A New Approach for Transmission Expansion Planning in Competitive Electricity Markets

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Abstract: Expansion planning strategies in power systems have been seriously influenced by disintegration of these systems during the past decade. Transmission Expansion Planning (TEP) is one of the most important parts of expansion planning of power systems. In this paper, a new algorithm for TEP in these environments is presented. The method is based on probabilistic Locational Marginal Price (LMP) considering electrical losses, transmission tariffs and transmission congestion costs. Incorporating modeling of reactive power and uncertainty of generation and load are other important features of the proposed algorithm.

Keywords:Transmission expansion planning, Electricity markets, Locational marginal price.

1. INTRODUCTION

Wold-wide focus on competition and open access transmission networks has resulted in restructuring of power systems. Different goals such as providing resources, increasing efficiency and customer choice have been considered in numerous countries and therefore, various strategies have been adopted [1, 2]. As a result, power industry and its related aspects have experienced dominant changes.

However, transmission network, as the interface between generation and load sections, has preserved its monopolistic characteristics in most power systems. It plays a vital role in order to facilitate competition in the generation and electricity retailing. While reliability and stability of a system as the most important features should be considered in TEP, the plan should also provide fair and no discriminatory access to the system for all consumers.

TEP can be considered as a computational tool which provides us with one or more quasi-optimal transmission plans. It consists of defining the time and the place where new lines should be installed to serve the growing electric energy market in an optimal way, considering a set of electrical, economic, financial, social and environmental constraints. General structure of TEP problem consists of input data, objective function, constraints and planning criteria.

Deregulation of electric industry has resulted in essential changes in power system planning. Some of the most important differences between TEP in conventional vertically integrated and in new power systems are as follows:

1) In vertically integrated systems, TEP is considered only as a part of general expansion planning for the whole integrated systems, while in competitive environments it is normally an isolated expansion planning for the transmission systems [3-6].

2) Comparing deregulated power systems with vertically integrated ones, there are a lot of uncertainties in input data of the new systems.

Therefore, TEP in these systems should be robust against those uncertainties [7, 8].

3) In the new environments, transmission service pricing has more impact on TEP [9].

4) In spite of conventional systems, disproportionate transmission expansion in new power systems may not only violate competition, but also may increase the investment risk.

Different models for solving TEP have been proposed. Generally, planning models are classified into three different categories including mathematical, heuristic and meta-heuristic optimization models [7].

Each of these approaches may be modelled statically or dynamically. In dynamic approaches, the long study period (20 or 30 years) of system planning is disaggregated into shorter time periods. Then different states of the system are investigated in any of these periods. Dynamic methods answer to different questions including, where, when and with which specifications a new line should be constructed. On the other hand, static approaches do not specify the time that a line should be constructed.

As the TEP problem is stated as a large-scale, non-linear and non-convex optimization problem, heuristic or meta-heuristic optimization algorithms should be applied. This is due to the fact that such algorithms can reach to better solutions as compared with those obtained through classical techniques [10].

Based on the above mentioned description, various methods such as multi-objective planning [11], fuzzy algorithm [12], cooperative game theory [13], multi-agent coalition formation [14], non-linear mixed integer programming [15], Genetic Algorithm (GA) [10,16], and LMP [8, 18, 19] have been proposed so far for TEP in competitive environments. It should be noted that due to uncertainties, most publications emphasize on probabilistic approaches for TEP in competitive environments [16, 17]. However, methods based on LMP are of more important. LMP is the price of supplying an additional MW of load at each bus in the system. The major factors affecting the LMP values are generator bidding prices, the transmission system components experiencing congestion, losses and the electrical characteristics of the system.

Various TEP models have been proposed based on LMP. Buygi and others have proposed an algorithm based on LMP, which considers operation and congestion costs [8]. The method uses a probabilistic tool for analyzing different transmission plans. Applying dc load flow, line operational and congestion costs are minimized. Maximizing the differences between energy transfer benefits and transmission investment based on LMP is emphasized by Gill and his colleagues [18]. Run has used a function consisting of energy prices to find the minimal non-congested plan, after specifying probable congested areas [19].

Hong and Hsiao have proposed application of neural networks and fuzzy systems for forecasting LMPs [20, 21].

In all proposed approaches, major simplifications have been applied for modelling of the TEP problem. Some of these simplifications are as follows:

Not simultaneous consideration of loss and congestion costs, ignoring transmission tariffs, not modelling of complete uncertainties and ignoring reactive power.

In this paper a new algorithm for TEP based on probabilistic LMP in competitive electricity markets is presented.

The proposed method considers loss, congestion, transmission tariffs and uncertainties in generators and loads using AC load flow. The algorithm is based on a nonlinear, meta-heuristic optimization model.

Application of the proposed approach on an 8-bus test system confirms its advantages.

2. Fundamentals of the Proposed Method

TEP in competitive electricity markets must encourage the competition and provide fair access to cheap generation which is useful for all customers. On the other hand, to facilitate a perfect and fair competition, electricity suppliers and consumers should have no constraint for bidding and offering the energy.

Therefore, to facilitate a fair competition, a good approach for TEP is to expand the network in a way which flats the LMP profile as much as possible.

In the proposed method, first, LMPs in all network buses are calculated. Due to many of uncertainties in competitive electricity markets, a probabilistic method should be used for calculation of LMPs. To do this, a probabilistic approach has been used to calculate pdfs for LMPs. The approach is based on a recursive application of an Optimal Power Flow (OPF) algorithm which utilizes the probability density functions (pdfs) of the input variables instead of their deterministic values (Fig. 1). As a result, the pdfs of output variables are calculated instead of their accurate values. Then using the pdfs and optimization techniques, the LMPs in all buses of the system are calculated in their probabilistic forms as pdfs.

After calculation of LMPs in all network buses, they are divided into source and sink sets based on their LMP mean values. Those buses at which, mean of LMPs are smaller than the total mean value of LMPs

(LMPave) are grouped in the source set and the rest are considered as the set of sink buses.

To reduce the number of lines nominated for expansion planning; only few buses among each of the above mentioned sets are selected to be connected through new lines. The criteria for choosing the nominated buses are as follows:

- Buses in the source set:

$$LMP_{ave} - LMP_k > \alpha\sigma_k \qquad (1)$$

- Buses in the sink set:

$$LMP_k - LMP_{ave} > \alpha\sigma_k \qquad (2)$$

Finally, the option plan for expansion is characterized by new lines that should be constructed between any of the nominated buses from sink and source sets for flatting the LMP profile.

To specify the flatness of a price profile, some indices are defined. In a network with n buses, the pdf of LMPs have been computed for a given pdf for each input. Consider MLMP is a $1 \times n$ vector such that its kth element is the mean of LMP at bus k, and VLMP is a $1 \times n$ vector such that its kth element is the variance of LMP at bus k. The following parameters can be defined for determining the flatness of price profile:

- Mean of MLMP or LMP_{ave}: The less mean of MLMP indicates that cheaper generators are dispatched. This means a better condition for competition. It should be noted that transmission planning may result in dispatching of all cheap marginal generators and therefore more expensive generators become marginal. In this case LMPs at all buses and consequently mean of MLMP may increase. Therefore, a bigger value for mean of MLMP does not necessarily indicate a bad condition for competition.
- Variance of MLMP: the smaller variance of MLMP indicates the flatter price profile and consequently better competition.
- Variance of VLMP: the smaller variance of VLMP indicates the more similar volatility of LMP at different buses and consequently the more similar risk in purchasing the power from different buses.

3. TEP Problem Formulation

To formulate the TEP problem in a competitive environment, important features should be simulated and considered in their mathematical forms. The most important parameters are as follows:

3.1. Uncertainties

Generation bid prices and load quantities are the major input uncertainties which have been modelled as normal pdfs.

3.2. Probabilistic LMP

As explained before, probabilistic LMPs are calculated in their probabilistic forms using OPF. To do this, utilizing the pdfs of the input variables instead of their deterministic values, the pdfs of output variables are calculated instead of their accurate values. This is achieved based on a proposed recursive solution of an OPF algorithm in which, the objective function is minimized subject to equality and inequality constraints considering load, voltage and generation limits. The objective function of costs includes generation bids and transmission costs based on transmission tariffs.

The proposed model is as follows:

$$Min \quad LMP_k = \sum_{i=1}^{G} C_i(P_{gi} + \Delta P_{gi}) - \sum_{i=1}^{G} C_i(P_{gi}) + \sum_{\ell=1}^{L} T_\ell \Delta P_{flow\,\ell} \qquad (3)$$

S.T:

$$\sum_{i=1}^{G} P_{gi} - \sum_{\ell=1}^{L} P_{loss\,\ell} - \sum_{k=1}^{N} P_{dk=0} \qquad (4)$$

$$P_{gi} - P_{di} - P_i(v,\delta) = 0 \qquad i \in N \qquad (5)$$

$$Q_{gi} - Q_{di} - Q_i(v,\delta) = 0 \qquad i \in C \qquad (6)$$

$$P_{gi}^{m} \le P_{gi} \le P_{gi}^{M} \qquad (7)$$

$$Q_{gi}^{m} \le Q_{gi} \le Q_{gi}^{M} \qquad (8)$$

$$\left| P_{flow\,\ell} \right| \le P_{flow\,\ell}^{M} \qquad (9)$$

$$v_k^{m} \le v_k \le v_k^{M} \qquad (10)$$

$$\delta_k^{m} \le \delta_k \le \delta_k^{M} \qquad (11)$$

It should be noted that Eqs. (4) to (11) must be satisfied if the active load at bus k is increased ($\Delta P_{dk} = 1MW$).

Fig. 1 shows the general structure of the proposed OPF for calculating the pdfs of LMPs based on the random values for the probabilistic input data.

Transmission Tariffs

Capital investment for new lines is the most important parameter for TEP. Here, we have used transmission tariffs for investment modelling. Transmission tariff is calculated according to the Levelized Transmitted Energy Cost (LTEC) as follows [22, 23]:

$$AC_{\ell} = (1+g)^{c_{\ell}}(PVC_{\ell}\ FCR_{\ell} + PVL_{\ell}\ FCRL_{\ell} + PVO_{\ell} CRF_{\ell}) \tag{12}$$

$$TE_{\ell} = 8760\ P^{M}_{flow\,\ell} \tag{13}$$

$$LTEC_{\ell} = \frac{AC_{\ell}}{TE_{\ell}} \tag{14}$$

$$T_{\ell} = b * LTEC_{\ell} \tag{15}$$

At first, the annual cost of line ℓ is calculated using Eq. (12) based on economic parameters. Then, $LTEC_{\ell}$ is calculated dividing the annual cost by total energy transmitted for line ℓ. Finally, the transmission tariff for line ℓ is obtained using Eq. (15).

3.3. Transmission Line Limits

Modelling of transmission line limits in OPF is very important. Transmitted power flow in each line is calculated as follows:

$$S_{qr} = V_q I^{*}_{qr} \tag{16}$$

$$P_{flow\,\ell} = Max(\mathrm{Real}(S_{qr}), \mathrm{Real}(S_{rq})) \tag{17}$$

However, the calculated values should be adjusted to be within their specified limits. The method for adjusting power flows in overloaded lines is based on linear programming utilizing generation shift factors [24] which is described as follows:

$$Min \quad \sum_{i=1}^{G}(K\Delta P^{+}_{gi} + K\Delta P^{-}_{gi}) \tag{18}$$

S.T:

$$\sum_{i=1}^{G} a_{\ell i}(\Delta P^{+}_{gi} - \Delta P^{-}_{gi}) \le P^{M}_{flow\,\ell} - P_{flow\,\ell} \tag{19}$$

$$\sum_{i=1}^{G} a_{\ell i}(\Delta P^{+}_{gi} - \Delta P^{-}_{gi}) \ge -P^{M}_{flow\,\ell} - P_{flow\,\ell} \tag{20}$$

$$0 \le \Delta P^{+}_{gi} \le P^{M}_{gi} - P_{gi} \tag{21}$$

$$0 \le \Delta P^{-}_{gi} \le P_{gi} - P^{m}_{gi} \tag{22}$$

$$\theta = [X]P \tag{23}$$

$$a_{\ell i} = \frac{dP_{flow\,\ell}}{dP_{gi}} = \frac{1}{x_{\ell}}(X(q,i) - X(r,i)) \tag{24}$$

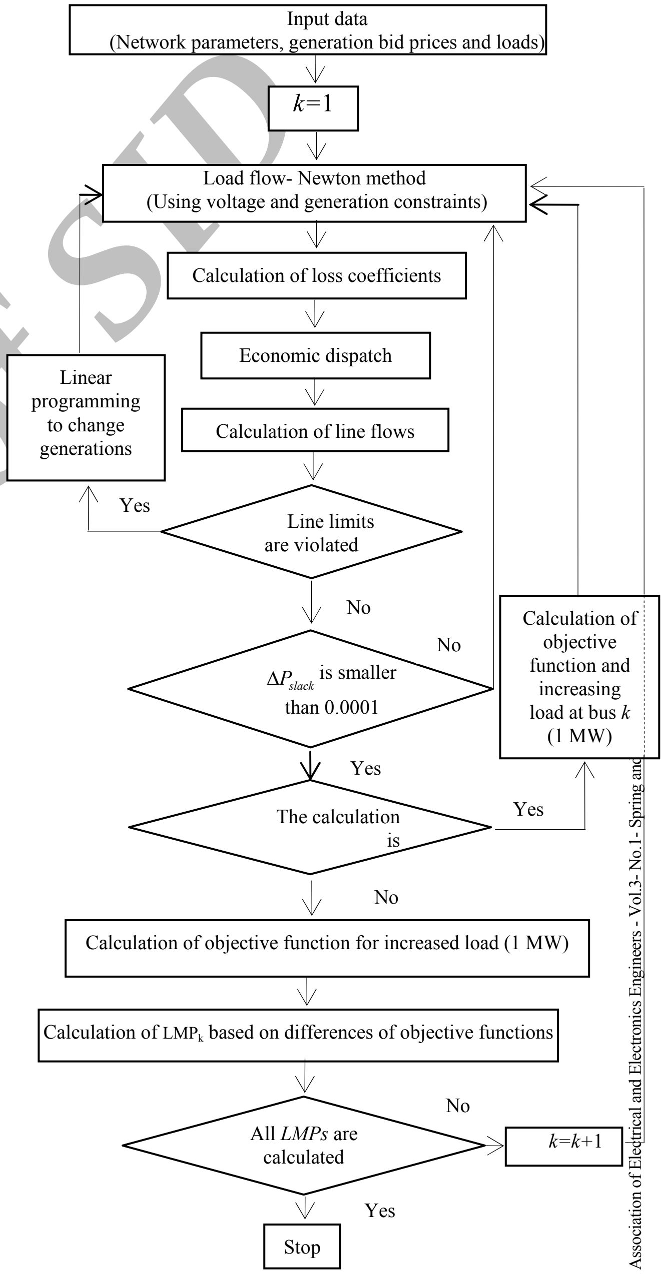


Fig.1: General structure of the recursive OPF for calculating pdfs of LMPs

3.4. Power Loss Modeling

A major step in optimal dispatch of generation is to express the system loss in terms of generator's real power outputs. In our problem formulation, the well known loss coefficient method, developed by Kron and adopted by Kirchmayer has been applied for loss calculations [25].

4. The Proposed Algorithm

The proposed algorithm for TEP in competitive electricity markets is described in Fig. 2. It should be mentioned that simulating normal random variables and selecting the magnitudes for input pdfs are performed applying the method described in Appendix A.

5. Case Study

The proposed algorithm has been applied to a typical 8-bus power system introduced in Ref. [8]. The network structure and its related parameters are presented in Appendix B. However, R and X parameters of the lines have been modified to be more realistic.

Using 500 random generated samples from the pdfs of loads and generation bid prices, MLMP and VLMP vectors for the base case are found as follows:
MLMP= [18.905 24.531 18.629 22.082 17.233 18.354 23.149 25.326] [$/MWh]

VLMP= [10.100 10.782 12.607 8.9144 12.505 9.727 13.426 14.815] [$/MWh]
Also, the mean value of MLMP for all buses of the network (LMPave) are calculated as 21.0261 [$/MWh].

Comparing MLMP values at any bus in the network with its average value sink and source buses and therefore, candidate lines are specified.

Set of source buses: {1, 3, 5, and 6}.
- Set of sink buses: {2, 4, 7, and 8}.
- Transmission line candidates: {1-2,1-4,1-7,1-8,3-2, 3-4,3-7,3-8,5-2,5-4,5-7,5-8,6-2,6-4,6-7,6-8}.

Finally, based on the proposed algorithm, the optimal candidate lines for expansion are specified as: {1-8, 3-4, 5-4 and 6-8} (Table I).

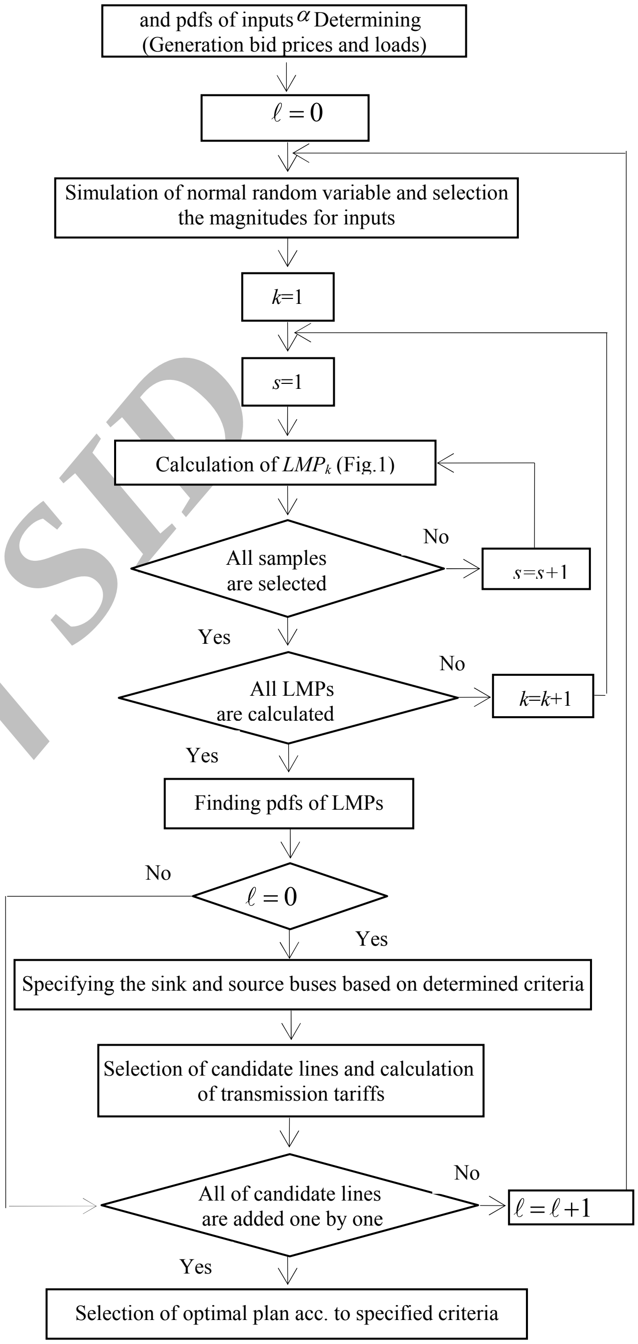


Fig.2: Flowchart of the proposed algorithm for TEP

It should be mentioned that as much as α is smaller, the candidate set of expansion buses will be bigger (Table II).
To investigate the validity of the proposed algorithm, all possible candidates are selected in another approach without applying any screening procedure (Table I). The results show that final optimal plans obtained by non-screened procedure [plans (1-8), (3-

4), (5-4) and (6-8)] are the same as those obtained using screened approach applying α=0.25. Furthermore, the run time is decreased about 60% in the latter one. The results confirm that none of the optimal candidates are among those filtered candidates. The major conclusions of the proposed method are as follows:

1) Comparing the results with those of Ref. [8] (which are presented in Table III), it can be observed that considering transmission tariffs and realistic line resistance will result in more monotonous values for LMPs at different buses. Furthermore, simplifying the models chosen for optimization procedure will result in finding sub-optimal or even non-optimal solutions. As it is observed the results obtained applying our proposed algorithm to the sample network of Ref. [8] differs from the results obtained in Ref. [8]. This is due to the fact that our proposed algorithm considers resistance of transmission lines, reactive powers of generators and loads, and transmission tariffs which are neglected in Ref. [8]. However, neglecting the above mentioned parameters in our approach results in obtaining the same plans as reported in Ref. [8].

2) Choosing an appropriate value for α and numbers of samples for input data, the calculation time can be reduced drastically, while it may not affect the solutions seriously.

3) Partial changes in some parameters such as the profit factor may lead to essential changes of the results. Based on calculations, it can be estimated as below:

$$\frac{d(LMP_{ave})}{d(b)} \approx 5.0 \qquad (25)$$

This shows that LMP_{ave} should be very sensitive to variation of this parameter. However, to compare and investigate on the credibility of our results, we have used b= 1.2 which is a commonly used value for profit factor in other references.

4) In cases where the network includes different levels of voltage, the electrical calculations are performed in the p.u. base. However, based on the voltage level at ending buses for the candidate lines, the voltage level for each candidate can be determined. Therefore, it is possible to consider the cost of transmission line at different levels using a correction factor in the objective function.

7. Conclusion

Expansion planning for networks in competitive environments relies seriously on parameters which are not deterministic. In such power systems, there may be many uncertainties about load and generation in the network. Therefore, probabilistic and heuristic methods instead of classic approaches may be applied to get better solutions.

Table 1: Final Results

Line	Mean of MLMP ($/MWh)	Variance of MLMP ($/MWh)	Variance of VLMP ($/MWh)
1-2	20.9538	3.9800	49.5549
1-4	20.9925	3.9086	4.8503
1-7	21.0588	4.9366	6.6343
1-8	20.9125	3.8316	2.0108
3-2	21.1863	3.8372	7.6887
3-4	20.6038	3.9237	4.4382
3-7	21.5088	4.4816	42.9999
3-8	20.7663	4.8428	10.6796
5-2	20.3250	2.7620	3.6992
5-4	20.2713	2.8295	2.0665
5-7	20.8450	3.9317	1.5732
5-8	20.4113	2.2360	0.8331
6-2	20.9863	3.1817	3.4456
6-4	21.3688	4.2670	4.1954
6-7	21.0525	4.1499	2.2607
6-8	20.5338	2.7085	0.9166
Other feasible candidate lines			
1-3	21.8143	5.2022	1.0007
1-5	21.9547	5.0124	8.0076
1-6	21.0207	4.5565	13.8966
2-4	21.6741	5.4219	0.4531
2-7	21.6109	5.3909	0.2233
2-8	21.6573	5.1677	0.3987
3-5	21.4084	4.8034	1.2227
3-6	21.6183	5.0499	2.1217
4-7	21.0039	3.2660	44.5071
4-8	21.5231	6.5671	13.9812
5-6	21.3273	4.1896	0.4850
7-8	21.9871	6.9801	10.2214

Table 2: Different Values of α and Optimal Options for TEP

α	All feasible candidates	0.0	0.25	0.50	0.75	1.0
Sink buses		1, 3, 5, 6	1, 3, 5, 6	1, 3, 5, 6	5, 6	5
Source buses		2, 4, 7, 8	2, 4, 7, 8	2, 7, 8	2, 8	2
Candidate lines	28	16	16	12	4	1
Optimal options for each sink bus	1-8, 3-4, 5-4, 6-8	1-8, 3-4, 5-4, 6-8	1-8, 3-4, 5-4, 6-8	1-8, 3-8, 5-2, 6-8	5-2, 6-8	5-2
Run time(Sec.)	198,120	125,116	125,116	16,577	2,250	1,080

In this paper a new algorithm for TEP has been proposed. While the method uses pdfs of the parameters instead of their accurate values, the priority criteria for the expansion plan is based on LMPs at different buses. This is due to the fact that as much as LMP values are uniform in a network, the competition is more fair and non-discriminatory.

The investigation confirms that while the method benefits acceptably small calculation time, the results are reliable and optimal.

Table 3: Results of Ref. [8]

<table>
<tr><th colspan="4">Base case results</th></tr>
<tr><td colspan="4">MLMP=[19.23 30.19 30.06 29.79 20.56 19.97 29.86 29.97]</td></tr>
<tr><td colspan="4">VLMP=[4.47 5.82 2.30 3.35 5.04 4.56 2.03 1.49]</td></tr>
<tr><td colspan="4">Source buses: 1, 5, 6 Sink buses: 2, 3, 4, 7, 8</td></tr>
<tr><td colspan="4">Candidate lines: {1-2, 1-3, 1-4, 1-7, 1-8, 5-2, 5-3, 5-4, 5-7, 5-8, 6-2, 6-3, 6-4, 6-7, 6-8}</td></tr>
<tr><th colspan="4">Final optimal plans</th></tr>
<tr><th>Line</th><th>Mean of MLMP ($/MWh)</th><th>Variance of MLMP ($/MWh)</th><th>Variance of VLMP ($/MWh)</th></tr>
<tr><td>1-3</td><td>24.639</td><td>16.396</td><td>1.912</td></tr>
<tr><td>5-3</td><td>24.290</td><td>15.764</td><td>1.234</td></tr>
<tr><td>6-3</td><td>24.285</td><td>22.009</td><td>42.146</td></tr>
</table>

Appendices

A. Simulation of normal random variable

For simulation of normal random variable following algorithm is used [26]:

A1: Random variables Y_1, Y_2 with mean 1.0 are generated.

$$Y_1 = e^{-t} \qquad 0 < t < \infty \tag{27}$$

$$Y_2 = e^{-t} \qquad 0 < t < \infty \tag{28}$$

A2: If

$$Y_2 - \frac{(Y_1 - 1)^2}{2} > 0 \tag{29}$$

Then

$$Y = Y_2 - \frac{(Y_1 - 1)^2}{2} \tag{30}$$

Go to stage (A3), otherwise back to stage (A1).

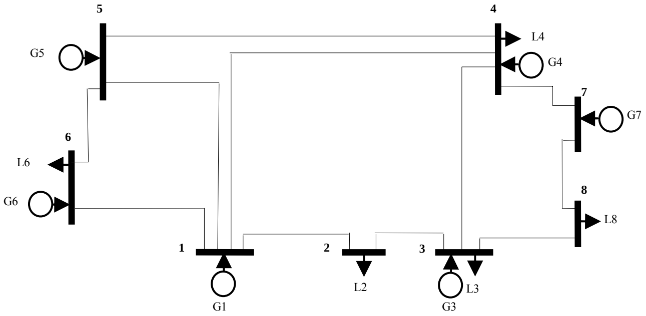
A3: Random number U is generated and

$$\begin{aligned} Z &= Y && \text{if } U \le 1/2 \\ Z &= -Y && \text{if } U > 1/2 \end{aligned} \tag{31}$$

Finally a normal random variable is generated in the form $\mu + \sigma Z$.

B. Network information

The information of case study network is as follows:



ig.3: Case study network

Table 4: Loads Data ($\cos\phi$ =0.95)

Load No.	Bus No.	Load (MW)(μ , σ^2)
1	2	(300 , 10)
2	3	(300 , 12)
3	4	(300 , 15)
4	6	(300 , 5)
5	8	(250 , 9)

Table 5: Generators Data

Gen. No.	Bus No.	P_{max} (MW)	Q_{max} (MVAr)	Bid ($/MWh)
1	1	100	50	(15 , 1.8)
2	3	520	300	(30 , 1.5)
3	4	250	150	(30 , 2)
4	5	600	400	(10 , 3)
5	6	400	200	(20 , 2.1)
6	7	200	150	(20 , 1.5)

Table 6: Lines Data (R, X in P.U. on 100 MVA Base)

Line	R(p.u.)	X(p.u.)	Limit (MW)	Tariff ($/MWh)
1-2	0.01675	0.06750	400	1.236
1-4	0.01122	0.09520	190	1.014
1-5	0.01340	0.05400	390	1.014
2-3	0.02364	0.09864	130	1.825
3-4	0.01770	0.12000	230	1.289
4-5	0.01340	0.11260	330	1.198
5-6	0.00680	0.05134	350	0.960
6-1	0.02400	0.15280	250	1.582
7-4	0.03480	0.22156	250	2.293
7-8	0.00800	0.06040	340	1.163
8-3	0.03240	0.20628	240	2.224

Table 7: Economic Parameters

Description	Qty. for 1 sample line 230 kV
Construction cost of line	50,000 $/Km
Price of land	25,000 $/Km in width of right of way
Operation cost of line	1,000 $/Km per year
Inflation rate	0.15
Duration for line construction	1 Year

Table 8: Candidate Lines Data

Line	R(p.u.)	X(p.u.)	Limit (MW)	Length (Km)	Tariff ($/MWh)
1-3	0.03757	0.25370	140	330	4.689
1-7	0.03600	0.22840	185	400	4.275
1-8	0.03015	0.25200	190	450	4.683
2-4	0.01273	0.10697	200	190	1.878
2-5	0.02345	0.09450	390	250	1.267
2-6	0.02100	0.19985	185	350	3.740
2-7	0.02111	0.08505	400	315	1.557
2-8	0.03540	0.22710	140	300	4.237
3-5	0.01822	0.15232	190	270	2.830
3-6	0.03240	0.21636	180	360	3.954
3-7	0.02950	0.18925	140	250	3.351
4-6	0.02700	0.17460	180	300	3.259
4-8	0.02310	0.19600	190	350	3.642
5-7	0.03600	0.22840	185	400	4.275
5-8	0.02700	0.25695	320	450	2.780
6-7	0.09850	0.40950	225	500	4.394
6-8	0.03600	0.23920	180	200	2.197

C. NOMENCLATURE

ℓ	Line number between buses q and r
G	Set of generators
L	Set of lines
N	Set of network buses
C	Set of PQ buses
M	Index for maximum limit
m	Index for minimum limit
c_ℓ	Duration for construction of line ℓ [year]
g	General inflation
b	Profit factor, equal to 1.2
x_ℓ	Impedance of line ℓ [p.u.]
s	Sample number
K, t	Positive numbers [K is app. 100]
α	A variable coefficient
T_ℓ	Transmission tariff of line ℓ [$/MWh]
θ	Vector of bus voltage angles
$[X]$	Impedance matrix
P	Generated power vector
$X(q,i)$	Member of $[X]$
LMP_k	LMP at kth bus [$/MWh]
$a_{\ell i}$	Generation shift factor of line ℓ related to ith generator
LMP_{ave}	Total mean value of LMPs [$/MWh]
σ_k	Standard deviation of pdf of LMP_k [$/MWh]
σ^2	Variance of a defined pdf [$/MWh]
$C_i(P_{gi})$	Cost of generated power of bus i [$/h]
ΔP_{gi}	Change in active generated power at bus i [MW]
$\Delta P_{gi}^+, \Delta P_{gi}^-$	Changes in active power values at bus i [MW]
$\Delta P_{flow\,\ell}$	Change in power flow of line ℓ [MW]
P_{gi}, Q_{gi}	Active and reactive generated powers at bus i [MW, MVAr]
$P_{loss\,\ell}$	Loss of line ℓ at base case [MW]
P_{dk}, Q_{dk}	Active and reactive loads at bus k [MW, MVAr]
$P_{flow\,\ell}$	Power flow of line ℓ [MW]
v_k	Voltage magnitude of bus k [p.u.]
δ_k	Voltage angle of bus k [Degree]
ΔP_{Slack}	Load flow accuracy [MW]
S_{qr}	Transmitted apparent power from bus q to bus r [MVA]
V_q	Voltage of bus q [p.u.]
I_{qr}	Current flowing from bus q to bus r [p.u.]
AC_ℓ	Annual cost of line ℓ [$]
PVC_ℓ	Present value of construction for line ℓ [$]
PVL_ℓ	Present value of land for line ℓ [$]
PVO_ℓ	Present value of operation for line ℓ [$]
FCR_ℓ	Fixed charge rate of line ℓ
$FCRL_\ell$	Fixed charge rate of land for line ℓ
CRF_ℓ	Capital return factor for line ℓ
TE_ℓ	Total energy transmitted by line ℓ [MWh]
$LTEC_\ell$	Levelized transmitted energy cost for line ℓ [$/MWh]
Y_1, Y_2, Y	Random variables
U	Random number
Z	Normal random variable

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