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Numerical Analysis of Availability of the Serial Processes in Milk-Powder Processing Plant

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Abstract

In this paper availability analysis of a milk-powder plant with three statesgood, reduced and failed is given. Taking different failure rates and repair rates the availability of the system is evaluated. Mathematical formulation of the problem is carried out using probability considerations and the differential equations are solved using Runge-Kutta method of order four. Tables for various parameters are given which are useful to the management for predictions about the behavior of various equipments.

Keywords: Availability, Markov method and Runga-Kutta method.

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1 Introduction

A system is kept failure free (as far as possible) under the given operative conditions to achieve the goal of production. A probabilistic analysis of the system under given operative conditions is helpful for designing and maintenance purposes for optimization of the system working.

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Reliability techniques have been used extensively in various technological fields such as electronics systems design, power systems, mechanical systems design, aerospace, military combat problems and also been applied to a number of industrial and transportation problems, e.g., the sugar industry [4], [6], fertilizer industry [8-9], [16] and paper production industry [3], [5], [7] and [10].

A Probabilistic analysis of the system under given operative conditions is helpful in design modification for minimum failure and maintenance planning for optimization of the system working. Saki and Okumato [13] have studied two unit redundant system with two phase repairs. For an extensive view on the subject up to the presentation we refer [1-2],[11-12], [14-15] and [17].

The paper studies the effect of failure and repair rates upon availability of a milkpowder plant making use of Runage-Kutta method for differential equations.

2 The System and Assumptions

The system considered in this paper is a milk powder plant and consists of four subsystems, namely-balance tank, heater, vapor separator and dryer. Generally balance tank never fails. The mathematical modeling is carried out for the remaining three sub-systems that are prone to failure:

- (i) **Heater**: It is one of the important parts of the system consisting of three parallel units. It is used to heat the milk that comes from the balance tank. Failure of any one of these units reduces the capacity of the system and hence loss in the production. Complete failure of the system occurs only when all the three units fail.
- (ii) **VaporSeparator**: After heating the milk in the heater, water vapor from the milk is separated.
- (iii) **Dryer**: This subsystem is used to dry the milk. Here all the water from the milk is absorbed and we get the milk powder.
 - Let, H, V and D, respectively, denote the good condition of the subsystems- heater,

vapor separator and dryer.

 $\underline{\mathbf{H}}$, $\underline{\mathbf{V}}$ and $\underline{\mathbf{D}}$ indicate that the subsystems H, V and D are working in the reduced state. The symbols h, v and d, respectively, represent the failed states of the subsystems H, V and D.

Let λ_i , $i=1, 2, \ldots, 6$, represents the respective failure rate of subsystems H, V, D, \underline{H} , \underline{V} and \underline{D} ; μ_i , $i=1, 2, \ldots, 6$, represents the respective repair rate of subsystems H, V, D, \underline{H} , \underline{V} and \underline{D} ; $P_j(t)$, $j=1, 2, 3 \ldots 20$; represents the probability of the system in the jth state at time t. Furthermore, we adopt the following assumptions:

- (i) Repair and failure are independent of each other;
- (ii) There is no simultaneous failure among the subsystems;
- (iii) Repaired components are like new components.

Following the above notations and assumptions, the transition diagram for the system is given in Figure-1.

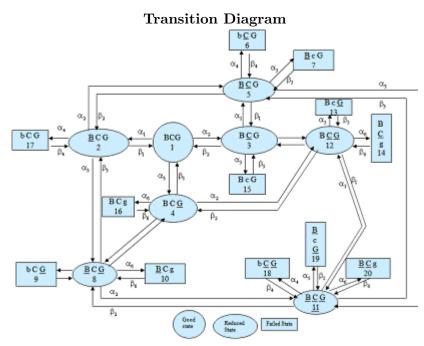


Figure-1

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3 Mathematical Modeling

With the help of Figure-1 we construct the ordinary differential equations of first order governing the milk-powder plant as follows.

$$P_1^{/}(t) + \left(\sum_{i=1}^{3} \lambda_i\right) P_1(t) = \sum_{i=1, j=2}^{3, 4} \mu_i P_j(t)$$
 (1)

$$P_2^{\prime}(t) + \left(\sum_{i=2}^4 \lambda_i + \mu_1\right) P_2(t) = \lambda_1 P_1(t) + \mu_2 P_5(t) + \mu_3 P_8(t) + \mu_4 P_{17}(t)$$
 (2)

$$P_3^{\prime}(t) + (\lambda_1 + \lambda_3 + \lambda_5 + \mu_2) P_3(t) = \lambda_2 P_1(t) + \mu_1 P_5(t) + \mu_3 P_{12}(t) + \mu_5 P_{15}(t)$$
 (3)

$$P_4^{\prime}(t) + \left(\sum_{i=1}^{2} \lambda_i + \lambda_6 + \mu_3\right) P_4(t) = \lambda_3 P_1(t) + \mu_1 P_8(t) + \mu_2 P_{12}(t) + \mu_6 P_{16}(t)$$
 (4)

$$P_5'(t) + \left(\sum_{i=3}^{5} \lambda_i + \sum_{j=1}^{2} \mu_j\right) P_5(t) = \lambda_1 P_3(t) + \lambda_2 P_2(t) + \mu_3 P_{11}(t) + \sum_{i=4,j=6}^{5,7} \mu_i P_j(t)$$
 (5)

$$P_8^{\prime}(t) + (\lambda_2 + \lambda_4 + \lambda_6 + \mu_1 + \mu_3) P_8(t) = \lambda_1 P_4(t) + \lambda_2 P_2(t) + \mu_2 P_{11}(t) + \mu_4 P_{10}(t) + \mu_6 P_9(t)$$
(6)

$$P_{11}^{/}(t) + \left(\sum_{i=4}^{6} \lambda_i + \sum_{j=1}^{3} \mu_j\right) P_{11}(t) = \lambda_1 P_{12}(t) + \lambda_2 P_{8}(t) + \lambda_3 P_{5}(t) + \sum_{i=4, j=18}^{6,20} \mu_i P_{j}(t)$$
(7)

$$P_{12}^{\prime}(t) + (\lambda_1 + \sum_{i=5}^{6} \lambda_i + \sum_{i=2}^{3} \mu_i) P_{12}(t) = \lambda_2 P_4(t) + \lambda_3 P_3(t) + \mu_1 P_{11}(t) + \mu_5 P_{13}(t) + \mu_6 P_{14}(t)$$
(8)

$$P_i^{\prime}(t) + \mu_4 P_i(t) = \lambda_4 P_j(t); i = 6, 10, 17 \text{ and } j = 5, 8, 2, respectively.$$
 (9)

$$P_i^{\prime}(t) + \mu_6 P_i(t) = \lambda_6 P_j(t); i = 9, 14, 16 \text{ and } j = 8, 12, 4, respectively.$$
 (10)

$$P_i^{\prime}(t) + \mu_5 P_i(t) = \lambda_5 P_j(t); i = 7, 13, 15 \text{ and } j = 5, 12, 3, respectively.$$
 (11)

$$P_i^{/}(t) + \mu_j P_i(t) = \lambda_j P_{11}(t); i = 18, 19, 20 \text{ and } j = 4, 5, 6, respectively.$$
 (12)

With initial conditions

$$P_i(0) = \begin{cases} 1, & i = 1 \\ 0, & i \neq 1 \end{cases}$$
 (13)

The equations from (1) to (12) with initial condition (13) have been solved numerically using Runge-Kutta method of 4th order. The availability of the system is given by

$$A(t) = \left(\sum_{i=1}^{5} P_i(t)\right) + P_8(t) + P_{11}(t) + P_{12}(t)$$
(14)

The availability of the system, as given in equation (14) is computed for a combination of values of the repair and failure rates. If the failure and repair rates are altered, the availability is affected. This effect is shown in the tables from 1 to 11.

(i) Effect of failure rate of Heater (λ_1) on availability of the system.

Taking $\lambda_2 = 0.1$, $\lambda_3 = 0.05$, $\lambda_4 = 0.033$, $\lambda_5 = 0.016$, $\lambda_6 = .02$ and $\mu_1 = 0.1$, $\mu_2 = 0.1$, $\mu_3 = 0.08$, $\mu_4 = 0.033$, $\mu_5 = 0.01$, $\mu_6 = 0.02$ we have

$\lambda_1 \rightarrow$	0.05	0.1	0.15	0.20	0.25
Time (in year)↓					
20	0.999963	0.999948	0.999914	0.999890	0.999865
40	0.999854	0.999758	0.999663	0.999568	0.999475
60	0.999680	0.999467	0.999257	0.999051	0.998847
80	0.999443	0.999071	0.998707	0.998350	0.998000
100	0.999148	0.998578	0.998022	0.997480	0.996950

Table-1

The table shows that the availability of the system decreases by approximately 0.0002476 with the increase of the failure rate of the Heater subsystem from 0.05 to 0.25 and availability decreases by approximately 0.0003588 with increase in the time from 20 years to 100 years. The effect of failure rate of heater on availability of the system is shown in Figure-2.

(ii) Effect of failure rate of Vapor Separator (λ_2) on availability of the system.

Taking $\lambda_1 = 0.05$, $\lambda_3 = 0.05$, $\lambda_4 = 0.033$, $\lambda_5 = 0.02$, $\lambda_6 = 0.02$ and $\mu_1 = 0.1$, $\mu_2 = 0.1$, $\mu_3 = 0.08$, $\mu_4 = 0.033$, $\mu_5 = 0.02$ and $\mu_6 = 0.02$ we have

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$\lambda_2 \rightarrow$	0.1	0.2	0.3	0.4	0.5
Time (in year)↓					
20	0.999961	0.999951	0.999942	0.999933	0.999923
40	0.999847	0.999811	0.999776	0.999741	0.999707
60	0.999662	0.999587	0.999513	0.999440	0.999369
80	0.999412	0.999286	0.999163	0.999043	0.998926
100	0.999101	0.998916	0.998736	0.998562	0.998394

Table-2

The table shows that the availability of the system decreases by approximately 0.0000832 with the increase of the failure rate of the Vapor Separator subsystem from 0.1 to 0.5 and availability decreases by approximately 0.00030007 with increase in the time from 20 years to 100 years.

(iii) Effect of failure rate of Dryer (λ_3) on availability of the system.

Taking λ_1 = 0.05, λ_2 = 0.1, λ_4 = 0.033, λ_5 = 0.02, λ_6 = 0.02 and μ_1 = 0.1, μ_2 = 0.1, μ_3 = 0.08, μ_4 = 0.033, μ_5 = 0.02 and μ_6 =0.02 we have

$\lambda_3 { ightarrow}$	0.05	0.1	0.15	0.2	0.25
Time (in year)↓					
20	0.999961	0.999932	0.999903	0.999874	0.999845
40	0.999847	0.999734	0.999622	0.999511	0.999402
60	0.999662	0.999416	0.999173	0.998934	0.998698
80	0.999412	0.998988	0.998571	0.998163	0.997762
100	0.999101	0.998458	0.997830	0.997217	0.996619

Table-3

The table shows that the availability of the system decreases by approximately 0.000283 with the increase of the failure rate of the Dryer subsystem from 0.05 to 0.25 and availability decreases by approximately 0.0005144 with increase in the time from 20 years to 100 years.

(iv) Effect of failure rate of reduced state of Vapor Separator (λ_5) on availability of the system.

Taking $\lambda_1 = 0.05$, $\lambda_2 = 0.04$, $\lambda_3 = 0.05$, $\lambda_4 = 0.033$, $\lambda_6 = 0.02$ and $\mu_1 = 0.1$, $\mu_2 = 0.033$, $\mu_3 = 0.08$, $\mu_4 = 0.033$, $\mu_5 = 0.01$ and $\mu_6 = 0.02$ we have

$\lambda_5 { ightarrow}$	0.02	0.04	0.06	0.08	0.1
Time (in year)↓					
20	0.999966	0.999963	0.999959	0.999955	0.999951
40	0.999868	0.999852	0.999837	0.999821	0.999805
60	0.999708	0.999673	0.999638	0.999603	0.999568
80	0.999489	0.999427	0.999365	0.999304	0.999243
100	0.999214	0.999117	0.999022	0.998927	0.998832

Table-4

The table shows that the availability of the system decreases by approximately 0.0000423 with the increase of the failure rate of the Vapor Separator subsystem from 0.02 to 0.1 and availability decreases by approximately 0.0002341 with increase in the time from 20 years to 100 years.

(v) Effect of failure rate of reduced state of Dryer (λ_6) on availability of the system.

Taking $\lambda_1 = 0.05$, $\lambda_2 = 0.04$, $\lambda_3 = 0.05$, $\lambda_4 = 0.033$, $\lambda_5 = 0.02$ and $\mu_1 = 0.1$, $\mu_2 = 0.033$, $\mu_3 = 0.08$, $\mu_4 = 0.033$, $\mu_5 = 0.01$ and $\mu_6 = 0.02$ we have

$\lambda_6 \rightarrow$	0.02	0.04	0.06	0.08	0.1
Time (in year)↓					
20	0.999966	0.999962	0.999957	0.999952	0.999947
40	0.999868	0.999849	0.999830	0.999810	0.999791
60	0.999708	0.999665	0.999623	0.999580	0.999538
80	0.999489	0.999414	0.999340	0.999266	0.999193
100	0.999214	0.999099	0.998985	0.998872	0.998760

Table-5

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The table shows that the availability of the system decreases by approximately 0.0000508 with the increase of the failure rate of the Dryer subsystem from 0.02 to 0.1 and availability decreases by approximately 0.0002427 with increase in the time from 20 years to 100 years.

(vi) Effect of repair rate of Heater subsystem (μ_1) on availability of the system.

Taking
$$\lambda_1 = 0.05$$
, $\lambda_2 = 0.1$, $\lambda_3 = 0.05$, $\lambda_4 = 0.05$, $\lambda_5 = 0.016$, $\lambda_6 = 0.02$ and $\mu_2 = 0.1$, $\mu_3 = 0.05$, $\mu_4 = 0.04$, $\mu_5 = 0.01$, $\mu_6 = 0.02$ we have

$\mu_1 \rightarrow$	0.1	0.4	0.7	1
Time (in year)↓				
20	0.999963	0.999963	0.999964	0.999965
40	0.999854	0.999860	0.999866	0.999871
60	0.999678	0.999697	0.999717	0.999736
80	0.999439	0.999484	0.999529	0.999574
100	0.999140	0.999228	0.999315	0.999401

Table-6

The table shows that the availability of the system increases by approximately 0.000315 with the increase of the repair rate of the Heater subsystem from 0.1 to 1 and availability decreases by approximately 0.0001733 with increase in the time from 20 years to 100 years.

(vii) Effect of repair rate of Vapor Separator subsystem (μ_2) on availability of the system.

Taking
$$\lambda_1 = 0.1$$
, $\lambda_2 = 0.5$, $\lambda_3 = 0.5$, $\lambda_4 = 0.1$, $\lambda_5 = 0.2$, $\lambda_6 = 0.1$ and $\mu_1 = 0.1$, $\mu_3 = 0.2$, $\mu_4 = 0.1$, $\mu_5 = 0.1$, $\mu_6 = 0.1$ we have

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$\mu_2 \rightarrow$	0.1	0.3	0.5	0.7	0.9
Time (in year)↓					
20	0.999193	0.999194	0.999195	0.999195	0.999196
40	0.996938	0.996942	0.996947	0.996951	0.996955
60	0.993459	0.993472	0.993485	0.993498	0.993510
80	0.988954	0.988983	0.989010	0.989037	0.989063
100	0.983600	0.983651	0.983699	0.983745	0.983789

Table-7

The table shows that the availability of the system increases by approximately 0.00001845 with the increase of the repair rate of the Vapor Separator subsystem from 0.1 to 0.9 and availability decreases by approximately 0.003875 with increase in the time from 20 years to 100 years.

(viii) Effect of repair rate of Dryer subsystem (μ_3) on availability of the system.

Taking
$$\lambda_1 = 0.1$$
, $\lambda_2 = 0.1$, $\lambda_3 = 0.5$, $\lambda_4 = 0.1$, $\lambda_5 = 0.2$, $\lambda_6 = 0.1$ and $\mu_1 = 0.1$, $\mu_2 = 0.2$, $\mu_4 = 0.1$, $\mu_5 = 0.1$, $\mu_6 = 0.1$ we have

$\mu_3 \rightarrow$	0.1	0.3	0.5	0.7	0.9
Time (in year)↓					
20	0.999190	0.999197	0.999204	0.999210	0.999216
40	0.996915	0.996965	0.997014	0.997062	0.997110
60	0.993386	0.993544	0.993698	0.993850	0.993998
80	0.988791	0.989143	0.989487	0.989821	0.990147
100	0.983298	0.983947	0.984576	0.985184	0.985773

Table-8

The table shows that the availability of the system increases by approximately 0.000233 with the increase of the repair rate of the Dryer subsystem from 0.1 to 0.9 and availability decreases by approximately 0.003662 with increase in the time from 20 years to 100 years.

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(ix) Effect of repair rate of Heater subsystem in reduced state (μ_4) on availability of the system.

Taking $\lambda_1 = 0.1$, $\lambda_2 = 0.1$, $\lambda_3 = 0.5$, $\lambda_4 = 0.1$, $\lambda_5 = 0.2$, $\lambda_6 = 0.1$ and $\mu_1 = 0.1$, $\mu_2 = 0.2$, $\mu_3 = 0.2$, $\mu_5 = 0.1$, $\mu_6 = 0.1$ we have

$\mu_4 \rightarrow$	0.1	0.3	0.5	0.7	0.9
Time (in year)↓					
20	0.999194	0.999197	0.999200	0.999203	0.999206
40	0.996940	0.996963	0.996987	0.997010	0.997033
60	0.993465	0.993538	0.993611	0.993684	0.993757
80	0.988968	0.989130	0.989291	0.989453	0.989615
100	0.983625	0.983920	0.984214	0.984509	0.984803

Table-9

The table shows that the availability of the system increases by approximately 0.000111 with the increase of the repair rate of the Heater subsystem in reduced state from 0.1 to 0.9 and availability decreases by approximately 0.00375 with increase in the time from 20 years to 100 years.

(x) Effect of repair rate of Vapor Separator subsystem in reduced state (μ_5) on the availability of the system.

Taking $\lambda_1 = 0.1$, $\lambda_2 = 0.1$, $\lambda_3 = 0.5$, $\lambda_4 = 0.1$, $\lambda_5 = 0.2$, $\lambda_6 = 0.1$ and $\mu_1 = 0.1$, $\mu_2 = 0.2$, $\mu_3 = 0.2$, $\mu_4 = 0.3$, $\mu_6 = 0.1$ we have

$\mu_5 \rightarrow$	0.1	0.4	0.7	1
Time (in year)↓				
20	0.999197	0.999197	0.999197	0.999197
40	0.996963	0.996964	0.996964	0.996964
60	0.993538	0.993539	0.993540	0.993541
80	0.989130	0.989133	0.989135	0.989138
100	0.983920	0.983926	0.983932	0.983938

Table-10

The table shows that the availability of the system increases by approximately 0.0000033 with the increase of the repair rate of the Vapor Separator subsystem in reduced state from 0.1 to 0.9 and availability decreases by approximately 0.0038171 with increase in the time from 20 years to 100 years.

(xi) Effect of repair rate of Dryer subsystem in reduced state (μ_6) on the availability of the system.

Taking $\lambda_1 = 0.1$, $\lambda_2 = 0.1$, $\lambda_3 = 0.5$, $\lambda_4 = 0.1$, $\lambda_5 = 0.2$, $\lambda_6 = 0.1$ and $\mu_1 = 0.1$, $\mu_2 = 0.2$, $\mu_3 = 0.2$, $\mu_4 = 0.3$, $\mu_5 = 0.1$ we have

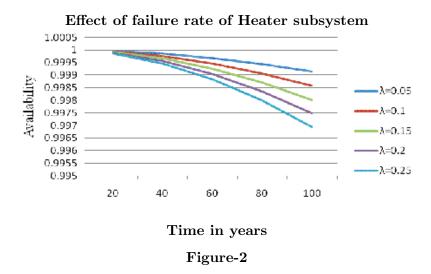
$\mu_6 \rightarrow$	0.1	0.4	0.7	1
Time (in year)↓				
20	0.999197	0.999199	0.999202	0.999204
40	0.996963	0.996983	0.997003	0.997022
60	0.993538	0.993604	0.993669	0.993735
80	0.989130	0.989283	0.989437	0.989591
100	0.983920	0.984216	0.984512	0.984809

Table-11

The table shows that the availability of the system increases by approximately 0.000108 with the increase of the repair rate of the Dryer subsystem in reduced state from 0.1 to 1 and availability decreases by approximately 0.003709 with increase in the time from 20 years to 100 years.

4 Conclusions

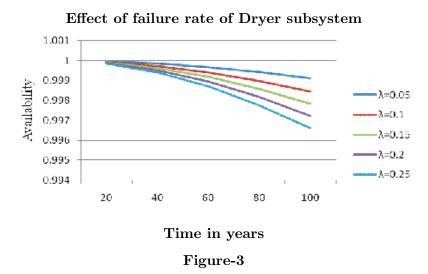
The tables from 1 to 11 show the effect of failure rates and repair rates of different subsystems when the whole system is in good as well as in reduced condition. We conclude from the above tables that the failure rate of Dryer and Heater subsystems affect highly on the availability of the system which has been shown in Figure-1 and Figure-2. Consequently, we should take extra care of these subsystems. Furthermore,



the repair rate of Vapor Separator shows that we must have a good repair facility for vapor separator to minimize the failure condition of the system and also to achieve maximum profit from the milk powder plant.

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