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Ant colony optimization techniques for the hamiltonian p-median problem

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Abstract

Location-Routing problems involve locating a number of facilities among candidate sites and establishing delivery routes to a set of users in such a way that the total system cost is minimized. A special case of these problems is Hamiltonian p-Median problem (HpMP). This research applies the metaheuristic method of ant colony optimization (ACO) to solve the HpMP. Modifications are made to the ACO algorithm used to solve the traditional vehicle routing problem (VRP) in order to allow the search of the optimal solution of the HpMP. Regarding this metaheuristic algorithm a computational experiment is reported as well.

Keywords: combinatorial optimization, Hamiltonian p-Median Problem (HpMP), Ant Colony Optimization (ACO), Vehicle Routing Problem (VRP).

1 Introduction

Combinatorial optimization problem (COP) deals with finding optimal combinations or permutations of available problem components. Hence, it is required that the problem is partitioned into a finite set of components, and the combinatorial optimization algorithm attempts to find their optimal combination or permutation.

Many real world optimization problems may be represented as COPs in a straightforward way. Due to the practical importance of CO problems, many algorithms to

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tackle them have been developed. These algorithms can be classified as either exact or approximate methods. The fundamental property of CO problems is that they are often NP-hard and exact algorithms often lead to computation times too high for practical purposes. Thus, the development of approximate methods, in which we sacrifice the guarantee of finding optimal solutions for the sake of getting good solutions in a significantly reduced amount of time, has received more and more attention in the last 30 years.

In the early years, specialized heuristics were typically developed to solve complex CO problems (Glover coined in 1986 the term metaheuristics for such methods). Ant colony optimization (ACO) is one of the most recent metaheuristic techniques. The inspiring source of ACO algorithms are real ant colonies. More specifically, ACO is inspired by the ants' foraging behavior. At the core of this behavior is the indirect communication between the ants by means of chemical pheromone trails, which enables them to find short paths between their nest and food sources. This characteristic of real ant colonies is exploited in ACO algorithms in order to solve, for example, CO problems.

In this paper we present the application of ACO algorithms to the hamiltonian p-median problem (HpMP), which has been introduced by Branco and Coelho (1990). The HpMP arises in practical applications belonging to the location theory, in particular, to the location-routing problems, e.g. milk collecting. Therefore, HpMP can be considered as a useful tool of supply chain process.

The plan for the rest of this paper is as follows. In section 2 we introduce the HpMP and transformation of it to the vehicle routing problem (VRP). In section 3 modifications are made to the ACO algorithm used to solve the VRP in order to allow the search of the optimal solution of the HpMP. An empirical analysis is reported in section 4 and finally, section 5 draws the conclusive remarks.

2 The HpMP and modification of it to the VRP

The HpMP can be formulated through a graph theory model as follows. Let G =(V,A) be a complete directed graph with cost c_{ij} associated with each arc (i,j). Then the HpMP consists of selecting p vertices (facilities) from V and assigning vertices in V (customers) to p disjoint directed circuits, each of which consisting exactly one of these p vertices, such that the total distribution cost is minimized. It is also clear that each vertex of a circuit can be chosen as a facility location. In summary, the HpMP is equivalent to determining p disjoint directed circuits covering the vertex set V, with respect to some objective function.

Regarding the ILP-formulations of the HpMP, we refer to Glaab and Pott (2000) and Zohrehbandian (2007). In this section, we will show that the HpMP problem can be transformed to the VRP problem. Hence, we can modify the suggested approaches for solving VRP to solve the HpMP. ACO algorithms are one of these approaches.

For transforming HpMP to VRP, suppose that vertex 0 is a virtual depot. Let $c_{io} = c_{oi} = 0$ for all i=1,...,n. Furthermore, for each vertex i (customer) we assign a non-negative demand $q_i = 1$ and for each vehicle, in transformed problem, we assign a non-negative capacity $Q = n$. Figure 1 shows that each solution of the transformed VRP problem is corresponds to a solution of the original HpMP. Note that the objective value of the HpMP problem is equal to the addition of objective value of the VRP problem and the cost of arcs (i,j), which both vertex i and j are adjacent to virtual depot in a same circuit.

3 ACO metaheuristics for the Hamiltonian p-median problem

Our general approach of using ACO for the HpMP follows the ant colony algorithms of Bell and McMullen (2004) used for the VRP. Initially, each ant starts at the virtual

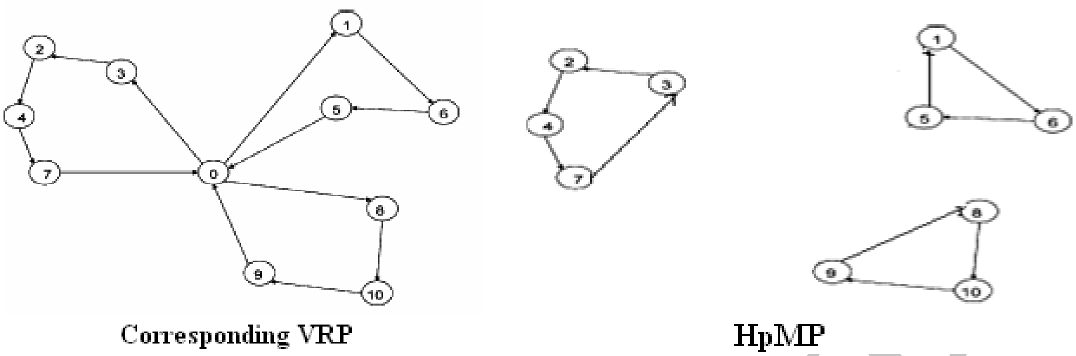

Figure 1. A feasible solution for the HpMP and the corresponding VRP.

depot and the set of vertices (customers) included in its solution is empty. The ant selects the next customer to visit from the list of feasible locations with a probabilistic formula and those vertices already visited by the ant are sorted in a working memory. When being at a vertex i, an ant k chooses to go to a still unvisited vertex j uses the following probabilistic formula:

$$j = \begin{cases} ArgMax\{(\tau_{iu})(\eta_{iu})^{\beta}\} & q \leq q_o, \\ s & q > q_o. \end{cases} \quad (1)$$

where τ_{iu} is equal to the amount of pheromone on the arc between the current vertex i and a still unvisited vertex u. The value η_{iu} is defined as the inverse of the distance between the two customer locations, and the parameter $\beta > 0$ establishes the importance of distance in comparison to pheromone quantity in the selection algorithm. Vertices already visited by the ant k are sorted in the ants working memory M_k and are not considered for the next selections. The value q is a random uniform variable [0,1] and the value q_o is a parameter. When each selection decision is made, the ant selects the arc with the highest value from equation 1, unless q is greater than q_o. In this case, the ant selects a random variable s to be the next vertex to visit based on the probability distribution of P_{ij}, which favors short arcs with high levels of pheromone:

$$P_{i,j} = \begin{cases} \frac{(\tau_{iu})(\eta_{iu})^{\beta}}{\sum_{u \notin M_k}(\tau_{iu})(\eta_{iu})^{\beta}} & j \notin M_k, \\ \\ 0 & j \in M_k. \end{cases} \quad (2)$$

Using formulas (1) and (2), k-th ant may either follows the most favorable path already established or may randomly select a path to follow based on a probability distribution established by distance and pheromone accumulation. Furthermore, each time the ant k visits the depot, number of circuits which are constructed by ant k adds by one. Meanwhile, this selection process continues until at least 3(p-t) vertices remain to visit, where t is the number of circuits which are constructed by ant k. This is for constructing a feasible solution of the HpMP problem. In this case, the ant k ends its circuit at the depot and makes a new circuit by starting from it. This process is continued until each customer is visited and the circuits are completed.

Our ACO algorithm constructs a complete VRP solution for the first ant prior to the second ant starting its movement, and as we mentioned in section 2, the total distribution cost (objective value of the HpMP) is computed by addition of the cost of arcs (i,j), which both customer i and j are adjacent to virtual depot in a same circuit.

In order to improve future solutions, the pheromone trails of the ants must be updated to reflect the ant's performance and the quality of the solutions found. This updating is a key element to the adaptive learning technique of ACO and helps to ensure improvement of subsequent solutions. Trail updating includes local updating of trails after individual solutions have been generated and global updating of the best solution route after a predetermined number of solutions have been accomplished.

First, local updating is conducted by reducing the amount of pheromone on all visited arcs in order to simulate the natural evaporation of pheromone and to ensure that no one path becomes too dominant. This is done with the following local trail updating equation $\tau_{ij} = (1-\alpha)\tau_{ij} + \alpha\tau_o$, where α is a parameter that controls the speed of evaporation and τ_o is an initial pheromone value assigned to all arcs.

After a predetermined number of ants construct feasible routes, global trail updating is performed by adding pheromone to all of the arcs included in the best route founds by one of the ants. Global trail updating is accomplished according to the following relationship $\tau_{ij} = (1-\alpha)\tau_{ij} + \alpha D^{-1}$, where D^{-1} is the inverse value of the best solution founds by one of the ants.

This updating encourages the use of shorter routes and increases the probability that future routes will use the arcs contained in the best solutions. This process is repeated for a predetermined number of iterations and the best solution from all of the iterations is presented as an output of the model and should represent a good approximation of the optimal solution for the problem.

Furthermore, for attainment of improved solutions to the problem, we can use the route improvement strategies in the algorithms. One of these strategies involves the inclusion of a local exchange procedure to act as an improvement heuristic within the routes found by individual ants. One of the techniques used for this purpose is the common 2-opt heuristic where all possible pairwise exchanges of customer locations visited by individual vehicle are tested to see if an overall improvement in the objective function can be attained.

4 Numerical example

In this section we will present the results of our numerical analysis. In order to motivate our approach we apply a simple network example involving just twelve vertices. The costs assigned to each arc of this network (c_{ij}'s) have been depicted in table 1. For p=3 (number of facilities), result of the execution of our approach is as follows which is the same as the optimal solution of the problem (computed based on the branch and bound algorithm).

$x_{2,5} = x_{5,6} = x_{6,10} = x_{10,2} = 1$,

$x_{1,3} = x_{3,9} = x_{9,8} = x_{8,11} = x_{11,1} = 1,$

$x_{7,12} = x_{12,4} = x_{4,7} = 1.$

	1	2	3	4	5	6	7	8	9	10	11	12
1	0	12	11	10	25	18	9	16	26	17	32	24
2	8	0	10	13	7	21	8	9	8	12	7	16
3	20	15	0	7	8	14	5	6	5	11	8	25
4	10	10	12	0	10	13	4	5	10	17	15	28
5	31	17	28	24	0	13	12	14	29	32	19	20
6	17	30	22	10	15	0	12	16	10	9	25	11
7	14	16	19	15	12	14	0	20	13	10	24	13
8	19	8	10	7	10	24	8	0	7	20	6	13
9	25	11	18	20	9	15	3	2	0	13	6	14
10	15	9	20	9	9	13	12	18	11	0	10	15
11	12	13	19	21	18	14	20	11	16	12	0	17
12	18	27	14	6	37	25	7	21	15	9	7	0

Table 1. Costs assigned to each arc of the network

5 Conclusion

In this paper we present an ant colony optimization for solving the hamiltonian p-median problem, which is an applicable algorithmic approach for solving NP-complete problems. Our general approach of using ACO for the HpMP is based on the modification of the ant colony algorithm used for the VRP in order to allow the search of the optimal solution of the HpMP. This is due to the fact that in the context of routing problems, VRP has been one of the widely studied topics, where goods must be picked up and/or delivered for a geographically dispersed set of customers.

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