Mathematical Sciences *Vol. 5, No. 4 (2011) 307-320*

A sixth-order compact finite difference method for free vibration analysis of Euler-Bernoulli beams

R. Ansari [a], **R. Gholami** [a], **K. Hosseini** [b,1]

[a] *Department of Mechanical Engineering, University of Guilan, P.O. Box 3756, Rasht, Iran*

[b] *Department of Mathematics, Lahijan Branch, Islamic Azad University, P.O. Box 1616, Lahijan, Iran*

Abstract

This paper presents an efficient method for solving free vibration problems for an Euler-Bernoulli beam under various supporting conditions. The method is based on the implementation of the sixth-order compact finite difference method (CFDM) for discretizing the governing differential equation to obtain the natural frequencies of beam corresponding to two commonly used boundary conditions namely simply supported-simply supported and clamped-free. A very good agreement is found between the natural frequencies obtained using the sixth-order compact finite difference scheme and exact natural frequencies, which confirms the validity of the present sixth-order discretization.

Keywords: Euler-Bernoulli beam, Free vibration, Sixth-order compact finite difference method, Natural frequencies.

1 Introduction

In recent years, many approaches have been adopted for solving vibration problems for an Euler-Bernoulli beam under various supporting conditions. For example, Liu et al. [1] presented a way of using He's variational iteration method [2-6] to solve

[1] Corresponding Author. E-mail Address: kamyar_hosseini@yahoo.com (K. Hosseini)

free vibration problems for an Euler-Bernoulli beam corresponding to various sets of boundary conditions. By applying this technique, the beam's natural frequencies and mode shapes was obtained and a rapidly convergent sequence was derived during the solution. The authors of [7] also presented an innovative eigenvalue problem solver for free vibration of Euler-Bernoulli beam by using the Adomian decomposition method [8-10]. Similarly, using this method the beam's natural frequencies and mode shapes was successfully obtained. The compact finite difference method is a numerical technique with excellent capability and resolution leading to it being in demand for application in different kinds of engineering problems. To mention just some of the engineering applications of this well-established numerical technique, Sari [11] utilized a sixth-order compact finite difference method in space and a low-storage total variation diminishing third-order Runge-Kutta scheme in time for solving the porous media equation. Dehghan and Mohebbi [12] used a compact finite difference approximation of fourth order to discretize spatial derivatives and a boundary value method of fourth order for the time integration of the resulting linear system of ordinary differential equations for solving the two-dimensional unsteady convection-diffusion equation. They also applied a compact finite difference approximation of fourth order for discretizing the spatial derivative and a fourth order A-stable DIRKN method for the time integration of the resulting nonlinear second-order system of ordinary differential equations for solving the one-dimensional nonlinear sine-Gordon equation [13]. In general, this method has been successfully applied for solving a wide variety of problems [14-22]. Therefore, in the present paper, a sixth-order compact finite difference method is utilized to obtain the beam's natural frequencies corresponding to two commonly used boundary conditions namely simply supported-simply supported and clamped-free. The rest of this paper has been organized as follows: In Section 2, the sixth-order compact finite difference scheme is employed for discretizing the Euler-Bernoulli beam equation under various supporting conditions. In Section 3, the matrix form of the difference scheme is given. In Section 4, the numerical results are illustrated to demonstrate the efficiency of the method and finally conclusion is provided in Section 5.

2 Euler-Bernoulli beam and compact finite difference method

Ignoring shear deformation and rotary inertia effect, the equation of motion for lateral vibrations of a uniform Euler-Bernoulli beam can be written as [1]

$$EI\frac{\partial^4 y(x,t)}{\partial x^4} + \rho A\frac{\partial^2 y(x,t)}{\partial t^2} = 0, \qquad 0 < x < l, \tag{1}$$

with the following boundary conditions

$$[c_{r3}\frac{\partial^3 y(x,t)}{\partial x^3} + c_{r2}\frac{\partial^2 y(x,t)}{\partial x^2} + c_{r1}\frac{\partial y(x,t)}{\partial x} + c_{r0}y(x,t)]|_{x=0} = 0, \qquad r = 1,2 \tag{2}$$

$$[d_{r3}\frac{\partial^3 y(x,t)}{\partial x^3} + d_{r2}\frac{\partial^2 y(x,t)}{\partial x^2} + d_{r1}\frac{\partial y(x,t)}{\partial x} + d_{r0}y(x,t)]|_{x=l} = 0, \qquad r = 1,2 \tag{3}$$

where $y(x,t)$ is the lateral deflection at distance x along the length of the beam and time t. EI, ρ and A are the flexural rigidity, the density and the cross-sectional area of the beam, respectively. c_{ri} and d_{ri} are constants coming from different boundary conditions for Euler-Bernoulli beams, where $i = 0,1,2,3$ and $r = 1,2$. For any mode of vibration, the lateral deflection $y(x,t)$ can be written in the form

$$y(x,t) = Y(x)h(t),$$

where $Y(x)$ is the model deflection and $h(t)$ is a harmonic function of time. If ω denotes the frequency of $h(t)$, then one has

$$\frac{\partial^2 y(x,t)}{\partial t^2} = -\omega^2 Y(x)h(t).$$

Therefore, Eq. (1) is reduced to the following differential equation

$$\frac{d^4 Y(x)}{dx^4} - \alpha Y(x) = 0, \qquad 0 < x < l, \tag{4}$$

where $\alpha = \frac{\rho A\omega^2}{EI}$ is the eigenvalue for this problem.

Similarly, the boundary conditions (Eqs. (2) and (3)) can be expressed as follows

$$[c_{r3}\frac{d^3Y(x)}{dx^3}+c_{r2}\frac{d^2Y(x)}{dx^2}+c_{r1}\frac{dY(x)}{dx}+c_{r0}Y(x)]|_{x=0}=0, \qquad r=1,2$$

$$[d_{r3}\frac{d^3Y(x)}{dx^3}+d_{r2}\frac{d^2Y(x)}{dx^2}+d_{r1}\frac{dY(x)}{dx}+d_{r0}Y(x)]|_{x=l}=0, \qquad r=1,2.$$

Now, a sixth-order compact finite difference method is applied for Eq. (4) under various supporting conditions.

Case 1. Consider Eq. (4) with the simply supported-simply supported boundary conditions

$$x=0: \ Y(x)=0, \ \frac{d^2Y(x)}{dx^2}=0,$$

$$x=l: \ Y(x)=0, \ \frac{d^2Y(x)}{dx^2}=0.$$

We discretize the region $S=\{x|x\in[0,l]\}$ with grid points located at $x_i=ih$, $i=0,1,\ldots,N+1$, where $h=\frac{l}{N+1}$ is the space grid step size. To solve the problem above, we apply the following sixth-order compact finite difference formula

$$\frac{1}{h^4}Y_{i-2}-\frac{4}{h^4}Y_{i-1}+\frac{6}{h^4}Y_i-\frac{4}{h^4}Y_{i+1}+\frac{1}{h^4}Y_{i+2}$$
$$-\alpha(-\frac{1}{720}Y_{i-2}+\frac{31}{180}Y_{i-1}+\frac{79}{120}Y_i+\frac{31}{180}Y_{i+1}-\frac{1}{720}Y_{i+2})=0, \qquad (5)$$

$i=3,\ldots,N-2$ for interior points. Also, when $i=1$, $i=2$, $i=N-1$ and $i=N$, we utilize the following formulae, respectively

$$\frac{60}{11h^4}Y_1-\frac{48}{11h^4}Y_2+\frac{12}{11h^4}Y_3-\alpha(\frac{29}{44}Y_1+\frac{19}{110}Y_2-\frac{1}{660}Y_3)=0, \qquad (6)$$

$$-\frac{4}{h^4}Y_1+\frac{6}{h^4}Y_2-\frac{4}{h^4}Y_3+\frac{1}{h^4}Y_4-\alpha(\frac{31}{180}Y_1+\frac{79}{120}Y_2+\frac{31}{180}Y_3-\frac{1}{720}Y_4)=0, \qquad (7)$$

$$\frac{1}{h^4}Y_{N-3}-\frac{4}{h^4}Y_{N-2}+\frac{6}{h^4}Y_{N-1}-\frac{4}{h^4}Y_N-\alpha(-\frac{1}{720}Y_{N-3}+\frac{31}{180}Y_{N-2}+\frac{79}{120}Y_{N-1}+\frac{31}{180}Y_N)=0, \qquad (8)$$

$$\frac{12}{11h^4}Y_{N-2}-\frac{48}{11h^4}Y_{N-1}+\frac{60}{11h^4}Y_N-\alpha(-\frac{1}{660}Y_{N-2}+\frac{19}{110}Y_{N-1}+\frac{29}{44}Y_N)=0. \qquad (9)$$

The details of the derivation of above formulae have been shown in Appendix A.

Case 2. Consider Eq. (4) with the clamped-free boundary conditions

$$x = 0: \ \ Y(x) = 0, \ \ \frac{dY(x)}{dx} = 0,$$

$$x = l: \ \ \frac{d^2Y(x)}{dx^2} = 0, \ \ \frac{d^3Y(x)}{dx^3} = 0.$$

We discretize the region $S = \{x|x \in [0, l]\}$ with grid points located at $x_i = ih$, $i = 0, 1, \ldots, N$, where $h = \frac{l}{N}$ is the space grid step size. Similarly, to solve the boundary value problem above, we implement the following sixth-order compact finite difference formula

$$\frac{1}{h^4}Y_{i-2} - \frac{4}{h^4}Y_{i-1} + \frac{6}{h^4}Y_i - \frac{4}{h^4}Y_{i+1} + \frac{1}{h^4}Y_{i+2}$$
$$-\alpha(-\frac{1}{720}Y_{i-2} + \frac{31}{180}Y_{i-1} + \frac{79}{120}Y_i + \frac{31}{180}Y_{i+1} - \frac{1}{720}Y_{i+2}) = 0, \quad (10)$$

$i = 3, \ldots, N-2$ for interior points. Moreover, when $i = 1$, $i = 2$, $i = N-1$ and $i = N$, we employ the following formulae, respectively

$$\frac{12}{h^4}Y_1 - \frac{6}{h^4}Y_2 + \frac{4}{3h^4}Y_3 - \alpha(\frac{13}{20}Y_1 + \frac{7}{40}Y_2 - \frac{1}{540}Y_3) = 0, \quad (11)$$

$$-\frac{4}{h^4}Y_1 + \frac{6}{h^4}Y_2 - \frac{4}{h^4}Y_3 + \frac{1}{h^4}Y_4 - \alpha(\frac{31}{180}Y_1 + \frac{79}{120}Y_2 + \frac{31}{180}Y_3 - \frac{1}{720}Y_4) = 0, \quad (12)$$

$$\frac{1}{h^4}Y_{N-3} - \frac{27}{7h^4}Y_{N-2} + \frac{33}{7h^4}Y_{N-1} - \frac{13}{7h^4}Y_N$$
$$-\alpha(-\frac{1}{720}Y_{N-3} + \frac{289}{1680}Y_{N-2} + \frac{1109}{1680}Y_{N-1} + \frac{853}{5040}Y_N) = 0, \quad (13)$$

$$\frac{12}{7h^4}Y_{N-2} - \frac{24}{7h^4}Y_{N-1} + \frac{12}{7h^4}Y_N - \alpha(\frac{3}{140}Y_{N-2} - \frac{3}{70}Y_{N-1} + \frac{143}{140}Y_N) = 0. \quad (14)$$

The details of the derivation of above formulae have been shown in Appendix B.

3 Matrix form of the difference scheme

If we consider the relations (5)-(9) and (10)-(14), then two systems of linear equations will be obtained as follows

$$([A] - \alpha[B])\{Y\} = 0, \tag{15}$$

$$([C] - \alpha[D])\{Y\} = 0, \tag{16}$$

where $\{Y\} = [Y_1, Y_2, \ldots, Y_{N-1}, Y_N]^T$ and A, B, C and D are the diagonal matrices. To obtain a non-trivial solution of (15) and (16), it is required that the determinant of the coefficients matrix of (15) and (16) vanish, namely

$$det([A] - \alpha[B]) = 0, \tag{17}$$

$$det([C] - \alpha[D]) = 0, \tag{18}$$

thus, one can determine the eigenvalues from (17) and (18) and then obtain the beam's natural frequencies for the cases of 1 and 2.

4 Numerical results and discussion

In this section, comparisons are made to illustrate the performance of the proposed method. The computations associated with the cases of 1 and 2 have been performed using Matlab software. It is assumed that

$$\rho = 1.2, \qquad A = 0.2 \times 0.1, \qquad E = 3 \times 10^7, \qquad I = \frac{0.1 \times (0.2)^3}{12}.$$

The natural frequencies generated by the CFDM for the case of 1, have been shown in Table 1. Also, Table 2 demonstrates the percentage of the relative errors of the natural frequencies obtained by the CFDM and exact natural frequencies for the case of 1. From Table 1, it can be observed that the natural frequencies obtained by the CFDM are in good agreement with the exact natural frequencies. Moreover, Table 2 reveals that the percentage of the relative errors of the natural frequencies obtained by the CFDM and the exact natural frequencies lie within the interval [0.0012,1.6922]. In particular, when $N = 20$ the percentage of relative errors of the natural frequencies obtained by the CFDM and the exact natural frequencies restrict to the interval [0.0176,1.2394]. This subject shows that the CFDM is reliable, even for a small number of nodes. Similarly, Table 3 indicates the natural frequencies generated by the CFDM for the case of 2. Also, the percentage of the relative errors of the natural frequencies obtained by the CFDM and exact natural frequencies for the case of 2, have been demonstrated in Table 4. From Table 3, it is clearly seen that the natural frequencies generated by the CFDM are in good agreement with the exact natural frequencies. Besides, Table 4 illustrates that the percentage of the relative errors of the natural frequencies obtained via the CFDM and the exact natural frequencies lie in the interval [0.0257,4.5390]. Especially, when $N = 20$ the percentage of relative errors of the natural frequencies obtained by the CFDM and the exact natural frequencies restrict to the interval [0.1724,1.3226]. This reveals that the CFDM is reliable, even for a small number of nodes.

5 Conclusion

In this paper, a sixth-order compact finite difference method was adopted for solving free vibration problems for an Euler-Bernoulli beam under various supporting conditions. By making use of this method, the natural frequencies of beam corresponding to two commonly used boundary conditions namely simply supported-simply supported and clamped-free were successfully obtained. From the results, it was obsereved that

1) The natural frequencies obtained by the CFDM are in good agreement with the exact natural frequencies.

2) The CFDM is reliable, even for a small number of nodes.

Table 1: Natural frequencies obtained by CFDM in comparison with the exact natural frequencies for the case of 1

Length	N	$\omega^{(1)}$	$\omega^{(2)}$	$\omega^{(3)}$	$\omega^{(4)}$
1 (m)	**10**	2852.541	11447.08	25864.45	46150.74
	20	2849.612	11404.3	25680.29	45700.35
	30	2849.266	11398.92	25654.34	45623.83
	50	2849.145	11397	25644.81	45594.61
	Exact	2849.11	11396.44	25641.98	45585.75
5 (m)	**10**	114.1016	457.883	1034.578	1846.029
	20	113.9845	456.1719	1027.212	1828.014
	30	113.9706	455.9568	1026.173	1824.953
	50	113.9658	455.88	1025.793	1823.784
	Exact	113.9644	455.8575	1025.679	1823.43
10 (m)	**10**	28.52541	114.4708	258.6445	461.5074
	20	28.49612	114.043	256.8029	457.0035
	30	28.49266	113.9892	256.5434	456.2383
	50	28.49145	113.97	256.4481	455.9461
	Exact	28.4911	113.9644	256.4198	455.8575

Appendix A

Let's consider the Eq. (4) with the simply supported-simply supported boundary conditions. We discretize the region $S = \{x|x \in [0, l]\}$ with grid points located at $x_i = ih$, $i = 0, 1, \ldots, N+1$, where $h = \frac{l}{N+1}$ is the space grid step size. If we apply the central difference approximation, we can obtain the following relation for Eq. (4) at the point x_i

$$\delta_x^4 Y_i - \alpha Y_i = \tau_i, \tag{19}$$

Table 2: Percentage of the relative errors of the natural frequencies obtained by CFDM and exact natural frequencies for the case of 1

Length	N	$\omega^{(1)}$	$\omega^{(2)}$	$\omega^{(3)}$	$\omega^{(4)}$
1 (m)	10	0.1204	0.4443	0.8676	1.6922
	20	0.0176	0.0690	0.1494	1.2394
	30	0.0058	0.0218	0.0482	0.0835
	50	0.0012	0.0049	0.0110	0.0194

Table 3: Natural frequencies obtained by CFDM in comparison with the exact natural frequencies for the case of 2

Length	N	$\omega^{(1)}$	$\omega^{(2)}$	$\omega^{(3)}$	$\omega^{(4)}$
1 (m)	10	1008.0697	6215.1073	17179.092	33317.2018
	20	1013.2438	6323.1512	17639.2858	34439.7797
	30	1014.2108	6343.9813	17733.4189	34691.2597
	50	1014.7069	6354.7359	17782.5657	34824.9567
	Exact	1014.9862	6360.8098	17810.4517	34901.3769
5 (m)	10	40.3228	248.6043	687.1637	1332.6881
	20	40.5298	252.9260	705.5714	1377.5912
	30	40.5684	253.7593	709.3368	1387.6504
	50	40.5883	254.1894	711.3026	1392.9983
	Exact	40.5994	254.4324	712.4181	1396.0551
10 (m)	10	10.0807	62.1511	171.7909	333.1720
	20	10.1324	63.2315	176.3929	344.3978
	30	10.1421	63.4398	177.3342	346.9126
	50	10.1471	63.5474	177.8257	348.2496
	Exact	10.1499	63.6081	178.1045	349.0138

Table 4: Percentage of the relative errors of the natural frequencies obtained by CFDM and exact natural frequencies for the case of 2

Length	***N***	$\omega^{(1)}$	$\omega^{(2)}$	$\omega^{(3)}$	$\omega^{(4)}$
1 (*m*)	**10**	0.6818	2.2906	3.5449	4.5390
	20	0.1724	0.5921	0.9610	1.3226
	30	0.0764	0.2646	0.4325	0.6020
	50	0.0257	0.0955	0.1565	0.2190

which

$$\delta_x^4 Y_i = \frac{Y_{i-2} - 4Y_{i-1} + 6Y_i - 4Y_{i+1} + Y_{i+2}}{h^4},$$

and τ_i is the truncation error and can be shown as follows

$$\tau_i = \alpha(\frac{h^4}{80}\delta_x^4 Y_i + \frac{h^2}{6}\delta_x^2 Y_i) + O(h^6),$$

where

$$\delta_x^2 Y_i = \frac{-Y_{i-2} + 16Y_{i-1} - 30Y_i + 16Y_{i+1} - Y_{i+2}}{12h^2}.$$

Now, substituting τ_i into Eq. (19), yields the following relation

$$\delta_x^4 Y_i - \alpha(\frac{h^4}{80}\delta_x^4 Y_i + \frac{h^2}{6}\delta_x^2 Y_i + Y_i) = 0. \tag{20}$$

Also, by considering the grid point x_0 and the fictitious point x_{-1}, the boundary conditions for the case of 1 can be written as

$$x = 0: \ \ Y_0 = 0, \ \ \frac{11Y_{-1} - 20Y_0 + 6Y_1 + 4Y_2 - Y_3}{12h^2} = 0,$$

and therefore

$$Y_{-1} = -\frac{6}{11}Y_1 - \frac{4}{11}Y_2 + \frac{1}{11}Y_3, \ \ Y_0 = 0. \tag{21}$$

Similarly, by considering the grid point x_{N+1} and the fictitious point x_{N+2}, the following relations can be obtained

$$x = l: \ \ Y_{N+1} = 0, \ \ Y_{N+2} = \frac{1}{11}Y_{N-2} - \frac{4}{11}Y_{N-1} - \frac{6}{11}Y_N. \tag{22}$$

Now, from (20) one has

$$\frac{1}{h^4}Y_{i-2} - \frac{4}{h^4}Y_{i-1} + \frac{6}{h^4}Y_i - \frac{4}{h^4}Y_{i+1} + \frac{1}{h^4}Y_{i+2}$$

$$-\alpha(-\frac{1}{720}Y_{i-2} + \frac{31}{180}Y_{i-1} + \frac{79}{120}Y_i + \frac{31}{180}Y_{i+1} - \frac{1}{720}Y_{i+2}) = 0, \quad i = 3, \dots, N-2,$$

when $i = 1$ and $i = 2$, by considering (20) and (21) we will obtain

$$\frac{60}{11h^4}Y_1 - \frac{48}{11h^4}Y_2 + \frac{12}{11h^4}Y_3 - \alpha(\frac{29}{44}Y_1 + \frac{19}{110}Y_2 - \frac{1}{660}Y_3) = 0,$$

$$-\frac{4}{h^4}Y_1 + \frac{6}{h^4}Y_2 - \frac{4}{h^4}Y_3 + \frac{1}{h^4}Y_4 - \alpha(\frac{31}{180}Y_1 + \frac{79}{120}Y_2 + \frac{31}{180}Y_3 - \frac{1}{720}Y_4) = 0.$$

Similarly when $i = N-1$ and $i = N$, from (20) and (22) we will obtain

$$\frac{1}{h^4}Y_{N-3} - \frac{4}{h^4}Y_{N-2} + \frac{6}{h^4}Y_{N-1} - \frac{4}{h^4}Y_N - \alpha(-\frac{1}{720}Y_{N-3} + \frac{31}{180}Y_{N-2} + \frac{79}{120}Y_{N-1} + \frac{31}{180}Y_N) = 0,$$

$$\frac{12}{11h^4}Y_{N-2} - \frac{48}{11h^4}Y_{N-1} + \frac{60}{11h^4}Y_N - \alpha(-\frac{1}{660}Y_{N-2} + \frac{19}{110}Y_{N-1} + \frac{29}{44}Y_N) = 0.$$

Appendix B

Again consider the Eq. (4) with the clamped-free boundary conditions. We discretize the region $S = \{x | x \in [0, l]\}$ with grid points located at $x_i = ih$, $i = 0, 1, \dots, N$, where $h = \frac{l}{N}$ is the space grid step size. By considering the grid point x_0 and the fictitious point x_{-1}, the boundary conditions for the case of 2 can be written in the form

$$x = 0: \quad Y_0 = 0, \quad \frac{-3Y_{-1} - 10Y_0 + 18Y_1 - 6Y_2 + Y_3}{12h} = 0,$$

and so

$$Y_{-1} = 6Y_1 - 2Y_2 + \frac{1}{3}Y_3, \quad Y_0 = 0. \tag{23}$$

Similarly, by considering the fictitious points x_{N+1} and x_{N+2}, the following relations can be obtained

$$Y_{N+1} = \frac{1}{7}Y_{N-2} - \frac{9}{7}Y_{N-1} + \frac{15}{7}Y_N, \tag{24}$$

$$Y_{N+2} = \frac{9}{7}Y_{N-2} - \frac{32}{7}Y_{N-1} + \frac{30}{7}Y_N. \tag{25}$$

Now, from (20) one can obtain

$$\frac{1}{h^4}Y_{i-2} - \frac{4}{h^4}Y_{i-1} + \frac{6}{h^4}Y_i - \frac{4}{h^4}Y_{i+1} + \frac{1}{h^4}Y_{i+2}$$
$$-\alpha(-\frac{1}{720}Y_{i-2} + \frac{31}{180}Y_{i-1} + \frac{79}{120}Y_i + \frac{31}{180}Y_{i+1} - \frac{1}{720}Y_{i+2}) = 0, \quad i = 3, \ldots, N-2,$$

when $i = 1$ and $i = 2$, by considering (20) and (23) we will have

$$\frac{12}{h^4}Y_1 - \frac{6}{h^4}Y_2 + \frac{4}{3h^4}Y_3 - \alpha(\frac{13}{20}Y_1 + \frac{7}{40}Y_2 - \frac{1}{540}Y_3) = 0,$$
$$-\frac{4}{h^4}Y_1 + \frac{6}{h^4}Y_2 - \frac{4}{h^4}Y_3 + \frac{1}{h^4}Y_4 - \alpha(\frac{31}{180}Y_1 + \frac{79}{120}Y_2 + \frac{31}{180}Y_3 - \frac{1}{720}Y_4) = 0.$$

Also when $i = N-1$, from (20) and (24) one has

$$\frac{1}{h^4}Y_{N-3} - \frac{27}{7h^4}Y_{N-2} + \frac{33}{7h^4}Y_{N-1} - \frac{13}{7h^4}Y_N$$
$$-\alpha(-\frac{1}{720}Y_{N-3} + \frac{289}{1680}Y_{N-2} + \frac{1109}{1680}Y_{N-1} + \frac{853}{5040}Y_N) = 0,$$

and finally, when $i = N$ from (20), (24) and (25) we obtain

$$\frac{12}{7h^4}Y_{N-2} - \frac{24}{7h^4}Y_{N-1} + \frac{12}{7h^4}Y_N - \alpha(\frac{3}{140}Y_{N-2} - \frac{3}{70}Y_{N-1} + \frac{143}{140}Y_N) = 0.$$

References

[1] Liu Y., Gurram C.S. (2009)" The use of He's variational iteration method for obtaining the free vibration of an Euler-Bernoulli beam," Mathematical and Computer Modelling, 50, 1545-1552.

[2] He J.H. (1997)" A new approach to non-linear partial differential equations," Communications in Non-linear Science and Numerical Simulation, 2, 230-235.

[3] Ansari R., Hemmatnezhad M. (2011)" Nonlinear vibrations of embedded multi-walled carbon nanotubes using a variational approach," Mathematical and Computer Modelling, 53, 927-938.

[4] Biazar J., Gholamin P., Hosseini K. (2010)" Variational iteration method for solving Fokker-Planck equation," Journal of the Franklin Institute, 347, 1137-1147.

[5] Geng F., Li X. (2009)" Variational iteration method for solving tenth-order boundary value problems," Mathematical Sciences, 3, 161-172.

[6] Geng F. (2010)" Variational iteration method for a class of singular boundary value problems," Mathematical Sciences, 4, 359-370.

[7] Lai H.Y., Hsu J.C., Chen C.K. (2008)" An innovative eigenvalue problem solver for free vibration of Euler-Bernoulli beam by using the Adomian decomposition method," Computers and Mathematics with Applications, 56, 3204-3220.

[8] Adomian G. (1976)" Nonlinear stochastic differential equations," Journal of Mathematical Analysis and Applications, 55, 441-452.

[9] Adomian G. (1991)" A review of the decomposition method and some recent results for nonlinear equations," Computers and Mathematics with Applications, 21, 101-127.

[10] Nouri K., Garshasbia M., Damirchi J. (2008)" Application of Adomian decomposition method to solve a class of diffusion problem arises during MRI," Mathematical Sciences, 2, 207-218.

[11] Sari M., Solution of the porous media equation by a compact finite difference method, Mathematical Problems in Engineering, doi:10.1155/2009/912541.

[12] Dehghan M., Mohebbi A. (2008)" High-order compact boundary value method for the solution of unsteady convection-diffusion problems," Mathematics and Computers in Simulation, 79, 683-699.

[13] Mohebbi A., Dehghan M. (2010)" High-order solution of one-dimensional sine-Gordon equation using compact finite difference and DIRKN methods," Mathematical and Computer Modelling, 51, 537-549.

[14] Shang J.S. (1999)" High-order compact difference schemes for time-dependent Maxwell equations," Journal of Computational Physics, 153, 312-333.

[15] Cui M. (2009)" Compact finite difference method for the fractional diffusion equation," Journal of Computational Physics, 228, 7792-7804.

[16] Gürarslan G. (2010)" Numerical modelling of linear and nonlinear diffusion equations by compact finite difference method," Applied Mathematics and Computation, 216, 2472-2478.

[17] Zhao J., Davison M., Corless R.M. (2007)" Compact finite difference method for American option pricing," Journal of Computational and Applied Mathematics, 206, 306-321.

[18] Wang Y.M., Zhang H.B. (2009)" Higher-order compact finite difference method for systems of reaction-diffusion equations," Journal of Computational and Applied Mathematics, 233, 502-518.

[19] Dehghan M., Taleei A. (2010)" A compact split-step finite difference method for solving the nonlinear Schrdinger equations with constant and variable coefficients," Computer Physics Communications, 181, 43-51.

[20] Ge Y. (2010)" Multigrid method and fourth-order compact difference discretization scheme with unequal meshsizes for 3D Poisson equation," Journal of Computational Physics, 229, 6381-6391.

[21] Sari M., Gürarslan G. (2009)" A sixth-order compact finite difference scheme to the numerical solutions of Burger's equation," Applied Mathematics and Computation, 208, 475-483.

[22] Sari M., Gürarslan G., Dag I. (2010)" A compact finite difference method for the solution of the generalized Burgers-Fisher equation," Numerical Methods in Partial Differential Equations, 26, 125-134.