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# Computing of Z- valued Characters for some Mathieu, McLaughlin and Higman-Sims groups

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**Abstract**

According to the main result of W. Feit and G. M. Seitz (see, Illinois J. Math. 33 (1), 103-131, 1988), the sporadic Mathieu $M_{11}$, $M_{12}$, $M_c$Laughlin ($M^C L$) and Higman-Sims (HS) groups are unmatured. In this paper, all the dominant classes and Q- conjugacy characters for the above groups are derived.

**Keywords:** Sporadic group, $M_{11}$, $M_{12}$, $M_C L$, HS, Conjugacy class, Q-conjugacy character.





## 1 Introduction

In recent years, the problems over group theory have drawn the wide attention of researchers in mathematics, physics and chemistry. Many problems of the computational group theory have been researched, such as the classification, the symmetry, the topological cycle index, etc. It is not only on the property of finite group, but also its wide-ranging connection with many applied sciences, such as Nanoscience, Chemical Physics and Quantum Chemistry, for instant see[[1]-[12]].

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S. Fujita suggested a new concept called the markaracter table, which enables us to discuss marks and characters for a finite group on a common basis, and then introduced tables of integer-valued characters and dominant classes, which are acquired for such groups. A dominant class is defined as a disjoint union of conjugacy classes corresponding the same cyclic subgroups, which is selected as a representative of conjugate cyclic subgroups. Moreover, the cyclic (dominant) subgroup selected from a non-redundant set of cyclic subgroups of G is used to compute the Q-conjugacy characters of G as demonstrated[[5],[6]].

The sporadic Mathieu $M_{11}$, $M_{12}$, McLaughlin and Higman-Sims groups with orders 7920, 95040, 898128000 and 44352000 respectively [13], are unmatured groups according to the main result of W. Feit and G. M. Seitz in [14]. The motivation for this study is outlined in[[7]-[12]]and the reader is encouraged to consult these papers and [15-18] for background material as well as basic computational techniques.

This paper is organized as follows: In Section 2, we introduce some necessary concepts, such as the maturity and Q-conjugacy character of a finite group. In Section 3, we provide all the dominant classes and Q- conjugacy characters for the sporadic Mathieu $M_{11}$, $M_{12}$, McLaughlin and Higman-Sims groups.

## 2 Preliminaries

Throughout this paper we adopt the same notations as in [[10],[11]]. For instance, we will use the ATLAS notations for conjugacy classes. Thus, nx, n is an integer and $x = a, b, c, \cdots, d$ enotes an arbitrary conjugacy class of G of elements of order n.

**Definition 2.1** *Let G be an arbitrary finite group and $h_1, h_2 \in G$, we say $h_1$ and $h_2$ are Q-conjugate if $t \in G$ exists such that $t^{-1} < h_1 > t =< h_2 >$ which is an equivalence relation on group G and generates equivalence classes that are called dominant classes. Therefore G is partitioned into dominant classes [[2]].*

**Definition 2.2** *Suppose H be a cyclic subgroup of order n of a finite group G. Then,* *the maturity discriminant of H denoted by m(H), is an integer number delineated by* $|N_G(H) : C_G(H)|$ *in addition, the dominant class of* $K \bigcap H$ *in the normalizer* $N_G(H)$ *is the union of* $t = \frac{m(H)}{\Phi(H)}$ *conjugacy classes of G where* $\phi$ *is Euler function, i.e. the maturity of G is clearly defined by examining how a dominant class corresponding to H contains conjugacy classes. The group G should be matured group if* $t = 1$*, but if* $t \in 2$*, the group G is an unmatured concerning subgroup H see [[5]-[12]]. For some properties of the maturity see the following theorem:*

**Theorem 2.3** *The wreath products of the matured groups again is a matured group, but the wreath products of at least one unmatured group is an unmatured group[7].*

**Definition 2.4** *Let C be a matrix of the character table for an arbitrary finite group G. Then, C is transformed into a more concise form called the Q-Conjugacy character table denoted by* $C_G^Q$ *containing integer-valued characters. By Theorem 4 in [[5]], the dimension of a Q-conjugacy character table,* $C_G^Q$ *is equal to its corresponding markaracter table, i.e.* $C_G^Q$ *is a* $m \times m$ *-matrix where m is the number of dominant classes or equivalently the number of non-conjugate cyclic subgroups denoted by denoted by* $SCS_G$*, see [[6]].*

**Definition 2.5** *If* $\chi_1, ..., \chi_k$ *are all the irreducible characters of a finite group H, let* $Q(H) = Q(\chi_1, ..., \chi_k)$ *be the field generated by all* $\chi_i(x), x \in H, 1 \leq i \leq k$*. A character is rational if* $Q(\chi) = Q$*. A group H is a rational group if* $Q(H) = Q$ *(e.g. every Weyl group is a rational group [[14]]).*

**Theorem 2.6** *Let G be a noncyclic finite simple group. Then G is a composition factor of a rational group if and only if G is isomorphic to an alternating group or one of the following groups:*$PSp_4(3), Sp_6(2), O_8^+(2), PSL_3(4), PSU_4(3)$.

# 3 Results and Discussions

According to the Theorem 2.6, the Mathieu $M_{11}$, $M_{12}$, McLaughlin and Higman-Sims groups are unmatured. Now we are equipped to compute all the dominant classes and Q-conjugacy characters for the above groups with aid GAP program http://www.gap-system.org [[11], [15], [16]].

## 3.1 Proposition

1. The Mathieu group $M_{11}$ with $SCSG_{M_{11}} = 8$, has two unmatured dominant classes with $t = 2$ in definition 2.2. Furthermore, there are eight Q- conjugacy characters for $M_{11}$ with the following degrees: 1, 10, 11, 20, 32, 44, 45 and 55.

2. The Mathieu group M12 with $SCSG_{M_{12}} = 14$, has one unmatured dominant class with $t = 2$. Furthermore, there are fourteen Q- conjugacy characters for $M_{12}$ with the following degrees: 1, 11, 32, 45, 54, 55, 66, 99, 120, 144 and 176.

**proof** Here, because of similar discussions we verify just (i) for $M_{11}$ of order 7920. To find all the number of dominant classes for $M_{11}$ at first, we calculate the markaracter table for $M_{11}$ [17, 18] via GAP system, see definition 2.2 and GAP programs in [[7]-[10]] for more details.

Hence, see the markaracter table for $M_{11}$ (i.e.$M_{(M_{11})})^C$) in Table 1, corresponding to eight non-conjugate cyclic subgroups( i.e. $G_i \in SCS_{M_{11}}$)) of orders 1, 2, 3, 4, 5, 6, 8 and 11 respectively, as follow:

| | $G_1$ | $G_2$ | $G_3$ | $G_4$ | $G_5$ | $G_6$ | $G_7$ | $G_8$ |
|---|---|---|---|---|---|---|---|---|
| $(M_{11}/G_1)$ | 7920 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $(M_{11}/G_2)$ | 3960 | 24 | 0 | 0 | 0 | 0 | 0 | 0 |
| $(M_{11}/G_3)$ | 2640 | 0 | 12 | 0 | 0 | 0 | 0 | 0 |
| $(M_{11}/G_4)$ | 1980 | 12 | 0 | 4 | 0 | 0 | 0 | 0 |
| $(M_{11}/G_5)$ | 1584 | 0 | 0 | 0 | 4 | 0 | 0 | 0 |
| $(M_{11}/G_6)$ | 1320 | 8 | 6 | 0 | 0 | 2 | 0 | 0 |
| $(M_{11}/G_7)$ | 990 | 6 | 0 | 0 | 2 | 0 | 2 | 0 |
| $(M_{11}/G_8)$ | 720 | 0 | 0 | 0 | 0 | 0 | 0 | 5 |

Table 1: The markarecter table of $M_{11}$ (i.e. $M^C_{(M_{11})}$)

$G_1 = id, G_2 =< (4,10)(5,8)(6,7)(9,11) >, G_3 =< (3,4,10)(5,11,6)(7,9,8) >,$

$G_4 =< (4,10)(5,8)(6,7)(9,11), (4,6,10,7)(5,11,8,9) >, G_5 =< (2,3,4,11,6)(5,7,10,8,9) >,$

$G_6 =< (4,10)(5,8)(6,7)(9,11), (1,2,3)(5,9,6)(7,8,11) >, G_7 =< (4,10)(5,8)(6,7)(9,11), (4,6,10,7)$

$(5,11,8,9), (2,3)(4,8,6,9,10,5,7,11) > \quad and \quad G_8 =< (1,3,7,2,4,11,5,9,10,6,8) >$

. Therefore, $|SCS_{M_{11}}| = 8$ and its dominant classes are $1a, 2a, 3a, 4a, 5a, 6a, A_8 = 8a8bandA_{11} = 11a \bigcup 11b$ which has two unmatured dominant classes with t = 2. Furthermore, $M_{11}$ has two unmatured Q-conjugacy characters 3 and 5 which are the sum of two irreducible characters respectively [9]. Therefore, there are two column-reductions (similarly two row-reductions) in the character table of $M_{11}$ [[5]-[9]]. There are eight Q- conjugacy characters for the Mathieu $M_{11}$ group with the following degrees: 1, 10, 11, 20, 32, 44, 45 and 55, we afford all Q-conjugacy characters of $M_{11}$ in Table 2.

| | 1a | 2a | 3a | 4a | 5a | 6a | $A_8$ | $A_{11}$ |
|---|---|---|---|---|---|---|---|---|
| $\chi_1$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\chi_2$ | 10 | 2 | 1 | 2 | 0 | -1 | 0 | -1 |
| $\chi_3$ | 20 | -4 | 2 | 0 | 0 | 2 | 0 | -2 |
| $\chi_4$ | 11 | 3 | 2 | -1 | 1 | 0 | -1 | 0 |
| $\chi_5$ | 32 | 0 | -4 | 0 | 2 | 0 | 0 | -1 |
| $\chi_6$ | 44 | 4 | -1 | 0 | -1 | 1 | 0 | 0 |
| $\chi_7$ | 45 | -3 | 0 | 1 | 0 | 0 | -1 | 1 |
| $\chi_8$ | 55 | -1 | 1 | -1 | 0 | -1 | 1 | 0 |

Table 2: The Q- conjugacy character table of (i.e. $M^Q_{(M_1 1)}$)

Similar discussions show that in the Mathieu group $M_{12}$ with $SCSGM_{12} = 14$, there are fourteen Q- conjugacy characters with the following degrees: 1, 11, 32, 45, 54, 55, 66, 99, 120, 144 and 176.

Besides, its dominant classes are 1a, 2a, 2b, 3a, 3b, 4a, 4b,5a, 5b, 6a, 6b, 8a, 8b, 10a and $B_{11} = 11a \bigcup 11b$, we afford all Q-conjugacy characters of $M_{12}$ in Table 3. As matter of fact in Table 3, $\mu_4$ is an unmatured Q- conjugacy character which is the sum of two irreducible characters, see Table 3.

| | 1a | 2a | 2b | 3a | 3b | 4a | 4b | 5a | 6a | 6b | 8a | 8b | 10a | $B_{11}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\mu_1$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\mu_2$ | 11 | -1 | 3 | 2 | -1 | -1 | 3 | 1 | -1 | 0 | -1 | 1 | -1 | 0 |
| $\mu_3$ | 11 | -1 | 3 | 2 | -1 | 3 | -1 | 1 | -1 | 0 | 1 | -1 | -1 | 0 |
| $\mu_4$ | 32 | 8 | 0 | -4 | 2 | 0 | 0 | 2 | 2 | 0 | 0 | 0 | -2 | -1 |
| $\mu_5$ | 45 | 5 | -3 | 0 | 3 | 1 | 1 | 0 | -1 | 0 | -1 | -1 | 0 | 1 |
| $\mu_6$ | 54 | 6 | 6 | 0 | 0 | 2 | 2 | -1 | 0 | 0 | 0 | 0 | 1 | -1 |
| $\mu_7$ | 55 | -5 | 7 | 1 | 1 | -1 | -1 | 0 | 1 | 1 | -1 | -1 | 0 | 0 |
| $\mu_8$ | 55 | -5 | -1 | 1 | 1 | 3 | -1 | 0 | 1 | -1 | -1 | 1 | 0 | 0 |
| $\mu_9$ | 55 | -5 | -1 | 1 | 1 | -1 | 3 | 0 | 1 | -1 | 1 | -1 | 0 | 0 |
| $\mu_{10}$ | 66 | 6 | 2 | 3 | 0 | -2 | -2 | 1 | 0 | -1 | 0 | 0 | 1 | 0 |
| $\mu_{11}$ | 99 | -1 | 3 | 0 | 3 | -1 | -1 | -1 | -1 | 0 | 1 | 1 | -1 | 0 |
| $\mu_{12}$ | 120 | 0 | -8 | 3 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | -1 |
| $\mu_{13}$ | 144 | 4 | 0 | 0 | -3 | 0 | 0 | -1 | 1 | 0 | 0 | 0 | -1 | 1 |
| $\mu_{14}$ | 176 | -4 | 0 | -4 | -1 | 0 | 0 | 1 | -1 | 0 | 0 | 0 | 1 | 0 |

Table 3: The Q- conjugacy character table of $M_{12}$ (i.e. $C^{Q}_{M_{12}}$), wherein $B_{11} = 11a \bigcup 11b$

### 3.2 Proposition

The sporadic McLaughlin group, $M^C L$ has six unmatured dominant classes with t =2. Furthermore, there are eighteen Q- conjugacy characters for $M^C L$ with the following degrees: 1, 22, 231, 252, 1540, 1750, 1792, 3520, 4500, 4752, 5103, 5544, 9625, 16038, 16500, 19172 and 20790 .

**proof** : Similar discussions like Proposition 3.1 show that in the sporadic McLaughlin group, $M^C L$ with SCSG(MCL) = 18, has eighteen Q- conjugacy characters with the following degrees: 1, 22, 231, 252, 1540, 1750, 1792, 3520, 4500, 4752, 5103, 5544, 9625, 16038, 16500, 19172 and 20790.

Besides, its dominant classes are 1a, 2a, 3a, 3b, 4a, 5a, 5b, 6a, 6b, $C_7 = 7a \bigcup 7b$, 8a, $C_9 = 9a \bigcup 9b$, 10a and $C_{11} = 11a \bigcup 11b$, 12a, $C_{14} = 14a \bigcup 14b$, $C_{15} = 15a \bigcup 15b$ and $C_{30} = 30a \bigcup 30b$, we afford all Q-conjugacy characters of $M^C L$ in Table 4. As matter of fact in Table 4, i is an unmatured Q- conjugacy character for $i = 5, 6, 14, 15, 17, 18$ which is the sum of two irreducible characters, see Table 4.

| | 1a | 2a | 3a | 3b | 4a | 5a | 5b | 6a | 6b | $C_7$ | 8a | $C_9$ | 10 | $C_{11}$ | 12a | $C_{14}$ | $C_{15}$ | $C_{30}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\eta_1$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\eta_2$ | 22 | 6 | −5 | 4 | 2 | −3 | 2 | 3 | 0 | 1 | 0 | 1 | 1 | 0 | −1 | −1 | 0 | −2 |
| $\eta_3$ | 231 | 7 | 15 | 6 | −1 | 6 | 1 | 7 | −2 | 0 | −1 | 0 | 2 | 0 | −1 | 0 | 0 | 2 |
| $\eta_4$ | 252 | 28 | 9 | 9 | 4 | 2 | 2 | 1 | 1 | 0 | 0 | 0 | −2 | −2 | 1 | 0 | −1 | 1 |
| $\eta_5$ | 1540 | −28 | −26 | 10 | −4 | −10 | 0 | 14 | 2 | 0 | 0 | −2 | 1 | 0 | 2 | 0 | −1 | −1 |
| $\eta_6$ | 1792 | 0 | 64 | −8 | 0 | −8 | 2 | 0 | 0 | 0 | 0 | −2 | 0 | −1 | 0 | 0 | 4 | 0 |
| $\eta_7$ | 1750 | 70 | −5 | 13 | 2 | 0 | 0 | −5 | 1 | 0 | 0 | −2 | 0 | 1 | −1 | 0 | 0 | 0 |
| $\eta_8$ | 3520 | 64 | −44 | 10 | 0 | −5 | 0 | 4 | −2 | −1 | 0 | 1 | −1 | 0 | 0 | 1 | 1 | −1 |
| $\eta_9$ | 3520 | −64 | 44 | 10 | 0 | −5 | 0 | −4 | 2 | −1 | 0 | 1 | 1 | 0 | 0 | −1 | 1 | 1 |
| $\eta_{10}$ | 4500 | 20 | 45 | −9 | 4 | 0 | 0 | 5 | −1 | −1 | 0 | 0 | 0 | 1 | 1 | −1 | 0 | 0 |
| $\eta_{11}$ | 4752 | −48 | 54 | 0 | 0 | 2 | 2 | −6 | 0 | −1 | 0 | 0 | 2 | 0 | 0 | 1 | −1 | −1 |
| $\eta_{12}$ | 5103 | 63 | 0 | 0 | 3 | 3 | −2 | 0 | 0 | 0 | 1 | 0 | 3 | −1 | 0 | 0 | 0 | 0 |
| $\eta_{13}$ | 5544 | −56 | 36 | 9 | 0 | 19 | −1 | 4 | 1 | 0 | 0 | 0 | −1 | 0 | 0 | 0 | 1 | −1 |
| $\eta_{14}$ | 16038 | −90 | 0 | 0 | 6 | −12 | −2 | 0 | 0 | 1 | −2 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| $\eta_{15}$ | 16500 | 20 | 30 | 12 | −4 | 0 | 0 | −10 | −4 | 1 | 0 | 0 | 0 | 0 | 2 | −1 | 0 | 0 |
| $\eta_{16}$ | 9625 | 105 | 40 | −5 | −3 | 0 | 0 | 0 | 3 | 0 | −1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\eta_{17}$ | 19172 | 0 | −16 | −16 | 0 | 12 | 2 | 0 | 0 | 0 | 0 | −1 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\eta_{18}$ | 20790 | −42 | 54 | 0 | −2 | −10 | 0 | 6 | 0 | 0 | 2 | 0 | −2 | 0 | −2 | 0 | −1 | 1 |

Table 4: The integer-valued character table of McLaughlin group (i.e. $C^Q_{M^cL}$ ), where $C_k = ka \bigcup kb$ is an unmatured dominant class for k=7,9,11,14,15,30.

### 3.3 Proposition

The sporadic Higman-Sims group, HS has two unmatured dominant classes with t =2. Furthermore, there are twenty two Q- conjugacy characters for HS with the following degrees: 1, 22, 77, 154, 175, 231, 693, 770, 825, 1056, 1386, 1408, 1540, 1750, 1792, 1925, 2520, 2750 and 3200.

**proof** The sporadic Higman-Sims group, HS has twenty two Q- conjugacy characters with the following degrees: 1, 22, 77, 154, 175, 231, 693, 770, 825, 1056, 1386, 1408, 1540, 1750, 1792, 1925, 2520, 2750 and 3200. Besides, its dominant classes are$1a, 2a, 2b, 3a, 4a, 4b, 4c, 5a, 5b, 5c, 6a, 6b, 7a, 8a, 8b, 8c, 10a, 10b, D_{11} = 11a \bigcup 11b$

$, 12a, 15a \quad and \quad D_{20} = 20a \bigcup 20b$ with SCSG(HS) = 22, we afford all Q-conjugacy characters of HS in Table 5.

As matter of fact in Table 5, $\Phi_{11}$ and $\Phi_{13}$ are unmatured Q- conjugacy character which are the sum of two irreducible characters, see Table 5.

| | 1a | 2a | 2b | 3a | 4a | 4b | 4c | 5a | 5b | 5c | 6a |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\phi_1$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\phi_2$ | 22 | 6 | −2 | 4 | −6 | 2 | 2 | −3 | 2 | 2 | −2 |
| $\phi_3$ | 77 | 13 | 1 | 5 | 5 | 5 | 1 | 2 | −3 | 2 | 1 |
| $\phi_4$ | 154 | 10 | 10 | 1 | −2 | 6 | −2 | 4 | 4 | −1 | 1 |
| $\phi_5$ | 154 | 10 | −10 | 1 | −10 | −2 | 2 | 4 | 4 | −1 | −1 |
| $\phi_6$ | 154 | 10 | −10 | 1 | −10 | −2 | 2 | 4 | 4 | −1 | −1 |
| $\phi_7$ | 175 | 15 | 11 | 4 | 15 | −1 | 3 | 0 | 5 | 0 | 2 |
| $\phi_8$ | 231 | 7 | −9 | 6 | 15 | −1 | −1 | 6 | 1 | 1 | 0 |
| $\phi_9$ | 693 | 21 | 9 | 0 | 21 | 5 | 1 | −7 | 3 | −2 | 0 |
| $\phi_{10}$ | 770 | 34 | −1 | 5 | −14 | 2 | −2 | −5 | 0 | 0 | −1 |
| $\phi_{11}$ | 1540 | −28 | 20 | 10 | −20 | −4 | −4 | −10 | 0 | 0 | 2 |
| $\phi_{12}$ | 825 | 25 | 9 | 6 | −15 | 1 | 1 | 0 | −5 | 0 | 0 |
| $\phi_{13}$ | 1792 | 0 | 32 | −8 | 0 | 0 | 0 | −8 | 2 | 2 | −2 |
| $\phi_{14}$ | 1056 | 32 | 0 | −6 | 0 | 0 | 0 | 6 | −4 | 1 | 0 |
| $\phi_{15}$ | 1386 | −6 | 18 | 0 | 6 | −2 | −2 | 11 | 6 | 1 | 0 |
| $\phi_{16}$ | 1408 | 0 | 16 | 4 | 0 | 0 | 0 | 8 | −7 | −2 | −2 |
| $\phi_{17}$ | 1750 | −10 | 10 | −5 | −10 | 6 | 2 | 0 | 0 | 0 | 1 |
| $\phi_{18}$ | 1925 | 5 | −19 | −1 | 5 | 5 | −3 | 0 | 5 | 0 | −1 |
| $\phi_{19}$ | 1925 | 5 | 1 | −1 | −35 | −3 | 1 | 0 | 5 | 0 | 1 |
| $\phi_{20}$ | 2520 | 24 | 0 | 0 | 24 | −8 | 0 | −5 | 0 | 0 | 0 |
| $\phi_{21}$ | 2750 | −50 | −10 | 5 | 10 | 2 | 2 | 0 | 0 | 0 | −1 |
| $\phi_{22}$ | 3200 | 0 | −16 | −4 | 0 | 0 | 0 | 0 | −5 | 0 | 2 |

Table 5: The integer-valued character table of Higman-Sims group (i.e. $C^Q_{HS}$)

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