[d(x_0,y_0)]\varphi_{n-(i+2)}(y_0)+$$
$$+\int_{y_0}^{y}\{-\sum_{i\le n-2}C_{n-2}^{i}D_x^{i}[b_\eta(x_0,\eta)]\varphi_{n-(i+2)}(\eta)-$$
$$-\sum_{i\le n-2}C_{n-2}^{i}D_x^{i}[d_\eta(x_0,\eta)]\varphi_{n-(i+2)}(\eta)+$$
$$+\sum_{i\le n-2}C_{n-2}^{i}D_x^{i}[e(x_0,\eta)]\varphi_{n-(i+2)}(\eta)\}\,d\eta=0,$$

where

$$\varphi_{n-i}(y)=D_x^{n-i}u(x_0,y).$$

The latter equation can be rewritten in the following form:

$$D_x^{n-2}[d(x_0, y)]\varphi(y) + \int_{y_0}^{y} D_x^{n-2}[e(x_0,\eta) -$$
$$- d_\eta(x_0,\eta)]\varphi(\eta)d\eta = r_n(y),$$

where

$$\begin{aligned} r_n(y) = -\varphi_n(y) + \varphi_n(y_0) - \int_{y_0}^{y}\sum_{i\le n-2} C_{n-2}^i D_x^i[e(x_0,\eta) - d_\eta(x_0,\eta)]\varphi_{n-(i+2)}(\eta)d\eta - \\ - \int_{y_0}^{y}\sum_{i\le n-2} C_{n-2}^i D_x^i[a(x_0,\eta)]\varphi_{n-i}(\eta)d\eta - \\ - \sum_{i\le n-2} C_{n-2}^i D_x^i[b(x_0, y)]\varphi_{n-(i+1)}(y) + \\ + \int_{y_0}^{y}\sum_{i\le n-2} C_{n-2}^i D_x^i[b_\eta(x_0,\eta)]\varphi_{n-(i+1)}(\eta)d\eta - \\ - \int_{y_0}^{y}\sum_{i\le n-2} C_{n-2}^i D_x^i[c(x_0,\eta)]\varphi_{n-(i+1)}(\eta)d\eta - \\ - \sum_{i\le n-2} C_{n-2}^i D_x^i[d(x_0, y)]\varphi_{n-(i+2)}(y) + \Omega_n(x_0, y_0), \end{aligned} \tag{4}$$

and

$$\Omega_n(x_0, y_0) = \sum_{i\le n-2} C_{n-2}^i \{D_x^i[b(x_0, y_0)]\varphi_{n-(i+1)}(y_0) +$$
$$+ D_x^i[d(x_0, y_0)]\varphi_{n-(i+2)}(y_0)\},$$

when $\varphi_i\,;\,(2 \le i \le n-1)$ are defined through the previous equation.

Thus,

$$\varphi_{n-1}(y) + \int_{y_0}^{y} a(x_0,\eta)\varphi_{n-1}(\eta)d\eta = l_{n-1}(y),$$

where

$$l_{n-1}(y) = \varphi_{n-1}(y_0) - \int_{y_0}^{y}\sum_{i\le n-3} C_{n-3}^i D_x^i[e(x_0,\eta) - d_\eta(x_0,\eta)]\varphi_{n-(i+3)}(\eta)d\eta -$$
$$- \int_{y_0}^{y}\sum_{i\le n-3} C_{n-3}^i D_x^i[a(x_0,\eta)]\varphi_{n-i-1}(\eta)d\eta -$$

$$-\sum_{i\le n-3} C_{n-3}^{i} D_x^i[b(x_0,y\ )]\varphi_{n-(i+2)}(y\ )+$$

$$+\int_{y_0}^{y}\sum_{i\le n-3} C_{n-3}^{i} D_x^i[b_\eta(x_0,\eta)]\varphi_{n-(i+2)}(\eta)d\eta-$$

$$-\int_{y_0}^{y}\sum_{i\le n-3} C_{n-3}^{i} D_x^i[c(x_0,\eta)]\varphi_{n-(i+2)}(\eta)d\eta-$$

$$-\sum_{i\le n-2} C_{n-3}^{i} D_x^i[d(x_0,y)]\varphi_{n-(i+3)}(y)+\Omega_{n-1}(x_0,y_0),$$

similarly,

$$\phi_{n-j}(y)+\int_{y_0}^{y} a(x_0,\eta)\phi_{n-j}(\eta)d\eta = l_{n-j}(y), \tag{5}$$

where

$$l_{n-j}(y)=\varphi_{n-j}(y_0)-\int_{y_0}^{y}\sum_{i\le n-2-j} C_{n-2-j}^{i} D_x^i[e(x_0,\eta)-d_\eta(x_0,\eta)]\varphi_{n-(i+2+j)}(\eta)d\eta-$$

$$-\int_{y_0}^{y}\sum_{i\le n-2-j} C_{n-2-j}^{i} D_x^i[a(x_0,\eta)]\varphi_{n-i-j}(\eta)d\eta-$$

$$-\sum_{i\le n-2-j} C_{n-2-j}^{i} D_x^i[b(x_0,y\ )]\varphi_{n-(i+1+j)}(y\ )+$$

$$+\int_{y_0}^{y}\sum_{i\le n-2-j} C_{n-2-j}^{i} D_x^i[b_\eta(x_0,\eta)]\varphi_{n-(i+j+1)}(\eta)d\eta-$$

$$-\int_{y_0}^{y}\sum_{i\le n-2-j} C_{n-2-j}^{i} D_x^i[c(x_0,\eta)]\varphi_{n-(i+j+1)}(\eta)d\eta-$$

$$-\sum_{i\le n-2-j} C_{n-2-j}^{i} D_x^i[d(x_0,y)]\varphi_{n-(i+j+2)}(y)+\Omega_{n-j}(x_0,y_0),$$

the analysis of the formula (4) indicates that $\varphi(y)$ depends on $\varphi_1(y),\ \varphi_2(y),....\varphi_n(y)$, where $\varphi_{n-1},\ \varphi_{n-2},....\varphi_2$ can be written through the following with the help of (5). It is clear that in the end, we obtain the integral equation with respect the $\varphi(y)$. In order to simplify the understanding of its construction in this paper, we will calculate the coefficient under $\varphi(y)$.

Let us introduce the following notations:

$$\left.\begin{aligned}
&\sum_{i_1\le n-4} C_{n-3}^{i_1} D_x^{i_1}[b(x_0,y)] = S_1(n-4,b),\\
&\sum_{i_1+i_2\le n-5} C_{n-i_1-4}^{i_2} D_x^{i_2}[b(x_0,y)] = S_2(n-5,b),\\
&\sum_{i_1\le n-5} C_{n-3}^{i_1} D_x^{i_1}[d(x_0,y)] = V_1(n-5,d)
\end{aligned}\right\} \tag{6}$$

after that, we write $l_{n-1}(y)$ through $\varphi(y)$, leaving only those addends that do not contain integrals. Then we obtain:

$$l_{n-1}(y)=\Delta_{n-1}(y)-S_1(n-4,b)\varphi_{n-2-i_1}(y)-$$
$$-V_1(n-5,d)\varphi_{n-3-i_1}(y)-D_x^{n-3}(d)\varphi(y).$$

Taking into account the formula (5) we have:

$$l_{n-1}(y)=\Delta_{n-1}(y)-S_1(n-4,b)\{-\sum_{i_1+i_2\le n-4}C_{n-i_1-4}^{i_2}D_x^{i_2}(b)\varphi_{n-3-(i_1+i_2)}(y)-$$
$$-\sum_{i_1+i_2\le n-4}C_{n-i_1-4}^{i_2}D_x^{i_2}(d)\varphi_{n-4-(i_1+i_2)}(y)\}-$$
$$-V_1(n-5,d)\{-\sum_{i_1+i_2\le n-5}C_{n-i_1-5}^{i_2}D_x^{i_2}(b)\varphi_{n-4-(i_1+i_2)}(y)-$$
$$-\sum_{i_1+i_2\le n-5}C_{n-i_1-5}^{i_2}D_x^{i_2}(d)\varphi_{n-5-(i_1+i_2)}(y)\}-D_x^{n-3}(d)\varphi(y)=$$
$$=\Delta_{n-1}(y)-S_1(n-4,b)[-S_2(n-5,b)]\varphi_{n-3-(i_1+i_2)}(y)-$$
$$-S_1(n-4,b)[-V_2(n-6,d)]\varphi_{n-4-(i_1+i_2)}(y)-$$
$$-V_1(n-5,d)[-S_2(n-6,b)]\varphi_{n-4-(i_1+i_2)}(y)-$$
$$-V_1(n-5,d)[-V_2(n-7,d)]\varphi_{n-5-(i_1+i_2)}(y)+$$
$$+S_1(n-4,b)D_x^{n-4-i_1}(d)\varphi(y)+$$
$$+V_1(n-5,d)D_x^{n-5-i_1}(d)\varphi(y)-D_x^{n-3}(d)\varphi(y).$$

It is clear that the substitution (5) should be continued until $n-j\ge 1$. As a result we obtain:

$$l_{n-1}(y)=\Delta_{n-1}(y)-S_1(n-4,b)[-S_2(n-5,b)]\bullet$$
$$\bullet\{-\sum_{i_1+i_2+i_3\le n-5}C_{n-(i_1+i_3)-5}^{i_3}D_x^{i_3}(b)\varphi_{n-4-(i_1+i_2+i_3)}(y)-$$
$$-\sum_{i_1+i_2+i_3\le n-5}C_{n-(i_1+i_3)-5}^{i_3}D_x^{i_3}(b)\varphi_{n-5-(i_1+i_2+i_3)}(y)\}-$$
$$-S_1(n-4,b)[-V_2(n-6,d)]\bullet$$
$$\bullet\{-\sum_{i_1+i_2+i_3\le n-6}C_{n-(i_1+i_3)-6}^{i_3}D_x^{i_3}(b)\varphi_{n-5-(i_1+i_2+i_3)}(y)-$$
$$-\sum_{i_1+i_2+i_3\le n-6}C_{n-(i_1+i_3)-6}^{i_3}D_x^{i_3}(b)\varphi_{n-6-(i_1+i_2+i_3)}(y)\}-$$
$$-V_1(n-5,d)[-S_2(n-6,b)]\bullet$$
$$\bullet\{-\sum_{i_1+i_2\le n-6}C_{n-i_1-6-i_2}^{i_2}D_x^{i_3}(b)\varphi_{n-5-(i_1+i_2+i_3)}(y)-$$

$$-\sum_{i_1+i_2+i_3\le n-6} C^{i_3}_{n-(i_1+i_3)-6} D^{i_3}_x(d)\varphi_{n-6-(i_1+i_2+i_3)}(y)\}-$$
$$-V_1(n-5,d)[-V_2(n-6,d)]\bullet$$
$$\bullet\{-\sum_{i_1+i_2+i_3\le n-7} C^{i_3}_{n-(i_1+i_3)-7} D^{i_3}_x(b)\varphi_{n-6-(i_1+i_2+i_3)}(y)-$$
$$-\sum_{i_1+i_2+i_3\le n-6} C^{i_3}_{n-(i_1+i_3)-6} D^{i_3}_x(d)\varphi_{n-6-(i_1+i_2+i_3)}(y)\}-$$
$$-V_1(n-5,d)[-V_2(n-7,d)]\bullet$$
$$\bullet\{-\sum_{i_1+i_2+i_3\le n-7} C^{i_3}_{n-(i_1+i_3)-7} D^{i_3}_x(b)\varphi_{n-6-(i_1+i_2+i_3)}(y)-$$
$$-\sum_{i_1+i_2+i_3\le n-7} C^{i_3}_{n-(i_1+i_3)-7} D^{i_3}_x(d)\varphi_{n-7-(i_1+i_2+i_3)}(y)\}+$$
$$+S_1(n-4,b)D^{n-4-i_1}_x(d)\varphi\ (y)+$$
$$+V_1(n-5,d)D^{n-5-i_1}_x(d)\varphi(y)-D^{n-3}_x(d)\varphi\ (y).$$

Let us note that the index in the formulas of types (6) S_1, S_2, V_1 and others means that in the sum of the left part (6), the index with this number (for example, i_1, i_2) is used. Besides, $\Delta_{n-1}(y)$ is changed at each step of this process and is simply introduced to facilitate the writing of this formula.

Using the abbreviations (6), we write the last formula as:

$$l_{n-1}(y)=\Delta_{n-1}(y)+(-1)^3 S_1(n-4,b)S_2(n-5,b)\bullet$$
$$\bullet S_3(n-6,b)\varphi_{n-4-(i_1+i_2+i_3)}(y)+(-1)^3 S_1(n-4,b)S_2(n-5,b)\bullet$$
$$\bullet V_3(n-7,d)\varphi_{n-5-(i_1+i_2+i_3)}(y)+$$
$$+(-1)^3 S_1(n-4,b)V_2(n-6,d)S_3(n-7,b)\varphi_{n-5-(i_1+i_2+i_3)}(y)+$$
$$+(-1)^3 S_1(n-4,b)V_2(n-6,d)V_3(n-8,d)\varphi_{n-6-(i_1+i_2+i_3)}(y)+$$
$$+[\ (-1)^3 S_1(n-4,b)S_2(n-5,b)D^{n-5-(i_1+i_3)}_x(d)+$$
$$+(-1)^3 V_1(n-5,d)S_2(n-6,b)S_3(n-7,b)\varphi_{n-5-(i_1+i_2+i_3)}(y)+$$
$$+(-1)^3 V_{1_1 1}(n-5,d)S_2(n-6,b)V_3(n-8,d)\varphi_{n-6-(i_1+i_2+i_3)}(y)+$$
$$+(-1)^3 V_1(n-5,d)V_2(n-7,d)S_3(n-8,b)\varphi_{n-6-(i_1+i_2+i_3)}(y)+$$
$$+(-1)^3 V_1(n-5,d)V_2(n-7,d)V_3(n-9,d)\varphi_{n-7-(i_1+i_2+i_3)}(y)+$$

$$+(-1)^3 S_1(n-4,b)V_2(n-6,d)D_x^{n-6-(i_1+i_3)}(d)+$$
$$+(-1)^3 S_1(n-4,b)V_2(n-6,d)D_x^{n-6-(i_1+i_3)}(d)+$$
$$+(-1)^3 V_1(n-5,d)S_2(n-6,b)D_x^{n-6-(i_1+i_3)}(d)+$$
$$+(-1)^3 V_1(n-5,d)V_2(n-7,d)D_x^{n-7-(i_1+i_3)}(d)+$$
$$+S_1(n-4,b)D_x^{n-4-i_1}(d)+V_1(n-5,d)D_x^{n-5-i_1}(d).\phi(y).$$

Then, if we denote

$$d = a_{01}\,,\ b = a_{11}\,,\ S_0 = V\,,\ S_1 = S,$$

the coefficient under $\varphi(y)$ can be written in the following form:

$$\sum_{k=0} \prod_{p=1}^{k} (-1)^{k+1} S_{\alpha_p}(n-3-2p+ \\ +\sum_{p=1}^{k}\alpha_p\,, a_{\alpha_p,1}) D_x^{n-3-2p+\sum_{p=1}^{k}\alpha_p-\sum_{p=1}^{k} i_p}(d), \tag{7}$$

where α_p can take the values of either 0 or 1, and $\prod_{p=1}^{k}$ is a product of corresponding terms. If k = 0, then S = 1. For $\varphi_{n-j}(y)$, where j > 1 we obtain a formula similar to (7):

$$l_{n-j}(y) = \Delta_{n-j}(y) - S_1(n-3-j,b)\varphi_{n-1-i_1-j}(y) -$$
$$-V_1(n-4-j,d)\varphi_{n-2-i_1-j}(y) - D_x^{n-2-j}(d)\varphi(y) =$$
$$= \Delta_{n-j}(y) - S_1(n-3-j,b)\bullet$$
$$\bullet\{-\sum_{i_1+i_2\le n-3-j} C_{n-i_1-3-j}^{i_2} D_x^{i_2}(b)\varphi_{n-2-j-(i_1+i_2)}(y) -$$
$$-\sum_{i_1+i_2\le n-2-j} C_{n-i_1-3-j}^{i_2} D_x^{i_2}(d)\varphi_{n-3-j-(i_1+i_2)}(y)\} -$$
$$-V_1(n-4-j,d)\{-\sum_{i_1+i_2\le n-4-j} C_{n-i_1-4-j}^{i_2} D_x^{i_2}(b)\varphi_{n-3-j-(i_1+i_2)}(y) -$$
$$-\sum_{i_1+i_2\le n-4-j} C_{n-i_1-4-j}^{i_2} D_x^{i_2}(d)\varphi_{n-4-j-(i_1+i_2)}(y)\} - D_x^{n-2-j}(d)\varphi(y) =$$
$$= \Delta_{n-j}(y) + S_1(n-3-j,b)S_2(n-3-j,b)\varphi_{n-2-j-(i_1+i_2)}(y) +$$
$$= S_1(n-3-j,b)V_2(n-5-j,d)]\varphi_{n-3-j-(i_1+i_2)}(y) +$$
$$+V_1(n-3-j,d)S_2(n-5-j,b)]\varphi_{n-3-j-(i_1+i_2)}(y) +$$
$$+V_1(n-4-j,d)V_2(n-6-j,d)]\varphi_{n-4-j-(i_1+i_2)}(y) +$$
$$+S_1(n-3-j,b)D_x^{n-3-j-i_1}(d)\varphi(y) +$$
$$+V_1(n-4-j,d)D_x^{n-4-j-i_1}(d)\varphi(y) - D_x^{n-2-j}(d)\varphi(y).$$

Then, we obtain:

$$\sum_{k=0,\dots}\prod_{p=1}^{k}(-1)^{k+1}S_{\alpha_p}(n-2-j-2p+ \\ +\sum_{p=1}^{k}\alpha_p, a_{\alpha_p,1})D_x^{n-2-j-2p+\sum_{p=1}^{k}\alpha_p-\sum_{p=1}^{k}i_p}(d), \tag{8}$$

it is clear that when $j = 1$, we obtain the formula (7). Thus, (8) is true for $j \geq 1$. Substituting the obtained results in (4), we extract the coefficient under $\varphi(y)$

$$D_x^{n-2}(d(x_0,y))\varphi(y)+\sum_{i\leq n-3}C_{n-2}^{i}D_x^{i}(b(x_0,y))\varphi_{n-i-1}(y)+ \\ +\sum_{i\leq n-4}C_{n-2}^{i}D_x^{i}(d(x_0,y))\varphi_{n-i-2}(y)= \\ =\Delta_n(y)+D_x^{n-2}(d(x_0,y))\varphi(y)+ \\ +\sum_{i\leq n-3}C_{n-2}^{i}D_x^{i}(b(x_0,y))[\sum_{k=0,\dots}\prod_{p=1}^{k}(-1)^{k+1}S_{\alpha_p}(n-3-i-2p+ \\ +\sum_{p=1}^{k}\alpha_p, a_{\alpha_p,1})D_x^{n-3-i-2p+\sum_{p=1}^{k}\alpha_p-\sum_{p=1}^{k}i_p}(d)]\varphi(y)+ \\ +\sum_{i\leq n-4}C_{n-2}^{i}D_x^{i}(d(x_0,y))[\sum_{k=0,\dots}\prod_{p=1}^{k}(-1)^{k+1}S_{\alpha_p}(n-4-i-2p+ \\ +\sum_{p=1}^{k}\alpha_p, a_{\alpha_p,1})D_x^{n-4-i-2p+\sum_{p=1}^{k}\alpha_p-\sum_{p=1}^{k}i_p}(d)]\varphi(y).$$

This formula can also be written as:

$$\sum_{k=0}\prod_{p=1}^{k}(-1)^{k+1}S_{\alpha_p}(n-2-2p+ \\ +\sum_{p=1}^{k}\alpha_p, a_{\alpha_p,1})D_x^{n-2-2p+\sum_{p=1}^{k}\alpha_p-\sum_{p=1}^{k}i_p}(d(x_0,y))=\Delta_n(y). \tag{9}$$

Based on the analysis (9) we determine

Theorem: If the coefficients of the equation (1) belong to the class of the unknown (desired) solutions and, besides that, the coefficient under $\varphi(y)$ in the left part of (9) is different from zero, while $b, d \in C^{(n-2)+1}(D \cup P)$, then the problem (1) is reduced to the Goursat problem. The arbitrary constants are $\varphi_1(y_0), \varphi_2(y_0), \dots, \varphi_n(y_0)$.

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