



A New Approach for Ranking of Fuzzy numbers With Continuous Weighted Quasi-Arithmetic Means

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Abstract

Many ranking methods have been proposed so far. The fuzzy number defuzzification method with continuous weighted quasi-arithmetic means was proposed by Yoshida in the domain $(-\infty, +\infty)$. Recently in [22], the researcher used this method with the ordinary fuzzy set in a compact interval, that all the conclusions can be extended to the ordinary fuzzy set in [24] directly. This parameterized defuzzification can be used as a crisp approximation with respect to a fuzzy quantity. In this article, the researchers will use this defuzzification for ordering fuzzy numbers. This method can effectively rank various fuzzy numbers, their images and overcome the shortcomings of the previous techniques. The proposed model is studied for a broad class for fuzzy numbers. The calculation of this method is far simpler than the other approaches. This article also used some comparative examples to illustrate the advantage of the proposed method.

Keywords: Ranking, Fuzzy numbers, Defuzzification, Continuous weighted quasi-arithmetic means, Function generators.

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1 Introduction

In decision analysis under fuzzy environment, ranking of fuzzy numbers is a very important decision-making procedure. Since Jain, Dubis and Prade [14] introduced the

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relevant concepts of fuzzy numbers, many researchers proposed the related methods or applications for ranking fuzzy numbers. For instance, Bortolan and Degani [4] reviewed some methods to rank fuzzy numbers in 1985, Chen and Hwang [5] proposed fuzzy multiple attribute decision making in 1992, Choobineh and Li [6] proposed an index for ordering fuzzy numbers in 1993, Dias [7] ranked alternatives by ordering fuzzy numbers in 1993, Lee [12] ranked fuzzy numbers with a satisfaction function in 1994, Requena [8] utilized artificial neural networks for the automatic ranking of fuzzy numbers in 1994, Fortemps [10] presented ranking and defuzzification methods based on area compensation in 1996 and Raj [11] investigated maximizing and minimizing sets to rank fuzzy alternatives with fuzzy weights in 1999. However, Chu [9] argued that some of these above methods are difficult to implement on grounds of computational complexity, and others are counterintuitive or not discriminating enough. Chu also considered that many methods different outcomes on the same problem. Chu and Tsao's method originated from the concepts of Lee and Li [12] and Cheng [13]. In 1988, Lee and Li proposed the comparison of fuzzy numbers, for which they considered mean and standard deviation values for fuzzy numbers based on the uniform and proportional probability distributions. Then, Cheng proposed the coefficient of variance (*CV* index) in 1998 to improve Lee and Li's method based on two comments presented as follows.

- (a) The mean and standard deviation values cannot be the sole basis to compare two fuzzy numbers.
- (b) It is difficult to rank fuzzy numbers, as higher mean value is associated with higher spread or lower mean value is associated with lower spread.

Although, Cheng overcame the problems from these comments and also proposed a new distance index to improve the method [13] proposed by Murakami et al., Chu and Tsao's still believed that Cheng's method contained some shortcomings. For instance, they illustrated a ranking example shown as below. For the two triangular fuzzy numbers in their example, $A = (0.9, 1, 1.1)$ and $B = (1.2, 2, 3)$. Intuitively, A should be smaller

than B . However, A is bigger than B on the basis of CV index. Furthermore, Cheng also proposed a distance method to rank fuzzy numbers for improving the method of Murakami et al., and the distance method often contradicts the CV index on ranking fuzzy numbers. To overcome these above problems, Chu and Tsao proposed a method to rank fuzzy numbers with an area between their centroid and original points. The method can avoid the problems Chu and Tsao mentioned; however, the researchers find other problems in their method. But, this method for some fuzzy numbers is unreasonable. Also, Wang et al. [21] proposed an approach to ranking fuzzy numbers based on lexicographic screening procedure and summarized some limitations of the existing methods.

This article proposes here a method to use the concept parameterized defuzzification of a fuzzy number, so as to find the order of fuzzy numbers. This method can distinguish the alternatives clearly. The main purpose of this article is that, this defuzzification of a fuzzy number can be used as a crisp approximation of a fuzzy number. Therefore, by the means of this defuzzification, this article aims to present a new method for ranking of fuzzy numbers. In addition to its ranking features, this method removes the ambiguous results and overcome the shortcomings from the comparison of previous ranking. The paper is organized as follows: In Section 2, the researchers recall some fundamental results on fuzzy numbers. Parameterized defuzzification with continuous weighted quasi-arithmetic means is explicated in Section 3. Proposed method for ranking fuzzy numbers is in the Section 4. Discussion and comparison of this work and other methods are carried out in Section 5. The paper ends with conclusions in Section 6.

2 Preliminaries

The basic definitions of a fuzzy number are given in [15, 16] as follows.

Definition 2.1 *Let U be a universe set. A fuzzy set A of U is defined by a membership function $\mu_A(x) \rightarrow [0, 1]$, where $\mu_A(x)$ indicates the degree of x in A .*

Definition 2.2 A fuzzy subset A of universe set U is normal iff $\sup_{x \in U} \mu_A(x) = 1$.

Definition 2.3 A fuzzy subset A of universe set U is convex iff $\mu_A(\lambda x + (1 - \lambda)y) \geq (\mu_A(x) \wedge \mu_A(y))$, $\forall x, y \in U, \forall \lambda \in [0, 1]$, where \wedge denotes the minimum operator.

Definition 2.4 A fuzzy set A is a fuzzy number iff A is normal and convex on U .

The set of all fuzzy numbers is denoted by F .

Definition 2.5 A triangular fuzzy number A is a fuzzy number with a piecewise linear membership function μ_A defined by :

$$\mu_A = \begin{cases} \frac{x-a_1}{a_2-a_1}, & \text{when } a_1 \leq x \leq a_2, \\ \frac{a_3-x}{a_3-a_2}, & \text{when } a_2 \leq x \leq a_3, \\ 0, & \text{otherwise .} \end{cases} \quad (1)$$

which can be denoted as a triplet (a_1, a_2, a_3) .

Definition 2.6 A trapezoidal fuzzy number A is a fuzzy number with a membership function μ_A defined by :

$$\mu_A = \begin{cases} \frac{x-a_1}{a_2-a_1}, & \text{when } a_1 \leq x \leq a_2, \\ 1, & \text{when } a_2 \leq x \leq a_3, \\ \frac{a_4-x}{a_4-a_3}, & \text{when } a_3 \leq x \leq a_4, \\ 0, & \text{otherwise .} \end{cases} \quad (2)$$

which can be denoted as a quartet (a_1, a_2, a_3, a_4) .

Definition 2.7 An extended fuzzy number A is described as any fuzzy subset of the universe set U with membership function μ_A defined as follows:

- (a) μ_A is a continuous mapping from U to the closed interval $[0, \omega]$, $0 < \omega \leq 1$.
- (b) $\mu_A(x) = 0$, for all $x \in (-\infty, a_1]$.
- (c) μ_A is strictly increasing on $[a_1, a_2]$.
- (d) $\mu_A(x) = \omega$, for all $x \in [a_2, a_3]$, as ω is a constant and $0 < \omega \leq 1$.

(e) μ_A is strictly decreasing on $[a_3, a_4]$.

(f) $\mu_A(x) = 0$, for all $x \in [a_4, +\infty)$.

In these above situations a_1, a_2, a_3 and a_4 are real numbers. If $a_1 = a_2 = a_3 = a_4$, A becomes a crisp real number.

Definition 2.8 The membership function μ_A of extended fuzzy number A is expressed by

$$\mu_A = \begin{cases} \mu_A^L(x), & \text{when } a_1 \leq x \leq a_2, \\ \omega, & \text{when } a_2 \leq x \leq a_3, \\ \mu_A^R(x), & \text{when } a_3 \leq x \leq a_4, \\ 0, & \text{otherwise .} \end{cases} \quad (3)$$

where $\mu_A^L : [a_1, a_2] \rightarrow [0, \omega]$ and $\mu_A^R : [a_3, a_4] \rightarrow [0, \omega]$.

Based on the basic theories of fuzzy numbers, A is a normal fuzzy number if $\omega = 1$, whereas A is a non-normal fuzzy number if $0 < \omega \leq 1$. Therefore, the extended fuzzy number A in Definition (3) can be denoted as $(a_1, a_2, a_3, a_4; \omega)$. The image $-A$ of A can be expressed by $(-a_4, -a_3, -a_2, -a_1; \omega)$ [15].

With Zadeh's extension principle, the arithmetic operation of fuzzy sets especially the fuzzy numbers can be defined. Here, this article recalls the two simplest cases of scalar addition and scalar multiplication. For the fuzzy set with membership function $\mu_A(x)$, the membership function of scalar addition $A + c$ and scalar multiplication kA ($k \neq 0$) are $\mu_{A+c}(x) = \mu_A(x - c)$ and $\mu_{kA} = \mu_A(x/k)$, respectively.

3 Defuzzification with continuous weighted quasi-arithmetic mean operator

The fuzzy number defuzzification method with continuous weighted quasi-arithmetic means was proposed by Yoshida in the domain $(-\infty, +\infty)$. Recently in [22] researchers used this method with the ordinary fuzzy set in a compact interval, that all the conclusions can be extended to the ordinary fuzzy set directly.

Let us introduce some definition which this article needs in this Section.

Definition 3.1 [22]. Let f be a continuous strictly monotonic mapping on $[a, b]$. For aggregated elements vector $X = (x_1, x_2, \dots, x_n) \in [a, b]^n$, a quasi-arithmetic mean can be defined as the aggregation operator $M_f : [a, b]^n \rightarrow [a, b]$ with

$$\mu_f(x_1, x_2, \dots, x_n) = f^{-1} \left(\frac{1}{n} \sum_{i=1}^n f(x_i) \right) \quad (4)$$

where f^{-1} is its inverse function and f is called the generator of the quasi-arithmetic mean M_f . This article will denote this with $M_f(x)$.

Definition 3.2 [22]. Let f be a continuous strictly monotonic mapping on $[a, b]$. For aggregated elements vector $X = (x_1, x_2, \dots, x_n) \in [a, b]^n$ and weighting vector $w = (w_1, w_2, \dots, w_n)$ with $w_i \in [0, 1]$ and $\sum_{i=1}^n w_i = 1$. A weighted quasi-arithmetic mean is defined the aggregation operator

$$M_{w,f}(x_1, x_2, \dots, x_n) = f^{-1} \left(\sum_{i=1}^n w_i f(x_i) \right), \quad (5)$$

where f^{-1} is the inverse function of f and $w = (w_1, w_2, \dots, w_n)$ is a weighting vector of dimension n that represents the importance of x_i 's. This article will denote this with $M_{w,f}(X)$. Also, $f(x)$ is called the generator of $M_{w,f}(X)$.

With the definition of Riemann integral, (5) can extend to the continuous case with infinite dimension:

$$M_{w,f} = f^{-1} \left(\frac{\int_a^b w(x) f(x) dx}{\int_a^b w(x) dx} \right), \quad (6)$$

where $f(x)$ is a continuous strictly monotonic mapping on $[a, b]$ and $w(x)$ is a continuous weighting function with $w(x) \geq 0$ and $\int_a^b w(x) dx \neq 0$.

Remark 1. [23] If the weighting function is replaced with the membership function of a fuzzy set $\mu_A(x)$, this article can get the defuzzification of a fuzzy set A with the continuous weighted quasi-arithmetic mean:

$$M_f(A) = f^{-1} \left(\frac{\int_a^b \mu_A(x) f(x) dx}{\int_a^b \mu_A(x) dx} \right). \quad (7)$$

Remark 2. In (7), if $f(x) = x$, therefore $M_f(A)$ is the center of gravity defuzzification method, namely $M_f(A) = \frac{\int_a^b \mu_A(x)x dx}{\int_a^b \mu_A(x) dx} = COG(A)$. For more detailed discussion see [23].

4 Comparison of fuzzy numbers using parameterized quasi-arithmetic means

In this Section, the researchers will propose the ranking of fuzzy numbers associated with the parameterized defuzzification quasi-arithmetic means.

This article discusses a family of parameterized quasi-arithmetic means with power function generators for fuzzy set defuzzification. As all the conclusions are the direct extension from the discrete case to the weighted continuous case, the researchers give the main results as briefly as possible.

From Corollary (3) in [22], for a quasi-arithmetic mean with generating function $\varphi(x)$, if this article want to make it more or-like or and-like only need to increase or to decrease the value of expression $\frac{f''(x)}{f'(x)}$. The simplest case is $\frac{f''(x)}{f'(x)} = \frac{r-1}{x}$, then $f(x) = C_1 + C_2x^r$ ($r \neq 0$). From Theorem (2) in [22], this article needs to consider $f(x)$ in the form $f(x) = x^r$ that $r \neq 0$, then

$$M_f(A) = \left(\frac{\int_a^b \mu_A(x)x^r dx}{\int_a^b \mu_A(x) dx} \right)^{\frac{1}{r}},$$

where $x_i > 0$. The researcher will call this quasi-arithmetic mean with power function generator. In particular case, consider a trapezoidal fuzzy number $A = (a_1, a_2, a_3, a_4)$. Its membership function is in the form of (2), then

$$M_f(A) = \left(\frac{2(a_1^{r+2} - a_2^{r+2})(a_3 - a_4) - 2(a_3^{r+2} - a_4^{r+2})(a_1 - a_2)}{(r + 2)(r + 1)(a_1 - a_2)(a_3 - a_4)(a_1 + a_2 - a_3 - a_4)} \right)^{\frac{1}{r}}.$$

Definition 4.1 Let A be a fuzzy number characterized by (3) and $M_f(A)$ is the parameterized defuzzification quasi-arithmetic means with power function generators of fuzzy number A .

Since this article wants to approximate a fuzzy number by a scalar value. Thus, the researchers have to use an operator $M_f : F \rightarrow \mathfrak{R}$ which transforms fuzzy numbers into family of real line. Operator M_f is an crisp approximation operator. Since ever parameterized defuzzification can be used as a crisp approximation of a fuzzy number, therefor the resulting value is used to rank the fuzzy numbers. Then $M_f(A)$ is used to rank fuzzy numbers. The larger $M_f(A)$ the larger fuzzy number.

Let $A, B \in F$ be two arbitrary fuzzy numbers. Define the ranking of A and B by $M_f(\cdot)$ on F as follows:

$$(1) M_f(A) > M_f(B) \quad \text{if only if} \quad A \succ B,$$

$$(2) M_f(A) < M_f(B) \quad \text{if only if} \quad A \prec B,$$

$$(3) M_f(A) = M_f(B) \quad \text{if only if} \quad A \sim B.$$

This article formulates the order \succeq and \preceq as $A \succeq B$ if and only if $A \succ B$ or $A \sim B$, $A \preceq B$ if and only if $A \prec B$ or $A \sim B$.

This study considers the following reasonable axioms that Wang and Kerre [17] proposed for fuzzy quantities ranking.

Remark 3. If $\inf \text{supp}(A) \geq 0$, then $M_f(A) \geq 0$.

Remark 4. If $\sup \text{supp}(A) \leq 0$, then $M_f(A) \leq 0$.

Remark 5.[22] If $M_f(A)$ be the parameterized defuzzification quasi-arithmetic means with power function generators $f(x) = x^r$ with $r \neq 0$, when r approaches $-\infty$, the defuzzification value is near to its lower bound, and when r approaches $+\infty$, the defuzzification value is near to its upper bound.

Let MI be an ordering method and S the set of fuzzy quantities for which the method MI can be applied and \mathcal{A} a finite subset of S . The axioms as the reasonable properties of ordering fuzzy quantities for an ordering approach MI are [17] as follows:

A-1 For an arbitrary finite subset \mathcal{A} of S and $A \in \mathcal{A}$; $A \succeq A$.

A-2 For an arbitrary finite subset \mathcal{A} of S and $(A, B) \in \mathcal{A}^2$; $A \succeq B$ and $B \succeq A$, we

should have $A \sim B$.

A-3 For an arbitrary finite subset \mathcal{A} of S and $(A, B, C) \in \mathcal{A}^3$; $A \succeq B$ and $B \succeq C$, we should have $A \succeq C$.

A-4 For an arbitrary finite subset \mathcal{A} of S and $(A, B) \in \mathcal{A}^2$; $\inf \text{supp}(A) > \sup \text{supp}(B)$, we should have $A \succeq B$.

A'-4 For an arbitrary finite subset \mathcal{A} of S and $(A, B) \in \mathcal{A}^2$; $\inf \text{supp}(A) > \sup \text{supp}(B)$, we should have $A \succ B$.

A-5 Let S and S' be two arbitrary finite sets of fuzzy quantities in which MI can be applied and A and B are in $S \cap S'$. We obtain the ranking order $A \succeq B$ by MI on S' if then if $A \succeq B$ by MI on S .

A-6 Let $A, B, A+C$ and $B+C$ be elements of S . If $A \succeq B$ and B , then $A+C \succeq B+C$ by MI on $A+C$ and $B+C$.

A'-6 Let $A, B, A+C$ and $B+C$ be elements of S . If $A \succ B$ by MI on A and B , then $A+C \succ B+C$ by MI on $A+C$ and $B+C$.

A-7 Let A, B, AC and BC be elements of S and $C \geq 0$. If $A \preceq B$ by MI on $\{A, B\}$, then $AC \preceq BC$ by MI on $\{AC, BC\}$.

Remark 6. The function MI has the properties (A-1), (A-2), ..., (A-5).

Remark 7. If $A \preceq B$, then $-A \succeq -B$.

Hence, this article can infer ranking order of the images of the fuzzy numbers.

5 Numerical Examples

In this section, the researchers compare proposed method with the others in [6, 18, 19, 20]. Throughout this section the researcher assumed that $f(x) = x^r$ with $r = 3$.

Example 5.1 Consider the following sets.

Set1: $A=(0.4,0.5,1)$, $B=(0.4,0.7,1)$, $C=(0.4,0.9,1)$.

Set2: $A=(0.3,0.4,0.7,0.9)$ (trapezoidal fuzzy number), $B=(0.3,0.7,0.9)$, $C=(0.5,0.7,0.9)$.

Set3: $A=(0.3,0.5,0.7)$, $B=(0.3,0.5,0.8,0.9)$ (trapezoidal fuzzy number), $C=(0.3,0.5,0.9)$.

Set4: $A=(0.3,0.5,0.8,0.9)$ (trapezoidal fuzzy number), $B=(0.2,0.5,0.9)$, $C=(0.1,0.6,0.8)$.

To compare with other methods researchers refer the reader to Table (5.1).

Table (5.1)

Comparative results of Example (5.1)

Authors	Fuzzy Number	Set1	Set2	Set3	Set4
Proposed method	A	0.6603	0.5650	0.5129	0.5226
	B	0.7208	0.6565	0.6510	0.5698
	C	0.7878	0.7093	0.5934	0.5386
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec C \prec B$
Sing Distance method with $p=1$	A	1.2000	1.1500	1.0000	0.0950
	B	1.4000	1.3000	1.2500	1.0500
	C	1.6000	1.4000	1.1000	1.0500
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec B \sim C$
Sing Distance method with $p=2$	A	0.8869	0.8756	0.7257	0.7853
	B	1.0194	0.9522	0.9416	0.7958
	C	1.1605	1.0033	0.8165	0.8386
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec B \prec C$
Distance Minimization	A	0.6	0.575	0.5	0.475
	B	0.7	0.65	0.625	0.525
	C	0.9	0.7	0.55	0.525
Result		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec B \sim C$
Abbasbandy and Hajjari (Magnitude method)	A	0.5334	0.5584	0.5000	0.5250
	B	0.7000	0.6334	0.6416	0.5084
	C	0.8666	0.7000	0.5166	0.5750
Result		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$B \prec A \prec C$
Choobineh and Li	A	0.3333	0.5480	0.3330	0.5000
	B	0.5000	0.5830	0.4164	0.5833
	C	0.6670	0.6670	0.5417	0.6111
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$
Baldwin and Guild	A	0.3000	0.2700	0.2700	0.4000

Table (5.1)(continued)

Authors	Fuzzy Number	Set1	Set2	Set3	Set4
	<i>B</i>	0.3300	0.2700	0.3700	0.4200
	<i>C</i>	0.4400	0.3700	0.4500	0.4200
Results		$A \prec B \prec C$	$A \sim B \prec C$	$A \prec B \prec C$	$A \prec B \sim C$
Chu and Tsao	<i>A</i>	0.2990	0.2847	0.2500	0.2440
	<i>B</i>	0.3500	0.3247	0.3152	0.2624
	<i>C</i>	0.3993	0.3500	0.2747	0.2619
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec C \prec B$
Yao and Wu	<i>A</i>	0.6000	0.5750	0.5000	0.4750
	<i>B</i>	0.7000	0.6500	0.6250	0.5250
	<i>C</i>	0.8000	0.7000	0.5500	0.5250
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec B \sim C$
Cheng distance	<i>A</i>	0.7900	0.7577	0.7071	0.7106
	<i>B</i>	0.8602	0.8149	0.8037	0.7256
	<i>C</i>	0.9268	0.8602	0.7458	0.7241
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec C \prec B$
CV uniform distribution	<i>A</i>	0.0272	0.0328	0.0133	0.0693
	<i>B</i>	0.0214	0.0246	0.0304	0.0385
	<i>C</i>	0.0225	0.0095	0.0275	0.0433
Results		$A \prec C \prec B$	$A \prec B \prec C$	$B \prec C \prec A$	$A \prec C \prec B$
CV proportional distribution	<i>A</i>	0.01830	0.0260	0.0080	0.0471
	<i>B</i>	0.0128	0.0146	0.0234	0.0236
	<i>C</i>	0.0137	0.0057	0.0173	0.0255
Results		$A \prec C \prec B$	$A \prec B \prec C$	$B \prec C \prec A$	$A \prec C \prec B$

Note that, in Table (5.1) and in set 4, for Sign Distance($p=1$) [1], Distance Minimization [3], Chu-Tsao and Yao-Wu methods, the ranking order for fuzzy numbers *B* and *C* is $B \sim C$, which is unreasonable. But this method has the same result as other techniques(Cheng distribution).

Example 5.2 The two symmetric triangular fuzzy numbers $A = (1, 3, 5)$ and $B =$

(2, 3, 4) shown in Fig. (5.2), taken from [20]. To compare with other methods this article refer the reader to Table (5.2).

In this Table, $A \sim B$ is the results of Sign Distance method with $p=1$, Magnitude method [2], Distance Minimization and Chen method, which is unreasonable. The results of proposed method is the same as Sign Distance method with $p=2$, i.e. $A \succ B$.

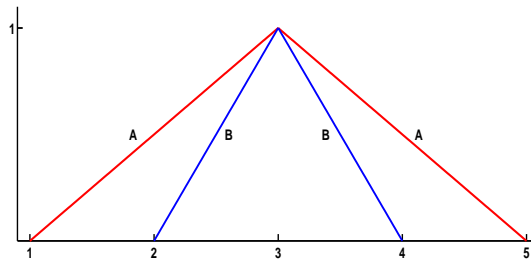


Fig. 5.2.

Table (5.2)

Comparative results of Example (5.2)

fuzzy number	new approach	Magnitude method	Sign Distance $p=1$	Sign Distance $p=2$	Distance Minimization	Chen Max-Min
A	3.2075	3	6	4.546	3	0.5
B	3.0545	3	6	4.32	3	0.5
Results	$A \succ B$	$A \sim B$	$A \sim B$	$A \succ B$	$A \sim B$	$A \sim B$

Example 5.3 Consider the three fuzzy numbers $A = (1, 2, 5)$, $B = (0, 3, 4)$ and $C = (2, 2.5, 3)$, (see Fig. (5.3)).

By using this new approach we have $M_f(A) = 2.9240$, $M_f(B) = 2.5962$ and $M_f(C) = 2.5165$. Hence, the ranking order is $C \prec B \prec A$ too. Obviously, the results obtained by "Sign distance" and "Distance Minimization" methods are unreasonable. To compare with some of the other methods in [20], the reader can refer to Table (5.3). Furthermore, to aforesaid example $M_f(-A) = -2.9240$, $M_f(-B) = -2.5962$ and $M_f(-C) = -2.5165$, consequently the ranking order of the images of three fuzzy

number is $-A \prec -B \prec -C$. Clearly, this proposed method has consistency in ranking fuzzy numbers and their images, which could not be guaranteed by CV-index method.

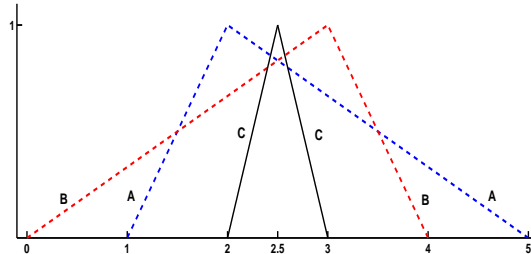


Fig. 5.3.

Table (5.3)

Comparative results of Example (5.3)

FN	new approach	Sign distance		Distance Minimization	Chu and Tsao
		$p=1$	$p=2$		
A	2.9240	5	3.9157	2.5	0.7407
B	2.5962	5	3.9157	2.5	0.7407
C	2.5165	5	3.5590	2.5	0.7500
Results	$C \prec B \prec A$	$C \sim B \sim A$	$C \prec B \sim A$	$C \sim B \sim A$	$A \sim B \prec C$

All the above examples show that the results of this new method are reasonable results. This method can be overcome the shortcoming of "Magnitude" method and "Distance Minimization" method.

6 Conclusion

The fuzzy number defuzzification method with continuous weighted quasi-arithmetic means was proposed in [24] and [22]. This parameterized defuzzification can be used as a crisp approximation with respect to a fuzzy quantity. In this article, the researchers will use this for ordering fuzzy numbers. The method can effectively rank various fuzzy numbers, their images(normal/ nonnormal/trapezoidal/general) and overcome

the shortcomings are found in the other techniques. The calculation of this method is far simpler than the other approaches.

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