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Application of Homotopy Perturbation Method for Solving Brinkman Momentum Equation for Fully Developed Forced Convection in a Porous Saturated Channel

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Abstract

In this paper an analytical solution is presented to solve Darcy-Brinkman equation for fully developed forced convection in a porous channel surrounded by two isoflux parallel plates, using homotopy perturbation method (HPM). The results show an excellent agreement between HPM and exact solution of the equation in various values of key parameters.

Keywords: Homotopy Perturbation Method, Porous Media, Brinkman Momentum Eqaution.

1 Introduction

Forced convection in porous media is so important for researchers due to its wide applications such as electronic cooling, solar collectors and ground water studies. There is convenient information about this topic in [1-8]. Fully developed Forced convection in channels has been considered by many authors to investigate fluid flow and heat transfer characteristics in channels filled with porous media with various cross sections.

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For instance, Kaviany [9] studied laminar flow in a porous channel bounded by two isoflux parallel plates numerically by use of Brinkman extended Darcy model. Vafai and Kim [10] using this model, analytically considered forced convection in thermally fully developed flow between two flat plates. Amiri and Vafai [11] considered the effects of non-thermal equilibrium and dispersion on the fully develop flow and heat transfer characteristic in a channel filled with a porous medium with variable porosity numerically. Hung et al. [12] studied on fully developed forced convection in a homogeneous porous medium and a closed form solutions obtained for the temperature distributions in the transverse direction. Nield et al. [13] considered forced convection in a channel filled with fluid-saturated porous medium with isothermal or isoflux boundaries. Haji-Sheikh and Vafai [14] used Brinkman's model to analyze flow and heat transfer in porous media imbedded inside various-shaped ducts and presented an exacta solution for both rectangular and circular ducts. Nield [15] considered, thermally developing forced convection in a porous medium inside a parallel plate channel with uniform wall temperature and axial conduction and viscous dissipation effects. Hooman et al [16] examined first and the second law characteristics of fully developed forced convection inside a porous-saturated duct of rectangular cross-section by use of Darcy–Brinkman flow model.

Homotopy Perturbation Method (HPM) is a brilliant methods that introduced by He [17, 18]. These days it has been utilized by many researchers to find an analytical solution of linear and non-linear ordinary and partial differential equations (ODEs and PDEs). For example, Dehghan and Shakeri [19] by use of this method solved Partial differential equation arising in modeling of flow in porous media. Siddiqui et al. Ganji and sadighi [20] solved nonlinear heat transfer and porous media equations by use of HPM and variational iteration method, and found a convincable agreement between these methods and exact solution. Biazar et al. [21] solved general form of porous medium equation by HPM and compared with the Adomian decomposition with a very good agreement between them. There are many other related works in [22-25].

The main goal of this paper is to solve Darcy-Brinkman momentum equation for fully developed forced convection in a porous channel bounded by two isoflux parallel plates, using homotopy perturbation method. Further more the effect of some existed parameters in the equation are considered on the dimensionless velocity profile and Nusselt number.

2 The Basic of Homotopy Perturbation Method

Following equations denotes the fundamental of the HPM

$$A(u) = f(r) \qquad r \in \Omega, \tag{1}$$

with the following boundary conditions:

$$B(u, \frac{\partial u}{\partial n}) = 0 \quad r \in \Gamma. \tag{2}$$

where $f(r)$ is a known analytical function, A is a general differential operator, B is a boundary operator, $\frac{\partial}{\partial n}$ symbolize differentiation along the normal vector drawn outward from Ω, and Γ is boundary of the domain Ω.

$A(r)$ would be separated to two linear and nonlinear parts. Hence, Eq. 1 can be rewritten as follows:

$$L(u) + N(u) - f(r) = 0. \tag{3}$$

Then the Homotopy perturbation structure is shown as follows:

$$D(g, p) = (1 - p)[L(g) - L(u_0)] + p[A(g) - f(r)] = 0, \qquad r \in \Omega, \quad p \in [0, 1], \tag{4}$$

where

$$g(r, p) : \Omega \times [0, 1] \to R \tag{5}$$

In Eq. 4, $p \in [0, 1]$ is an embedding parameter which increases from 0 to 1 and is the initial guess of Eq. 1 that satisfies boundary conditions. Due to Eq. 4, we have:

$$D(g, 0) = L(g) - L(u_0) = 0, \tag{6}$$

$$D(g,1) = A(g) - f(r) = 0. \tag{7}$$

By using the perturbation technique, we assume that the solution of Eq. 4 can be written as a power series in p, as following:

$$g = g_0 + g_1 p + g_2 p^2 + g_3 p^3 + ... \tag{8}$$

By assuming $p = 1$, we will have:

$$u = \lim_{p \to 1} g = g_0 + g_1 + g_2 + g_3 + ... \tag{9}$$

3 Governing equations

Figure 1 shows a schematic of fully developed flow in porous channel bounded by two isoflux parallel plates.

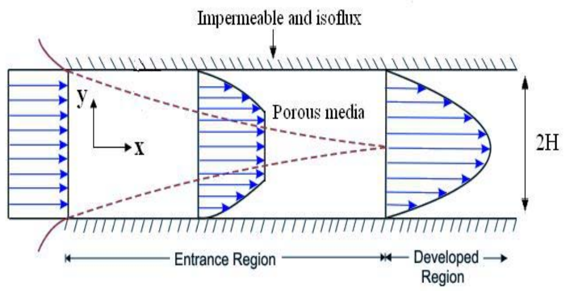

Figure 1. Schematic of a flow in a channel filled with porous media.

The governing equations of a viscous and incompressible fluid in fully developed flow and heat transfer conditions introduces as:

$$-\frac{dp}{dx} + \mu_{eff}\frac{d^2u}{dy^2} - \frac{\mu u}{K} = 0, \tag{10}$$

where μ and ρ are viscosity and density of fluid, respectively. Also, μ_{eff} is the effective viscosity of the fluid in the porous medium. K, u and p are permeability, form coefficient, velocity and pressure, respectively. So, the dimensionless variables are:

$$\bar{y} = \frac{y}{H}, \quad \bar{x} = \frac{x}{PeH}, \quad \bar{u} = \frac{\mu u}{GH^2}. \tag{11}$$

By substitution Eq. (11) in Eq. 10, we will have the dimensionless form of the Eq. 10 with following boundry conditions:

$$1 + M\frac{d^2\bar{u}}{d\bar{y}^2} - \frac{\bar{u}}{Da} = 0. \tag{12}$$

No-slip boundary condition at top wall and symmetry conditions at centerline are defined as

$$\overline{u}(1) = 0, \ \ \overline{u}'(0) = 0. \tag{13}$$

Here, we have

$$M = \frac{\mu_{eff}}{\mu}, \ \ Da = \frac{K}{H^2}. \tag{14}$$

And also, the Peclet number Pe is defined as:

$$Pe = \frac{\rho C_P HU}{K}. \tag{15}$$

The bulk mean temperature and mean velocity are defined as:

$$T_m = \frac{1}{HU}\int_0^H uT\,dy, \quad U = \frac{1}{H}\int_0^H u\,dy. \tag{16}$$

Further dimensionless variable are introduced as:

$$\overline{U} = \frac{\bar{u}}{U}, \quad \theta = \frac{T - T_w}{T_m - T_w}' \tag{17}$$

where T_m and T_w are the bulk mean temperature and downstream wall temperature respectively. The Nusselt number is determined as:

$$Nu = \frac{2Hq''}{K_\varphi(T_w - T_m)}, \tag{18}$$

where q" is wall heat flux. Homogeneity and local thermal equilibrium is assumed. The steady-state energy equation is:

$$\rho\,C_P u\frac{\partial T}{\partial x} = k\frac{\partial}{\partial y}(\frac{\partial T}{\partial y}), \tag{19}$$

where k, C_p are thermal conductivity and specific heat, respectively. Due to the first law of thermodynamics we have:

$$\frac{\partial T}{\partial x} = \frac{2q''}{\rho C_P H U}. \tag{20}$$

By substituting Eq.20 into Eq. 19 we have:

$$\frac{2q''u}{HU} = K\frac{\partial}{\partial y}(\frac{\partial T}{\partial y}). \tag{21}$$

Finally, the dimensionless form of Eq. 18 becomes:

$$2\frac{d^2\theta}{d\bar{y}^2} + \overline{U}Nu = 0, \tag{22}$$

where the boundary conditions are:

$$\left.\frac{d\theta}{d\bar{y}}\right|_{\bar{y}=0} = 0, \quad \theta|_{\bar{y}=1} = 0. \tag{23}$$

4 Method of solution

To solve Eq. 12 the linear operator which is the linear part of the Eq. 8 is determined as:

$$L(\zeta) = M\frac{d^2\zeta}{d\bar{y}^2} - \frac{\zeta}{Da}, \tag{24}$$

where ζ is an auxiliary function. The second step is to guess an arbitrary initial approximation as follows:

$$\bar{u}_{ini}(\bar{y}) = \bar{y}^2 - 1, \tag{25}$$

where subscript *ini* refers to an initial approximation of Eq. 12. According Eq. 8 and HPM the following homotopy equation would be constructed as:

$$D\left(g,p\right) =$$

$$(1-p)\left[M\frac{d^2g(\bar{y})}{d\bar{y}^2} - \frac{g(\bar{y})}{Da} - (M\frac{d^2\bar{u}_{ini}(\bar{y})}{d\bar{y}^2} - \frac{\bar{u}_{ini}(\bar{y})}{Da})\right] + p\left[1 + M\frac{d^2g(\bar{y})}{d\bar{y}^2} - \frac{g(\bar{y})}{Da}\right] = 0 \tag{26}$$

Substituting Eq. 4 into Eq. 22 and equating the coefficient with identical power of p we obtain a system of equations with $n+1$ differential equations to be solved simultaneously; as follows where n is the order of p in Eq. 4.

Zeroth-order:

$$\frac{d^2 g_0(\bar{y})}{d\bar{y}^2} - \frac{g_0(\bar{y})}{MDa} - 2 + \frac{\bar{y}^2 - 1}{MDa} = 0. \tag{27}$$

First order:

$$-\frac{g_1(\bar{y})}{MDa} + \frac{d^2}{dY^2} g_1(\bar{y}) + \frac{1}{M} - \frac{\bar{y}^2 - 1}{MDa} + 2 = 0. \tag{28}$$

Second order:

$$\frac{d^2}{d\bar{y}^2} g_2(\bar{y}) - \frac{g_2(\bar{y})}{MDa} = 0. \tag{29}$$

Third order:

$$\frac{d^2}{d\bar{y}^2} g_3(\bar{y}) - \frac{g_3(\bar{y})}{MDa} = 0. \tag{30}$$

And the boundary conditions are:

$$g_i\,(1) = 0, \quad g_i'\,(0) = 0 \qquad i \geq 0 \tag{31}$$

Solving Eqs. 27- 30 with corresponding boundary conditions, the following functions can be obtained successively:

$$g_0(\bar{y}) = \bar{y}^2 - 1, \tag{32}$$

$$g_1(\bar{y}) = -\frac{Da\, e^{\frac{\bar{y}}{M\sqrt{Da}}} - Da\, e^{-\frac{\bar{y}}{\sqrt{M}\sqrt{Da}}} - e^{\frac{1}{\sqrt{M}\sqrt{Da}}} - e^{\frac{-1}{\sqrt{M}\sqrt{Da}}} + R(\bar{y})}{e^{\frac{1}{\sqrt{M}\sqrt{Da}}} + e^{-\frac{1}{\sqrt{M}\sqrt{Da}}}}, \tag{33}$$

where

$$R(\bar{y}) = -Da\, e^{\frac{1}{\sqrt{M}\sqrt{Da}}} - Da\, e^{-\frac{1}{\sqrt{M}\sqrt{Da}}} + \bar{y}^2 e^{\frac{1}{\sqrt{M}\sqrt{Da}}} + \bar{y}^2 e^{-\frac{1}{\sqrt{M}\sqrt{Da}}},$$

$$g_2(\bar{y}) = g_3(\bar{y}) = 0. \tag{34}$$

Finally, by summing up the results, and $p \to 1$ we write the velocity profile as:

$$\bar{u}(\bar{y}) = \sum_{i=0}^{1} g_i(\bar{y}) = \frac{Da\left(-e^{\frac{\bar{y}}{\sqrt{M}\sqrt{Da}}} - e^{-\frac{\bar{y}}{\sqrt{M}\sqrt{Da}}} + e^{\frac{1}{\sqrt{M}\sqrt{Da}}} + e^{-\frac{1}{\sqrt{M}\sqrt{Da}}}\right)}{e^{\frac{1}{\sqrt{M}\sqrt{Da}}} + e^{-\frac{1}{\sqrt{M}\sqrt{Da}}}} \tag{35}$$

Eq. 35 is the analytical solution of problem by use of HPM.

5 Results

Due to Fig.2 and table 1, it is clear that there is a very good agreement between HPM and exact solution of the Darcy Brinkman equation for fully developed forced convection in a parallel plats channel imbedded with homogeneous porous medium i.e. HPM is a reliable solution for this equation by convenience assumed initial approximation and suitable choose of linear part.

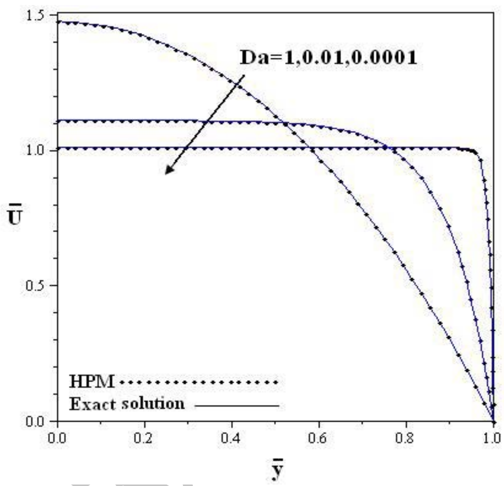

Figure 2. Comparison of HPM results with exact solution in different values of Darcy number (Da)

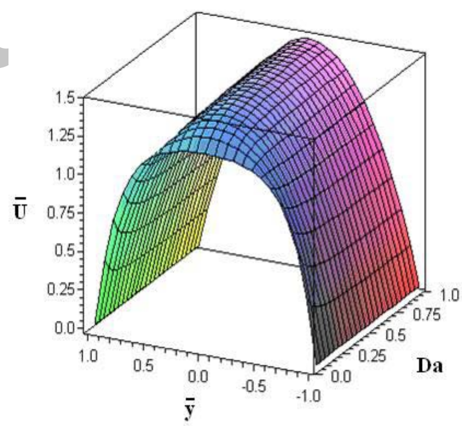

Figure 3. Velocity profile in various values of Da along the half channel height.

As seen in the Fig.3, velocity profile becomes more flat by decreasing Da due to low permeability that resists the flow.

Figure 4 illustrates the variation of Nusselt number versus Darcy number as well as M. As seen, it is obvious from the figure that by increasing the value of M and Da the value of Nusselt decreases.

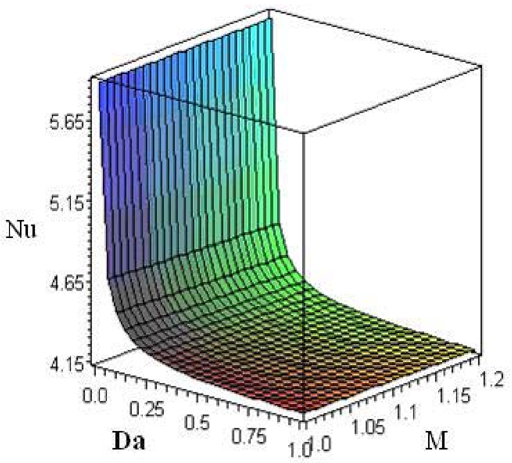

Figure 4. variation of Nusselt number versus Darcy number and M.

6 Conclusion

Darcy Brinkman equation for fully developed forced convection in a porous saturated parallel plate channel is solved analytically by use of HPM and compared with exact solution which has been presented by Haji-Sheikh et al [14]. The results show a suitable agreement between HPM and exact solution of the equation. Also it can be concluded from the results that decreasing the Darcy number makes the velocity profile more flat and decreases the velocity maxima while it leads to increasing the Nusselt.

Table1. Validity of HPM solution versus exact solution when $\bar{y} = 0$

	Da=1			Da=0.01			Da=0.0001		
M	Exact	HPM	Error	Exact	HPM	Error	Exact	HPM	Error
1	1.476246	1.476246	0	1.11101	1.11101	0	1.010101	1.010101	0
2	1.487819	1.487819	0	1.162737	1.162737	0	1.014345	1.014345	0
3	1.49181	1.49181	0	1.201965	1.201965	0	1.017626	1.017626	0
4	1.493831	1.493831	0	1.233128	1.233128	0	1.020408	1.020408	0
5	1.495052	1.495052	0	1.258491	1.258491	0	1.022872	1.022872	0
6	1.495869	1.495869	0	1.279517	1.279517	0	1.02511	1.02511	0
7	1.496455	1.496455	0	1.297216	1.297216	0	1.027177	1.027177	0
8	1.496895	1.496895	0	1.312311	1.312311	0	1.029108	1.029108	0
9	1.497238	1.497238	0	1.325331	1.325331	0	1.030928	1.030928	0
10	1.497513	1.497513	0	1.336675	1.336675	0	1.032655	1.032655	0

References

[1] Vafai K. and Tien C.L., (1980) "Boundary and Inertia Effects on Convection Mass Transfer In Porous Media," Int. J. Heat Mass Transfer, 25(8), 1183-1190.

[2] Amiri A. and Vafai K., (1994) "Analysis of Dispersion Effects and Non-thermal Equilibrium, Non-Darcian, Variable Porosity Incompressible Flow Through Porous Media," Int. J. Heat Mass Transfer, 37(6), 939-954.

[3] Iyer S.V. and Vafai K., (1999) "Passive Heat Transfer Augmentation in a Cylindrical Annual Utilizing a Porous Perturbation," Numerical Heat Transfer, Part A, 36, 115-128.

[4] Alazmi B. and Vafai K., (1999) "Analysis of Variants within the Porous Media Transport Models,' ASME J. Heat Transfer, 122, 303-326.

[5] Kuznestov A.V. and Xiong M., (2000) "Numerical Simulation of the Effect of Thermal Dispersion on Forced Convection in a Circular Duct Partly Filled With a Brinkman-Forchheimer Porous Medium," Int. J. Numerical Methods for Heat & Fluid Flow, 10(5), 488-501.

[6] Kuznetsov A.V., placeCityRaleigh (2000) "Investigation of the Effect of Transverse Thermal Dispersion on Forced Convection in Porous Media," Acta Mechanica, 145, 35-43.

[7] Seyf H.R. and Layeghi M. (2010) "Numerical Analysis of Convective Heat Transfer From an Elliptic Pin Fin Heat Sink With and Without Metal Foam Insert," ASME Journal of Heat Transfer, 132(7), 071401- 071410.

[8] Chen W. and Liu W. (2004) "Numerical analysis of heat transfer in a composite wall solar-collector system with a porous absorber," Applied Energy, 78(2), 137-149.

[9] Kaviany M., (1985) "Laminar flow through a porous channel bounded by isothermal parallel plates", Int. J. Heat Mass Transfer, 28, 851-858.

[10] Vafai K., Kim S., (1989) "Forced convection in a channel filled with porous medium: an exact solution," ASME J. Heat Transfer, 111(4), 1103-1106.

[11] Amiri A., Vafai K. (1994) "Analysis of dispersion effects and nonthermal equilibrium, non-Darcian, variable porosity incompressible flow through porous media," Int. J. Heat Mass Transfer, 37(6) , 939-954.

[12] Hung Y.M. and Tso C.P., (2009) "Effects of viscous dissipation on fully developed forced convection in porous media," International Communications in Heat and Mass Transfer, 36(6), 597-603.

[13] Nield D.A., Junqueira S.L.M, Lage J.L. (1996) "Forced convection in a fluid-saturated porous-medium channel with isothermal or isoflux boundaries," J. Fluid Mech, 322, 201-214.

[14] Haji-Sheikh A., Vafai K., (2004) "Analysis of flow and heat transfer in porous media Imbedded inside various-shaped ducts," Int. J. Heat and Mass Transfer, 47, 1889-1905.

[15] Nield D.A., Kuznetsov A.V., Xiong M. (2003) "Thermally developing forced convection in a porous medium: parallel plate channel with walls at uniform temperature with axial conduction and viscous dissipation effects," Int. J. Heat Mass Transfer, 46(4), 643-651.

[16] Gurgenci H. and Merrikh A.A., (2007) "Heat transfer and entropy generation optimization of forced convection in porous-saturated ducts of rectangular cross-section," International Journal of Heat and Mass Transfer, 50(11,12), 2051-2059.

[17] He J.H. (1999) "Homotopy perturbation technique," 17bud Comput. Meth. Appl. Mech. Eng. 178 (3-4), 257-262.

[18] He J.H. (2003) "Homotopy perturbation method: a new nonlinear analytical technique," Appl. Math. Comput, 135, 73-79.

[19] Dehghan M. and Shakeri F. (2008) "Use of He's Homotopy Perturbation Method for Solving a Partial Differential Equation Arising in Modeling of Flow in Porous Media," Journal of Porous Media, 11, 765-778.

[20] Ganji D.D. and sadighi (2008) "Application of homotopy perturbation and variational iteration Methods to nonlinear heattransfer and porous media equations," J. Computional and Applied Mathematics, 207, 24-34.

[21] Biazar J., Ayati Z. and Ebrahimi H.(2009) "Homotopy Perturbation Method for General Form of Porous Medium Equation," Journal of Porous Media, 12, 1121-1127.

[22] Rafei M., Vaseghi J. and Ganji D. (2007) "Application of Homotopy-Perturbation Method for Systems of Nonlinear Momentum and Heat Transfer Equations," Heat Transfer Research, 38, 361-379

[23] Jafari H., Zabihi M. and Saidy M. (2008) "Application of Homotopy Perturbation Method for Solving Gas Dynamics Equation," Applied Mathematical Sciences, 2, 2393-2396

[24] Ganj D. and Esmaeilpour M. (2008) "A Study on Generalized Couette Flow by He's Methods and Comparison with the Numerical Solution," World Applied Sciences Journal, 4, 470-478

[25] Siddiqui A.M., Mahmood R., Ghori Q.K. (2006) "Homotopy Perturbation Method for Thin Film Flow of Third Grade Fluid down an Inclined Plane," Chaos Solitons and Fractals, 35, 140-147

[26] Abbasbandy S. (2007) "Application of He's homotopy perturbation method to functional integral equation," Chaos, Solitons and Fractals, 31, 1243-1247.

[27] Abbasbandy S. (2007) "A new application of He's variational iteration method for quadratic Riccati differential equation by Adomian's polynomials," J. Comput. Appl. Math. 207, 59-63.

[28] Ezzati R., Tajdini F. (2010) "A method for solving nonlinear wave equations with Homotopy Perturbation Method," Proceeding of Second International Conference on Computer Research and Development.