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Remarks On Tornado Dynamics

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Abstract

Mathematical modeling of Tornado paved the way in understanding the cause [1], basic structure [2], and the dynamical behavior aspect [3] of the Tornado in the present model. Modeling in Tornado have been basically focused onto estimating the flow parameters such as velocity [radial, Azimuthal, and axial] and pressure across the rotational axis so that, one can estimate the speed with which Tornado is moving and amount of pressure it can induce onto the structure. Studies from mathematical aspects have been incorporated to include nonlinear aspect of the flow phenomena in the Tornado and studying rotational velocity and radial velocity aspect [4].

Keywords: Tornado dynamics, Swirl velocity, Pressure, Azimuthal velocity.

1 Introduction

Thunder storms act as earth's cooling agent by drawing hot, moist from the ground. When temperatures vary greatly between the ground and atmosphere this air rises rapidly condenses and forms thunder heads. This heated updraft collides with higher cold air and creates turbulent winds surrounding it. These are forced into a violent upward spin and referred as Tornadoes.Cyclones or Tornadoes are quite destructive in nature. The exact cause of such phenomena is still under investigations however by

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and large the phenomenon is attributed due to atmospheric and metrological variations in the region it gets affected. Once the Epicenter of this Tornado is located, the speed with which it proceeds and the destruction it may cause can be estimated and forecasted which will then help the evacuation teams for getting prepared in case of eventualities. It is estimated that, the destruction of property in USA alone exceeds few million dollar every year due to this Tornado in addition to the casualties to human lives. Studies pertaining to Tornadoes help in understanding the basic causes and helps in protective measures. The other models on Tornadoes [5] have studies effects of axial and radial velocity over a spread angle (r/z). The model also aims at estimating of pressure variations across the Tornado. The method adopted basically of series solution approach to the order of variation principles. Rotational (Azimuthal) aspect of velocity has not been studied here in Takhar's [5] Works however, the concept of swirl velocity has been introduced based on theory of Circulation. One of the basic motivations behind the present studies is to develop a model which accounts numerical approach incorporating most of the futures of the non-linear models [6] and also incorporates effects of exact solutions in it. The model has been developed using order of magnitude analysis developed by [7] and computational aspect by numerical approach. The structure and dynamics of Tornado has been modeled in the present investigations. The flow parameters such as velocities (Radial, Azimuthal, and Axial), pressure, and swirl velocity have been analyzed by the method of order of magnitude analysis. Use of MATLAB software has been made use of for the numerical computation of flow variables. The results have been compared with other theoretical and experimental works. The shortcomings in the literature have been cited.

2 Analysis

Steady axi-symmetrical vortex motion in a cylindrical frame of reference (r, θ, z) with the origin fixed in a position relative to the source has been considered in the present analysis. The flow diagram is depicted in Figure1.

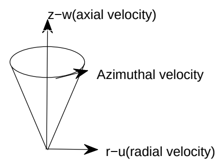

Figure 1 : Azimutal velocity with distance.

The velocity with respect to r,θ, and z-direction are assumed to be u, v, and w respectively. The presence of Coriolis force is accounted (initially at the time of development of the model) due to rotational aspect of the fluid at the model. However this is neglected at later stages when compared with the rotational motion of the Tornado. The velocity is assumed to be the function of r and z due to axi-symmetric nature of the Tornado. The governing equations of motion are

$$\partial\rho/\partial(t) + \nabla.(\rho v) = 0 \tag{1}$$

$$\rho\frac{Dv}{Dt} = -\nabla P + F + \mu\nabla^2 v \tag{2}$$

Where ρ, P, and F , u, v, w, are air density, pressure , body force , radial velocity, Azimuthal and vertical velocity respectively. The equation (2) can be solved by numerical methods.

$$(1/r)(ru)_r + w_z = 0 \tag{3}$$

$$uu_r + wu_z - v^2/r - 2\Omega v = -(1/\rho)p_r + \nu(u_{rr} + 1/ru_r - u/r^2 + u_{zz}) \tag{4}$$

$$uv_r + wv_z + uv/r + 2\Omega v = \nu(v_{rr} + 1/rv_{rr} - v/r^2 + v_{zz}) \tag{5}$$

$$uw_r + ww_z + = -(1/\rho)p_z + \nu(w_{rr} + 1/rw_r + w_{zz}) \tag{6}$$

Where u, v, and w are called Radial, Azimuthal, and Axial velocity respectively, ρ,ν, and Ω are air density, kinematic viscosity, and angular velocity respectively and $u_r =$

$\partial u/\partial r$. Initial conditions required for solving the equations (3) to (6) are assumed as

$$at\ r = 0\ we\ have\ u = v = 0,\ w \neq 0(finite), w_r = 0, p_r = 0(finite), p_r = 0 \quad (7)$$

Conservation of mass and momentum have been related by order of magnitude analysis approached proposed by Makofski [7]. Following magnitude analysis has been adopted in the present model. In the order of magnitude we define:

$$u \simeq U, v \simeq V, w \simeq W, r \simeq R \quad (8)$$

equation (3) simplifies to

$$(1/R).(R.U/R) + W/Z = 0 \Longrightarrow U/R + W/Z = 0 \quad (9)$$

Ro= Rossby number= Inertial force/Coriolis force= $(V.W/Z)/\Omega.U$= $V/(R.\Omega)$. It is to be noted that when $Ro \gg 1$, $C_f = 0$ where C_f is Coriolis force (=$\Omega.U$). Under the assumption of order of magnitude analysis, governing equations (3-6) can be simplified to

$$i)P = (\rho.M^2/\nu^2.z^2).h(\eta), ii)\eta = (M^{1/2}/\nu.z).r, iii)\Gamma = r.v = M^{1/2}.g(\eta), iv)\Psi = \nu.z.f(\eta) \quad (10)$$

Analytical approach adopted by Takhar [5] has been incorporated while arriving onto equation (10).Where P,Γ,ν, andρ are pressure, swirl velocity, air viscosity, and air density respectively. And

$$M = \int_0^\infty (W^2 + P/\rho - \nu.W_z)rdr \quad (11)$$

We have also assumed Ψ is stream function and is related to u and w as

$$u = -(1/r).\Psi_z,\ w = (1/r).\Psi_r \quad (12)$$

equation (10) for Ψ is used for getting radial(u), and axial velocity (w)components. That is

$$u = -(1/r)[\nu.f - \nu.z.M^{1/2}f'/(\nu.z^2)] \Rightarrow f' - f/\eta = u.z/M^{1/2} \quad (13)$$

$$w = (1/r).\Psi_r = [M^{1/2}/(\nu.z)].f'/\eta \tag{14}$$

$$v = M^{1/2}.g/r \Rightarrow g/\eta = (\nu.z/M).v \tag{15}$$

Substituting eqn.(9) eqn.(4), we get [It is assumed that $C_f = 0 and (R/Z) \ll 1$]

$$U^2/R + W.U/Z - V^2/R - 0 = -P/(\rho.R) + 0 \tag{16}$$

Assuming $\nu\mathrm{U}/R^2 \gg \nu\mathrm{U}/Z^2$, eqn.(16) reduces to

$$V^2/R = P/(\rho.R) \tag{17}$$

Equation (17) can be written in differential form as

$$V^2/r = (1/\rho).p_r \tag{18}$$

Governing equations (3)-(6) simplifies to

$$(1/r)(ru)_r + w_z = 0 \tag{19}$$

$$-v^2/r - 2\Omega v = -(1/\rho)p_r \tag{20}$$

$$uv_r + wv_z + uv/r = \nu(v_{rr} + 1/rv_{rr} - v/r^2) \tag{21}$$

$$uw_r + ww_z + = -(1/\rho)p_z + \nu(w_{rr} + 1/rw_r) \tag{22}$$

It is assumed that, Coriolis force is small in comparison to other forces, and

$$V \simeq W = \nu z/r^2 \tag{23}$$

The other comparative analysis has also been used (which is referred in Takhar's [5] research works) while arriving onto eqs. (21) and (22).

3 Estimation of velocity:

Substituting for p from eqn.(10)(i) in eqn.(20), we get

$$g^2 = \eta^3.h' \tag{24}$$

Where h' = $dh/d\eta$. Similarly substituting for v from eqn.(10)(iii) in eqn.(21) and also u and w from eqs.(13) and (14) in eqn.(10) and substituting we get

$$\eta . g'' - g' + fg' = 0 \tag{25}$$

Similarly equation (22) can be simplified with the help of eqn.(13) and (14) to

$$f''' = (1/\eta^2)[(f' - f/\eta)(-f' + \eta f'') - \eta^2 f' f'' - 2\eta^3 h - \eta^4 h' - 2f + f' + \eta f''] \tag{26}$$

We need to solve three equations [eqs.(23)-(25)] in three unknowns [f, g, and h]. Use of MATLAB Software has been made use of while solving system of Equations [23-25]. Initial condition [eqn.(7)] has been made use while solving for f, g, and h. Having known f, g, and h by numerical approach, u, v, and w can be computed by using equation (13) and eqn.(14) respectively. The variation of u, v, and w has been observed withη. It is assumed that the value of M=1, z =1000m for the computational purpose. Density and kinetic viscosity which have been listed in Table.1 has been used for the computation purpose in the present model. The data on air density and kinetic viscosity used are for the temperature of 40c.

4 Results

The results of proportional radial (u), Azimuthal (v), and of axial velocity (w) are shown in Fig.2 [the results need to be multiplied or divided by respective parameters for getting velocities]. The variation has been observed with η (rate of r/z). It is observed that, radial velocity has fluctuations over increased η (Fig.2a). The magnitude of velocity is very negligible and almost approximates to zero. The results on Azimuthal velocity (Fig.2b) indicates that, velocity increases to maximum value at η=10 and then decrease to 0.22 as η approaches to 50. The maximum Azimuthal velocity related with the present model is 20m/s.

The result on axial velocity (Fig.2c) indicates that velocity decreases with increase in η. The maximum axial velocity (non dimensional) found is 0.1 along the z-axis. The results of radial and axial velocities have been compared with the results of Takhar [5] and shown in respective figures of radial and Azimuthal velocities [Fig.2a and 2c]. The results on radial velocity have found no significance with present approach whereas the results on axial velocity have been found to be lower in comparison to Takhar [5] works. The discrepancy in the results are mainly attributed due to approach adopted by Takhar [5] is somewhat approximate in nature (adaptation of Blasius series approach) whereas the present model, it accounts for numerical approach.

The results on axial velocity could not be compared due to non-availability of Takhar works on Azimuthal velocity. In order to compare our model, the results of the present model has been compared to that of experimental results on Tornado captured through satellite images of Stockton (USA) Tornado [8] [Fig.3]. The results of the present findings have been lower in comparison to the experimental observations. However the trend of Azimuthal velocity of the present model have found to be matching very nice to that observed experimentally. This justifies that, the approach adopted in the present model is acceptable. The lower values can be justified that, the model can be improved by adding parameters like Coriolis force, Turbulence parameters. It could be said here that additional experimentations is also required to justify the accuracy of the experimental results also.

In addition to the velocities, the model has also been computed for swirl velocity and for the pressure variations. The computed results have been shown in Fig.4a and Fig.4b. The results indicate that swirl velocity and pressure increases with increase in η and found that the present computed results is greater than the one reported by Takhar's model [5] on swirl velocity and on pressure. The simulated graph of Tornado in 3D for the present model has been drawn by using equation of $\eta = (M^{1/2}/(\eta.z)).r$ and MATLAB Software and shown in Fig.4. The simulated graph depicts exact replica of the Tornado with all the relevant parameters accounted for the computational purpose.

5 Conclusion

Steady axi-symmetric Tornado flow has been modeled in the present investigations. Order of magnitude analysis has been used for the solving the flow variables. Numerical method has been chosen for the computational purpose. The computed results have been compared with other theoretical and experimental findings. The results of present findings have found to be in good agreement with the experimental findings however they found to be lower in comparison to that of other theoretical works.

Table.1: Data of flow parameters.

$Temperature$	$\rho(kg/m^3)$	$kinetic\ viscosity(\nu \times 10^{-6})$
0	1.293	13.3
20	1.205	15.11
40	1.127	16.97

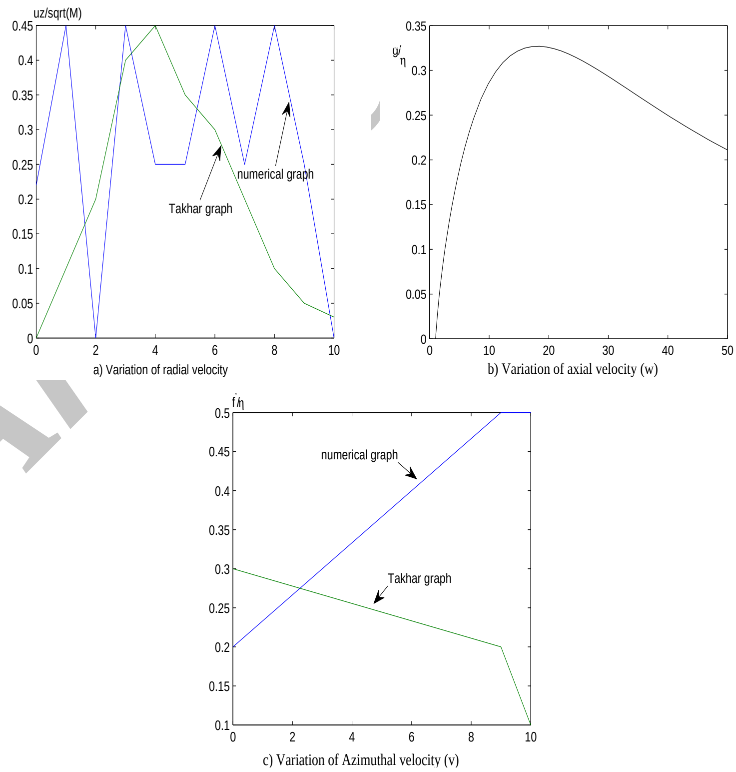

Figure 2(a-c) : Variation of non-dimensional radial, Azimuthal, and axial velocity.

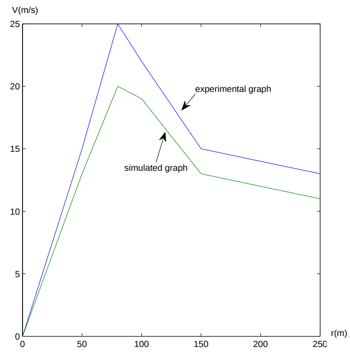

Figure 3 : Variation of Azimuthal velocity with radius.

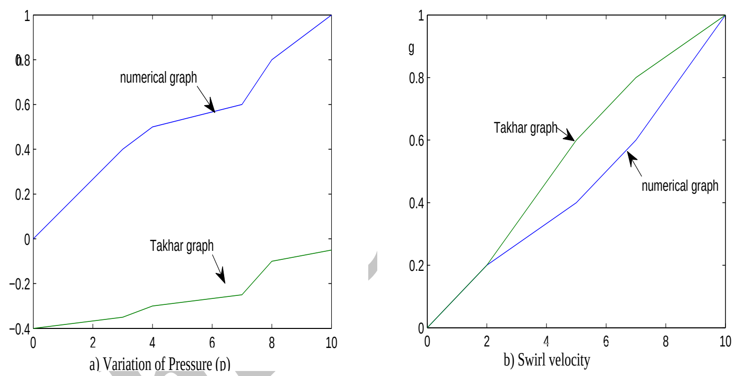

Figure 4(a-b) : Variation of pressure and swirl velocity.

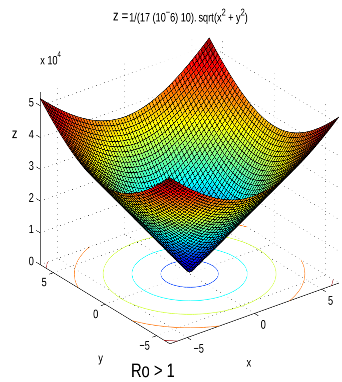

Figure 5 : The graph of simulated tornado.

References

[1] Nolan D.S. (2008) "A new scaling for tornado-like vortices," J. Atmos. Sci, 62.

[2] Lewellen D.C, Lewellen W.S., Xia J. (2007) "The influence of a local swirl ratio on tornado intensification near the surface," J. Atmos. Sci, 57.

[3] Nolan D.S., Almgren A.S., Bell J.B. (2000) "Studies of the relationship between the structure and dynamics of tornado-like vortices and their environmental forcing," Lawrence Berkeley National Laboratory, Report no. LBNL-47554.

[4] Ward N.B (1972) "The exploration of certain features of tornado dynamics using a laboratory model," J. Atmos. Sci, 29, 1194-1204.

[5] Takhar H.s., The reports on mathematical models of the geophysical vortices, Manchester metropolitan university, UK, 1990.

[6] Burgers J.M. (1948) "A mathematical model illustrating the theory of turbulence," Advances in Applied Mechanics, 1(197), Academic Press, New York.

[7] Makofski R.A., An order of magnitude analysis of the navier-stokes equations in the viscous layer regime in hypersonic flow, DTIC publication, Virginia, U.S.A, 1959.

[8] Robin L. Tanamachi, Research Laboratory of Meteorology School, University of Oklahoma, Norman, Oklahoma, U.S.A, 1999.