Int.J.Nano Dim. 2(4): 227-232, Spring 2012
ISSN: 2008-8868

Contents list available at **IJND**

International Journal of Nano Dimension

Journal homepage: *www.IJND.ir*

Buckling analysis of graphene nanosheets based on nonlocal elasticity theory

A. Tourki Samaei[1,2*]
Sh. Hosseini Hashemi[2]

[1]Young Researchers Club, Chalous Branch, Islamic Azad University, Chalous, Iran.
[2]School of Mechanical Engineering, Iran University of Science and Technology, Tehran 16844, Iran.

Received: 18 May 2011
Accepted: 25 July 2011

ABSTRACT

This paper proposed analytical solutions for the buckling analysis of rectangular single-layered graphene sheets under in-plane loading on all edges simply is supported. The characteristic equations of the graphene sheets are derived and the analysis formula is based on the nonlocal Mindlin plate. This theory is considering both the small length scale effects and transverse shear deformation effect. Nonlocal elasticity theory takes into account the small length scale effects as examining nanostructures such as nanoplates. It is presented graphically that the small scale or nonlocal effects on the nondimensional buckling loads in the presence of aspect ratio and buckling modes.

Keywords: *Buckling; Graphene nanosheet; Nonlocal elasticity*.

INTRODUCTION

Since the discovery of carbon nanostructures like graphene sheets, fullerenes and carbon nanotubes, numerous researches have been conducted on their superior mechanical and electronic properties [1,2]. In this paper, an analysis of the mechanical properties for a two-dimensional (2D) allotrope of the carbon family comprising of self-aligned graphene sheets is studied. Graphene sheets can be applied in micro nanoscale technologies like micro/nano-electromechanical systems (MEMS and NEMS) and atomic dust detectors as a platform for nano-electronic devices and biological sensors [3,4]. A few investigations concerning the classical elasticity theory for interpretation of the mechanical characteristics of graphene sheets or nanoplates have been reported in the literature [5-7]. The classical elasticity theory does not enable to compare of fundamental size-dependence in the elastic solutions of micro/nanostructures [6].

* Corresponding author:
Arash Tourki Samaei
Young Researchers Club, Chalous Branch, Islamic Azad University, Chalous, Iran.
Tel +98 9356315945
Fax +98 2177240488
Email *faramarz.samaei@gmail.com*

Hence, the nonlocal elasticity theory, which is the modified version of the local elasticity theory through considering the small scale effect of the entire micro/nanostructure of materials, provides us with a current theory to cope with small scale influence [9]. These size-effects are associated with atoms and molecules that form the materials. The effects of long-range interatomic and intermolecular cohesive forces on the static, buckling and dynamic properties cannot be neglected as the length scales are reduced. A nonlocal plate model was first reported to examine the small length scale effect on the nanoplates and displayed that the nonlocal elasticity theory would play a significant role in the study of micro/nanoscale structures [8,10]. Duan and Wang [11] studied the axisymmetric bending of micro- and nanoscale circular plates using a nonlocal plate theory. Their results showed that the deflections, shear forces and moments became larger than a local plate model. Recently, Murmu and Pradhan [12] investigated buckling behavior of orthotropic small length scale plates under biaxial compression based on nonlocal Kirchhoff plate theory. In this paper, the major objective is to extend a nonlocal plate model for the single layer graphene sheet under in-plane loading and applied it to find the small length scale influences on nondimensional loads. Explicit relations for buckling loads are obtained. It has been shown that nonlocal effect is quite significant in buckling analysis studies and needs to be included in the continuum model of single-layered graphene sheets.

EXPERIMENTAL

Nonlocal Mindlin plate model

Nonlocal elasticity theory states that the stress at a reference point χ in an elastic body relies not only on strain at aforementioned point but also on the strains at all other points in the same body [12]. The nonlocal constitutive relations can be simplified as follows

$$(1-\tau^2 l^2 \nabla^2)\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl} \qquad (1)$$

Where σ_{ij} and C_{ij} are the stress tensor of the nonlocal elasticity and the classical stress tensor, which is related to the linear strain tensor ε_{kl}, respectively. $\tau = e_0 a/l$ is nonlocal parameter which is material constant that depends on a and l the internal (such as the carbon-carbon bond length) and external (graphene nano-sheet length) characteristics length of the system, respectively. e_0 is nonlocal elasticity constant appropriate to each material [12]. In this approach, a single layer graphene sheet is modeled as a thick rectangular plate with thickness h, length l and width b. A coordinate system (x, y, z) is used for the graphene sheet (Figure 1) with the x, y and z axes along the length, width, and thickness of plate respectively. On the basis of the Mindlin plate theory, the displacement components can be expressed as

$$u_x = u(x,y,z) + z\psi_x \qquad \textbf{(2.a)}$$

$$u_y = u(x,y,z) + z\psi_y \qquad \textbf{(2.b)}$$

$$u_z = w(x,y,z) \qquad \textbf{(2.c)}$$

Here,ψ_x, ψ_y, ψ_z and t are the rotational displacement about the y axis, rotational displacement about the x axis, transverse displacement, and the time variable, respectively. Using nonlocal constitutive relations, the stress constitutive relations can be written as

$$(1-(e_0 a)^2\nabla^2)\sigma_{xx} = \frac{E}{(1-\upsilon^2)}\left(\varepsilon_{xx} + \upsilon\varepsilon_{yy}\right) \qquad (3.a)$$

$$(1-(e_0 a)^2\nabla^2)\sigma_{yy} = \frac{E}{(1-\upsilon^2)}\left(\varepsilon_{yy} + \upsilon\varepsilon_{xx}\right) \qquad (3.b)$$

$$(1-(e_0 a)^2\nabla^2)\sigma_{xy} = 2G\varepsilon_{xy} \qquad (3.c)$$

$$(1-(e_0 a)^2\nabla^2)\sigma_{xz} = 2G\varepsilon_{xz} \qquad (3.d)$$

$$(1-(e_0 a)^2\nabla^2)\sigma_{yz} = 2G\varepsilon_{yz} \qquad (3.e)$$

Where,E, G and υ are the elastic moduli, shear modulus and poisson's ratio, respectively. The stress resultant-displacement relations can be derived by

$$M_{ij} = \int_{-h/2}^{h/2} \sigma_{ij}\, z dz\,, \quad i,j = x,y \qquad (4.a)$$

$$Q_f = k^2 \int_{-h/2}^{h/2} \sigma_{ij}\, dz, \quad f = x,y \qquad (4.b)$$

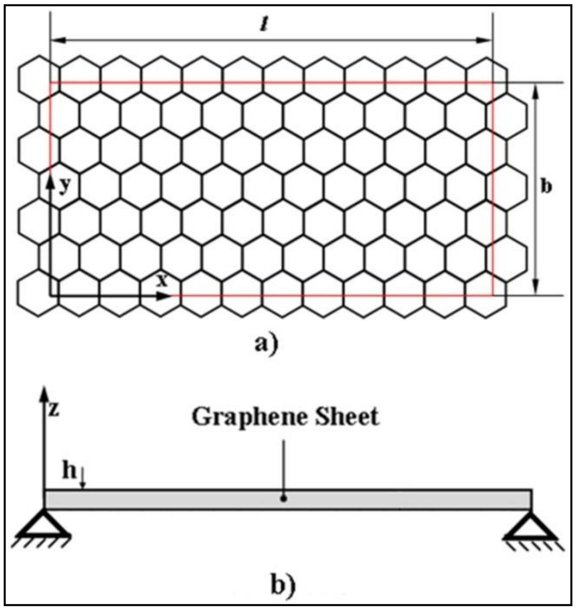

Fig. 1. A continuum plate model of a single-layered graphene nanosheet.

Here M_{xx} and M_{yy} are the resultant moments per unit length, M_{xy} is the twisting moment per unit length, and Q_x and Q_y are the transverse shear forces per unit length and also here k^2 is the transverse shear correction coefficient. For this study, the biaxial compression loads are $N_{xx} = N$,$N_{yy} = \mu N$, $N_{xy} = 0$ and $\mu = N_{xx}/N_{yy}$, Where N and μ are the in-plane loading per unit length and the compression ratio, respectively. The governing differential equations of motion of the nonlocal Mindlin plate theory can be taken by

$$\nabla^2\psi_x + \frac{1+\upsilon}{1-\upsilon}\left(\frac{\partial^2\psi_x}{\partial x} + \frac{\partial^2\psi_y}{\partial x\partial y}\right) - \frac{2k^2Gh}{D(1-\upsilon)}\psi_x = \frac{2k^2Gh}{D(1-\upsilon)}\frac{\partial w}{\partial x}$$

(5.a)

$$\nabla^2\psi_y + \frac{1+\upsilon}{1-\upsilon}\left(\frac{\partial^2\psi_y}{\partial y} + \frac{\partial^2\psi_x}{\partial x\partial y}\right) - \frac{2k^2Gh}{D(1-\upsilon)}\psi_y = \frac{2k^2Gh}{D(1-\upsilon)}\frac{\partial w}{\partial y}$$

(5.b)

$$\left(\frac{\partial\psi_x}{\partial x} + \frac{\partial\psi_y}{\partial y}\right) = -\nabla^2 w - \frac{N_{xx}}{k^2Gh}\frac{\partial^2 w}{\partial x^2} - \frac{N_{yy}}{k^2Gh}\frac{\partial^2 w}{\partial y^2} + \frac{(e_0a)^2}{k^2Gh}\nabla^2\left(N_{xx}\frac{\partial^2 w}{\partial x^2} + N_{yy}\frac{\partial^2 w}{\partial y^2}\right)$$

(5.c)

In which $D = Eh^3/12(1-\upsilon^2)$ and $G = E/2(1+\upsilon)$ are the flexural rigidity and the shear modulus, respectively. The governing differential equations (5) reduce to that of the classical Mindlin plate model when the nonlocal parameter (e_0a) is set to zero [13].

Solution problem

The simply supported boundary conditions for the rectangular plate are

$$w = 0, \psi_y = 0 \text{ and } M_{xx} = 0 \text{ at } x = 0, l \quad (6.a)$$

$$w = 0, \psi_x = 0 \text{ and } M_{yy} = 0 \text{ at } y = 0, b \quad (6.b)$$

For the present problem, we assume the solution of governing equations (5) with satisfaction of equations (6) as

$$\psi_x = ACos(\alpha x)Sin(\beta y) \quad (7.a)$$

$$\psi_y = BSin(\alpha x)Cos(\beta y) \quad (7.b)$$

$$w = WSin(\alpha x)Sin(\beta y) \quad (7.c)$$

Here $\alpha = m\pi/l$ and $\beta = n\pi/b$. m and n are the half wave numbers. By substituting equations (7) into equations (5) we have

$$[A]_{3\times3}[u]_{3\times1} = \bar{N}[u]_{3\times1} \quad (8)$$

By solving the characteristic equation, which obtains from setting the determinant of the coefficient matrix $[A]_{3\times3}$ equal to zero, the buckling loads in equation (8) are obtained as $\bar{N} = -Na^2/D$.

RESULTS AND DISCUSSION

To illustrate the effect of small length scale on the buckling load of single-layered graphene sheet, buckling behavior is analyzed for different lengths, nonlocal parameter, buckling modes and aspect ratios.

Considering the graphene sheet with Poisson's ratio, $\upsilon = 0.25$, the transverse shear correction coefficient, $k^2 = 0.86667$, Young's modulus, $E = 1765\ GPa$, compression ratio, $\mu =$

4, and the thickness, $h = 34\,nm$. The length L of the graphene is assumed as between 5 to $50nm$. For the choice of nonlocal parameter, we take, $e_0a = 0.0, 0.5, 1.0, 1.5$ and $2.0nm$.

The buckling analysis of moderately thick rectangular plates with all the four edges simply supported under uniformly distributed in-plane loads N represents for various geometries using nonlocal Mindlin plate theory. Figure 2 depicts the nondimensional buckling load, $\bar{N}$ versus the variation of length for a rectangular nanoplate at different nonlocal parameters(e_0a). The buckling modes numbers for this analysis is assumed as$(m = 1, n = 1)$.

From Figure 2, it is observed that nonlocal solution for buckling load is smaller than the classical solutions. This is attributed to the effect of small length scale. In addition, increasing the nonlocal parameter decreases the buckling load. This implies that increasing the nonlocal parameter leads to a decrease in stiffness of structure. In addition, as the length of the graphene sheets increases, the buckling load increases. This is due to with increase of length; the influence of nonlocal effect reduces. Furthermore, with further increase of length the curves become flat in nature. Approximately at $L \geq 50nm$ all results converge to the classical buckling load$(e_0a = 0)$. This implies that the nonlocal effect diminishes with increase of the graphene sheet length and vanishes after a certain length.

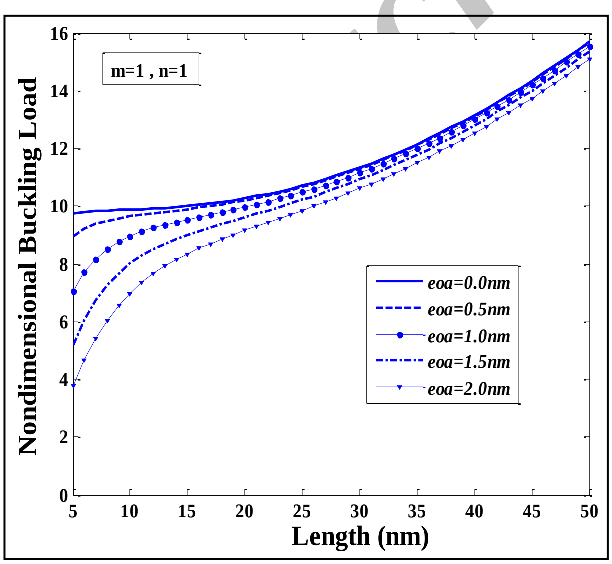

Fig. 2. Variety of nondimensional buckling load with length of graphene sheet for different nonlocal parameters.

To indicate the effect of higher buckling modes, nondimensional buckling load versus the variation of length at different nonlocal parameters is plotted in Figure 3 and 4. Figure 3 and 4 are shown for buckling modes, $(m = 2, n = 1)$ and$(m = 3, n = 1$, respectively. Similar buckling load analysis as that of $(m = 1, n = 1)$ is observed in these figures, i.e. nonlocal solutions are smaller than the corresponding classical solutions. However, nonlocal effects are highly prominent in buckling load of higher buckling modes.

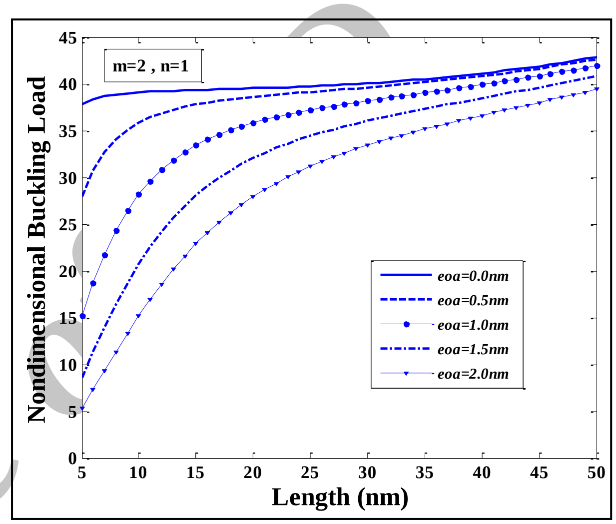

Fig. 3. Variety of nondimensional buckling load with length of graphene sheet for different nonlocal parameters.

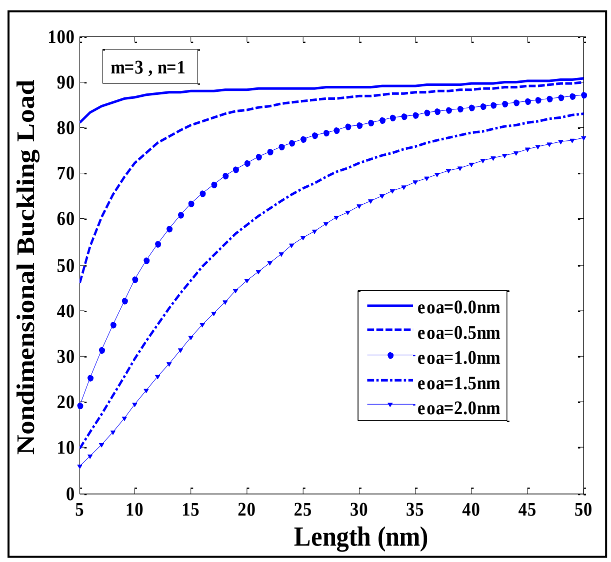

Fig. 4. Variety of nondimensional buckling load with length of graphene sheet for different nonlocal parameters.

Nondimensional buckling loads $\overline{N}$ versus various nonlocal parameters (e_0a) for the rectangular Mindlin micro/nanoscale plate have been plotted in Figure 5. It can be observed that the small scale effects on nondimensional buckling load for biaxially compressed micro/nanoplates at higher modes. The figure represents the small scale coefficient has extreme effects on the nondimensional buckling loads and that increases the value of nondimensional buckling load of the single-layered graphene sheet decreases for all modes.

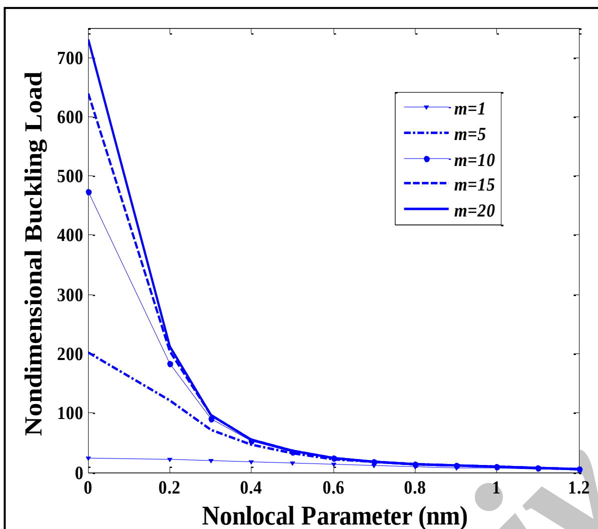

Fig. 5. Influence of small scale effects on the nondimensional buckling load of Mindlin nanoplate for different mode number.

To illustrate the effect of small scale on the buckling load of nanoplates with different aspect ratios l/b has been considered. Four aspect ratios of the graphene sheet are considered and nondimensional buckling load has been plotted against nonlocal parameter in Figure 6. It can be observed that for a give nonlocal parameter, nondimensional buckling load is closer to 10 for higher aspect ratio. Further nonlocal effect would be associated with decreasing in aspect ratios.

CONCLUSION

In this study, nonlocal Mindlin plate model for buckling analysis of single-layered graphene sheets is presented using nonlocal continuum mechanics. Explicit relations for nondimensional buckling loads are obtained through direct separation of variables. The present nonlocal Mindlin plate model addresses the size effects through nonlocality. Both the small scale effect and the transverse shear deformation influence are considered to examine buckling behavior of graphene nanosheet by nonlocal Mindlin plate theory. Nonlocal effect and transverse shear deformation are less important in lower buckling modes, and they are significantly important in higher buckling modes.

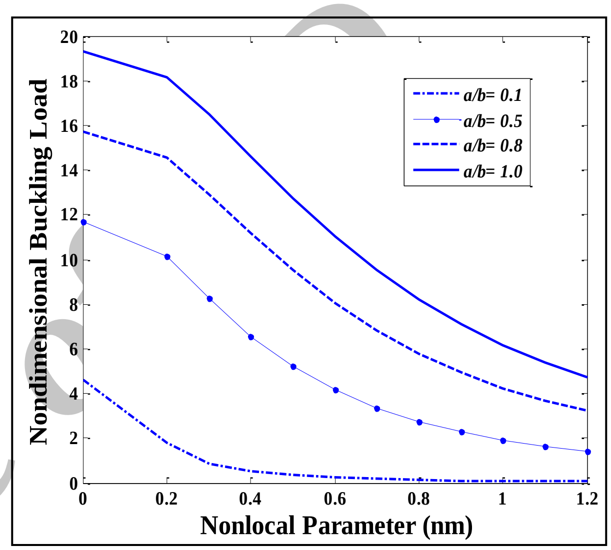

Fig. 6. Influence of small scale effects on the non-dimensional buckling load of Mindlin nanoplate for different aspect ratio.

REFERENCES

[1] Lourie O., Cox D.M., Wagner H.D., (1998), Buckling and collapse of embedded carbon nanotubes, *Phys. Rev. Lett.* 81:1638-1647.

[2] Ruoff R.S., Lorents D.C., (1995), Mechanical and thermal properties of carbon nanotubes, *Carbon*, 33(7):925-930.

[3] Lavrik N.V., Sepaniak M.J., Datskos P.G., (2004), Cantilever transducers as a platform for chemical and biological sensors, Review of Scienti c Instruments, 75:2229-2253.

[4] Dai H., Hafner J.H., Rinzler A.G., Colbert D.T., Smalley R. E., (1996), Nanotubes as nanoprobes in scanning probe microscopy, *Nature*, 384:147-156.

[5] Iijima S., (1991), Helical microtubules of graphitic carbon, *Nature*, 354:56-61.

[6] Tanaka K., Aoki H., Yamabe T., Okahara K., (1997), Interlayer interaction of two graphene sheets as a model of double-layer carbon nanotubes, *Carbon*, 35:121-129.

[7] Stankovich S., et al., (2006), Graphene-based composite materials, *Nature Lett.* 442:282 286.

[8] Sakhaee-Pour A., (2009), Elastic buckling of single-layered graphene sheet, Comput. *Mat. Sci*. 45:266 270.

[9] Eringen A.C., (1972), Linear theory of nonlocal elasticity and dispersion of plane waves, *Inter. J. Eng. Sci.* 10:425-434.

[10] Lu P., Zhang P.Q., Lee H.P., Wang C.M., Reddy J.N., (2007), Non-local elastic plate theories, Procs. *Roy. Soc. A.* 463:3225-3231.

[11] Duan W.H., Wang C.M., (2007), Exact solutions for axisymmetric bending of micro/nanoscale circular plates based on nonlocal plate theory, *Nanotechnology*, 18:385704-385711.

[12] Pradhan S.C., Murmu T., (2009), Small scale effect on the buckling of single-layered graphene sheets under biaxial compression via nonlocal continuum mechanics, Comp. *Mat. Scie.* 47:268-274.

[13] Akhavan H., Hashemi Sh.H., Damavandi H.R., Alibeigloo A., Vahabi Sh., (2009), Exact solutions for rectangular Mindlin plates under in-plane loads resting on Pasternak elastic foundation. Part I: Buckling analysis, *Com. Mat. Scie*. 44:968-974.