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Computing of Z- valued Characters for some Mathieu, McLaughlin and Higman-Sims groups

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Abstract

According to the main result of W. Feit and G. M. Seitz (see, Illinois J. Math. 33 (1), 103-131, 1988), the sporadic Mathieu M_{11}, M_{12}, M_cLaughlin ($M^C L$) and Higman-Sims (HS) groups are unmatured. In this paper, all the dominant classes and Q- conjugacy characters for the above groups are derived.

Keywords: Sporadic group, M_{11}, M_{12}, $M_C L$, HS, Conjugacy class, Q-conjugacy character.

1 Introduction

In recent years, the problems over group theory have drawn the wide attention of researchers in mathematics, physics and chemistry. Many problems of the computational group theory have been researched, such as the classification, the symmetry, the topological cycle index, etc. It is not only on the property of finite group, but also its wide-ranging connection with many applied sciences, such as Nanoscience, Chemical Physics and Quantum Chemistry, for instant see[[1]-[12]].

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S. Fujita suggested a new concept called the markaracter table, which enables us to discuss marks and characters for a finite group on a common basis, and then introduced tables of integer-valued characters and dominant classes, which are acquired for such groups. A dominant class is defined as a disjoint union of conjugacy classes corresponding the same cyclic subgroups, which is selected as a representative of conjugate cyclic subgroups. Moreover, the cyclic (dominant) subgroup selected from a non-redundant set of cyclic subgroups of G is used to compute the Q-conjugacy characters of G as demonstrated[[5],[6]].

The sporadic Mathieu M_{11}, M_{12}, McLaughlin and Higman-Sims groups with orders 7920, 95040, 898128000 and 44352000 respectively [13], are unmatured groups according to the main result of W. Feit and G. M. Seitz in [14]. The motivation for this study is outlined in[[7]-[12]]and the reader is encouraged to consult these papers and [15-18] for background material as well as basic computational techniques.

This paper is organized as follows: In Section 2, we introduce some necessary concepts, such as the maturity and Q-conjugacy character of a finite group. In Section 3, we provide all the dominant classes and Q- conjugacy characters for the sporadic Mathieu M_{11}, M_{12}, McLaughlin and Higman-Sims groups.

2 Preliminaries

Throughout this paper we adopt the same notations as in [[10],[11]]. For instance, we will use the ATLAS notations for conjugacy classes. Thus, nx, n is an integer and $x = a, b, c, \cdots, d$ enotes an arbitrary conjugacy class of G of elements of order n.

Definition 2.1 *Let G be an arbitrary finite group and $h_1, h_2 \in G$, we say h_1 and h_2 are Q-conjugate if $t \in G$ exists such that $t^{-1} < h_1 > t =< h_2 >$ which is an equivalence relation on group G and generates equivalence classes that are called dominant classes. Therefore G is partitioned into dominant classes [[2]].*

Definition 2.2 *Suppose H be a cyclic subgroup of order n of a finite group G. Then,* *the maturity discriminant of H denoted by m(H), is an integer number delineated by* $|N_G(H) : C_G(H)|$ *in addition, the dominant class of* $K \bigcap H$ *in the normalizer* $N_G(H)$ *is the union of* $t = \frac{m(H)}{\Phi(H)}$ *conjugacy classes of G where* ϕ *is Euler function, i.e. the maturity of G is clearly defined by examining how a dominant class corresponding to H contains conjugacy classes. The group G should be matured group if* $t = 1$*, but if* $t \in 2$*, the group G is an unmatured concerning subgroup H see [[5]-[12]]. For some properties of the maturity see the following theorem:*

Theorem 2.3 *The wreath products of the matured groups again is a matured group, but the wreath products of at least one unmatured group is an unmatured group[7].*

Definition 2.4 *Let C be a matrix of the character table for an arbitrary finite group G. Then, C is transformed into a more concise form called the Q-Conjugacy character table denoted by* C_G^Q *containing integer-valued characters. By Theorem 4 in [[5]], the dimension of a Q-conjugacy character table,* C_G^Q *is equal to its corresponding markaracter table, i.e.* C_G^Q *is a* $m \times m$ *-matrix where m is the number of dominant classes or equivalently the number of non-conjugate cyclic subgroups denoted by denoted by* SCS_G*, see [[6]].*

Definition 2.5 *If* $\chi_1, ..., \chi_k$ *are all the irreducible characters of a finite group H, let* $Q(H) = Q(\chi_1, ..., \chi_k)$ *be the field generated by all* $\chi_i(x), x \in H, 1 \le i \le k$*. A character is rational if* $Q(\chi) = Q$*. A group H is a rational group if* $Q(H) = Q$ *(e.g. every Weyl group is a rational group [[14]]).*

Theorem 2.6 *Let G be a noncyclic finite simple group. Then G is a composition factor of a rational group if and only if G is isomorphic to an alternating group or one of the following groups:*$PSp_4(3), Sp_6(2), O_8^+(2), PSL_3(4), PSU_4(3)$.

3 Results and Discussions

According to the Theorem 2.6, the Mathieu M_{11}, M_{12}, McLaughlin and Higman-Sims groups are unmatured. Now we are equipped to compute all the dominant classes and Q-conjugacy characters for the above groups with aid GAP program http://www.gap-system.org [[11], [15], [16]].

3.1 Proposition

1. The Mathieu group M_{11} with $SCSG_{M_{11}} = 8$, has two unmatured dominant classes with $t = 2$ in definition 2.2. Furthermore, there are eight Q- conjugacy characters for M_{11} with the following degrees: 1, 10, 11, 20, 32, 44, 45 and 55.

2. The Mathieu group M12 with $SCSG_{M_{12}} = 14$, has one unmatured dominant class with $t = 2$. Furthermore, there are fourteen Q- conjugacy characters for M_{12} with the following degrees: 1, 11, 32, 45, 54, 55, 66, 99, 120, 144 and 176.

proof Here, because of similar discussions we verify just (i) for M_{11} of order 7920. To find all the number of dominant classes for M_{11} at first, we calculate the markaracter table for M_{11} [17, 18] via GAP system, see definition 2.2 and GAP programs in [[7]-[10]] for more details.
Hence, see the markaracter table for M_{11} (i.e.$M_{(M_{11})})^C$) in Table 1, corresponding to eight non-conjugate cyclic subgroups(i.e. $G_i \in SCS_{M_{11}}$)) of orders 1, 2, 3, 4, 5, 6, 8 and 11 respectively, as follow:

	G_1	G_2	G_3	G_4	G_5	G_6	G_7	G_8
(M_{11}/G_1)	7920	0	0	0	0	0	0	0
(M_{11}/G_2)	3960	24	0	0	0	0	0	0
(M_{11}/G_3)	2640	0	12	0	0	0	0	0
(M_{11}/G_4)	1980	12	0	4	0	0	0	0
(M_{11}/G_5)	1584	0	0	0	4	0	0	0
(M_{11}/G_6)	1320	8	6	0	0	2	0	0
(M_{11}/G_7)	990	6	0	0	2	0	2	0
(M_{11}/G_8)	720	0	0	0	0	0	0	5

Table 1: The markarecter table of M_{11} (i.e. $M^C_{(M_{11})}$)

$$G_1 = id, G_2 =< (4,10)(5,8)(6,7)(9,11) >, G_3 =< (3,4,10)(5,11,6)(7,9,8) >,$$

$$G_4 =< (4,10)(5,8)(6,7)(9,11), (4,6,10,7)(5,11,8,9) >, G_5 =< (2,3,4,11,6)(5,7,10,8,9) >,$$

$$G_6 =< (4,10)(5,8)(6,7)(9,11), (1,2,3)(5,9,6)(7,8,11) >, G_7 =< (4,10)(5,8)(6,7)(9,11), (4,6,10,7)$$

$$(5,11,8,9), (2,3)(4,8,6,9,10,5,7,11) > \quad and \quad G_8 =< (1,3,7,2,4,11,5,9,10,6,8) >$$

. Therefore, $|SCS_{M_{11}}| = 8$ and its dominant classes are $1a, 2a, 3a, 4a, 5a, 6a, A_8 = 8a8bandA_{11} = 11a \bigcup 11b$ which has two unmatured dominant classes with t = 2. Furthermore, M_{11} has two unmatured Q-conjugacy characters 3 and 5 which are the sum of two irreducible characters respectively [9]. Therefore, there are two column-reductions (similarly two row-reductions) in the character table of M_{11} [[5]-[9]]. There are eight Q- conjugacy characters for the Mathieu M_{11} group with the following degrees: 1, 10, 11, 20, 32, 44, 45 and 55, we afford all Q-conjugacy characters of M_{11} in Table 2.

	1a	2a	3a	4a	5a	6a	A_8	A_{11}
χ_1	1	1	1	1	1	1	1	1
χ_2	10	2	1	2	0	-1	0	-1
χ_3	20	-4	2	0	0	2	0	-2
χ_4	11	3	2	-1	1	0	-1	0
χ_5	32	0	-4	0	2	0	0	-1
χ_6	44	4	-1	0	-1	1	0	0
χ_7	45	-3	0	1	0	0	-1	1
χ_8	55	-1	1	-1	0	-1	1	0

Table 2: The Q- conjugacy character table of (i.e. $M^Q_{(M_11)}$)

Similar discussions show that in the Mathieu group M_{12} with $SCSGM_{12} = 14$, there are fourteen Q- conjugacy characters with the following degrees: 1, 11, 32, 45, 54, 55, 66, 99, 120, 144 and 176.

Besides, its dominant classes are 1a, 2a, 2b, 3a, 3b, 4a, 4b,5a, 5b, 6a, 6b, 8a, 8b, 10a and $B_{11} = 11a \bigcup 11b$, we afford all Q-conjugacy characters of M_{12} in Table 3. As matter of fact in Table 3, μ_4 is an unmatured Q- conjugacy character which is the sum of two irreducible characters, see Table 3.

	1a	2a	2b	3a	3b	4a	4b	5a	6a	6b	8a	8b	10a	B_{11}
μ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
μ_2	11	-1	3	2	-1	-1	3	1	-1	0	-1	1	-1	0
μ_3	11	-1	3	2	-1	3	-1	1	-1	0	1	-1	-1	0
μ_4	32	8	0	-4	2	0	0	2	2	0	0	0	-2	-1
μ_5	45	5	-3	0	3	1	1	0	-1	0	-1	-1	0	1
μ_6	54	6	6	0	0	2	2	-1	0	0	0	0	1	-1
μ_7	55	-5	7	1	1	-1	-1	0	1	1	-1	-1	0	0
μ_8	55	-5	-1	1	1	3	-1	0	1	-1	-1	1	0	0
μ_9	55	-5	-1	1	1	-1	3	0	1	-1	1	-1	0	0
μ_{10}	66	6	2	3	0	-2	-2	1	0	-1	0	0	1	0
μ_{11}	99	-1	3	0	3	-1	-1	-1	-1	0	1	1	-1	0
μ_{12}	120	0	-8	3	0	0	0	0	0	1	0	0	0	-1
μ_{13}	144	4	0	0	-3	0	0	-1	1	0	0	0	-1	1
μ_{14}	176	-4	0	-4	-1	0	0	1	-1	0	0	0	1	0

Table 3: The Q- conjugacy character table of M_{12} (i.e. $C^{Q}_{M_{12}}$), wherein $B_{11} = 11a \bigcup 11b$

3.2 Proposition

The sporadic McLaughlin group, $M^C L$ has six unmatured dominant classes with t =2. Furthermore, there are eighteen Q- conjugacy characters for $M^C L$ with the following degrees: 1, 22, 231, 252, 1540, 1750, 1792, 3520, 4500, 4752, 5103, 5544, 9625, 16038, 16500, 19172 and 20790 .

proof : Similar discussions like Proposition 3.1 show that in the sporadic McLaughlin group, $M^C L$ with SCSG(MCL) = 18, has eighteen Q- conjugacy characters with the following degrees: 1, 22, 231, 252, 1540, 1750, 1792, 3520, 4500, 4752, 5103, 5544, 9625, 16038, 16500, 19172 and 20790.

Besides, its dominant classes are 1a, 2a, 3a, 3b, 4a, 5a, 5b, 6a, 6b, $C_7 = 7a \bigcup 7b$, 8a, $C_9 = 9a \bigcup 9b$, 10a and $C_{11} = 11a \bigcup 11b$, 12a, $C_{14} = 14a \bigcup 14b$, $C_{15} = 15a \bigcup 15b$ and $C_{30} = 30a \bigcup 30b$, we afford all Q-conjugacy characters of $M^C L$ in Table 4. As matter of fact in Table 4, i is an unmatured Q- conjugacy character for $i = 5, 6, 14, 15, 17, 18$ which is the sum of two irreducible characters, see Table 4.

	1a	2a	3a	3b	4a	5a	5b	6a	6b	C_7	8a	C_9	10	C_{11}	12a	C_{14}	C_{15}	C_{30}
η_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
η_2	22	6	−5	4	2	−3	2	3	0	1	0	1	1	0	−1	−1	0	−2
η_3	231	7	15	6	−1	6	1	7	−2	0	−1	0	2	0	−1	0	0	2
η_4	252	28	9	9	4	2	2	1	1	0	0	0	−2	−2	1	0	−1	1
η_5	1540	−28	−26	10	−4	−10	0	14	2	0	0	−2	1	0	2	0	−1	−1
η_6	1792	0	64	−8	0	−8	2	0	0	0	0	−2	0	−1	0	0	4	0
η_7	1750	70	−5	13	2	0	0	−5	1	0	0	−2	0	1	−1	0	0	0
η_8	3520	64	−44	10	0	−5	0	4	−2	−1	0	1	−1	0	0	1	1	−1
η_9	3520	−64	44	10	0	−5	0	−4	2	−1	0	1	1	0	0	−1	1	1
η_{10}	4500	20	45	−9	4	0	0	5	−1	−1	0	0	0	1	1	−1	0	0
η_{11}	4752	−48	54	0	0	2	2	−6	0	−1	0	0	2	0	0	1	−1	−1
η_{12}	5103	63	0	0	3	3	−2	0	0	0	1	0	3	−1	0	0	0	0
η_{13}	5544	−56	36	9	0	19	−1	4	1	0	0	0	−1	0	0	0	1	−1
η_{14}	16038	−90	0	0	6	−12	−2	0	0	1	−2	0	0	0	0	1	0	0
η_{15}	16500	20	30	12	−4	0	0	−10	−4	1	0	0	0	0	2	−1	0	0
η_{16}	9625	105	40	−5	−3	0	0	0	3	0	−1	1	0	0	0	0	0	0
η_{17}	19172	0	−16	−16	0	12	2	0	0	0	0	−1	0	0	0	0	0	0
η_{18}	20790	−42	54	0	−2	−10	0	6	0	0	2	0	−2	0	−2	0	−1	1

Table 4: The integer-valued character table of McLaughlin group (i.e. $C^Q_{M^cL}$), where $C_k = ka \bigcup kb$ is an unmatured dominant class for k=7,9,11,14,15,30.

3.3 Proposition

The sporadic Higman-Sims group, HS has two unmatured dominant classes with t =2. Furthermore, there are twenty two Q- conjugacy characters for HS with the following degrees: 1, 22, 77, 154, 175, 231, 693, 770, 825, 1056, 1386, 1408, 1540, 1750, 1792, 1925, 2520, 2750 and 3200.

proof The sporadic Higman-Sims group, HS has twenty two Q- conjugacy characters with the following degrees: 1, 22, 77, 154, 175, 231, 693, 770, 825, 1056, 1386, 1408, 1540, 1750, 1792, 1925, 2520, 2750 and 3200. Besides, its dominant classes are$1a, 2a, 2b, 3a, 4a, 4b, 4c, 5a, 5b, 5c, 6a, 6b, 7a, 8a, 8b, 8c, 10a, 10b, D_{11} = 11a \bigcup 11b$

$, 12a, 15a \;\; and \;\; D_{20} = 20a \bigcup 20b$ with SCSG(HS) = 22, we afford all Q-conjugacy characters of HS in Table 5.

As matter of fact in Table 5, Φ_{11} and Φ_{13} are unmatured Q- conjugacy character which are the sum of two irreducible characters, see Table 5.

	$1a$	$2a$	$2b$	$3a$	$4a$	$4b$	$4c$	$5a$	$5b$	$5c$	$6a$
ϕ_1	1	1	1	1	1	1	1	1	1	1	1
ϕ_2	22	6	−2	4	−6	2	2	−3	2	2	−2
ϕ_3	77	13	1	5	5	5	1	2	−3	2	1
ϕ_4	154	10	10	1	−2	6	−2	4	4	−1	1
ϕ_5	154	10	−10	1	−10	−2	2	4	4	−1	−1
ϕ_6	154	10	−10	1	−10	−2	2	4	4	−1	−1
ϕ_7	175	15	11	4	15	−1	3	0	5	0	2
ϕ_8	231	7	−9	6	15	−1	−1	6	1	1	0
ϕ_9	693	21	9	0	21	5	1	−7	3	−2	0
ϕ_{10}	770	34	−1	5	−14	2	−2	−5	0	0	−1
ϕ_{11}	1540	−28	20	10	−20	−4	−4	−10	0	0	2
ϕ_{12}	825	25	9	6	−15	1	1	0	−5	0	0
ϕ_{13}	1792	0	32	−8	0	0	0	−8	2	2	−2
ϕ_{14}	1056	32	0	−6	0	0	0	6	−4	1	0
ϕ_{15}	1386	−6	18	0	6	−2	−2	11	6	1	0
ϕ_{16}	1408	0	16	4	0	0	0	8	−7	−2	−2
ϕ_{17}	1750	−10	10	−5	−10	6	2	0	0	0	1
ϕ_{18}	1925	5	−19	−1	5	5	−3	0	5	0	−1
ϕ_{19}	1925	5	1	−1	−35	−3	1	0	5	0	1
ϕ_{20}	2520	24	0	0	24	−8	0	−5	0	0	0
ϕ_{21}	2750	−50	−10	5	10	2	2	0	0	0	−1
ϕ_{22}	3200	0	−16	−4	0	0	0	0	−5	0	2

Table 5: The integer-valued character table of Higman-Sims group (i.e. C^Q_{HS})

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References

[1] Ganief S., Moori J. (1999) "2-Generations of the Forth Janko Group J4," J. Algebra, 212, 305-322.

[2] Evans A.B. (2009) "The admissibility of sporadic simple groups," J. Algebra, 321, 105-116.

[3] Moghani A., Ashrafi A.R. (2006) "nX-complementary generations of the Fischer group Fi23," J. Appl. Math & Computing 21, 393-409.

[4] Smeyers Y.G. (1992) "Structure and Dynamics of Non-Rigid Molecular Systems," Adv. Quant. Chem., 24, 1-77.

[5] Fujita S. (1998) "Inherent Automorphism and Q-Conjugacy Character Tables of Finite Groups. An Application to Combinatorial Enumeration of Isomers," Bull. Chem. Soc. Jpn., 71, 2309-2321.

[6] Fujita S., Diagrammatical Approach to Molecular Symmetry and Enumeration of Stereoisomers, MCM, Kragujevac, 2007.

[7] Moghani A. (2009) "A New Simple Method for Maturity of Finite Groups and Application to Fullerenes and Fluxional Molecules," Bull. Chem. Soc. Jpn., 82, 1103-1106.

[8] Darafsheh M.R., Moghani A., Naghdi Sedeh S. (2008) "Group Theory for Tetramethylethylene II, Acta Chim. Slov., 55, 602-607.

[9] Moghani A. (2010) "Study of Symmetries on some Chemical Nanostructures, J. Nano Res., 11, 7-11.

[10] Moghani A., Ashrafi A.R., Naghdi S., Ahmadi A. (2007) "Automorphism group of some chemical graphs, London Math. Soc., 340, 630-639.

[11] Sharifi H., Moghani A., Sorouhesh M.R., The Integer-valued characters of some Mathieu groups, Group Theory Conference, Mashhad, Iran, 2010.

[12] Moghani A., Naghdi Sedeh S., Sorouhesh M.R. (2010) "Fujita's Combinatorial Enumeration for the Non-Rigid Group of 2,4-Dimethylbenzene," J. Serb. Chem. Soc., 75, 91-99.

[13] Aschbacher M., Sporadic Groups, Cambridge University Press, UK. 1997.

[14] Feit W., Seitz G.M. (1988) "On finite rational groups and related topics," Illinois J. Math., 33, 103-131.

[15] Conway J.H., Curtis R.T., Norton S.P., Parker R.A., Wilson R.A., ATLAS of finite groups, Oxford Univ. Press (Clarendon), Oxford, 1985.

[16] GAP, Groups, Algorithms and Programming, Lehrstuhl De for Mathematik, RWTH, Aachen, 1995.

[17] Kerbe A., Thurlings K., Combinatorial Theory, Springer, Berlin, 1982.

[18] Moghani A., Gilani A. "Integer-valued Characters for some Sporadic Groups," Amer. Inst. Phy. Conf. Proc., 1309, 321-328.