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Wave propagation in temperature rate dependent thermoelasticity with two temperatures

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Abstract

The present investigation is concerned with two problems. (i) Reflection and transmission of thermoelastic waves between two thermoelastic half-spaces with two temperature at an imperfect interface; and (ii) Propagation of Rayleigh waves at the free surface of thermoelastic solid with two temperature. In problem (i) the amplitude ratios for reflection and transmission coefficients are obtained and deduced for normal force stiffness, transverse force stiffness, thermal contact conductance and perfect bonding. The numerical results obtained have been illustrated graphically to understand the behavior of amplitude ratios versus angle of incidence of longitudinal wave (P-wave), thermal wave (T-wave) and SV-wave. It is found that the amplitude ratios of various reflected and transmitted waves are affected by the stiffness and two temperature effects. In problem (ii) the phase velocity and attenuation coefficient are obtained and presented graphically to depict the effect of two temperatures. Some special cases of interest have been deduced from the two problems also.

Keywords: Wave Propagation, Thermoelastic solid with two temperature, Amplitude ratios, Perfect bonding, Rayleigh waves.

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1 Introduction

The generalized theories of thermoelasticity have been developed to overcome the physically unrealistic prediction of the coupled dynamical theory (CDT) of thermoelasticity that thermal signal propagates with infinite speed. Lord and Shulman theory (LST) [1] of thermoelasticity is well-established theory of generalized thermoelasticity, which introduce the thermal relaxation parameters in the basic equations of the coupled dynamical thermoelasticity theory and admit the finite value of heat propagation speed. The finiteness of the speed of thermal signal has been found to have experiment evidence too. The generalized thermoelasticity theories are, therefore, more realistic and have aroused much interest in recent research.

Thermoelasticity with two temperature is one of the non-classical theories of thermoelasticity of elastic solids. The main difference of this theory with respect to classical one is in thermal dependence. Chen and Gurtin [2] and Chen, Gurtin and Willam [3] have formulated a theory of heat conduction in deformable bodies, which depend on two distinct temperature, the conductive temperature ϕ and the thermodynamic temperature θ. For time independent situations, the difference between these two temperature is proportional to the heat supply, the two temperature are identical. For time dependent problems, however and for wave propagation problem in particular, the two temperatures are in general different, regardless of the presence of a heat supply. The two temperature and the strain are found to have representation in the form of a traveling wave pulse a response, which occur instantaneously throughout the body [4].

Warren and Chen [5] investigated the wave propagation in the two temperature theory of thermoelasticity. Quitanilla [6,7] proved some theorems in thermoelasticity with two temperature. Youssef [8] presented a new theory of generalized thermoelasticity by taking into account the theory of heat conduction in deformable bodies, which depends on two distinct temperatures, the conductive temperature and the thermodynamics temperature. The difference between these two temperatures is proportional to the heat supply. He also established a uniqueness theorem for equation of two temperature generalized linear thermoelasticity for a homogeneous and isotropic body.Recently,Puri and Jordan[9] studied the propagation of plane waves under two temperature.

Imperfect bonding considered in the present investigation is to mean that the stress components are continuous and small displacement field is not. The small vector difference in the displacement is assumed to depend linearly on the traction vector. More precisely ,jumps in the displacement components are assumed to be proportional (in terms of "spring-factor-type"interface parameters) to their respective interface components. The infinite values of interface parameters imply vanishing of displacement jumps and therefore corresponds to perfect interface conditions.

On the other hand, zero values of the interface parameters imply vanishing of the corresponding interface tractions which corresponds to complete debonding. Any finite positive values of the interface parameters define an imperfect interface. Such interface parameters may be present due to the presence of an interphase layer or perhaps interface bond deterioration caused by, for example, fatigue damage or environmental and chemical effects. The values of interface parameters depend upon the material properties of the medium i.e microstructure as well as the bi-material properties.

Significant work has been done to describe the physical conditions on the interface by different mechanical boundary conditions by different investigators. Notable among them are (Jones [10],Murty[11],Nayfeh and Nassar[12], Rokhlin et.al.[13], Rokhlin[14], Baik and Thomson[15], Achenbach et.al.[16] , Lavrentyev and Rokhlin[17]).

Recently various authors have used the imperfect conditions at an interface to study various types of problems (Chen et al.[18], Shodja et al.[19], Samsam Shariat[20], Mitra and Krishanan S [21], Kumar and Rupender[22], Kumar and Chawala[23])

In the present investigation, we studied the reflection and transmission of thermoelastic waves between two thermoelastic half-spaces with two temperatures at an imperfect boundary and propagation of thermoelastic waves with two temperature at the free surface.

2 Basic equations

Following Youseff [8], the field equations and constitutive relations in thermoelastic body with two temperatures having one relaxation time can be written as:

$$(\lambda+2\mu)\nabla(\nabla\cdot\overrightarrow{u})-\mu(\nabla\times\nabla\times\overrightarrow{u})-\beta\nabla\theta=\rho\frac{\partial^2\overrightarrow{u}}{\partial t^2}, \tag{1}$$

$$K^*\nabla^2\phi=\rho C^*(1+\tau_0\frac{\partial}{\partial t})\frac{\partial\theta}{\partial t}+\beta\theta_0(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2})\nabla.\overrightarrow{u}, \tag{2}$$

$$t_{ij}=\lambda\ u_{k,k}\delta_{ij}+\mu(u_{i,j}+u_{j,i})-\beta\theta\delta_{ij}, \tag{3}$$

where

$$\theta=(1-a\nabla^2)\phi, \tag{4}$$

λ, μ- Lame's constants, t-time, $\beta=(3\lambda+2\mu)\alpha_t$, α_t- coefficient of linear thermal expansion, ρ-density, C^*-specific heat, K^*- thermal conductivity, ϕ-conductive temperature, θ-Temperature distribution, t_{ij} -components of stress tensor, τ_0- the relaxation time, δ_{ij}- Kronecker delta, $\overrightarrow{u}$- displacement vector, a-two temperature parameter, θ_0-reference temperature.

and

$$\nabla=\hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z},\quad \nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}.$$

3 Formulation and solution of the problem

We consider two homogeneous, isotropic thermally conducting elastic half-spaces with two temperature being in contact with each other at a plane surface which we designate as the plane $x_3=0$ of a rectangular cartesian co-ordinate $Ox_1x_2x_3$. We consider thermoelastic plane wave in x_1x_3-plane with wave front parallel to x_2-axis and all the field variables depend only on x_1, x_3 and t. For two dimensional problem, we take

$$\overrightarrow{u}=(u_1,0,u_3) \tag{5}$$

To facilitate the solution, following dimensionless quantities are introduced:

$$x_1' = \frac{\omega_1}{c_2}x_1,\ x_3' = \frac{\omega_1}{c_2}x_3,\ u_1' = \frac{\omega_1}{c_2}u_1,\ u_3' = \frac{\omega_1}{c_2}u_3,\ t_{33}' = \frac{t_{33}}{\lambda},\ t_{31}' = \frac{t_{31}}{\lambda},\ \phi' = \frac{\phi}{\theta_0},$$

$$\theta' = \frac{\theta}{\theta_0},\ t' = \omega_1 t,\ a' = \frac{\omega_1^2}{c_2^2}a,\ \acute{K_n} = \frac{c_2}{\lambda\omega_1}K_n,\ \acute{K_t} = \frac{c_2}{\lambda\omega_1}K_t,\ \acute{K_\phi} = \frac{c_2}{K^*\omega_1}K_\phi,\ \tau_o^{'} = \omega_1\tau_o, \tag{6}$$

where

$$c_2^2 = \frac{\mu}{\rho} \quad and \quad \omega_1 = \frac{\rho C^* c_2^2}{K^*}$$

The components of displacement u_1 and u_3 are related by the potential functions $q(x_1, x_3, t)$ and $\psi(x_1, x_3, t)$ as

$$u_1 = \frac{\partial q}{\partial x_1} - \frac{\partial \psi}{\partial x_3}, \quad u_3 = \frac{\partial q}{\partial x_3} + \frac{\partial \psi}{\partial x_1}. \tag{7}$$

Making use of equations (5)-(7) in equation (1) and (2) along with equation (4) (suppressing the primes for convenience), we obtain

$$(a_1\nabla^2 - \frac{\partial^2}{\partial t^2})q - a_2(1 - a\nabla^2)\phi = 0 \tag{8}$$

$$(\nabla^2 - a_4(\frac{\partial}{\partial t} - \tau_0\frac{\partial^2}{\partial t^2}))(1 - a\nabla^2)\phi = a_5(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2})\nabla^2 q \tag{9}$$

$$a_3\nabla^2\psi - \frac{\partial^2\psi}{\partial t^2} = 0 \tag{10}$$

where

$$a_1 = \frac{(\lambda + 2\mu)}{\rho c_2^2}, \quad a_2 = \frac{\beta\theta_0}{\rho c_2^2}, \quad a_3 = \frac{\mu}{\rho c_2^2}, \quad a_4 = \frac{\rho C^* c_2^2}{K^*\omega_1}, \quad a_5 = \frac{\beta c_2^2}{K^*\omega_1}.$$

Equation (10) is uncoupled, whereas equation (8) and (9) are coupled in ϕ and q. Solutions of equations (8)-(10) are sought in the form of harmonic traveling wave

$$(q, \phi, \psi) = (q_0, \phi_0, \psi_0)e^{\iota k(x_1 \sin\theta - x_3\cos\theta + vt)} \tag{11}$$

Where v is the phase speed, k is the wave number and $(\sin\theta_0, \cos\theta_0)$ denote the projection of the normal onto xz-plane. Inserting equation (11) in equations (8)-(10) and eliminating ϕ from (8)and (9), we obtain

$$(v^4 + v^2 A + B)q = 0 \tag{12}$$

$$(v_3^2 - a_3)\psi = 0 \tag{13}$$

where

$$A = \frac{aa_4\tau_{11}\omega^2 - 1 - a_1a_4\tau_{11} + a_2a_5\tau_{11}}{a_4\tau_{11}},\ B = \frac{aa_2a_5\tau_{11}\omega^2 - a_1 - a_1a_4\tau_{11}}{a_4\tau_{11}},\ \tau_{11} = \tau_0 - \frac{\iota}{\omega}.$$

Since equation (12) is quadratic in v^2 so it has two positive values of v i.e.v_1 and v_2 corresponds to the velocities of P-wave and T-wave, respectively whereas the root of equation (13) correspond to SV-wave having velocity v_3.

4 Reflection and Transmission

We consider thermoelastic plane wave(P-or T- or-SV- Wave) propagating through the medium M which we designate as the region $x_3 > 0$ and incident at the plane $x_3 = 0$ with its direction of propagating with angle θ_0 normal to the surface. Corresponding to each incident wave, we get waves in the medium M as reflected P-wave, T-wave and SV-wave and transmitted P-wave, T-wave and SV-wave in medium $\acute{M}$. We write all the variables without a prime in the region $x_3 > 0$ (medium M) and attach a prime to denote the variables in the region $x_3 < 0$ (medium $\acute{M}$) as shown in Figure (0) (geometry of the problem).

5 Boundary Conditions

We consider two thermically conducting elastic half spaces with two temperature as shown in the figure(a) (Geometry of the problem). If the bonding is imperfect and the size and spacing between the imperfection is much smaller than the wave length then at the interface, these can be described by using boundary condition at $x_3 = 0$ Lavrentyev and Rokhlin [17] as

$$(i)(t_{33})_{\acute{M}} = K_n[(u_3)_M - (u_3)_{\acute{M}}]\ , \tag{14}$$

$$(ii)(t_{31})_{\acute{M}} = K_t[(u_1)_M - (u_1)_{\acute{M}}]\ , \tag{15}$$

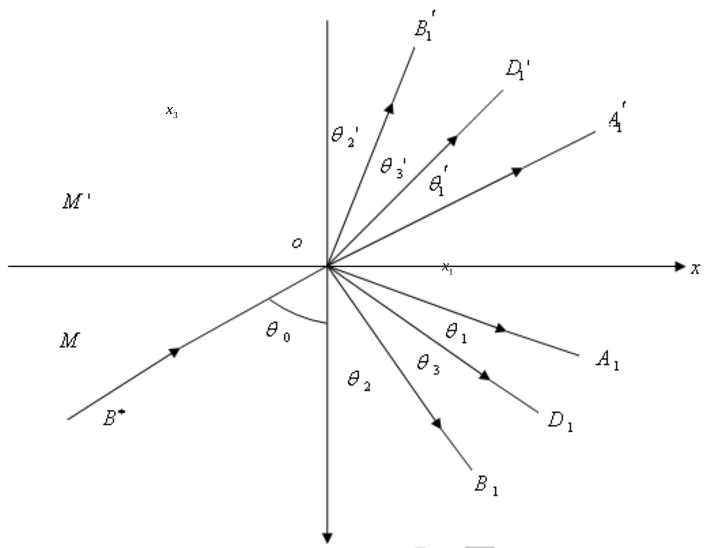

Figure 0) Geometry of the problem

$$(iii)K^{*'}(\frac{\partial\phi}{\partial x_3})_{\acute{M}} = K_\phi[(\phi)_M - (\phi)_{\acute{M}}] \;, \tag{16}$$

$$(iv)(t_{33})_M = (t_{33})_{\acute{M}} \;, \tag{17}$$

$$(v)(t_{31})_M = (t_{31})_{\acute{M}} \;, \tag{18}$$

$$(vi)K^*(\frac{\partial\phi}{\partial x_3})_M = K^{*'}(\frac{\partial\phi}{\partial x_3})_{\acute{M}} \;, \tag{19}$$

where K_n, K_t, K_ϕ are normal force stiffness, transverse force stiffness and thermal contact conductance coefficients of a unit layer thickness and have dimension $\frac{N}{m^3}$, $\frac{N}{m^3}$ and $\frac{W}{m^2c^0}$. Appropriate potentials satisfying the boundary equations (14)-(19) in M and M' can be written as

Medium M

$$q = A_0\exp[\iota k_1(x_1\sin\theta_0 - x_3\cos\theta_0) + \iota\omega t] + A_1\exp[\iota k_1(x_1\sin\theta_1 + x_3\cos\theta_1) + \iota\omega t]$$

$$+B_0 \exp[\iota k_1(x_1 \sin\theta_0 - x_3 \cos\theta_0) + \iota\omega t] + B_1 \exp[\iota k_1(x_1 \sin\theta_2 + x_3 \cos\theta_2) + \iota\omega t], \quad (20)$$

$$\phi = d_1 A_0 \exp[\iota k_1(x_1 \sin\theta_0 - x_3 \cos\theta_0) + \iota\omega t] + d_1 A_1 \exp[\iota k_1(x_1 \sin\theta_1 + x_3 \cos\theta_1) + \iota\omega t]$$

$$+d_2 B_0 \exp[\iota k_1(x_1 \sin\theta_0 - x_3 \cos\theta_0) + \iota\omega t] + d_2 B_1 \exp[\iota k_1(x_1 \sin\theta_2 + x_3 \cos\theta_2) + \iota\omega t], \quad (21)$$

$$\psi = D_0 \exp[\iota k_1(x_1 \sin\theta_0 - x_3 \cos\theta_0) + \iota\omega t] + D_1 \exp[\iota k_1(x_1 \sin\theta_3 + x_3 \cos\theta_3) + \iota\omega t] \quad (22)$$

Medium $M^{'}$

$$q^{'} = A_1^{'} \exp[\iota k_1^{'}(x_1 \sin\theta_1^{'} - x_3 \cos\theta_1^{'}) + \iota\omega t] + B_1^{'} \exp[\iota k_2^{'}(x_1 \sin\theta_2^{'} - x_3 \cos\theta_2)^{'} + \iota\omega t] \quad (23)$$

$$\phi^{'} = d_1^{'} A_1^{'} \exp[\iota k_1^{'}(x_1 \sin\theta_1^{'} - x_3 \cos\theta_1^{'}) + \iota\omega t] + d_2^{'} B_1^{'} \exp[\iota k_2^{'}(x_1 \sin\theta_2^{'} - x_3 \cos\theta_2^{'}) + \iota\omega t] \quad (24)$$

$$\psi^{'} = D_1^{'} \exp[\iota k_3^{'}(x_1 \sin\theta_3^{'} - x_3 \cos\theta_3^{'}) + \iota\omega t] \quad (25)$$

where

$B_0 = D_0 = 0, \quad \theta_0 = \theta_1$ for incident P-wave

$A_0 = D_0 = 0, \quad \theta_0 = \theta_2$ for incident T-wave

$A_0 = B_0 = 0, \quad \theta_0 = \theta_3$ for incident SV-wave

Snell's law is given as

$$\frac{\sin\theta_0}{v_0} = \frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2} = \frac{\sin\theta_3}{v_3} = \frac{\sin\theta_1^{'}}{v_1^{'}} = \frac{\sin\theta_2^{'}}{v_2^{'}} = \frac{\sin\theta_3^{'}}{v_3^{'}} \quad (26)$$

where

$$k_1 v_1 = k_2 v_2 = k_3 v_3 = k_1^{'} v_1^{'} = k_2^{'} v_2^{'} = k_3^{'} v_3^{'} = \omega \quad at \quad x_3 = 0 \quad (27)$$

$$v_0 = \begin{cases} v_1, & \text{for incident} \quad \text{P} - \text{wave} \\ v_2, & \text{for incident} \quad \text{T} - \text{wave} \\ v_3, & \text{for incident} \quad \text{SV} - \text{Wave} \end{cases}$$

Making use of potentials given by equations (20)-(25) in boundary conditions (14)-(19) and with the help of equations (3)-(7), we get a system of six non homogeneous equation, which can be written as

$$\sum_{m=1}^{6} a_{mn} Z_n = Y_m (n = 1, 2...6), \tag{28}$$

where

$$a_{1i} = \iota k_i S_1 K_n \frac{\upsilon_i}{\upsilon_0}[(\frac{\upsilon_0}{\upsilon_i})^2 - \sin^2\theta_0]^{\frac{1}{2}}, \quad a_{13} = \iota k_3 S_1 K_n \frac{\upsilon_3}{\upsilon_0}\sin\theta_0,$$

$$a_{1j} = h_1 k_i'^2 [1-(\frac{\upsilon_i'}{\upsilon_0})^2 \sin^2\theta_0'] + k_i'^2 (\frac{\upsilon_i'}{\upsilon_0})^2 \sin^2\theta_0' + \iota k_i' S_1 S_2 K_n \frac{\upsilon_i'}{\upsilon_0}[(\frac{\upsilon_0}{\upsilon_i'})^2 - \sin^2\theta_0']^{\frac{1}{2}} + g_3(1+ak_i'^2)d_i'$$

$$a_{16} = -k_3'^2 (h_1 - 1)(\frac{\upsilon_3'}{\upsilon_0})^2 [(\frac{\upsilon_0}{\upsilon_3'})^2 - \sin^2\theta_0]^{\frac{1}{2}} \sin\theta_0, \quad a_{2i} = -\iota k_i S_1 K_t \frac{\upsilon_i}{\upsilon_0}\sin\theta_0,$$

$$a_{23} = \iota k_3 S_1 K_t \frac{\upsilon_3}{\upsilon_0}[(\frac{\upsilon_0}{\upsilon_3})^2 - \sin^2\theta_0]^{\frac{1}{2}}, \; a_{2j} = h_2 k_i'^2 (\frac{\upsilon_i'}{\upsilon_0})^2 [(\frac{\upsilon_0}{\upsilon_i'})^2 - \sin^2\theta_0]^{\frac{1}{2}} \sin\theta_0 + \iota k_i' S_1 S_2 K_t \frac{\upsilon_i}{\upsilon_0}\sin\theta_0$$

$$a_{26} = h_2 k_3'^2 [1-2(\frac{\upsilon_3'}{\upsilon_0})^2 \sin^2\theta_0] - \iota k_3' S_1 S_2 K_t \frac{\upsilon_3}{\upsilon_0}[(\frac{\upsilon_0}{\upsilon_3})^2 - \sin^2\theta_0]^{\frac{1}{2}}, \quad a_{3i} = -S_3 K_\theta d_i, \quad a_{33} = 0$$

$$a_{3j} = -[\iota k_i' d_i' \frac{\upsilon_i'}{\upsilon_0}[(\frac{\upsilon_0}{\upsilon_i'})^2 - \sin^2\theta_0]^{\frac{1}{2}} - S_3 S_4 k_\theta d_1'], \quad a_{36} = 0$$

$$a_{4i} = -S_1[g_1 k_i^2 [1 - (\frac{\upsilon_i}{\upsilon_0})^2 \sin^2\theta_0] + k_i^2 (\frac{\upsilon_i}{\upsilon_0})^2 \sin^2\theta_0 + g_2 d_i (1 + ak_i^2)]$$

$$a_{43} = k_3^2 (1 + g_1)(\frac{\upsilon_3}{\upsilon_0})^2 [(\frac{\upsilon_0}{\upsilon_3})^2 - \sin^2\theta_0]^{\frac{1}{2}} \sin\theta_0$$

$$a_{4j} = h_1 k_1'^2 [1 - (\frac{\upsilon_i'}{\upsilon_0})^2 \sin^2\theta_0] + k_1'^2 (\frac{\upsilon_i'}{\upsilon_0})^2 \sin^2\theta_0 + g_3 d_1' (1 + ak_i'^2)$$

$$a_{46} = -k_3'^2 (h_1 - 1)(\frac{\upsilon_3'}{\upsilon_0})^2 [(\frac{\upsilon_0}{\upsilon_3'})^2 - \sin^2\theta_0]^{\frac{1}{2}} \sin\theta_0, \quad a_{5i} = 2g_4 k_i^2 S_1 (\frac{\upsilon_i}{\upsilon_0})^2 [(\frac{\upsilon_0}{\upsilon_i})^2 - \sin^2\theta_0]^{\frac{1}{2}} \sin\theta_0,$$

$$a_{53} = -g_4 k_3^2 S_1 [1 - 2(\frac{\upsilon_3}{\upsilon_0})^2 \sin^2\theta_0], \quad a_{5j} = 2h_2 k_i'^2 (\frac{\upsilon_i'}{\upsilon_0})^2 [(\frac{\upsilon_0}{\upsilon_i'})^2 - \sin^2\theta_0]^{\frac{1}{2}} \sin\theta_0,$$

$$a_{56} = h_2 k_3'^2 [1 - 2(\frac{\upsilon_3'}{\upsilon_0})^2 \sin^2\theta_0], \quad a_{6i} = -d_i S_3 \frac{\upsilon_i}{\upsilon_0}[(\frac{\upsilon_0}{\upsilon_i})^2 - \sin^2\theta_0]^{\frac{1}{2}} \quad a_{63} = 0,$$

$$a_{6j} = d_i' \frac{\upsilon_i'}{\upsilon_0}[(\frac{\upsilon_0}{\upsilon_i'})^2 - \sin^2\theta_0]^{\frac{1}{2}}, \quad a_{66} = 0, \quad h_1 = \frac{\lambda' + 2\mu'}{\lambda'}, \quad h_2 = \frac{\mu'}{\lambda'}, \quad g_1 = \frac{\lambda + 2\mu}{\lambda},$$

$$g_2 = \frac{\beta\theta_0}{\lambda}, \quad g_3 = \frac{\beta^{'}\theta_0^{'}}{\lambda^{'}}, \quad g_4 = \frac{\mu}{\lambda}, \quad S_1 = \frac{\lambda}{\lambda^{'}}, \quad S_2 = \frac{c_2^{'}\omega_1}{c_2\omega_1^{'}}, \quad S_3 = \frac{K^* c_2^{'}\omega_1\theta_0}{K^{*'} c_2\omega_1^{'}\theta_0^{'}} \quad S_4 = \frac{\theta_0^{'}}{\theta_0}$$
$$i = 1, 2 \quad \& \quad j = 3, 4$$

Considering the phase of the reflected waves can easily write using equations (24)-(25)

$$\frac{\cos\theta_i}{\upsilon_i} = [(\frac{\upsilon_0}{\upsilon_i})^2 - \sin^2\theta_0]^{\frac{1}{2}}, \quad \frac{\cos\theta_j}{\upsilon_j} = [(\frac{\upsilon_0}{\upsilon_j})^2 - \sin^2\theta_0]^{\frac{1}{2}} \qquad i = 1, 2 \quad \& \quad j = 3, 4$$

Following Schoenberg [24], if we write

$$\frac{\cos\theta_i}{\upsilon_i} = \frac{\cos\theta_i'}{\upsilon_i'} + \iota\frac{c_i}{2\pi\upsilon_0} \qquad (i = 1, 2, 3, 4, 5) \qquad then$$
$$\frac{\cos\theta_i'}{\upsilon_i'} = \frac{1}{\upsilon_0}R_e\{[(\frac{\upsilon_0}{\upsilon_i})^2 - \sin^2\theta_0]^{\frac{1}{2}}\}, \quad c_i = 2\pi I_m\{[(\frac{\upsilon_0}{\upsilon_i})^2 - \sin^2\theta_0]^{\frac{1}{2}}\}$$

where υ_i', the real phase speed and θ_i', the angle of reflection are given by

$$\frac{\upsilon_i'}{\upsilon_0} = \frac{\sin\theta_i'}{\sin\theta_0}[\sin^2\theta_0 + [R_e\{[(\upsilon_0/\upsilon_5)^2 - \sin^2\theta_0]^{\frac{1}{2}}\}]^2]^{-\frac{1}{2}}$$

and c_i, the attenuation in a depth is equal to the wavelength of incident wave i.e. $(2\pi\upsilon_0)/\omega$

$$Z_1 = \frac{A_1}{B^*}, \quad Z_2 = \frac{B_1}{B^*}, \quad Z_3 = \frac{D_1}{B^*}, \quad Z_4 = \frac{A_1^{'}}{B^*}, \quad Z_5 = \frac{B_1^{'}}{B^*}, \quad Z_6 = \frac{D_1^{'}}{B^*} \qquad (29)$$

For incident P-wave $B^* = A_0$

$$Y_1 = a_{11}, \quad Y_2 = -a_{21}, \quad Y_3 = -a_{31}, \quad Y_4 = -a_{41}, \quad Y_2 = a_{51}, \quad Y_6 = -a_{61} \qquad (30)$$

For incident T-wave $B^* = B_0$

$$Y_1 = a_{12}, \quad Y_2 = -a_{22}, \quad Y_3 = -a_{32}, \quad Y_4 = -a_{42}, \quad Y_2 = a_{52}, \quad Y_6 = a_{62} \qquad (31)$$

For incident SV-wave $B^* = D_0$

$$Y_1 = a_{13}, \quad Y_2 = a_{23}, \quad Y_3 = a_{33}, \quad Y_4 = a_{43}, \quad Y_2 = -a_{53}, \quad Y_6 = -a_{63} \qquad (32)$$

Where Z_1, Z_2,Z_3 are the amplitude ratio's of reflected P-wave, T-wave, SV-wave making angle θ_1, θ_2, θ_3 and Z_4, Z_5,Z_6 are amplitude ratio's of reflected P-wave, T-wave, SV-wave making angle $\theta_1^{'}$, $\theta_2^{'}$, $\theta_3^{'}$

6 Particular cases

We obtain the corresponding expressions for different boundaries by letting

CASE I: $K_n \neq 0$, $K_t \to \infty$, $K_\phi \to \infty$ for Normal Force Stiffness

CASE II: $K_n \to \infty$, $K_t \neq 0$, $K_\phi \to \infty$ for Transverse Force Stiffness

CASE III: $K_n \to \infty$, $K_t \to \infty$, $K_\phi \neq 0$ for Thermal contact conductance

CASE IV: $K_n \to \infty$, $K_t \to \infty$, $K_\phi \to \infty$ for Perfect bonding .

7 Special case

(i) If two temperature parameter vanishes i.e $a = 0$ in equations (8)-(10), we obtain the reflection and transmission coefficients in thermoelasticity with one temperature.

(ii) In the absence of medium M,our results result reduces to free surface i.e. x_3=0. The vanishing of normal force stress,tangential force stress and thermal contact conductance and hence the boundary conditions reduces to

$$(i) \quad t_{33} = 0, \quad (ii) \quad t_{31} = 0, \quad (iii) \quad \frac{\partial \phi}{\partial x_3} = 0. \tag{33}$$

Adopting the same procedure,we obtain three non-homogeneous equations

$$\sum_{m=1}^{6} c_{mn} Z_n = Y_m (n = 1, 2...6), \tag{34}$$

where

$$c_{1i} = g_1 k_i^2 [1 - (\frac{\upsilon_i}{\upsilon_0})^2 \sin^2 \theta_0] + k_i^2 (\frac{\upsilon_i}{\upsilon_0})^2 \sin^2 \theta_0 + g_2 d_1,$$

$$c_{13} = k_3^2 (1 + g_1)(\frac{\upsilon_3}{\upsilon_0})^2 [(\frac{\upsilon_0}{\upsilon_3})^2 - \sin^2 \theta_0]^{\frac{1}{2}} \sin \theta_0, \quad c_{21} = -2k_i^2 (\frac{\upsilon_3'}{\upsilon_0})^2 [(\frac{\upsilon_0}{\upsilon_3'})^2 - \sin^2 \theta_0]^{\frac{1}{2}} \sin \theta_0,$$

$$c_{23} = k_3^2 (\sin^2 \theta_3 + \cos^2 \theta_3), \quad c_{3i} = d_i \frac{\upsilon_i}{\upsilon_0} [(\frac{\upsilon_0}{\upsilon_i})^2 - \sin^2 \theta_0]^{\frac{1}{2}}, \quad c_{33} = 0 \qquad (i = 1, 2)$$

and

$$Z_1 = \frac{A_1}{B^*}, \quad Z_2 = \frac{B_1}{B^*}, \quad Z_3 = \frac{D_1}{B^*}. \tag{35}$$

For incident P-wave $B^* = A_0$

$$Y_1 = -c_{11}, \quad Y_2 = c_{21}, \quad Y_3 = c_{31}, \tag{36}$$

For incident T-wave $B^* = B_0$

$$Y_1 = -c_{12}, \quad Y_2 = c_{22}, \quad Y_3 = c_{32}, \tag{37}$$

For incident SV-wave $B^* = D_0$

$$Y_1 = c_{13}, \quad Y_2 = -c_{23}, \quad Y_3 = a_{33}, \tag{38}$$

where Z_1, Z_2, Z_3 are the amplitude ratio's of reflected P-wave, T-wave, SV-wave making angle θ_1, θ_2, θ_3.

II Rayleigh waves in a thermoelastic half-space with two temperature

In this case,we assume the solutions of eqs.(8)-(10) of the form

$$q = (A_1 e^{-\delta_1 x_3} + B_1 e^{-\delta_2 x_3}) e^{ik(x_1 - ct)}, \tag{39}$$

$$\phi = (b_1 A_1 e^{-\delta_1 x_3} + b_2 B_1 e^{-\delta_2 x_3}) e^{ik(x_1 - ct)}, \tag{40}$$

$$\psi = (D_1 e^{-\delta_3 x_3}) e^{ik(x_1 - ct)}, \tag{41}$$

where

$$\delta_i^2 = k^2(1 - c^2 v_i^2) \quad i = 1, 2, \quad \delta_3 = k^2(1 - \frac{c^2}{a_3})$$

and coupling constants are given as

$$b_1 = \frac{a_1(\delta_1^2 - k^2) + k^2 c^2}{a_2(1 - a(\delta_1^2 - k^2))}, \quad b_2 = \frac{a_1(\delta_2^2 - k^2) + k^2 c^2}{a_2(1 - a(\delta_2^2 - k^2))}$$

Making use of potentials given by (39)-(41) in the boundary conditions (33) and eliminating the unknowns A_1, B_1 and D_1 from the resulting expressions, we obtain the wave velocity equation for Rayleigh waves in the half-space with two temperature.

$$c^2 = a_3(2 - EX3) \tag{42}$$

where

$$EX3 = \frac{2\delta_3(a_3 - a_6)(\delta_1 b_2 - \delta_2 b_1)}{a_6(\delta_2^2 b_1 - \delta_1^2 b_2) - a_7 k^2 (b_1 - b_2) - a b_1 b_2 (\delta_1^2 - \delta_2^2)}$$

Particular case

If $a = 0$, equation (42) reduce to the frequency equation in thermally conducting half-space with one temperature.

8 Numerical results and discussion

In order to illustrate theoretical results obtained in the proceeding sections, we now present some numerical results. Materials chosen for this purpose are Magnesium crystal (medium M) (Eringen[25], Dhaliwal and Singh[26]) and copper material(medium M')(Sherief and Saleh[27]) , the physical data for which are given as

MAGNESIUM

$\lambda = 9.4 \times 10^{10} Nm^{-2},\ \mu = 4.0 \times 10^{10} Nm^{-2},\ \rho = 1.74 \times 10^{3} Kgm^{-3},\ \nu = 2.68 \times 10^{6} Nm^{-2} deg^{-1},\ T = 298K,\ K^{*} = 1.7 \times 10^{2} Wm^{-1}K^{-1},\ C^{*} = 1.04 \times 10^{3} JKg^{-1}deg^{-1}.$

COPPER

$\acute{\lambda} = 7.76 \times 10^{10} Nm^{-2},\ \acute{\mu} = 3.86 \times 10^{10} Nm^{-2},\ \acute{\rho} = 8954 \times 10^{3} Kgm^{-3},\ \alpha_t = 1.78 \times 10^{-5} K^{-1},\ \acute{T} = 293K,\ \acute{K^{*}} = 386 Wm^{-1}K^{-1},\ \acute{C^{*}} = 383.1 JKg^{-1}K^{-1}.$

with non-dimensional interface parameters and thermal relaxation times taken as $K_n = 20$, $K_t = 10$, $K_\phi = 5$, τ_0=0.3s and $\acute{\tau_0}$=0.2s.

A computer programme has been developed and amplitude ratios of various reflected and transmitted waves has been computed. The variations of amplitude ratios for normal force stiffness (NFS), transverse force stiffness(TFS) and thermal contact conductance (TCC) for $a = 0$ and $a = 0.0104$ have been shown .The solid line, small dashed line, dash dot dash line is for $a = 0$ and solid line with center symbol 'diamond', small dashed line with center symbol 'triangle' ,dash dot dash line with center symbol 'circle' for $a = 0.0104$ respectively. The variations of the amplitude ratios for NFS(a=0), TFS(a=0), TCC(a=0), NFS(a=0.0104), TFS(a=0.0104) and TCC(a=0.0104), with angle of incidence of the incident P-wave, T-wave and SV-wave are shown graphically in figures 1-18. These variations are shown from normal incidence to grazing incidence i.e. $0^0 \leq \theta \leq 90^0$.

8.1 Incident P-wave

Figs.1-6 show the variations of amplitude ratios $|Z_i|$ (i=1,...,6) with the angle of incidence for P-wave. In the initial range,the variations of amplitude ratios $|Z_i|$ (i=1,2) for all boundary stiffnesses are almost stationary or constant i.e.the effect of two temperature is almost negligible near the normal incidence except for $|Z_1|, |Z_4|$ and $|Z_6|$ in case of TFS($a = .0104$). As the angle of incidence increases further,the impact of TFS is more as compare to NFS and TCC for both values of a and attain the peak value. Near the grazing incidence,the trend of variations of $|Z_i|$ (i=1,...,6) is similar for all distribution curves i.e. fluctuating.

8.2 Incident T-wave

The variations of amplitude ratios $|Z_i|$ (i=1,...,3) for NFS, TFS and TCC are same i.e two temperature parameter a near the grazing incidence except for $|Z_1|$ it increases in-case of TFS ($a = 0$) and decreases in-case of TFS ($a = 0.0104$). This shows that the impact of two temperature is same on the amplitude ratios $|Z_i|$ (i=1,...,3) of NFS, TFS, TCC irrespective of the value of two temperature parameter and are shown in figures 7-9.

It is depicted from figures 10 and 11 that the values of $|Z_4|$ and $|Z_5|$ for NFS, TFS and TCC is decreasing from normal incidence to grazing incidence except at some pockets for $|Z_4|$ for both values of a.

The behavior of all distribution curves is similar for both $a = 0$ and $a = 0.0104$ i.e. increasing from normal incidence to intermediate range and decreasing from intermediate range to grazing incidence(figure 12).

8.3 Incident SV-wave

The values of $|Z_i|$ (i=1,...,6) for both values of two temperature parameter i.e $a = 0$ and $a = 0.0104$ in-case of NFS is small incomparable to other boundary stiffnesses. So, the variations are not seen very clear due to scale of graph. Here the role of stiffness seems more prominent than the impact of two temperature. As the disturbances travels through different constituents of the medium, it suffers sudden changes, resulting in an inconsistent/non-uniform pattern of curves. Therefore,the trend of curves exhibits the properties of of the medium. These variations are shown in figures 13-18.

Discussion for problem II

In general ,wave number and phase velocity of waves are complex quantities,therefore,the waves are attenuated in space.If we write

$$C^{-1} = V^{-1} + \iota\omega^{-1}Q \tag{43}$$

then $\xi = R + \iota Q$,where $R = \frac{\omega}{V}$ and Q are real numbers.This shows that V is the propagation speed and Q is attenuation coefficient of waves. Upon using the equation (41)in secular equation (40), the value of propagation speed V and attenuation coefficient Q for wave propagation can be obtained. Using the same parameters (COPPER), Figures 19 and 20 show the variations of phase velocity and attenuation coefficients versus wave number respectively. It is evident from figure 19 that the values of phase velocity $a = 0.0104$ are more as compare to $a = 0$ whereas the the values of attenuation coefficient are in the reverse order(figure 20). Further it is evident from fig.19 that in the initial range, the values of phase velocity decreases sharply whereas decrease slowly in the remaining for both values of two temperature parameter(i.e a). The values of attenuation coefficient increasing almost constantly in the whole range for both $a = 0$ and $a = 0.0104$.

9 Conclusion

Effect of two temperature and stiffness have significant impact on the amplitude ratios. It is depicted from the figures that the behavior and trend of variations for NFS, TFS and TCC is same for both values of a.In most of the figures the impact of TFS is more than NFS and TCC. It is observed from figures 19 and 20 that the phase velocity decreases with wave number whereas the attenuation coefficient increases with wave number. The research work is supposed to be useful in further studies,both theoretical and observational of wave propagation in more realistic models of the thermoelastic solids present in the earth's interior. The problem is of geophysical interest, particularly investigations concerned with earthquake and other phenomenon in seismology and engineering.

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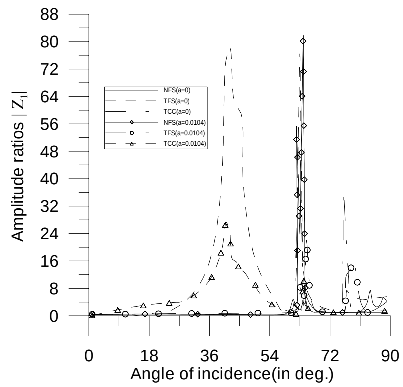


Fig.1

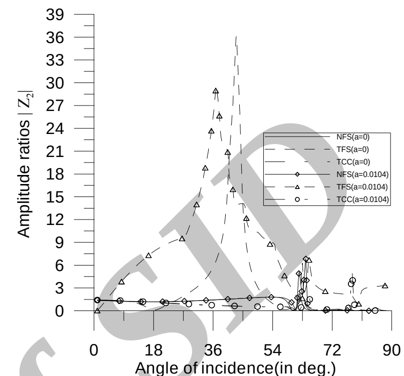
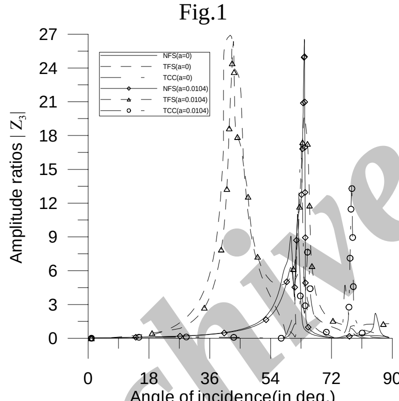


Fig.2



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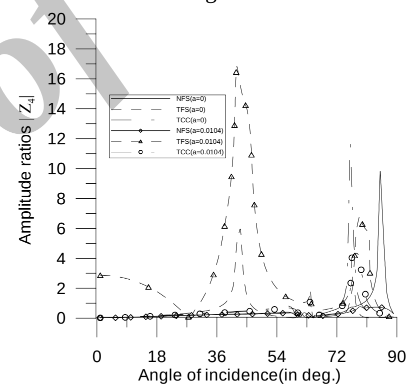

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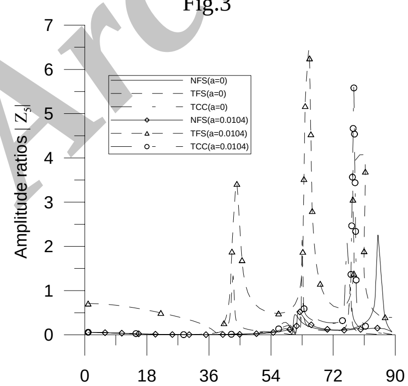

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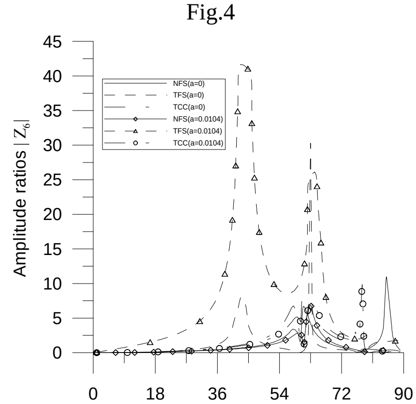

Fig.6

Figs. 1-6: VARIATIONS OF AMPLITUDE RATIOS WITH ANGLE OF INCIDENCE FOR P-WAVE

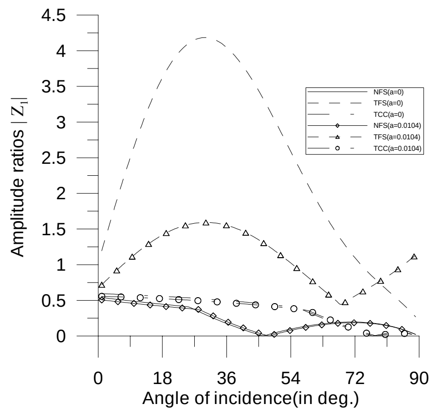

Fig.7

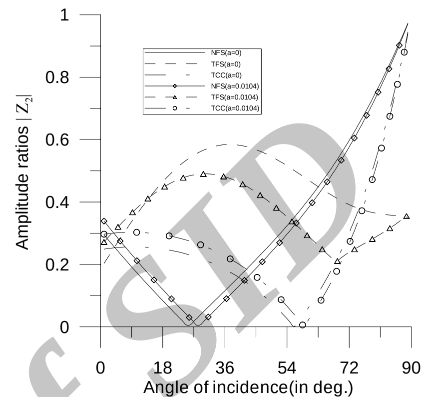

Fig.8

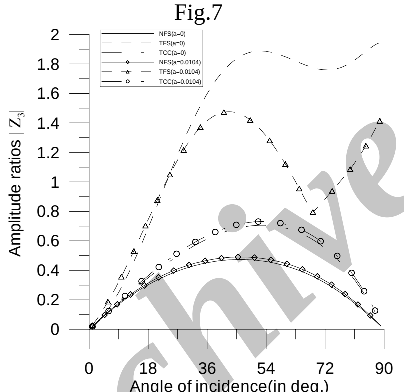

Fig.9

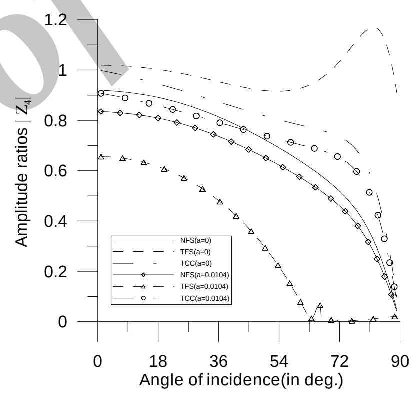

Fig.10

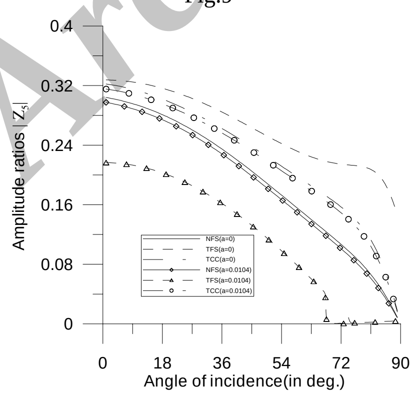

Fig.11

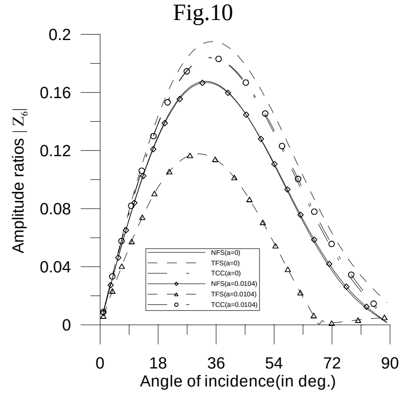

Fig.12

Figs. 7-12: VARIATIONS OF AMPLITUDE RATIOS WITH ANGLE OF INCIDENCE FOR T-WAVE

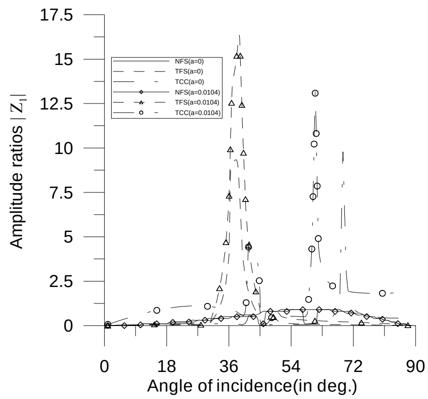

Fig.13

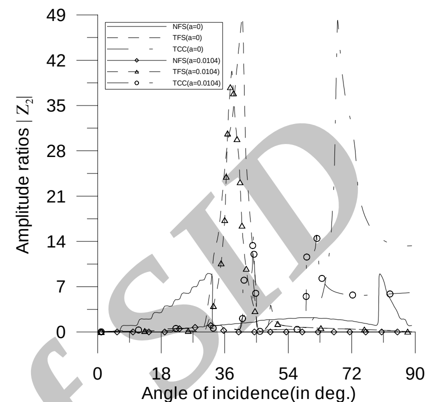

Fig.14

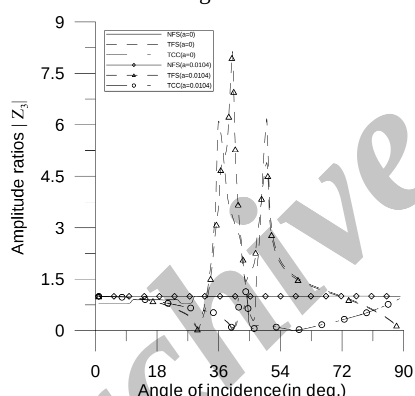

Fig.15

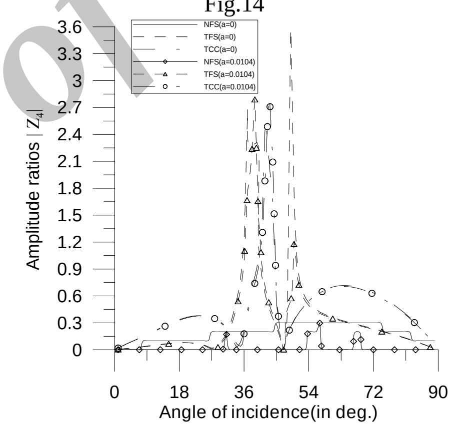

Fig.16

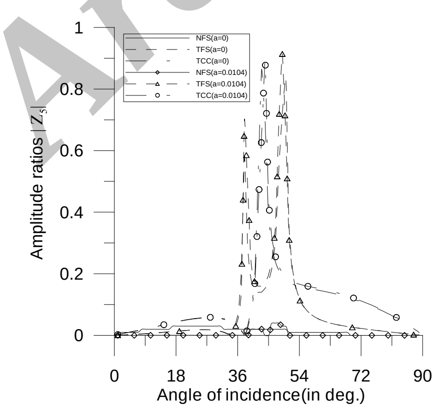

Fig.17

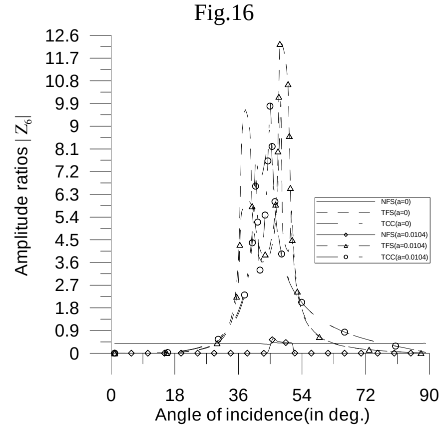

Fig.18

Figs. 13-18: VARIATIONS OF AMPLITUDE RATIOS WITH ANGLE OF INCIDENCE FOR SV-WAVE

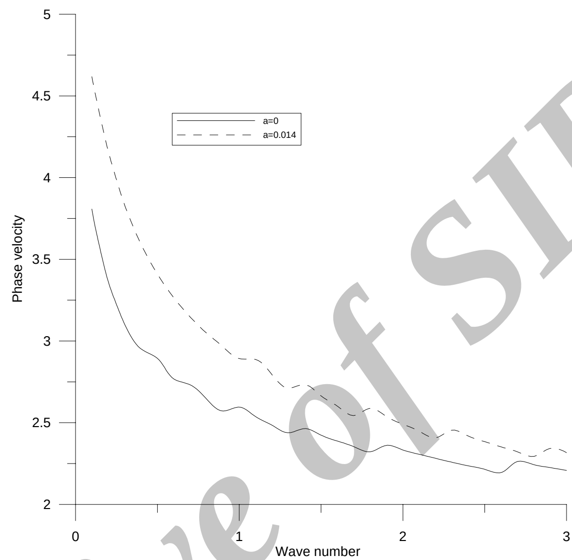

Figure 19) shows the variations of Phase velocity versus wave number

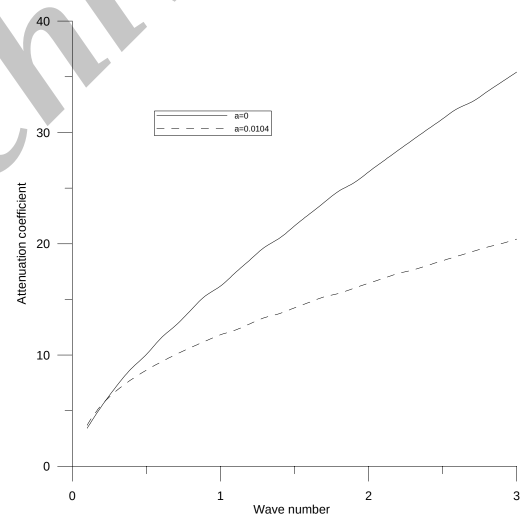

Figure 20) shows the variations of attenuation coefficient versus wave number