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A unique common fixed-point theorem for two maps under ψ-ϕ contractive condition in partial metric spaces

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Abstract

Purpose: The purpose of this paper is to study a common fixed point theorem for two maps in partial metric spaces.

Methods: To prove Cauchy sequence, we used $\Psi - \Phi$ contractive method and obtain common fixed points.

Results: We obtained a common fixed point result and illustrated with one example.

Conclusions: It is concluded from the present study that one can generalize some results from metric spaces to partial metric spaces.

Keywords: Partial metric, Weakly compatible maps, Complete space

Background

The notion of partial metric space was introduced by S.G. Matthews [1] as a part of the study of denotational semantics of data flow networks. In fact, it is widely recognized that partial metric spaces play an important role in constructing models in the theory of computation ([2-9], etc). Matthews [1], Oltra and Valero [10], Romaguera [11] and Altun et al. [12] proved fixed-point theorems in partial metric spaces for a single map.

In this paper, we obtain a unique common fixed-point theorem for two self mappings satisfying a generalized $\psi - \phi$ contractive condition in partial metric spaces. Our result generalizes and improves a theorem of Altun et al.[12] and some known theorems in partial metric spaces.

First, we recall some definitions and lemmas of partial metric spaces.

Definition 1.1. *[1] A partial metric on a non-empty set X is a function $p : X \times X \to R^+$ such that for all $x, y, z \in X$:*

(p_1) $x = y \Leftrightarrow p(x,x) = p(x,y) = p(y,y)$,
(p_2) $p(x,x) \leq p(x,y), p(y,y) \leq p(x,y)$,
(p_3) $p(x,y) = p(y,x)$,
(p_4) $p(x,y) \leq p(x,z) + p(z,y) - p(z,z)$.
(X,p) is called a partial metric space.

It is clear that $|p(x,y) - p(y,z)| \leq p(x,z) \forall x,y,z \in X$.

It is also clear that $p(x,y) = 0$ implies $x = y$ from (p_1) and (p_2). But if $x = y$, $p(x,y)$ may not be zero. A basic example of a partial metric space is the pair (R^+, p), where $p(x,y) = \max\{x,y\}$ for all $x,y \in R^+$.

Each partial metric p on X generates τ_0 topology τ_p on X which bases on the family of open p-balls $\{B_p(x,\epsilon)/ x \in X, \epsilon > 0\}$ for all $x \in X$ and $\epsilon > 0$, where $B_p(x,\epsilon) = \{y \in X / p(x,y) < p(x,x) + \epsilon\}$ for all $x \in X$ and $\epsilon > 0$.

If p is a partial metric on X, then the function $d_p : X \times X \to R^+$ given by $d_p(x,y) = 2p(x,y) - p(x,x) - p(y,y)$ is a metric on X.

Definition 1.2. *[1] Let (X,p) be a partial metric space.*

(a) *A sequence $\{x_n\}$ in (X,p) is said to converge to a point $x \in X$ if and only if $p(x,x) = \lim_{n\to\infty} p(x,x_n)$.*
(b) *A sequence $\{x_n\}$ in (X,p) is said to be Cauchy sequence if $\lim_{n,m\to\infty} p(x_n,x_m)$ exists and is finite.*
(c) *(X,p) is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges, w.r.t. to τ_p, to a point $x \in X$ such that $p(x,x) = \lim_{n,m\to\infty} p(x_n,x_m)$.*

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Lemma 1.3. *[1] Let (X, p) be a partial metric space.*

(a) *$\{x_n\}$ is a Cauchy sequence in (X, p) if and only if it is a Cauchy sequence in the metric space (X, d_p).*
(b) *(X, p) is complete if and only if the metric space (X, d_p) is complete. Furthermore, $\lim_{n\to\infty} d_p(x_n, x) = 0$ if and only if $p(x, x) = \lim_{n\to\infty} p(x_n, x) = \lim_{n,m\to\infty} p(x_n, x_m)$.*

Note 1.4. If $\{x_n\}$ converges to x in (X, p), then $\lim_{n\to\infty} p(x_n, y) \le p(x, y) \forall\, y \in X$.

Proof. Since $\{x_n\}$ converges to x, we have $p(x, x) = \lim_{n\to\infty} p(x_n, x)$. Now, $p(x_n, y) \le p(x_n, x) + p(x, y) - p(x, x)$. Letting $n \to \infty$, $\lim_{n\to\infty} p(x_n, y) \le \lim_{n\to\infty} p(x_n, x) + p(x, y) - p(x, x)$. Thus, $\lim_{n\to\infty} p(x_n, y) \le p(x, y)$. □

Results and discussion

Theorem 2.1. *Let (X, p) be a partial metric space and let $S, f : X \to X$ be such that*

(2.1.1) $\psi\left(p\left(Sx, Sy\right)\right) \le \psi\left(M\left(x, y\right)\right) - \phi\left(M\left(x, y\right)\right), \forall x, y \in X$, where $\psi : [0, \infty) \to [0, \infty)$ is continuous, non-decreasing and $\phi : [0, \infty) \to [0, \infty)$ is lower semi-continuous with $\phi(t) > 0$ for $t > 0$ and $M\left(x, y\right) = \max\left\{p(fx, fy), p(fx, Sx), p(fy, Sy), \frac{1}{2}\left[p\left(fx, Sy\right) + p(fy, Sx)\right]\right\}$,
(2.1.2) $S(X) \subseteq f(X)$ and $f(X)$ is a complete subspace of X, and
(2.1.3) the pair (f, S) is weakly compatible. Then, S and f have a unique common fixed point in X.

Proof. Let $x_0 \in X$. From (2.1.2), there exist sequences $\{x_n\}$ and $\{y_n\}$ in X such that $y_n = Sx_{n-1} = fx_n, n = 1, 2, \ldots\ldots.$

Case (i): Suppose $y_n = y_{n+1}$ for some n, then $fz = Sz$, where $z = x_n$. Let us denote $fz = Sz = \alpha$. From (2.1.3), we have $f\alpha = S\alpha$. Suppose $S\alpha \ne \alpha$, then

$$\begin{aligned}\psi(p(S\alpha, \alpha)) &= \psi(p(S\alpha, Sz)) \\ &\le \psi(M(\alpha, z)) - \phi(M(\alpha, z)).\end{aligned}$$

$$\begin{aligned}M(\alpha, z) &= \max\left\{p(S\alpha, \alpha), p(S\alpha, S\alpha), p(\alpha, \alpha), \frac{1}{2}\left[p(S\alpha, \alpha) + p(\alpha, S\alpha)\right]\right\} \\ &= p(S\alpha, \alpha), \text{ from } (p_2).\end{aligned}$$

Thus,

$$\begin{aligned}\psi(p(S\alpha, \alpha)) &\le \psi(p(S\alpha, \alpha)) - \phi(p(S\alpha, \alpha)) \\ &< \psi(p(S\alpha, \alpha)), \text{ since } \phi(t) > 0\ \forall\ t > 0.\end{aligned}$$

Hence, $S\alpha = \alpha$. Thus, α is a common fixed point of f and S. Suppose β is another common fixed point of f and S,

$$\begin{aligned}M(\alpha, \beta) &= \max\left\{p(\alpha, \beta), p(\alpha, \alpha), p(\beta, \beta), \frac{1}{2}\left[p(\alpha, \beta) + p(\beta, \alpha)\right]\right\} \\ &= p(\alpha, \beta), \text{ from } (p_2)\end{aligned}$$

$$\begin{aligned}\psi(p(\alpha, \beta)) &= \psi(p(S\alpha, S\beta)) \\ &\le \psi(M(\alpha, \beta)) - \phi(M(\alpha, \beta)) \\ &= \psi(p(\alpha, \beta)) - \phi(p(\alpha, \beta)) \\ &< \psi(p(\alpha, \beta)), \text{ since } \phi(t) > 0 \forall t > 0.\end{aligned}$$

Hence, $\beta = \alpha$. Thus, α is the unique common fixed point of S and f.

Case (ii): Assume that $y_n \ne y_{n+1}$ for all n. Denote $p_n = p(y_n, y_{n+1})$.

$$\begin{aligned}\psi\left(p_n\right) &= \psi\left(p\left(y_n, y_{n+1}\right)\right) \\ &= \psi\left(p\left(Sx_{n-1}, Sx_n\right)\right) \\ &\le \psi\left(M\left(x_{n-1}, x_n\right)\right) - \phi\left(M\left(x_{n-1}, x_n\right)\right).\end{aligned}$$

$$M(x_{n-1}, x_n) = \max\left\{\begin{array}{c} p(y_{n-1}, y_n), p(y_{n-1}, y_n), p(y_n, y_{n+1}), \\ \frac{1}{2}\left[p(y_{n-1}, y_{n+1}) + p(y_n, y_n)\right]\end{array}\right\}.$$

But

$$\begin{aligned}\frac{1}{2}\left[p(y_{n-1}, y_{n+1}) + p(y_n, y_n)\right] &\le \frac{1}{2}\left[p(y_{n-1}, y_n) + p(y_n, y_{n+1})\right] \\ &\le \max\left\{p_{n-1}, p_n\right\}.\end{aligned}$$

Hence, $M(x_{n-1}, x_n) = max\{p_{n-1}, p_n\}$. If p_n is maximum, then

$$\begin{aligned}\psi(p_n) &\le \psi(p_n) - \phi(p_n) \\ &< \psi(p_n), \text{ since } \phi(t) > 0\ \forall\ t > 0.\end{aligned}$$

Hence,

$$\begin{aligned}\psi(p_n) &\le \psi(p_{n-1}) - \phi(p_{n-1}) \\ &< \psi(p_{n-1}).\end{aligned} \tag{1}$$

Since ψ is non-decreasing, we have $p_n < p_{n-1}, n = 1, 2, 3, \ldots$ Thus, $\{p_n\}$ is a non-increasing sequence of non-negative real numbers and must converge to a real number, say, $k \ge 0$. Letting $n \to \infty$ in (1), we get $\psi(k) \le \psi(k) - \phi(k)$ so that $\phi(k) \le 0$. Hence, $k = 0$.

Thus,

$$\lim_{n\to\infty} p(y_n, y_{n+1}) = 0 \tag{2}$$

Hence,

$$\lim_{n\to\infty} p(y_n, y_n) = 0 = \lim_{n\to\infty} p(y_{n+1}, y_{n+1}) \text{ from } (p_2) \tag{3}$$

From the definition of d_p and (2) and (3), we have

$$\lim_{n\to\infty} d_p(y_n, y_{n+1}) = 0 \tag{4}$$

Now, we prove that $\{y_n\}$ is a Cauchy sequence in (X, d_p). On contrary, suppose that $\{y_n\}$ is not Cauchy, then there exists an $\epsilon > 0$ and monotone increasing sequences of natural numbers $\{m_k\}$ and $\{n_k\}$ such that $n_k > m_k$,

$$d_p(y_{m_k}, y_{n_k}) \geq \epsilon \tag{5}$$

and

$$d_p(y_{m_k}, y_{n_k-1}) < \epsilon \tag{6}$$

From (5),

$$\begin{aligned}\epsilon &\leq d_p(y_{m_k}, y_{n_k}) \\ &\leq d_p(y_{m_k}, y_{n_k-1}) + d_p(y_{n_k-1}, y_{n_k}) \\ &< \epsilon + d_p(y_{n_k-1}, y_{n_k}) \ \textit{from}\ \ (6).\end{aligned}$$

Letting $k \to \infty$ and using (4), we have

$$\lim_{k\to\infty} d_p(y_{m_k}, y_{n_k}) = \epsilon. \tag{7}$$

Letting $k \to \infty$ and using (7) and (4) in $|d_p(y_{m_k}, y_{n_k+1}) - d_p(y_{m_k}, y_{n_k})| \leq d_p(y_{n_k+1}, y_{n_k})$ we get

$$\lim_{k\to\infty} d_p(y_{m_k}, y_{n_k+1}) = \epsilon. \tag{8}$$

From the definition of d_p, (8) and (3), we have

$$\lim_{k\to\infty} p(y_{m_k}, y_{n_k+1}) = \frac{\epsilon}{2}. \tag{9}$$

Letting $k \to \infty$ and using (7) and (4) in $|d_p(y_{m_k-1}, y_{n_k}) - d_p(y_{m_k}, y_{n_k})| \leq d_p(y_{m_k-1}, y_{m_k})$ we get

$$\lim_{k\to\infty} d_p(y_{m_k-1}, y_{n_k}) = \epsilon. \tag{10}$$

Hence, we have

$$\lim_{k\to\infty} p(y_{m_k-1}, y_{n_k}) = \frac{\epsilon}{2}. \tag{11}$$

Letting $k \to \infty$ and using (10) and (4) in $|d_p(y_{n_k-1}, y_{m_k-1}) - d_p(y_{m_k-1}, y_{n_k})| \leq d_p(y_{n_k-1}, y_{n_k})$ we get

$$\lim_{k\to\infty} d_p(y_{n_k-1}, y_{m_k-1}) = \epsilon. \tag{12}$$

Hence, we have

$$\lim_{k\to\infty} p(y_{n_k-1}, y_{m_k-1}) = \frac{\epsilon}{2}. \tag{13}$$

Letting $k \to \infty$ and using (10) and (4) in $|d_p(y_{n_k}, y_{m_k-2}) - d_p(y_{n_k}, y_{m_k-1})| \leq d_p(y_{m_k-2}, y_{m_k-1})$ we get

$$\lim_{k\to\infty} d_p(y_{n_k}, y_{m_k-2}) = \epsilon. \tag{14}$$

Hence, we have

$$\lim_{k\to\infty} p(y_{n_k}, y_{m_k-2}) = \frac{\epsilon}{2}. \tag{15}$$

$$\begin{aligned}&\psi\left(p(y_{m_k}, y_{n_k+1})\right) \\ &= \psi\left(p(Sx_{m_k-1}, Sx_{n_k})\right) \\ &\leq \psi\left(\max\left\{\begin{array}{c} p(y_{m_k-1}, y_{n_k}), p(y_{m_k-1}, y_{m_k-2}), p(y_{n_k}, y_{n_k-1}), \\ \frac{1}{2}[p(y_{m_k-1}, y_{n_k-1}) + p(y_{n_k}, y_{m_k-2})] \end{array}\right\}\right) \\ &\quad - \phi\left(\max\left\{\begin{array}{c} p(y_{m_k-1}, y_{n_k}), p(y_{m_k-1}, y_{m_k-2}), p(y_{n_k}, y_{n_k-1}), \\ \frac{1}{2}[p(y_{m_k-1}, y_{n_k-1}) + p(y_{n_k}, y_{m_k-2})] \end{array}\right\}\right).\end{aligned}$$

Letting $k \to \infty$ and using (9), (11), (2), (13) and (15), we get

$$\begin{aligned}\psi\left(\frac{\epsilon}{2}\right) &\leq \psi\left(\max\left\{\tfrac{\epsilon}{2}, 0, 0, \tfrac{1}{2}\left[\tfrac{\epsilon}{2} + \tfrac{\epsilon}{2}\right]\right\}\right) \\ &\quad - \phi\left(\max\left\{\tfrac{\epsilon}{2}, 0, 0, \tfrac{1}{2}\left[\tfrac{\epsilon}{2} + \tfrac{\epsilon}{2}\right]\right\}\right) \\ \psi\left(\frac{\epsilon}{2}\right) &\leq \psi\left(\frac{\epsilon}{2}\right) - \phi\left(\frac{\epsilon}{2}\right) < \psi\left(\frac{\epsilon}{2}\right).\end{aligned}$$

It is a contradiction. Hence, $\{y_n\}$ is a Cauchy sequence in (X, d_p). Thus,

$$\lim_{n,m\to\infty} d_p(y_n, y_m) = 0. \tag{16}$$

Now $d_p(y_n, y_m) = 2p(y_n, y_m) - p(y_n, y_n) - p(y_m, y_m)$. Letting $n, m \to \infty$ and using (16) and (3), we get

$$\lim_{n,\, m\to\infty} p(y_n, y_m) = 0. \tag{17}$$

Suppose $f(X)$ is complete. Since $\{y_n\} \subseteq f(X)$ is a Cauchy sequence in the complete metric space $(f(X), d_p)$, it follows that $\{y_n\}$ converges in $(f(X), d_p)$. Thus, $\lim_{n\to\infty} d_p(y_n, v) = 0$ for some $v \in f(X)$. There exists $u \in X$ such that $v = fu$. From Lemma 1.3(b), we have

$$p(v, v) = \lim_{n\to\infty} p(y_n, v) = \lim_{n,\, m\to\infty} p(y_n, y_m) \tag{18}$$

Now, from (17) and (18),

$$p(v, v) = \lim_{n\to\infty} p(y_n, v) = 0. \tag{19}$$

Now, suppose $Su \neq v$

$$\begin{aligned}p(Su, v) &\leq p\left(Su, Sx_n\right) + p\left(Sx_n, v\right) - p\left(Sx_n, Sx_n\right) \\ &\leq p\left(Su, Sx_n\right) + p\left(y_{n+1}, v\right) \\ \psi(p(Su, v)) &\leq \psi\left[p(Su, Sx_n) + p(y_{n+1}, v)\right].\end{aligned}$$

Letting $n \to \infty$, we have

$$\begin{aligned}&\psi(p(Su, v)) \\ &\leq \psi\left(\lim_{n\to\infty} p(Su, Sx_n) + 0\right) \ \textit{from}\ \ (19) \\ &= \lim_{n\to\infty} \psi(p(Su, Sx_n)), \ \textit{since}\ \psi\ \textit{is continuous} \\ &\leq \lim_{n\to\infty} \left\{\begin{array}{c} \psi\left(\max\left\{\begin{array}{c} p(v, y_n), p(v, Su), p(y_n, y_{n+1}), \\ \frac{1}{2}[p(v, y_{n+1}) + p(y_n, Su)] \end{array}\right\}\right) \\ -\phi\left(\max\left\{\begin{array}{c} p(v, y_n), p(v, Su), p(y_n, y_{n+1}), \\ \frac{1}{2}[p(v, y_{n+1}) + p(y_n, Su)] \end{array}\right\}\right) \end{array}\right\}.\end{aligned}$$

$$= \max\left\{ \begin{array}{l} \lim\limits_{n\to\infty} p\left(v, y_n\right), p(v, Su), \lim\limits_{n\to\infty} p\left(y_n, y_{n+1}\right), \\ \frac{1}{2}\left[\lim\limits_{n\to\infty} p\left(v, y_{n+1}\right) + \lim\limits_{n\to\infty} p\left(y_n, Su\right)\right] \end{array} \right\}$$

$$= \max\left\{ 0, p(v, Su), 0, \tfrac{1}{2}\left[0 + \lim_{n\to\infty} p\left(y_n, Su\right)\right]\right\}$$

$$= p(v, Su),\ \textit{since}\ \frac{1}{2}\lim_{n\to\infty} p(y_n, Su)$$

$$\leq \frac{1}{2} p(v, Su)\ \textit{from Note 1.4}$$

Therefore,

$$\psi(p(Su, v)) \leq \psi(p(Su, v)) - \phi(p(Su, v)) < \psi(p(Su, v)).$$

Hence, $Su = v$. Thus, $fu = Su = v$.

As in case (i), we can prove that v is the unique common fixed point of S and f. The following example illustrates our Theorem 2.1. □

Example 2.2. *Let* $X = [0, 1]$ *and* $p(x, y) = \max\{x, y\}$ *for all* $x, y \in X$. *Let* $S, f : X \to X$ *be defined by*

$$Sx = \begin{cases} \frac{x}{4}, & \text{if } x \neq 1 \\ \frac{1}{8}, & \text{if } x = 1 \end{cases} \quad \text{and} \quad fx = \begin{cases} \frac{x}{2}, & \text{if } \ x \neq 1 \\ \frac{1}{4}, & \text{if } \ x = 1 \end{cases}$$

Define $\psi : [0, \infty) \to [0, \infty)$ *by* $\psi(t) = t$ *and* $\phi : [0, \infty) \to [0, \infty)$ *by* $\phi(t) = \frac{t}{2}$. *Then, all conditions (2.1.1),(2.1.2) and (2.1.3) are satisfied, and 0 is the unique common fixed point of S and f.*

Corollary 2.3. *Theorem 2.1 holds with the condition (2.1.1) is replaced by (2.3.1)* $p(Sx, Sy) \leq \varphi\left(\max \left\{ \begin{array}{l} p(fx, fy), p(fx, Sx), p(fy, Sy), \\ \frac{1}{2}\left[p(fx, Sy) + p(fy, Sx)\right] \end{array} \right\}\right)$ $\forall x, y \in X$, *where* $\varphi : [0, \infty) \to [0, \infty)$ *is continuous and* $\varphi(t) < t$ *for* $t > 0$.

Proof. Define $\psi(t) = t$ and $\phi(t) = t - \varphi(t)$ $\forall\, t \geq 0$. Then, the condition (2.3.1) implies the condition (2.1.1). □

Corollary 2.4. *Let* (X, p) *be a complete partial metric space and* $F : X \to X$ *be a map such that* $p(Fx, Fy) \leq \varphi\left(\max\left\{p(x, y), p(x, Fx), p(y, Fy), \frac{1}{2}\left[p(x, Fy) + p(y, Fx)\right]\right\}\right)$, $\forall x, y \in X$, *where* $\varphi : [0, \infty) \to [0, \infty)$ *is continuous and* $\varphi(t) < t$ *for* $t > 0$. *Then, F has a unique fixed point in X.*

Remark 2.5. *Altun et al. [12] proved the corollary 2.4 with an additional condition on* φ, *namely,* φ *is non-decreasing.*

Competing interests

The authors declare that they have no competing interests.

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Authors' contributions

KPRR formulated the problem and GNVK,KASNVP drafted and aligned the manuscript sequentially.The three authors read and approved the final manuscript.

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