New approach in sensitivity analysis and identification of the region of efficiency for an efficient DMU

M. Ghadimi[a,1]**, M. Ahadzadeh Namin**[b]

[a]Department of Mathematics, Islamic Azad University-Karaj Branch, P.O. Box 31485-313, Karaj, Iran.

[b]Department of Mathematics, Science & Research Branch, Islamic Azad University, Tehran-Iran.

Abstract

One of the topics of interests in data envelopment analysis (DEA) is sensitivity analysis of specific decision making unit (DMU), which is under evaluation. Changes in inputs or outputs of any DMU can alter its classification, i.e. an efficient DMU become inefficient and vise versa. In this paper, we develop a new sensitivity analysis approach for obtaining the region of efficiency of an efficient DMU. In this region changes in inputs and outputs (inputs expansion and outputs contraction) of a DMU will not alter its efficiency status.

Keywords: Data Envelopment Analysis; Sensitivity analysis; Parametric programming.

1   Introduction

Data envelopment analysis(DEA), originally proposed by Charnes et al.(1978), has became one of the most widely used methods in management sciences. DEA is a non parametric technique for measuring and evaluating the relative efficiency of DMUs which stand of decision making units with several inputs and outputs. One of the topics of interests in this field is sensitivity analysis.

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[1]Corresponding Author. E-mail Address: m_a_ghadimi@yahoo.com

The first DEA sensitivity analysis paper by Charnes et al.(1985) examined change in single output. This was followed by Charnes and Nerallic (1990) in which sufficient condition preserving efficiency was determined. Another type of DEA sensitivity analysis is based on supper-efficiency DEA approach in which an under evaluation DMU is not included in the reference set. Charnes et al.(1992,1996) developed a super-efficiency DEA sensitivity analysis technique for the situation where simultaneous proportional changes is assumed in all inputs and outputs for a specific DMU under consideration. This data variation condition was relaxed in Zhu (1996) and Seiford and Zhu (1998) to situation where inputs or outputs can be changed individually and the largest stability region which encompasses that of Charnes et al.(1992) is obtained.

The DEA sensitivity analysis methods we have just reviewed are all developed for the situation where data variation are only applied to the under evaluation efficient DMU and the data for the remaining DMUs are assumed fixed. Obviously, this assumption may not be realistic, since possible data errors may occur in each DMU. Seiford and Zhu (1998) generalize the technique in Zhu (1998) and Seiford and Zhu (1998) to the case where the efficiency of the under evaluation efficient DMU is deteriorating while the efficiencies of the other DMUs are improving.

In this paper, based on Boljunčić's approach (2006), we propose a procedure to obtain the complete region of efficiency (RE_o), assuming simultaneous inputs expansion and outputs contraction of an efficient DMU_o. We start with the extended DEA model which results a point DMU_o^* on the boundary of RE_o, by maximal input/output changes. Next, by using the optimal simplex tableau, and applying parametric programming with input/output changes as parameters, we obtain all possible input/output changes. This results with the complete RE_o, represented with hyperplanes which serve as boundary to input/output changes. Also, we can use coefficients of this hyperplanes as dual multipliers, thus making connection to the sensitivity approach as in Thompson et al.(1994).

The plan for the rest of this paper is as follows. Section 2 formalized the formal

expression of the concepts with which we deal. In section 3, we present a sensitivity analysis approach with a new point of view. In section 4 we describe the proposed approach by a simple example. Finally, section 5 draws our conclusive remarks.

2 Region of efficiency

We assume that there are n DMUs to be evaluated indexed by $j = 1, 2, ..., n$ and each DMU is assumed to produce s different outputs from m different inputs. Let the observed input and output vectors of DMU_j be $X_j = (x_{1j}, ..., x_{mj})$ and $Y_j = (y_{1j}, ..., y_{sj})$ respectively. The production possibility set of obviously most widely used DEA model, BCC, is defined as semi-positive vectors (X, Y) as follows:

$$PPS = \{(X, Y)^T | Y \leq \sum_{j=1}^{n} \lambda_j Y_j, \ X \geq \sum_{j=1}^{n} \lambda_j X_j, \ \sum_{j=1}^{n} \lambda_j = 1, \ \lambda_j \geq 0, j = 1, ..., n\} \quad (1)$$

Then $DMU_o = (X_o, Y_o)^T$ is efficient if and only if there is no other point $(X, Y)^T$ from PPS where $x_{io} \geq x_i, i = 1, ..., m$ and $y_{ro} \leq y_r, r = 1, ..., s$ with at least one strict inequality. As stated before, we are interested in sensitivity analysis of an efficient DMU_o and obtaining the region of efficiency for it. Since input contraction and output expansion of DMU_o will not decrease its efficiency, we consider only input/output changes of the form

$$x_{io}^* = x_{io}\beta_i, \quad \beta_i \geq 1 \quad i = 1, ..., m$$

$$y_{ro}^* = y_{ro}\alpha_r, \quad 0 < \alpha_r \leq 1 \quad r = 1, ..., s \quad (2)$$

$$DMU_o^* = (X_o^*, Y_o^*)^T$$

Region of efficiency for the efficient DMU_o, RE_o, is defined as the set of all possible values that DMU_o^* can obtain and still be efficient. As it seems from (2), we consider a new point of view for RE_o based on inputs expansion and outputs contraction of DMU_o. We have:

$RE_o = \{(X_o^*, Y_o^*)^T \mid X_o^* \geq X_o, \ Y_o^* \leq Y_o, (X_o^*, Y_o^*)^T$ is efficient compared to remaining n-1 DMUs$\}$

For dealing with RE$_o$ we introduce reduced production possibility set, $RPPS_o$, defined as PPS in (1), with the only difference that DMU_o is removed from reference set. In other words, $RPPS_o$ is a subset of PPS, and defined as follows (figures 1 and 2 represent $RPPS_2$, obtained from PPS by removing DMU_2):

$$RPPS_o = \{(X, Y)^T \mid Y \leq \sum_{j=1, j \neq o}^{n} \lambda_j Y_j, \ X \geq \sum_{j=1, j \neq o}^{n} \lambda_j X_j, \ \sum_{j=1, j \neq o}^{n} \lambda_j = 1, \ \lambda \geq 0\}$$

3 The procedure of sensitivity analysis.

We use an iterative procedure for sensitivity analysis of an extreme efficient DMU$_o$. We start with one of the facets of $RPPS_o$, which is also a boundary one for RE_o. We then proceed moving from one facet of $RPPS_o$ to the adjacent one, lead by possible input/output changes, until we obtained all the facets which serve as a boundary of RE_o. As a starting facet when applying this procedure, we can select any of the facets of $RPPS_o$ which contain any DMU_o^* as in (2). Good selection to start is to choose a facet of $RPPS_o$ which contains one of the projection points of DMU_o such as DMU_{22} which is a projection point along output, or DMU_{21} which is a projection point along input (see Figure 2). We can assume that this chosen input/output is the first input, x_1.

$$
\begin{aligned}
& Min \quad \beta_1 \\
& s.t. \quad -\sum_{j=1, j \neq o}^{n} \lambda_j x_{1j} \geq -\beta_1 x_{1o} \\
& \qquad -\sum_{j=1, j \neq o}^{n} \lambda_j x_{ij} \geq -x_{io} \qquad i = 2, ..., m \\
& \qquad \sum_{j=1, j \neq o}^{n} \lambda_j y_{rj} \geq y_{ro} \qquad\quad r = 1, ..., s \\
& \qquad \sum_{j=1, j \neq o}^{n} \lambda_j = 1 \\
& \qquad \beta_1 \geq 1 \qquad\qquad\qquad\qquad\qquad \lambda_j \geq 0 \quad j = 1, ..., n
\end{aligned}
\tag{3}
$$

Let $\hat{\beta}_1 = \beta_1 - 1 \geq 0$ then $\beta_1 = \hat{\beta}_1 + 1$, and we have

$$
\begin{aligned}
Min \quad & \hat{\beta}_1 + 1 \\
s.t. \quad & -\sum_{j=1, j \neq o}^{n} \lambda_j x_{1j} + \hat{\beta}_1 x_{1o} \geq -x_{1o} \\
& -\sum_{j=1, j \neq o}^{n} \lambda_j x_{ij} \geq -x_{io} \qquad i = 2, ..., m \\
& \sum_{j=1, j \neq o}^{n} \lambda_j y_{rj} \geq y_{ro} \qquad r = 1, ..., s \\
& \sum_{j=1, j \neq o}^{n} \lambda_j = 1 \\
& \lambda_j, \hat{\beta}_1 \geq 0 \qquad j = 1, ..., n
\end{aligned}
\tag{4}
$$

suppose that $(\lambda^*, \hat{\beta}_1^*)$ is optimal solution of LP model (4). Let $\beta_1^* = \hat{\beta}_1^* + 1$. Then

$$
DMU_0^* = (x_1 \beta_1^*, x_2, ..., x_m, y_1, ..., y_s) \in RE_o
\tag{5}
$$

The dual model of model (4) introduce the equation of an efficient facet contain DMU_o^*. In other words, for identifying the equation of an efficient facet containing DMU_o^*, we can use following model.

$$
\begin{aligned}
Max \quad & -\sum_{i=1}^{m} v_i x_{i0} + \sum_{r=1}^{s} \mu_r y_{ro} + w + 1 \\
s.t. \quad & -\sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} \mu_r y_{rj} + w \leq 0 \quad j = 1, ..., n \quad j \neq o \\
& v_1 x_{1o} \leq 1 \\
& v_i \geq 0 \qquad\qquad\qquad\qquad i = 1, ..., s \\
& \mu_r \geq 0 \qquad\qquad\qquad\qquad r = 1, ..., s
\end{aligned}
\tag{6}
$$

Where in optimality we have

$$
-\sum_{i=1}^{m} v_i^* x_{io} + \sum_{r=1}^{s} \mu_r^* y_{ro} + w^* = \beta_1^*
\tag{7}
$$

However, instead of solving model (6) we can obtain the negative values of optimal dual variables are in the reduced cost row under the slack variables in the optimal

simplex tableau of LP model (4). On the other hand from (7) we have $-\hat{\beta}_1^* - \sum\limits_{i=1}^{m} v_i^* x_{io} +$

$\sum\limits_{r=1}^{s} \mu_r^* y_{ro} + w^* = 0$ and since $v_1^* x_{1o} = 1$ (due to SCSC). Then $-v_1^* x_{1o} \hat{\beta}_1^* - \sum\limits_{i=1}^{m} v_i^* x_{io} +$

$\sum\limits_{r=1}^{s} \mu_r^* y_{ro} + w^* = 0$, and $-v_1^* x_{1o}(\hat{\beta}_1^* + 1) - \sum\limits_{i=2}^{m} v_i^* x_{io} + \sum\limits_{r=1}^{s} \mu_r^* y_{ro} + w^* = 0$. Therefore, we

have $-v_1^* x_{1o} \beta_1^* - \sum\limits_{i=2}^{m} v_i^* x_{io} + \sum\limits_{r=1}^{s} \mu_r^* y_{ro} + w^* = 0$

Hence, this is an equation of a supporting hyperplane at $DMU_0^* = (x_1 \beta_1^*, x_2, ..., x_m, y_1, ..., y_s)$ which identify a facet of PPS. After a facet is obtained either as a starting one, or during the iterative process, we assess the subset of input/output changes of type (2) defined with that facet. To obtain other changes, if any exist, we have to pivot from the obtained facet to the adjacent one and then repeat the procedure to assess input/output changes defined with the new facet. Pivoting from this facet to the adjacent one is done by applying parametric programming with input/output changes as parameters applied to the right hand side (RHS) of LP model (4).

We assess this changes such that the obtained optimal basis of LP model (4) remains optimal, i.e., nonnegativity and optimality is preserved because the introduced changes do not affect the optimal basis and the obtained optimal simplex tableau (except for the RHS, i.e., vector Γ_o). Since only the RHS has changed so the reduced cost row is not affected. Following this, nonnegativity should be investigated. Using notation $P_o = (-x_{1o}, ..., -x_{mo}, y_{1o}, ..., y_{so}, 1)^T$ where P_o corresponds to DMU_o, except for the negative sign in input and value 1 in the last entry of the vector. The input/output changes can be represented as Δ that is a diagonally matrix with diagonal $D = (\beta_1, \beta_2, ..., \beta_m, \alpha_1, ..., \alpha_s, 1)$. It also means the following changes of the right hand side vector in LP model (4): $p_o^* = \Delta p_o = (-\beta_1 x_{1o}, ..., -\beta_m x_{mo}, \alpha_1 y_{1o}, ..., \alpha_s y_{so}, 1)^T$, where p_o^* correspond to DMU_o^*. To preserve nonnegativity of the obtained optimal basis of LP model (4), the following condition has to be satisfied: $B^{-1} p_o^* \geq 0$, i.e.

$$-\sum_{i=1}^{m} b_{ki}^{-1} \beta_i x_{io} + \sum_{r=1}^{s} b_{km+r}^{-1} \alpha_r y_{ro} + b_{kn}^{-1} \geq 0, \quad k = 1, ..., m+s+1 \quad (8)$$

The input/ouput changes that satisfy (8) also define a subset of RE_o associated with the obtained facet (i.e., with the optimal basis corresponding to that facet). Coefficients of the first inequality in (8), the one corresponding to the basic variable β_1, are in fact optimal dual variables,

i.e. $b_{1i}^{-1} = v_i^*,\ i = 1, ..., m,\qquad b_{1m+r}^{-1} = \mu_r^*,\ r = 1, ..., s,\ \text{ and }\ b_{1n}^{-1} = w^*$, and thus coefficient of the obtained facet. Let us assume that DMU_o^* satisfies following inequality for input/output changes.

$$-\sum_{i=1}^{m} v_i^* \beta_i x_{io} + \sum_{r=1}^{s} \mu_r^* \alpha_r y_{ro} + w^* \geq 0 \tag{9}$$

$$\beta_i \geq 1 \quad i = 1, ..., m$$
$$0 < \alpha \leq 1 \quad r = 1, ..., s$$

Then by inputs expansion and outputs contraction by multipliers β and α, respectively, DMU_o^* is still efficient. To move to adjacent facet we have to perform a dual pivot step in optimal simplex of LP model (4), i.e. one variable leaves and new one enters the basis. In selecting variable which will leave the basis two conditions must be satisfied.

1: There must be negative coefficients in the corresponding row in the optimal simplex tableau.

2: There must exist such input/output changes that the corresponding inequality in (8) can be transformed into an equation.

By performing a dual pivot step, a new optimal basis (thus a new facet) is obtained and we repeat the procedure with a new facet to assess further input/output changes. We can use the following LP model to determine if a certain variable satisfies the second condition. Without loss of generality, we can consider the l-th variable (one of the λs

or slack variables, not the β_1)

$$
\begin{aligned}
Min \quad & -\sum_{i=1}^{m} b_{li}^{-1}\beta_i x_{io} + \sum_{r=1}^{s} b_{lm+r}^{-1}\alpha_r y_{ro} + b_{ln}^{-1} \\
s.t. \quad & -\sum_{i=1}^{m} b_{ki}^{-1}\beta_i x_{io} + \sum_{r=1}^{s} b_{km+r}^{-1}\alpha_r y_{ro} + b_{kn}^{-1} \geq 0 \quad k = 1, ..., m+s+1 \\
& 0 < \alpha_i \leq 1 \qquad\qquad\qquad\qquad\qquad\qquad i = 1, ..., m \\
& \beta_r \geq 1 \qquad\qquad\qquad\qquad\qquad\qquad\quad r = 1, ..., s
\end{aligned}
\tag{10}
$$

If the optimal value of the objective function of LP model (10) equals 0, the same as RHS of that inequality in (8), then we can transform this inequality into an equality for certain input/output changes. Thus, we can perform the pivot step.

After pivoting we repeat the process with the new simplex tableau associated with the new facet that are boundary of RE_o. The iteration part of the procedure is continued until the new produced basis is same as previous basis or in the next tableau we obtain $\hat{\beta}^* < 0$, which result contraction of input ($\beta^* = \hat{\beta}^* + 1 < 1$) and as mentioned before, in this case, projected point is indeed efficient. Hence, we obtain the complete RE_o.

4   Numerical example

Let us assume that we have 5 DMUs, each using one input to produce one output with data given in table 1. See Figure 1, where PPS defined with these DMUs is represented.

Table 1. The Raw Data set.

	DMU_1	DMU_2	DMU_3	DMU_4	DMU_5
X-input	1.5	3	9	3.5	5
Y-output	2	8	9	5.5	7

DMU_2 is extreme efficient and we can apply the above sensitivity analysis procedure. We start the procedure by changing the (first) input using LP model (4). Hence:

$$Min \quad \hat{\beta}$$

$$s.t. \quad 3\hat{\beta} - 1.5\lambda_1 - 9\lambda_3 - 3.5\lambda_4 - 5\lambda_5 \geq -3$$

$$2\lambda_1 + 9\lambda_3 + 5.5\lambda_4 + 7\lambda_5 \geq 8 \tag{11}$$

$$\lambda_1 + \lambda_3 + \lambda_4 + \lambda_5 = 1$$

$$\hat{\beta}, \lambda_j \geq 0 \qquad\qquad j = 1, ..., 5$$

The optimal value of LP model (11) is $\hat{\beta}^* = 4/3$ with variables $\hat{\beta}, \lambda_3$ and λ_5 in the optimal basis, $\lambda_3^* = 0.5$ and $\lambda_5^* = 0.5$. The projection point along the input using $\beta^* = (4/3) + 1 = 7/3$ is $DMU_2^* = (7, 8)$ and the optimal simplex tableau is as follows:

$\hat{\beta}$	$\hat{\beta}$	λ_1	λ_3	λ_4	λ_5	s^-	s^+	s_{a1}	s_{a2}	s_{a3}	Γ_o
$c_j - z_j$	0	2.167	0	0.5	0	0.333	0.667	M-0.333	M- 0.667	M+3	-1.333
$\hat{\beta}$	1	-2.167	0	-0.5	0	-0.333	-0.667	0.333	0.667	-3	1.333
λ_5	0	3.5	0	1.75	1	0	0.5	0	-0.5	4.5	0.5
λ_3	0	-2.5	1	-0.75	0	0	-0.5	0	0.5	-3.5	0.5

Tableau 1 Optimal simplex tableau of Step 1

The optimal dual variables, i.e., $v^* = 0.333, \mu^* = 0.667, w^* = -3$, are coefficients of the facet which contain the projection point (DMU_2^*). The equation of that facet is

$$-0.333x + 0.667y - 3 = 0 \tag{12}$$

Based on this equation we have a subset of RE_2 defined as

$$-0.333x + 0.667y - 3 \geq 0 \tag{13}$$

Equivalently, the input/output changes defined with this hyperplane must satisfy

$$-\beta + 5.336\alpha \geq 3$$

$$0 < \alpha \leq 1$$

$$\beta \geq 1$$

In the next step we should assess input/output changes which preserve nonnegativity of the obtained basis, thus the optimal basis will remain optimal $(-B^{-1}p_o^* \geq 0)$.

$$-\beta + 5.336\alpha \geq 3$$
$$-4\alpha \geq -4.5$$
$$4\alpha \geq 3.5$$

We should pivot to the adjacent facet for selecting variable that leave the basis. We have negative coefficients only in the third row of the optimal simplex tableau. We solve LP model (14) as follows:

$$
\begin{aligned}
Min \quad & 4\alpha \\
s.t. \quad & -\beta + 5.336\alpha = 3 \\
& -4\alpha \geq -4.5 \\
& 4\alpha \geq 3.5 \\
& 0 < \alpha \leq 1 \\
& \beta \geq 1
\end{aligned}
\tag{14}
$$

We obtain $\alpha^* = 0.875$ and 3.5 as the optimal value of the objective function. Since this value equals RHS we can perform the dual pivot step. This result with λ_3 (thus DMU_3) leaving the basis, and λ_4 (thus DMU_4) entering the basis, and the new basis as well as new facet are obtained. We repeat the process, using the new simplex tableau.

$\hat{\beta}$	$\hat{\beta}$	λ_1	λ_3	λ_4	λ_5	s^-	s^+	s_{a1}	s_{a2}	s_{a3}	Γ_o
$c_j - z_j$	0	0.5	0.667	0	0	0.333	0.333	M-0.333	M-0.333	M+0.667	-1
$\hat{\beta}$	1	-0.5	-0.667	0	0	-0.333	-0.333	0.333	0.333	-0.667	1
λ_5	0	-2.333	2.333	0	1	0	-0.667	0	0.667	-3.667	1.667
λ_4	0	3.333	-1.333	1	0	0	0.667	0	-0.667	4.667	-0.667

Tableau 2 Simplex tableau after pivoting in the first Step

From tableau 2 we obtain the inverse of a new basis matrix and new dual variables $v = 0.333$, $\mu = 0.333$, $w = -0.667$. then, the equation of the new facet is

$$-0.333x + 0.333y - 0.667 = 0 \tag{15}$$

Based on this equation we have a subset of RE_2 defined as

$$-0.333x + 0.333y - 0.667 \geq 0 \tag{16}$$

Equivalently, input/output changes must satisfy

$$-\beta + 2.664\alpha \geq 0.667$$
$$0 < \alpha \leq 1$$
$$\beta \geq 1$$

Nonnegativity of the obtained basis is preserved for input/outpot changes satisfying

$$-\beta + 2.664\alpha \geq 0.667$$
$$5.333\alpha \geq 3.667$$
$$-5.333\alpha \geq -4.667$$

To move to the adjacent facet we should consider the second row in tableau 2. The corresponding LP model is

$$
\begin{aligned}
Max \quad & 5.333\alpha \\
s.t. \quad & -\beta + 2.664\alpha = 0.667 \\
& 5.333\alpha \geq 3.667 \\
& -5.333\alpha \geq -4.667 \\
& 0 < \alpha \leq 1 \\
& \beta \geq 1
\end{aligned}
\tag{17}
$$

By solving (17) we obtain 3.667 as the optimal value of the objective function, which is equal to the RHS of the second inequality. Thus, we can pivot to the adjacent facet. This result with λ_5 leaving the basis, and λ_1 entering the basis, and the new basis as well as new facet are obtained. We repeat the process, using the new simplex tableau.

$\hat{\beta}$	$\hat{\beta}$	λ_1	λ_3	λ_4	λ_5	s^-	s^+	s_{a1}	s_{a2}	s_{a3}	Γ_o
$c_j - z_j$	0	0	1.167	0	0.214	0.333	0.190	M-0.333	M-0.190	M-0.119	-0.643
$\hat{\beta}$	1	0	-1.167	0	-0.214	-0.333	-0.190	0.333	0.190	0.119	0.643
λ_1	0	1	-1	0	-0.429	0	0.286	0	-0.286	1.571	-0.714
λ_4	0	0	2	1	1.429	0	-0.286	0	0.286	-0.571	1.714

Tableau 3 Optimal simplex tableau of Step 2

Dual variables are $v = 0.333$, $\mu = 190$, $w = 0.119$ and the equation of the new facet is

$$-0.333x + 0.190y + 0.119 = 0 \qquad (18)$$

Based on this equation we have a subset of RE_2 defined as

$$-0.333x + 0.190y + 0.119 \geq 0 \qquad (19)$$

Equivalently, input/output changes must satisfy

$$-\beta + 1.52\alpha \geq 0.119$$
$$0 < \alpha \leq 1$$
$$\beta \geq 1$$

Here, by entering λ_5 and leaving λ_1, possible dual pivoting will only reverse the previous pivoting. Moreover, by entering s^+ and leaving λ_4, we obtain $\hat{\beta}^* = -0.5 < 0$. This implies that we can not pivot to some new facet. Hence, we obtained all facets of $RPPS_2$ which serve as boundary for RE_2.

Input/output changes can be assessed such that at least one of the inequalities from the system of inequalities defined with these hyperplanes holds, i.e., DMU_o^* must satisfy at least one of the given inequalities. In our example these inequalities are as follows:

$$-0.333x + 0.667y - 3 \geq 0$$

$$-0.333x + 0.333y - 0.667 \geq 0$$

$$-0.333x + 0.190y + 0.119 \geq 0$$

Equivalently, when using input/output changes as follows:

$$-\beta + 5.336\alpha \geq 3$$

$$-\beta + 2.664\alpha \geq 0.667$$

$$-\beta + 1.52\alpha \geq 0.119$$

$$0 < \alpha \leq 1$$

$$\beta \geq 1$$

The areas associated to the above inequalities are depicted in figures 3, 4, and 5.

5   Conclusion

In this paper we have proposed an approach to sensitivity analysis where input/output change of type (2) are considered. We are interested in efficiency preservation of an extreme efficient DMU_o. The characterization of the complete region of efficiency for DMU_o, RE_o, is obtained. i.e. we give sufficient and necessary condition on input expansion or output contraction of DMU_o so that it remains efficient.

In our approach we have used one LP model (LP model (4)) and obtained projection of DMU_o on the $RPPS_o$, but sometimes LP model (4) can be infeasible this implies that we can not obtain the projection point on $RPPS_o$ along a chosen input or output. We can possibly bypass the infeasibility of LP model (4) by choosing another input/output which will not result with its infeasibility. But, when considering infeasibility we should focus on two aspects. First, we have to bypass the infeasibility in order to continue with the sensitivity analysis procedure. Second, we have to obtained the infeasibility region for a chosen input or output. Further research will concentrate on this problem and the situations where LP model (4) is infeasible along all chosen inputs or outputs.
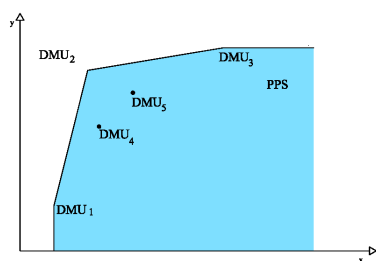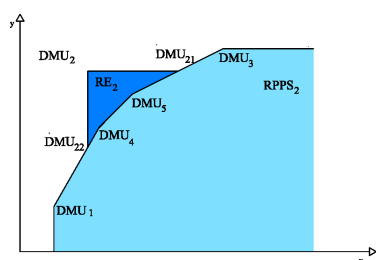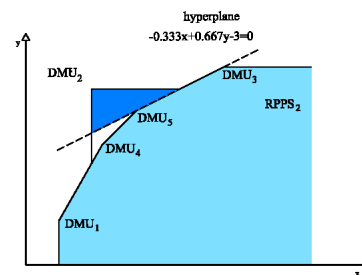
Fig.3 Part of RE$_2$ obtained after the first step

Fig.1 Region of efficiency for DMU$_2$ from example 1
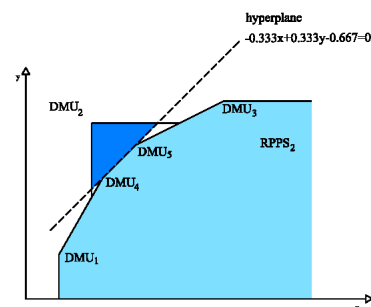
Fig.4 Part of RE$_2$ obtained after the second step

Fig.2 Region of efficiency for DMU$_2$ from example 1
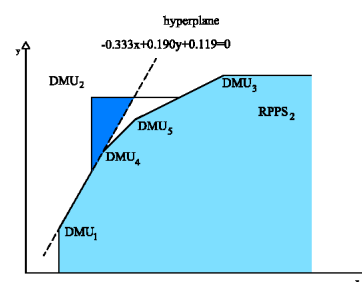
Fig.5 Part of RE$_2$ obtained after the third step

References

[1] Boljunčić V. (2006) "Sensitivity analysis of an efficient DMU in DEA model with variable returns to scale (VRS)," Journal of Productivity Analysis, 25, 173-192.

[2] Charnes A., Cooper W.W., Rhodes E. (1978) "Measuring the efficiency of decision making units," European Journal of Operational Research, 2, 429-444.

[3] Charnes A., Cooper W.W., Lewin A.Y., Morey R.C., Rousseau J. (1985) "Sensitivity and stability analysis in DEA," Annals of Operations Research, 2, 139-156.

[4] Charnes A., Neralic L. (1990) "Sensitivity analysis of the additive model in data envelopment analysis," European Journal of Operational Research, 48, 332-341.

[5] Charnes A., Haag S., Jaska P., Semple J. (1992) "Sensitivity of efficiency classifications in additive model of data envelopment analysis," International Journal of Systems Science, 23, 789-798.

[6] Charnes A., Rousseau J., Semple J. (1996) "Sensitivity and stability of efficiency classifications in data envelopment analysis," Journal of Productivity Analysis, 7, 5-18.

[7] Seiford L.M., Zhu J. (1998) "Stability regions for maintaining efficiency in data invelopment analysis," European Journal of Operational Research, 108(1), 127-139.

[8] Seiford L.M., Zhu J. (1998) "Sensitivity analysis of DEA models for Simultaneous changes in all the data," Journal of Operational Research Society, 49, 1060-1071.

[9] Thompson R.G., Dharmapala P.S., Thrall R.M., Sensitivity analysis of efficiency measures with application to Kansas farming and Illinois coal mining, In: Charnes et al. (eds) Data envelopment analysis: theory, methodology and applications, Kluwer Academic Pablishers, 1994.