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An approximate solution of a model for HIV infection of CD4$^+$ T cells

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Abstract

In this paper, the approximate solution of the differential system modeling HIV infection of CD4$^+$ T cells is obtained by a reliable algorithm based on an adaptation of the standard variational iteration method (VIM), which is called the multi-stage variational iteration method(MSVIM). A comparison between MSVIM and the fourth-order Runge-Kutta method (RK4-method) reveal that the proposed technique is a promising tool to solve the considered problem.

Keywords: HIV infection; variational iteration method; numerical solution

1. Introduction

As can be seen from the literature about infectious diseases, mathematical models have played important roles in modelling of infectious diseases and facilitated the understanding of them as well. One of these is a model in [1], which is given by the following system of differential equations:

$$\begin{aligned} \frac{dT}{dt} &= q - \alpha T + rT\left(1 - \frac{T+I}{T_{\max}}\right) - kVT, \\ \frac{dI}{dt} &= kVT - \beta I, \\ \frac{dV}{dt} &= N\beta I - \gamma V, \end{aligned} \tag{1}$$

subject to the following initial conditions

$$T(0) = r_1,\ I(0) = r_2,\ V(0) = r_3, \tag{2}$$

where, $T(t), I(t)$ and $V(t)$ denote the concentration of CD4$^+$ T cells, the concentration of infected CD4$^+$ T cells by the HIV viruses, and free HIV virus particles respectively. Throughout this paper, we set

$$\begin{aligned} &q = 0.1,\ \alpha = 0.02,\ \beta = 0.3,\ r = 3,\ \gamma = 2.4, \\ &k = 0.0027,\ N = 10,\ T_{\max} = 1500, r_1 = 0.1, \\ &r_2 = 0,\ r_3 = 0.1. \end{aligned} \tag{3}$$

The aim of this paper is to extend the application of the analytic variational iteration method [2-5] to solve a model for HIV infection of CD4$^+$ T cells given in (1).

The paper is organized as follows: A brief review of VIM and MSVIM are given in Section 2 and 3, respectively. The application of the proposed numerical scheme to model (1)-(2) is illustrated in Section 4. The conclusions are then given in the final Section 5.

2. Variational iteration method

According to the variational iteration method [2], we consider the following differential equation:

$$Lu + N(u) = g(t), \tag{4}$$

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where L is a linear operator, N is a non-linear operator, and $g(t)$ is an inhomogeneous term. Then, we can construct a correct functional as follows:

$$u_{i,n+1}(t) = u_{i,n}(t) + \int_{t_0}^{t} \lambda\{Lu_{i,n}(s) + N\tilde{u}_{i,n}(s) - g(s)\}ds, \quad (5)$$

where λ is a general Lagrangian multiplier [2] that can be identified optimally via variational theory. The second term on the right is called the correction and $\tilde{u}_{i,n}$ is considered as a restricted variation, i.e., $\delta\tilde{u}_{i,n} = 0.$

3. Multistage variational iteration method

For large *t*, VIM is not a good result to approximate the solution of some differential equations. In order to guarantee the validity of the approximation solution for large *t*, a new approach called the MSVIM, mentioned in [6-10], can be considered. According to this approach, the solution from $[t_0, t)$ can be reproduced by subdividing this interval into $[t_0, t), [t_1, t_2), ..., [t_{j-1}, t_j = t)$, and a recursive formula of (6) applied on each subinterval [11-14]:

$$u_{i,n+1}(t) = u_{i,n}(t) + \int_{t^*}^{t} \lambda\{Lu_{i,n}(s) + N\tilde{u}_{i,n}(s) - g(s)\}ds, \quad (6)$$

Notice that this strategy gives a new construction of the correction functional (6) with a variable t^* as the lower limit of the integration instead of a fixed lower limit of t_0 in (5). The fixed limits are a norm used in the classical VIM which can be seen in [15-18]. The initial approximation in each interval is taken from the solution in the previous interval,

$$u_{i,0}(t) = u_i(t^*) = c_i^*, \quad (7)$$

where t_i^* is the left-end point of each subinterval and c_i^* is denoted as the initial approximations for $i = 1,2,...,m.$ By knowing the first initial conditions, one would be able to solve (6) for all unknowns $u_{i,n}(t)$, $(i = 1,2,...,m; n = 0,1,...$. In order to carry out the iteration in every subinterval of equal length $\Delta(t)$, $[t_0, t), [t_1, t_2), ..., [t_{j-1}, t_j = t)$, we need to know the values of the following:

$$u^*_{i,0}(t) = u_i(t^*) = c_i^*, \quad i = 1,2,...,m. \quad (8)$$

This information is typically not directly attainable, but through the initial value $t^* = t_0$, we were able to derive all the initial approximations. This is done by taking the previous initial approximation from the nth-iterate of the preceding subinterval given by (5), i.e.

$$u^*_{i,0}(t) \cong u_{i,n}(t^*), \quad i = 1,2,...,m \text{ and } t^* \in (t_0, t_1). \quad (9)$$

4. Application

In this section, the variational iteration method is applied to nonlinear ordinary differential systems (1). According to the variational iteration method, a correct functional is derived as follows:

$$T_{n+1}(t) = T_n(t) + \int_{t^*}^{t} \lambda_1 \left\{ T'_n(\xi) - q + \alpha\tilde{T}_n - r\tilde{T}_n\left(1 - \frac{\tilde{T}_n + \tilde{I}_n}{T_{\max}}\right) + k\tilde{T}_n\tilde{V}_n \right\} d\xi, \quad (10)$$

$$I_{n+1}(t) = I_n(t) + \int_{t^*}^{t} \lambda_2 \left\{ I'_n(\xi) - k\tilde{T}_n\tilde{V}_n + \beta\tilde{I}_n \right\} d\xi,$$

$$V_{n+1}(t) = V_n(t) + \int_{t^*}^{t} \lambda_3 \left\{ V'_n(\xi) - N\beta\tilde{I}_n + \gamma\tilde{V}_n \right\} d\xi,$$

where λ_1, λ_2 and λ_3 are general Lagrange multipliers, $\tilde{T}_n(\xi)$, $\tilde{I}_n(\xi)$ and $\tilde{V}_n(\xi)$ denote restricted variations, i.e. $\delta\tilde{T}_n(\xi) = \delta\tilde{I}_n(\xi) = \delta\tilde{V}_n(\xi) = 0.$ Making the above correction functional stationary, following stationary conditions can be obtained:

$$\begin{aligned} &\lambda_1'(\xi) = 0, \\ &1 + \lambda_1(\xi)\big|_{\xi=t} = 0, \\ &\lambda_2'(\xi) = 0, \\ &1 + \lambda_2(\xi)\big|_{\xi=t} = 0, \\ &\lambda_3'(\xi) = 0, \\ &1 + \lambda_3(\xi)\big|_{\xi=t} = 0. \end{aligned} \quad (11)$$

The Lagrange multipliers, therefore, can be identified as

$$\lambda_1 = \lambda_2 = \lambda_3 = -1. \quad (12)$$

Substituting Eq. (12) into the correction functional Eq. (10) results in the following iteration formula:

$$T_{n+1}(t) = T_n(t) - \int_{t^*}^{t} \left\{ T'_n(\xi) - q + \alpha\tilde{T}_n - r\tilde{T}_n\left(1 - \frac{\tilde{T}_n + \tilde{I}_n}{T_{\max}}\right) + k\tilde{T}_n\tilde{V}_n \right\} d\xi, \quad (13)$$

$$I_{n+1}(t) = I_n(t) - \int_{t^*}^{t} \left\{ I'_n(\xi) - k\tilde{T}_n\tilde{V}_n + \beta\tilde{I}_n \right\} d\xi,$$

$$V_{n+1}(t) = V_n(t) - \int_{t^*}^{t} \left\{ V'_n(\xi) - N\beta\tilde{I}_n + \gamma\tilde{V}_n \right\} d\xi.$$

These results, obtained by MSVIM with time-step $h = 0.1$ and the fourth-order Runge Kutta method with time-step $h = 0.01$ for $T(t), I(t)$ and $V(t)$ are presented as follow. In Fig. 1, the local changes of $T(t), I(t)$ and $V(t)$ variables are given. Figure 1 presents the comparison between the MSVIM solution and the RK4 solution. Good agreement for the MSVIM solution with RK4 solution can be seen.

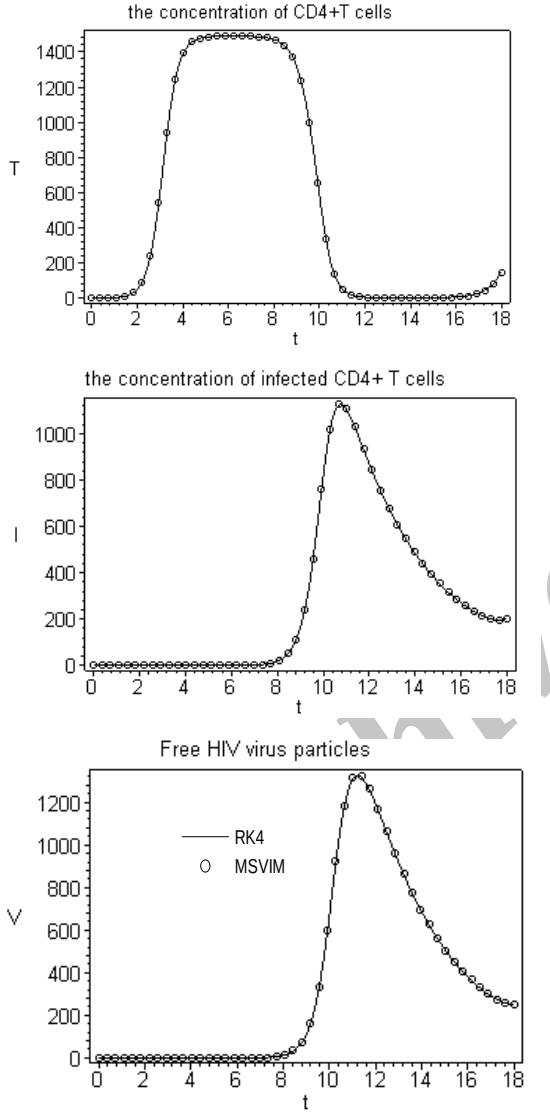

Fig. 1. Graphical comparisons between MSVIM with time-step $h = 0.1$ and RK4 method time-step $h = 0.01$ for $T(t), I(t)$ and $V(t)$ vs *t*, respectively

5. Conclusions

In this paper, multistage variational iteration method was used to find the solutions of nonlinear ordinary differential equation systems such as a model for HIV infection of $CD4^+$ T cells. We demonstrated the accuracy and efficiency of this method by solving for the considered ordinary differential equation system. The computations associated with the examples in this paper were performed using Maple7.

References

[1] Liancheng, W. & Michael, Y. L. (2006). Mathematical analysis of the global dynamics of a model for HIV infection of $CD4^+$ T cells. *Mathematical Biosciences*, 200 , 44-57, 2006.

[2] He, J. H. (1998). Approximate analytical solution for seepage flow with fractional derivatives in porous media. *Computer Methods in Applied Mechanics and Engineering*, 167, 57-68.

[3] He, J. H. (1998). Approximate solution of nonlinear differential equations with convolution product nonlinearities. *Computer Methods in Applied Mechanics and Engineering*, 67, 69-73.

[4] He, J. H. (1999). Variational iteration method-a kind of nonlinear analytical technique: some examples. *International Journal of Nonlinear Mechanics*, 34, 699-708.

[5] He, J. H. (2006). Some asymptotic methods for strongly nonlinear equations. *International Journal of Modern Physics B*, 20, 1141-1199.

[6] Batiha, B., Noorani, M. S. M., Hashim, I. & Ismail, E. S. (2007). The multistage variational iteration method for a class of nonlinear system of ODEs. *Phys. Scripta*, 76, 1-5.

[7] Goh, S. M., Noorani, M. S. M. & Hashim, I. (2009). Efficacy of variational iteration method for chaotic Genesio system-Classical and multistage approach. *Chaos, Solitons & Fractals*, 40, 2152-2159.

[8] Goh, S. M., Ismail, A. I. M., Noorani, M. S. M. & Hashim, I. (2009). Dynamics of the Hantavirus infection through variational iteration method. *Nonlinear Anal.*, 10, 2171-2176.

[9] Goh, S. M., Noorani, M. S. M., Hashim, I. & Al-Sawalha, M. M. (2009). Variational iteration method as a reliable treatment for the hyperchaotic Rössler system. *Int. J. Nonlinear Sci. Numer. Simul.*, 10, 363-371.

[10] Goh, S. M., Noorani, M. S. M. & Hashim, I. (2008). Prescribing a multistage analytical method to a prey-predator dynamical system. *Physics Letters A*, 373, 107-110.

[11] Al-Sawalha, M. M., Noorani, M. S. M. & Hashim, I. (2009). On accuracy of Adomian decomposition method for hyperchaotic Rössler system. *Chaos, Solitons & Fractals*, 40, 1801-1807.

[12] Hashim, I., Noorani, M. S. M., Ahmad, R., Bakar, S. A., Ismail, E. S. I. & Zakaria, A. M. (2006). Accuracy of the Adomian decomposition method applied to the Lorenz system. *Chaos, Solitons & Fractals*, 28, 1149-1158.

[13] Noorani, M. S. M., Hashim, I., Ahmad, R., Bakar, S. A., Ismail, E. S. I. & Zakaria, A. M. (2007). Comparing numerical methods for the solutions of the Chen system. *Chaos, Solitons & Fractals*, 32, 1296-1304.

[14] Al-Sawalha, M. M. & Noorani, M. S. M. (2009). Application of the differential transformation method for the solution of the hyperchaotic Rössler system. *Comm. Non. Sci. Num. Simul.*, 14, 1509-1514.

[15] He, J. H. (2007). Variational iteration method-some recent results and new interpretations. *J. Comp. Appl. Math.*, 207, 3-17.

[16] He, J. H. & Wu X. H. (2007). Variational iteration method: new development and applications. *Int. J. Comp. Math. Appl*., 54,881-894.

[17] He, J. H. (2008). An elementary introduction to recently developed asymptotic methods and nanomechanics in textile engineering. *Int. J. Mod. Phys. B*, 22, 3487-578.

[18] He, J. H. (2007). Some asymptotic methods for strongly nonlinear equations. *Int. J. Mod. Appl. Phys. B*, 20, 1141-99.