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Approximate symmetry reduction to the perturbed coupled KdV equations derived from two-layer fluids

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Abstract

Purpose: The purpose of this paper is to research the solution of the perturbed coupled Korteweg-de Vries (KdV) equations.

Methods: The results are obtained through researching a system of corresponding partial differential equations which approximates the perturbed coupled KdV equations and the symmetry method.

Results: The symmetry structure and the corresponding general approximate symmetry reduction are derived. Then, using the symmetry reduction, the infinite series solutions are obtained.

Conclusions: The approximate symmetry reduction approach can be used to search for similar results of other perturbed nonlinear differential equations with small parameters.

Keywords: Coupled KdV equations, Approximate symmetry reduction, Series reduction solutions

Background

It is well known that nonlinear dynamical systems emerge in various fields, such as fluid mechanics, plasma physics, biology, hydrodynamics and optical fibers. The nonlinear phenomena which occurred in such fields are often related to some nonlinear wave equations. In general, one often chooses to exactly solve the related unperturbed partial differential systems as the beginning point of research. However, many nonlinear partial differential equations (PDEs) are, of course, approximations in their derivation, i.e. higher order terms, and dissipation may have been neglected. In order to better understand such phenomena as well as further apply them in practical scientific research, it is important to study the properties of the perturbed PDEs. The approximate symmetry perturbation approach [1] is a powerful method for such substitutions and has been applied to many classical PDEs [2-7]. Maybe in theory it is not difficult, however, the concrete study shows that it, indeed, is not a trivial work to construct the different-order perturbation solutions and even give the general formulae. During the computations, one must make appropriate choices not only on the type of the solution functions but also on the parameters which occurred during the computations, such as some integral constants, to render the solutions physically meaningful and have correct asymptotic forms. Next, we take the coupled Korteweg-de Vries (KdV) equations [8,9] as an example, which reads

$$u_t + u_{xxx} + 2uu_x + (uv)_x = 0, \tag{1a}$$
$$v_t + v_{xxx} + 2vv_x + (uv)_x = 0. \tag{1b}$$

The model equation (Equation 1) is important to describe many kinds of physical systems and atmosphere [10], especially, for a two-layer fluid system. To describe the real two-layer fluid system, we have to introduce some types of physical effects, say, the viscosity. In this letter, we consider the above coupled KdV equations

$$u_t + u_{xxx} + 2uu_x + (uv)_x + \epsilon u_{xx} = 0, \tag{2a}$$
$$v_t + v_{xxx} + 2vv_x + (uv)_x + \epsilon v_{xx} = 0. \tag{2b}$$

in terms of approximate symmetry reduction method.

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Results and discussion

Substituting Equation 14 into Equation 15 and eliminating u_{jt} and v_{jt} $(j = 0, 1, \cdots)$ in terms of Equation 13 result in determining equations by vanishing all coefficients of different partial derivatives of u_j and v_j for the unknown functions X, T, U_j and V_j $(j = 0, 1, \cdots)$, which have the solution

$$\begin{aligned}
&X = ax + x_0,\ T = 3at + t_0,\ U_0 = (-2a-b)v_0 + cu_0,\\
&V_0 = bv_0 + (-c-2a)u_0,\ U_1 = c_1u_0 + (a+c)u_1\\
&\quad + d_1v_0 - (2a+b)v_1,\ V_1 = -c_1u_0 - (c+2a)u_1\\
&\quad - d_1v_0 + (a+b)v_1,\ U_2 = c_2u_0 + c_1u_1\\
&\quad + (c+2a)u_2 + d_2v_0 + d_1v_1 - (2a+b)v_2,\\
&V_2 = -c_2u_0 - c_1u_1 - (c+2a)u_2 - d_2v_0 - d_1v_1\\
&\quad + (2a+b)v_2,\ \cdots\ \cdots\ \cdots,\\
&U_j = (c+ja)u_j - (2a+b)v_j + \sum_{i=0}^{j-1}(c_{j-i}u_i + d_{j-i}v_i),\\
&V_j = -(c+2a)u_j + (b+ja)v_j\\
&\quad - \sum_{i=0}^{j-1}(c_{j-i}u_i + d_{j-i}v_i)\cdots,
\end{aligned} \tag{3}$$

where a, b, c, x_0, t_0, c_j and d_j are arbitrary constants. The sum items in the general formula of U_j and V_j are, respectively, the turn of u_j and v_j, so we just suppose $c_j = 0$ and $d_j = 0$. Subsequently, solving the characteristic equations

$$\frac{dx}{X} = \frac{dt}{T} = \frac{du_0}{U_0} = \frac{du_1}{U_1} = \frac{du_2}{U_2} = \cdots = \frac{du_j}{U_j} = \cdots \tag{4}$$

leads to the similarity solutions to Equation 2 which can be distinguished in the following two subcases:

Case 1. When $a \neq 0$, and to be convenient, we let $x_0 = 0$ and $t_0 = 0$, the similarity solutions are

$$\begin{aligned}
u_0 &= \frac{(2a+b)Q_0(\xi)}{t^{\frac{2}{3}}} + P_0(\xi)t^{\frac{b+c+2*a}{3a}},\\
v_0 &= \frac{(c+2a)Q_0(\xi)}{t^{\frac{2}{3}}} - P_0(\xi)t^{\frac{b+c+2a}{3a}},\\
u_1 &= t^{\frac{b+c+3a}{3a}}P_1(\xi) + \frac{(2a+b)Q_1(\xi)}{t^{\frac{1}{3}}},\\
v_1 &= -t^{\frac{b+c+3a}{3a}}P_1(\xi) + \frac{(2a+c)Q_1(\xi)}{t^{\frac{1}{3}}},\\
u_2 &= t^{\frac{b+c+4a}{3a}}P_2(\xi) + (2a+b)Q_2(\xi),\\
v_2 &= -t^{\frac{b+c+4a}{3a}}P_2(\xi) + (2a+c)Q_2(\xi),\\
u_3 &= t^{\frac{b+c+5a}{3a}}P_3(\xi) + (2a+b)Q_3(\xi)t^{\frac{1}{3}},\\
v_3 &= -t^{\frac{b+c+5a}{3a}}P_3(\xi) + (2a+c)Q_3(\xi)t^{\frac{1}{3}},\\
&\cdots\ \cdots\ \cdots\\
u_j &= t^{\frac{b+c+(j+2)a}{3a}}P_j(\xi) + (2a+b)Q_j(\xi)t^{\frac{j-2}{3}},\\
v_j &= -t^{\frac{b+c+(j+2)a}{3a}}P_j(\xi) + (2a+c)Q_j(\xi)t^{\frac{j-2}{3}},\\
&\cdots\ \cdots\ \cdots
\end{aligned} \tag{5}$$

with the similarity variable $\xi = \frac{x}{t^{\frac{1}{3}}}$. Hence, the perturbation series solution to Equation 2 is

$$u = \sum_{j=0}^{\infty}\epsilon^j\left[t^{\frac{b+c+(j+2)a}{3a}}P_j(\xi) + (2a+b)Q_j(\xi)t^{\frac{j-2}{3}}\right], \tag{6a}$$

$$v = \sum_{j=0}^{\infty}\epsilon^j\left[-t^{\frac{b+c+(j+2)a}{3a}}P_j(\xi) + (2a+c)Q_j(\xi)t^{\frac{j-2}{3}}\right]. \tag{6b}$$

The similarity reduction equations related to similarity solution equations are

$$O\left(\epsilon^0\right): P_{0\xi\xi\xi} = A(P_0Q_0)_\xi + \frac{\xi P_{0\xi}}{3} - \frac{(b+c+2a)P_0}{3a}, \tag{7a}$$

$$Q_{0\xi\xi\xi} = BQ_0Q_{0\xi} + \frac{\xi Q_{0\xi}}{3} + \frac{2}{3}Q_0, \tag{7b}$$

$$\begin{aligned}O\left(\epsilon^1\right): P_{1\xi\xi\xi} &= A\sum_{i=0}^{1}(Q_iP_{1-i})_\xi + \frac{\xi P_{1\xi}}{3}\\ &\quad + \frac{(c+b+3a)P_1}{3a} - P_{0\xi\xi},\end{aligned} \tag{7c}$$

$$\begin{aligned}Q_{1\xi\xi\xi} &= B\sum_{i=0}^{1}\frac{1}{2}(Q_iQ_{1-i})_\xi + \frac{\xi Q_{1\xi}}{3}\\ &\quad + \frac{\xi Q_1}{3} - Q_{0\xi\xi},\end{aligned} \tag{7d}$$

$$\begin{aligned}O\left(\epsilon^2\right): P_{2\xi\xi\xi} &= A\sum_{i=0}^{2}(Q_iP_{2-i})_\xi + \frac{\xi P_{2\xi}}{3}\\ &\quad + \frac{(c+b+4a)P_2}{3a} - P_{1\xi\xi},\end{aligned} \tag{7e}$$

$$Q_{2\xi\xi\xi} = B\sum_{i=0}^{2}\frac{1}{2}(Q_iQ_{2-i})_\xi + \frac{\xi Q_{2\xi}}{3} - Q_{1\xi\xi}, \tag{7f}$$

$$O\left(\epsilon^3\right): P_{3\xi\xi\xi} = A\sum_{i=0}^{3}(Q_iP_{3-i})_\xi + \frac{\xi P_{3\xi}}{3} + \frac{(c+b+5a)P_3}{3a} - P_{2\xi\xi}, \tag{7g}$$

$$Q_{3\xi\xi\xi} = B\sum_{i=0}^{3}\frac{1}{2}(Q_iQ_{3-i})_\xi + \frac{\xi Q_{3\xi}}{3} - \frac{1}{3}Q_3 - Q_{2\xi\xi}, \tag{7h}$$

$$\cdots\ \cdots\ \cdots,$$

$$O\left(\epsilon^j\right): P_{j\xi\xi} = A\sum_{i=0}^{j}(Q_iP_{j-i})_\xi + \frac{\xi P_{j\xi}}{3} + \frac{(c+b+(j+2)a)P_j}{3a} - P_{j-1\xi\xi}, \tag{7i}$$

$$Q_{j\xi\xi\xi} = B\sum_{i=0}^{j}\frac{1}{2}(Q_iQ_{j-i})_\xi + \frac{\xi Q_{j\xi}}{3} - \frac{j-2}{3}Q_j - Q_{j-1\xi\xi}, \tag{7j}$$

$$\cdots\ \cdots\ \cdots,$$

where $A = -b-c-4a$ and $B = -2b-2c-8a$ are integral constants, and $P_{-1} = Q_{-1} = 0$.

Case 2. (1). $a = 0,\ b+c \neq 0$: We let $t_0 = 1$, then the similarity solutions are

$$u_j = e^{(b+c)t}F_j(\xi) + bG_j(\xi),\ v_j = -e^{(b+c)t}F_j(\xi) + cG_j(\xi) \tag{8}$$

with the similarity variable $\xi = x - x_0t$, where F_j and G_j $(j = 0, 1, ..)$ are arbitrary functions of ξ. Accordingly, the similarity reduction equations related to similarity solution equations are

$$O(\epsilon^0): F_{0\xi\xi\xi} = -(b+c)(F_0G_0)_\xi + x_0F_{0\xi} - (b+c)F_0, \tag{9a}$$

$$G_{0\xi\xi\xi} = -2(b+c)G_0G_{0\xi} + x_0G_{0\xi} \tag{9b}$$

$$O(\epsilon^1): F_{1\xi\xi\xi} = -(b+c)(F_0G_1 + F_1G_0)_\xi + x_0F_{1\xi} - (b+c)F_1 - F_{0\xi\xi}, \tag{9c}$$

$$G_{1\xi\xi\xi} = -2(b+c)(G_0G_1)_\xi + x_0G_{1\xi} - G_{0\xi\xi}, \tag{9d}$$

$$O(\epsilon^2): F_{2\xi\xi\xi} = -(b+c)\sum_{i=0}^{2}(F_iG_{2-i})_\xi + x_0F_{2\xi} - (b+c)F_2 - F_{1\xi\xi}, \tag{9e}$$

$$G_{2\xi\xi\xi} = -2(b+c)(G_0G_2)_\xi - 2(b+c)G_1G_{1\xi} + x_0G_{2\xi} - G_{1\xi\xi}, \tag{9f}$$

$$O(\epsilon^3): F_{3\xi\xi\xi} = -(b+c)\sum_{i=0}^{3}(F_iG_{3-i})_\xi + x_0F_{3\xi} - (b+c)F_3 - F_{2\xi\xi}, \tag{9g}$$

$$G_{3\xi\xi\xi} = -2(b+c)(G_1G_2 + G_0G_3)_\xi + x_0G_{3\xi} - G_{2\xi\xi}, \tag{9h}$$

$$\cdots\ \cdots\ \cdots,$$

$$O(\epsilon^j): F_{j\xi\xi\xi} = -(b+c)\sum_{i=0}^{j}(F_iG_{j-i})_\xi + x_0F_{j\xi} - (b+c)F_j - F_{j-1\xi\xi}, \tag{9i}$$

$$G_{j\xi\xi\xi} = -(b+c)\sum_{i=0}^{j}(G_iG_{j-i})_\xi + x_0G_{j\xi} - G_{j-1\xi\xi}, \tag{9j}$$

$$\cdots\ \cdots\ \cdots,$$

with $F_{-1} = 0$ and $G_{-1} = 0$.

(2). $a = 0,\ b+c = 0$: In this case, the similarity solutions are

$$u_j = F_j(\xi),\ v_j = G_j(\xi) \tag{10}$$

with the similarity variable $\xi = x - mt$, where m is an arbitrary constant, and here, F_j and G_j $(j = 0, 1, \cdots.)$ are also arbitrary functions of ξ. The reduction equations related to similarity solution equations are

$$O(\epsilon^0): F_{0\xi\xi\xi} = mF_{0\xi} - (2F_0 + G_0)F_{0\xi} - F_0G_{0\xi}, \tag{11a}$$

$$G_{0\xi\xi\xi} = mG_{0\xi} - (F_0 + 2G_0)G_{0\xi} - F_{0\xi}G_0, \tag{11b}$$

$$O(\epsilon^1): F_{1\xi\xi\xi} = mF_{1\xi} - (2F_0 + G_0)F_{1\xi} - (2F_1 + G_1)F_{0\xi} - F_{0\xi\xi} - F_0G_{1\xi} - F_1G_{0\xi}, \tag{11c}$$

$$G_{1\xi\xi\xi} = mG_{1\xi} - (F_0 + 2G_0)G_{1\xi} - (F_1 + 2G_1)G_{0\xi} - G_{0\xi\xi} - F_{1\xi}G_0 - F_{0\xi}G_1, \tag{11d}$$

$$O(\epsilon^2): F_{2\xi\xi\xi} = mF_{2\xi} - F_{1\xi\xi} - \sum_{i=0}^{2}[\,(F_{2-i} + G_{2-i})G_i]_\xi\,, \tag{11e}$$

$$G_{2\xi\xi\xi} = mG_{2\xi} - G_{1\xi\xi} - \sum_{i=0}^{2}[(F_i + G_i)G_{2-i}]_\xi, \quad (11f)$$

$$O(\epsilon^3): F_{3\xi\xi\xi} = mF_{3\xi} - F_{2\xi\xi} - \sum_{i=0}^{3}[(F_{3-i} + G_{3-i})G_i]_\xi, \quad (11g)$$

$$G_{3\xi\xi\xi} = mG_{3\xi} - G_{2\xi\xi} - \sum_{i=0}^{3}[(F_i + G_i)G_{3-i}]_\xi, \quad (11h)$$

$$\cdots \ \cdots \ \cdots,$$

$$O(\epsilon^j): F_{j\xi\xi\xi} = mF_{j\xi} - F_{j-1\xi\xi} - \sum_{i=0}^{j}[(F_{j-i} + G_{j-i})G_i]_\xi, \quad (11i)$$

$$G_{j\xi\xi\xi} = mG_{j\xi} - G_{j-1\xi\xi} - \sum_{i=0}^{j}[(F_i + G_i)G_{j-i}]_\xi, \quad (11j)$$

$$\cdots \ \cdots \ \cdots,$$

with $F_{-1} = 0$ and $G_{-1} = 0$.

Conclusions

In summary, by applying the approximate symmetry reduction approach to perturbed coupled KdV equations which is an important physical model to describe many kinds of physical problems especially related to the two-layer fluids with unavoidable viscosity, we have unearthed that the similarity reduction solutions and similarity equations of different orders are coincident in their forms. Therefore, we summarize the infinite series solutions and general formulae of the similarity equations.

Nonetheless, we have not mentioned the convergence of infinite series solutions because of its difficulty. Actually, the convergence radius of the series is dependent on the concrete exact solutions. However, it is still very difficult to get exact solutions for infinitely many reduction equations. More details on this topic should be studied further. The approximate symmetry reduction approach can be used to search for similar results of other perturbed nonlinear differential equations with small parameters, and it is worthwhile to summarize a general principle for the perturbed nonlinear differential equations holding analogous results.

Methods

Approximate symmetry reduction approach to the coupled KdV equations

With the perturbation theory, solutions of perturbed partial differential equations can be expressed as a series containing a small parameter. Specifically, for Equation 2, the solution can be supposed to be of the general form

$$u = \sum_{j=0}^{\infty} \epsilon^j u_j, v = \sum_{j=0}^{\infty} \epsilon^j v_j, \quad (12)$$

with u_j being functions of x and t. Substituting Equation 12 into Equation 2 and vanishing the coefficients of ϵ, we obtain a system of partial differential equations

$$O(\epsilon^0): \ u_{0t} + u_{0xxx} + 2u_0u_{0x} + (u_0v_0)_x = 0, \quad (13a)$$

$$v_{0t} + v_{0xxx} + 2v_0v_{0x} + (u_0v_0)_x = 0, \quad (13b)$$

$$O(\epsilon^1): \ u_{1t} + u_{1xxx} + u_{0xx} + (u_0v_1)_x + (u_1v_0)_x + 2(u_0u_1)_x = 0, \quad (13c)$$

$$v_{1t} + v_{1xxx} + v_{0xx} + (v_0u_1)_x + (v_1u_0)_x + 2(v_0v_1)_x = 0, \quad (13d)$$

$$O(\epsilon^2): \ u_{2t} + u_{2xxx} + \left(\sum_{i=0}^{2} u_iu_{2-i}\right)_x + \left(\sum_{i=0}^{2} u_iv_{2-i}\right)_x + u_{1xx} = 0, \quad (13e)$$

$$v_{2t} + v_{2xxx} + \left(\sum_{i=0}^{2} v_iv_{2-i}\right)_x + \left(\sum_{i=0}^{2} u_iv_{2-i}\right)_x + v_{1xx} = 0, \quad (13f)$$

$$\cdots \ \cdots \ \cdots$$

$$O(\epsilon^j): \ u_{jt} + u_{jxxx} + \left(\sum_{i=0}^{j} u_iu_{j-i}\right)_x + \left(\sum_{i=0}^{j} u_iv_{j-i}\right)_x + u_{j-1xx} = 0, \quad (13g)$$

$$v_{jt} + v_{jxxx} + \left(\sum_{i=0}^{j} v_iv_{j-i}\right)_x + \left(\sum_{i=0}^{j} u_iv_{j-i}\right)_x + v_{j-1xx} = 0, \quad (13h)$$

$$\cdots \ \cdots \ \cdots.$$

where we just let $u_{-1} = 0$ and $v_{-1} = 0$. The next crucial step is to study the symmetry reduction of the above system. To that end, we search for the Lie point symmetries in the form

$$\sigma_j = Xu_{jx} + Tu_{jt} - U_j, \ (j = 0, 1, \cdots), \quad (14a)$$

$$\sigma'_j = Xv_{jx} + Tv_{jt} - V_j, \ (j = 0, 1, \cdots), \quad (14b)$$

where X, T, U_j and V_j are functions with respect to x, t, u_j and v_j $(j = 0, 1, \cdots)$. The linearized equations for Equation 2 are

$$\sigma_{0t} + \sigma_{0xxx} + (v_0\sigma_0)_x + \left(u_0\sigma_0'\right)_x + 2\,(\sigma_0 u_0)_x = 0, \tag{15a}$$

$$\sigma_{0t}' + \sigma_{0xxx}' + (v_0\sigma_0)_x + \left(u_0\sigma_0'\right)_x + 2\left(\sigma_0' v_0\right)_x = 0, \tag{15b}$$

$$\sigma_{1t} + \sigma_{1xxx} + \sigma_{0xx} + (v_0\sigma_1 + v_1\sigma_0)_x + \left(u_0\sigma_1' + u_1\sigma_0'\right)_x + 2\,(u_0\sigma_1 + u_1\sigma_0)_x = 0, \tag{15c}$$

$$\sigma_{1t}' + \sigma_{1xxx}' + \sigma_{0xx}' + (v_0\sigma_1 + v_1\sigma_0)_x + \left(u_0\sigma_1' + u_1\sigma_0'\right)_x + 2\left(v_0\sigma_1' + v_1\sigma_0'\right)_x = 0, \tag{15d}$$

$$\sigma_{2t} + \sigma_{2xxx} + \sigma_{1xx} + \left(\sum_{i=0}^{2} v_i\sigma_{2-i}\right)_x + \left(\sum_{i=0}^{2} u_i\sigma_{2-i}'\right)_x + 2\left(\sum_{i=0}^{2} u_i\sigma_{2-i}\right)_x = 0, \tag{15e}$$

$$\sigma_{2t}' + \sigma_{2xxx}' + \sigma_{1xx}' + \left(\sum_{i=0}^{2} v_i\sigma_{2-i}\right)_x + \left(\sum_{i=0}^{2} u_i\sigma_{2-i}'\right)_x + 2\left(\sum_{i=0}^{2} v_i\sigma_{2-i}'\right)_x = 0, \tag{15f}$$

$$\cdots\ \cdots\ \cdots,$$

$$\sigma_{jt} + \sigma_{jxxx} + \sigma_{j-1xx} + \left(\sum_{i=0}^{j} v_i\sigma_{j-i}\right)_x + \left(\sum_{i=0}^{j} u_i\sigma_{j-i}'\right)_x + 2\left(\sum_{i=0}^{j} u_i\sigma_{j-i}\right)_x = 0, \tag{15g}$$

$$\sigma_{jt}' + \sigma_{jxxx}' + \sigma_{j-1xx}' + \left(\sum_{i=0}^{j} v_i\sigma_{j-i}\right)_x + \left(\sum_{i=0}^{j} u_i\sigma_{j-i}'\right)_x + 2\left(\sum_{i=0}^{j} v_i\sigma_{j-i}'\right)_x = 0, \tag{15h}$$

$$\cdots\ \cdots\ \cdots,$$

which means that Equation 2 is invariant under the transformations $u_j \rightarrow u_j + \epsilon'\sigma_j$ and $v_j \rightarrow v_j + \epsilon'\sigma_j'$ $(j = 0, 1, \cdots)$ with an infinitesimal parameter ϵ'.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
XL drafted the manuscript. SP conceived the study and participated in its design and coordination. Both authors read and approved the final manuscript.

Authors' information
XL, with a Master's degree, is a teacher while SP, with a Doctor's degree, is a professor in Changshu Institute of Technology.

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