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Soft topological soft groups

Sk Nazmul[1*] and Syamal Kumar Samanta[2]

Abstract

Purpose: The purpose of this paper is to introduce several notions, such as soft topological soft groups, soft topological soft normal subgroups, and soft topological soft factor groups, and to study their properties.

Methods: We have adopted the analytical method.

Results: We have studied properties of soft topological soft groups, soft subgroups, soft normal subgroups, soft factor groups, and soft homomorphisms.

Conclusions: The fundamental homomorphism theorem is extended in a soft topological soft group setting.

Keywords: Soft sets, Soft groups, Topological groups, Soft topological groups, Soft topological soft groups, Soft topological soft normal subgroup, Soft topological soft factor group, Soft homomorphism

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Introduction

In 1999, Molodtsov [1] proposed a new approach, *viz.* soft set theory for modeling vagueness and uncertainties inherent in the problems of physical science, biological science, engineering, economics, social science, medical science, etc. After that, in 2001 to 2003, Maji et al. [2,3] worked on some mathematical aspects of soft sets and fuzzy soft sets. On the other hand, Biswas and Nanda [4], and Rosenfeld [5] worked on rough groups and fuzzy groups, respectively. In 2007, Aktas and Cagman [6] introduced a basic version of soft group theory which we further extended to fuzzy soft group [7] in 2011. Recently, in 2011, Shabir and Naz [8] introduced a notion of soft topological spaces. As a continuation of this, it is natural to investigate the behavior of topological structure or a combination of algebraic and topological structures in soft set theoretic form. In view of this and also considering the importance of topological group structure in developing Haar measure and Haar integral, we have introduced in this paper a notion of soft topological soft groups. In this connection, it is worth mentioning that in a fuzzy setting, some significant works have been done on fuzzy topological group structure by Foster [9], and Liang and Hai [10] and, in a soft setting, we have worked on soft topological groups [11]. In this paper, our aim is to introduce a notion of soft topological soft groups and its subsystems and morphisms, and to study their properties.

*Correspondence: sk.nazmul_math@yahoo.in
[1] Gov't. College of Education, Burdwan, Kazirhat, Lakurdi, Burdwan, West Bengal, 713102, India
Full list of author information is available at the end of the article

Preliminary

Following the works of Molodtsov [1], Maji et al. [2], and Aktas and Cagman [6], some definitions and preliminary results are presented in this section in our form. Unless otherwise stated, U will be assumed to be an initial universal set and A will be taken to be a set of parameters. Let $P(U)$ denote the power set of U, and $S(U,A)$ denote the set of all soft sets over U. In particular, if U is a group, then $P(U)$ will denote the set of all subgroups of U.

Soft sets

Definition 1. A pair (F,A), where F is a mapping from A to $P(U)$, is called a *soft set* over U.

Let (F_1,A) and (F_2,A) be two soft sets over a common universe U, then (F_1,A) is said to be a *soft subset* of (F_2,A) if $F_1(x) \subseteq F_2(x)$, *for all* $x \in A$. This relation is denoted by $(F_1,A)\tilde{\subseteq}(F_2,A)$. (F_1,A) is said to be *soft equal* to (F_2,A) if $F_1(x) = F_2(x)$, *for all* $x \in A$. It is denoted by $(F_1,A) = (F_2,A)$.

The *complement* of a soft set (F,A) is defined as $(F,A)^c = (F^c,A)$, where $F^c(x) = (F(x))^c = U - F(x)$, *for all* $x \in A$.

A soft set (F, A) over U is said to be *a null soft set (an absolute soft set)* if $F(x) = \phi$ $(F(x) = U)$, *for all* $x \in A$. This is denoted by $\widetilde{\Phi}$ $(\widetilde{A})$.

Definition 2. Let $\{(F_i, A); i \in I\}$ be a nonempty family of soft sets over a common universe U, and then the following are defined:

(a) Their *intersection*, denoted by $\widetilde{\cap}_{i\in I}$, is defined by $\widetilde{\cap}_{i\in I}(F_i, A) = (\widetilde{\cap}_{i\in I}F_i, A)$, where $(\widetilde{\cap}_{i\in I}F_i)(x) = \cap_{i\in I}(F_i(x))$, *for all* $x \in A$.
(b) Their *union*, denoted by $\widetilde{\cup}_{i\in I}$, is defined by $\widetilde{\cup}_{i\in I}(F_i, A) = (\widetilde{\cup}_{i\in I}F_i, A)$, where $(\widetilde{\cup}_{i\in I}F_i)(x) = \cup_{i\in I}(F_i(x))$, *for all* $x \in A$.

Definition 3. Let X and Y be two nonempty sets and $f : X \to Y$ be a mapping, and then the following are defined:

(i) The *image* of a soft set $(F, A) \in S(X, A)$ under the mapping f is defined by $f(F, A) = (f(F), A)$, where $[f(F)](x) = f[F(x)]$, *for all* $x \in A$.
(ii) The *inverse image* of a soft set $(G, A) \in S(Y, A)$ under the mapping f is defined by $f^{-1}(G, A) = (f^{-1}(G), A)$, where $[f^{-1}(G)](x) = f^{-1}[G(x)]$, *for all* $x \in A$.

Proposition 1. *Let X and Y be two nonempty sets and $f : X \to Y$ be a mapping. If (F_1, A), $(F_2, A) \in S(X, A)$, then*

(i) $(F_1, A)\widetilde{\subseteq}(F_2, A) \Rightarrow f[(F_1, A)]\widetilde{\subseteq}f[(F_2, A)]$.
(ii) $f[(F_1, A)\widetilde{\cup}(F_2, A)] = f[(F_1, A)]\widetilde{\cup}f[(F_2, A)]$.
(iii) $f[(F_1, A)\widetilde{\cap}(F_2, A)]\widetilde{\subseteq}f[(F_1, A)]\widetilde{\cap}f[(F_2, A)]$.
(iv) $f[(F_1, A)\widetilde{\cap}(F_2, A)] = f[(F_1, A)]\widetilde{\cap}f[(F_2, A)]$, *if f is injective.*

Proof. We give the proof for (ii). Proofs of other results are similar.
(ii) $[f(F_1\widetilde{\cup}F_2)](x)$
$= f[F_1(x) \cup F_2(x)]$
$= f[F_1(x)] \cup f[F_2(x)]$ (as $F_1(x)$ and $F_2(x)$ are ordinary sets)
$= [f(F_1)](x) \cup [f(F_2)(x)]$
$= [f(F_1)\widetilde{\cup}f(F_2)](x)$, *for all* $x \in A$.
Hence, $f[(F_1, A)\widetilde{\cup}(F_2, A)] = f[(F_1, A)]\widetilde{\cup}f[(F_2, A)]$. □

Proposition 2. *Let X and Y be two nonempty sets and $f : X \to Y$ be a onto mapping. If (G_1, A), $(G_2, A) \in S(Y, A)$, then*

(i) $(G_1, A)\widetilde{\subseteq}(G_2, A) \Rightarrow f^{-1}[(G_1, A)]\widetilde{\subseteq}f^{-1}[(G_2, A)]$.
(ii) $f^{-1}[(G_1, A)\widetilde{\cup}(G_2, A)] = f^{-1}[(G_1, A)]\widetilde{\cup}f^{-1}[(G_2, A)]$.
(iii) $f^{-1}[(G_1, A)\widetilde{\cap}(G_2, A)] = f^{-1}[(G_1, A)]\widetilde{\cap}f^{-1}[(G_2, A)]$.

Proof. We give the proof for (iii). Proofs of other results are similar.
(iii) $[f^{-1}(F_1\widetilde{\cap}F_2)](x)$
$= f^{-1}[F_1(x) \cap F_2(x)]$
$= f^{-1}[F_1(x)] \cap f^{-1}[F_2(x)]$ (as $F_1(x)$ and $F_2(x)$ are ordinary sets)
$= [f^{-1}(F_1)](x) \cap [f^{-1}(F_2)(x)]$
$= [f^{-1}(F_1)\widetilde{\cap}f^{-1}(F_2)](x)$, *for all* $x \in A$.
Hence, $f^{-1}[(F_1, A)\widetilde{\cap}(F_2, A)] = f^{-1}[(F_1, A)]\widetilde{\cap}f^{-1}[(F_2, A)]$. □

Proposition 3. *Let X and Y be two nonempty sets and $f : X \to Y$ be a mapping. If $(G, A) \in S(Y, A)$, then*

(i) $f[f^{-1}(G, A)]\widetilde{\subseteq}(G, A)$.
(ii) $f[f^{-1}(G, A)] = (G, A)$, *if f is surjective.*

Proof. We give the proof for (ii). Proof of part (i) is similar.
(ii) $f[f^{-1}(G)](x)$
$= f[f^{-1}(G(x))]$
$= G(x)$ if f is surjective (as $G(x)$ is a ordinary set), for all $x \in A$.
Hence, $f[f^{-1}(G, A)] = (G, A)$, if f is surjective. □

Proposition 4. *Let X and Y be two nonempty sets and $f : X \to Y$ be a mapping. If $(F, A) \in S(X, A)$, then*

(i) $(G, A)\widetilde{\subseteq}f^{-1}[f(G, A)]$.
(ii) $f^{-1}[f(G, A)] = (G, A)$, *if f is injective.*

Proof. We give the proof for (ii). Proof of part (i) is similar.
(ii) $f^{-1}[f(G)](x)$
$= f^{-1}[f(G(x))]$
$= G(x)$ if f is injective (as $G(x)$ is a ordinary set), for all $x \in A$.
Hence, $f^{-1}[f(G, A)] = (G, A)$, if f is injective. □

Soft groups

Let G, G_1, G_2, and K be groups and A be any nonempty set.

Definition 4. Let (F, A) be a soft set over G, and then (F, A) is said to be a soft group over G if $F(x)$ is a subgroup of G, *for all* $x \in A$, i.e., $F(x) \leq G$, *for all* $x \in A$ [6].

Theorem 1. *Let $\{(F_i, A); i \in I\}$ be a nonempty family of soft groups of G where I is an index set, and then $\widetilde{\bigcap}_{i\in I}(F_i, A)$ is a soft group over G [6].*

Definition 5. Let (F, A) be a soft group over G, and then [6]

(i) (F, A) is said to be a*n identity soft group* over G if *F(x) = {e}, for all* $x \in A$, where e is the identity element of G.

(ii) (F, A) is said to be *an absolute soft group* if $F(x) = G$, *for all* $x \in A$.

Theorem 2. *Let (F, A) be a soft group over G and $f : G \to K$ be a group homomorphism [6]:*

(i) If F(x) = Kerf, the kernel of f, for all x ∈ A, then (f(F),A) is an identity soft group over K.
(ii) If (F,A) be an absolute soft group over G and f be onto, then $(f(F), A)$ is an absolute soft group over K.

Definition 6. Let (F_1, A) and (F_2, A) be two soft groups over G, and then (F_1, A) is said to be a *soft subgroup (soft normal subgroup)* of (F_2,A), denoted by $(F_1,A) \,\tilde{\leq}\, (F_2,A)$ $((F_2,A) \,\tilde{\lhd}\, (F_1,A))$ if $F_1(x) \leq F_2(x)$ $(F_1(x) \lhd F_2(x))$, *for all* $x \in A$ [6].

Theorem 3. *Let (F, A) be a soft group over G and $\{(H_i, A); i \in I\}$ is a nonempty family of soft subgroups (soft normal subgroups) of (F, A), where I is an index set, and then $\widetilde{\bigcap}_{i\in I}(H_i, A)$ is a soft subgroup (normal soft subgroup) of (F, A) [6].*

Theorem 4. *Let (F_1, A) and (F_2, A) be two soft groups over G and (F_1, A) be a soft subgroup of (F_2, A). If $f : G \to K$ be a homomorphism, then $(f(F_1), A)$ and $(f(F_2), A)$ are both soft subgroups over K, and $(f(F_1), A)$ is a soft subgroup of $(f(F_2), A)$ [6].*

Definition 7. Let (F, A) be a soft group over G_1 and $f : G_1 \to G_2$ be a homomorphism. Define the function $K_f : A \to P(G_1)$ such that $K_f(x) = [Ker(f)]_{F(x)} = (Kerf) \cap F(x) = \{g \in F(x); f(g) = e_{G_2}\}$, for all $x \in A$. Therefore, (K_f, A) is a soft group over G_1. It is clear that (K_f, A) is a normal soft subgroup of (F, A) [7].

Definition 8. Let $\mathcal{G} = \{(G_i) : i \in \Delta\}$ be a nonempty collection of groups and A be a nonempty set. Let $F : A \to \mathcal{G}$ be a mapping, and then (F, A) will be called a *generalized soft group*. In fact, the direct product $\Pi_{i\in\Delta} G_i$ is a group, and as each $G_i (\in \mathcal{G})$ is embedded in $\Pi_{i\in\Delta} G_i$, the generalized soft group can be interpreted as a soft group $(\hat{F}, A)$ over $\Pi_{i\in\Delta} G_i$ such that $\hat{F}(a) = \widetilde{F(a)}$, where $\widetilde{F(a)}$ is the embedded subgroup of $\Pi_{i\in\Delta} G_i$ corresponding to the group $F(a)$ [7].

Definition 9. Let (N, A) and (F, A) be two soft groups over G such that (N, A) is a normal soft subgroup of (F, A). Define a mapping $\frac{F}{N}$ over A by $\frac{F}{N}(x)$ = the factor group $\frac{F(x)}{N(x)}$, *for all* $x \in A$, then the factor group $\frac{F(x)}{N(x)}$ is a group, for each $x \in A$. Thus, for each $x \in A$, we get a factor group $\frac{F(x)}{N(x)}$, and thus, it induces a generalized soft group which we call *soft factor group* and denote it by $(\frac{F}{N}, A)$ [7].

Definition 10. Let (F_1, A) and (F_2, A) be two soft groups over G_1 and G_2, respectively, and then (F_1, A) is said to be *soft homomorphic* to (F_2, A), denoted by $(F_1, A) \sim (F_2, A)$, if for each $x \in A$, there exists a homomorphism $\alpha_x : F_1(x) \to F_2(x)$ such that $\alpha_x(F_1(x)) = F_2(x)$ [7].

In this definition, if $\alpha_x : F_1(x) \to F_2(x)$ is an isomorphism for each $x \in A$, then (F_1, A) is said to be *soft isomorphic* to (F_2, A). This is denoted by $(F_1, A) \simeq (F_2, A)$.

Definition 11. Let (F_1, A) and (F_2, A) be two soft groups over G_1 and G_2, respectively. Also, let (F_1, A) be soft homomorphic to (F_2, A) [7]. Define $\alpha F_1 : A \to P(G_2)$ by $(\alpha F_1)(x) = (\alpha_x(F_1(x)))$, *for all* $x \in A$ and $\alpha^{-1} F_2 : A \to P(G_1)$ by $(\alpha^{-1} F_2)(x) = (\alpha_x^{-1}(F_2(x))$, *for all* $x \in A$ where α_x be the corresponding homomorphism of the Definition 10.

Theorem 5. *Let (F_1, A) and (F_2, A) be two soft groups over G_1 and G_2, respectively. Also, let (F_1, A) be soft homomorphic to (F_2, A) [7]:*

(i) If $\alpha_x : F_1(x) \to F_2(x)$ be the corresponding homomorphism for each $x \in A$, then $(\alpha F_1, A)$ and $(\alpha^{-1} F_2, A)$ are soft groups over G_2 and G_1, respectively.
(ii) If (F_3, A) be a soft normal subgroup of (F_1, A), then $(\alpha F_3, A)$ is a soft normal subgroup of $(\alpha F_1, A)$.
(iii) If (F_4, A) be a soft normal subgroup of (F_2, A), then $(\alpha^{-1} F_4, A)$ is a soft normal subgroup of $(\alpha^{-1} F_2, A)$.

Proof. (i) Since $\alpha_x : F_(x) \to F_2(x)$ is a homomorphism, it follows that $(\alpha F_1)(x) = (\alpha_x(F_1(x)))$ is a subgroup of $F_2(x)$ and hence subgroup of G_2 for each $x \in A$. Therefore, $(\alpha F_1, A)$ is a soft group over G_2. Again, $(\alpha^{-1} F_2)(x) = (\alpha_x^{-1}(F_2(x))$ is a subgroup of $F_1(x)$ and hence subgroup of G_1, for each $x \in A$, where $(\alpha_x^{-1}(F_2(x))$ is the inverse image of $F_2(x)$ under the mapping α_x. Thus, $(\alpha^{-1} F_2, A)$ is a soft group G_1.

(ii) Since α_x is a homomorphism from $F_1(x)$ onto $F_2(x)$, it follows that $\alpha_x(F_1(x))$ and $\alpha_x(F_3(x))$ are subgroups of $F_2(x)$, *for all* $x \in A$. Again, $F_3(x)$ is a normal subgroup of $F_1(x)$, and α_x is a homomorphism; $\alpha_x(F_3(x))$ is a subgroup of $\alpha_x(F_1(x))$. Let $y \in \alpha_x(F_1(x))$, then their exists $z \in F_1(x)$ such that $y = \alpha_x(z)$. Now, $y\, \alpha_x(F_3(x)) = \alpha_x(z)\, \alpha_x(F_3(x)) = \alpha_x(zF_3(x)) = \alpha_x(F_3(x)z) = \alpha_x(F_3(x))\, \alpha_x(z) = \alpha_x(F_3(x))\, y$, *for all* $x \in A$. Thus, $\alpha_x(F_3(x))$ is normal subgroup of $\alpha_x(F_1(x))$,*for all* $x \in A$. Therefore, $(\alpha F_3, A)$ is soft normal subgroup of $(\alpha F_1, A)$.

(iii) Proof is similar to that of part (ii).

□

Theorem 6. *Let (N, A) be a soft normal subgroup of (F, A), and then for each $x \in A$, the canonical mapping $\phi_x : F(x) \to F(x)/N(x)$, given by $\phi_x(\xi) = \xi N(x)$, $\xi \in F(x)$, is an onto homomorphism [7].*

Definition 12. Let (F_1, A) and (F_2, A) be two soft groups over G_1 and G_2, respectively, such that (F_1, A) is soft homomorphic to (F_2, A). Also, let for each $x \in A$, $\alpha_x : F_1(x) \to F_2(x)$ be the corresponding homomorphism and K_x be the kernel of α_x. Define a mapping $K : A \to P(G_1)$, such that $K(x) = K_x$. Clearly, (K, A) is a soft set over G_1 and is called *soft kernel* corresponding to $\{\alpha_x; x \in A\}$. Also, (K, A) is a soft normal subgroup of (F_1, A) [7].

Theorem 7. *Let (F_1, A) and (F_2, A) be two soft groups over G_1 and G_2, respectively, such that (F_1, A) is soft homomorphic to (F_2, A). Also, let that for each $x \in A$, $\alpha_x : F_1(x) \to F_2(x)$ be the corresponding homomorphism and (K, A) be the soft kernel corresponding to the family of homomorphisms $\{\alpha_x, x \in A\}$, then the soft group $(\frac{F_1}{K}, A)$ is isomorphic to the soft group (F_2, A) [7].*

Topological groups

In this section, the well-known definition of topological groups is taken, and some established results on topological groups are cited, which will be used in this paper.

Definition 13. Let G be a group and τ be a topology on G, and then (G, τ) is called topological group if the onto mappings are as follows [12]:

(i) $f : (G, \tau) \times (G, \tau) \to (G, \tau)$ defined by $f(x, y) = xy$, *for all* $x, y \in G$ and
(ii) $g : (G, \tau) \to (G, \tau)$ defined by $g(x) = x^{-1}$, *for all* $x \in G$ are continuous.

Theorem 8. *Let (G, τ) be a topological group [12]:*

(a) If H be an algebraic subgroup of G, then (H, τ_H) is a topological group, where τ_H is the relativized topology on H induced from τ.
(b) Let H be a normal subgroup of G and $\phi : G \to G/H$ be the canonical mapping. If $\tau' = \{A \subset G/H : \phi^{-1}(A) \in \tau\}$, then τ' is a quotient topology and $(G/H, \tau')$ is a topological group.
(c) If H be a normal subgroup of G, then the canonical mapping $\phi : (G, \tau) \to (\frac{G}{H}, \tau')$ defined by $\phi(x) = xH$, $x \in G$ is an open homomorphism.

Theorem 9. *Let α be an algebraic homomorphism from a topological group (G, τ) into a topological group (G_1, τ_1). Let H be the kernel of α and $\phi : (G, \tau) \to (\frac{G}{H}, \tau')$ be the canonical mapping. Let $\alpha = \alpha_0 \phi$ for some $\alpha_0 : (\frac{G}{H}, \tau') \to (G_1, \tau_1)$, and then $\alpha : (G, \tau) \to (G_1, \tau_1)$ is continuous (open) if $\alpha_0 : (\frac{G}{H}, \tau') \to (G_1, \tau_1)$ is continuous (open) [12].*

Theorem 10. *Let (G_α, τ_α) be a topological group, for all$\alpha \in A$, and then $G = \Pi_{\alpha \in A} G_\alpha$, endowed with the product topology $\Pi_{\alpha \in A} \tau_\alpha$, is a topological group [12].*

Soft topological spaces

In this section, some properties of soft topology are studied using the definition of soft topology by Shabir and Naz [8]. Unless otherwise stated, X is an initial universal set. A is the nonempty set of parameters, and $S(X, A)$ denotes the collection of all soft sets over X under the parameter set A.

Definition 14. Let τ be the collection of soft sets over X, and then τ is said to be a soft topology on X if the following conditions are met [8]:

(i) $(\widetilde{\phi}, A)$, $(\widetilde{X}, A) \in \tau$ where $\widetilde{\phi}(\alpha) = \phi$ and $\widetilde{X}(\alpha) = X$, *for all* $\alpha \in A$.
(ii) The intersection of any two soft sets in τ belongs to τ.
(iii) The union of any number of soft sets in τ belongs to τ.

The triplet (X, A, τ) is called a soft topological space over X.

Proposition 5. *Let (X, A, τ) be a soft topological space over X, and then the collection $\tau^\alpha = \{F(\alpha) : (F, A) \in \tau\}$ for each $\alpha \in A$ defines a topology on X [8].*

Proposition 6. *Let (X, A, τ_1) and (X, A, τ_2) be two soft topological spaces over X, and then $(X, A, \tau_1 \widetilde{\cap} \tau_2)$ where $\tau_1 \widetilde{\cap} \tau_2 = \{(F, A) : (F, A) \in \tau_1 \ \& \ (F, A) \in \tau_2\}$ is a soft topological space over X. However, the union of two soft topological spaces over X may not be a soft topological space over X [8].*

Definition 15. Let τ_1 and τ_2 be two soft topologies over X. If $\tau_1 \widetilde{\subseteq} \tau_2$, then τ_2 is said to be soft finer than τ_1.

Definition 16. Let τ and ν be two soft topologies on two nonempty sets X and Y, respectively, and $f : X \to Y$ be a mapping. The image of τ and the pre-image of ν under f are denoted by $f(\tau)$ and $f^{-1}(\nu)$, respectively, defined by the following:

(i) $f(\tau) = \{(G, A) \in S(Y, A) : f^{-1}(G, A) = (f^{-1}(G), A) \in \tau\}$ and
(ii) $f^{-1}(\nu) = \{f^{-1}(G, A) = (f^{-1}(G), A) : (G, A) \in \nu\}$.

Theorem 11. *Let τ and ν be two soft topologies on two nonempty sets X and Y, respectively, and $f : X \to Y$ be a mapping, and then*

(i) $f^{-1}(\nu)$ *is a soft topology on* X*, and*
(ii) $f(\tau)$ *is a soft topology on* Y.

Proof. Proof follows from Propositions 1, 2, and 3. □

Theorem 12. *Let* τ_1 *and* τ_2 *be two soft topologies over* X. *Let* $f : X \rightarrow Y$ *be a onto mapping. If* $\tau_1 \widetilde{\subseteq} \tau_2$*, then* $f(\tau_1) \widetilde{\subseteq} f(\tau_2)$.

Theorem 13. *Let* ν_1 *and* ν_2 *be two soft topologies over* Y. *Let* $f : X \rightarrow Y$ *be a mapping. If* $\nu_1 \widetilde{\subseteq} \nu_2$*, then* $f^{-1}(\nu_1) \widetilde{\subseteq} f^{-1}(\nu_2)$.

Soft topological soft groups

In this section, the definition of a soft topological soft group is introduced, and some of its properties are studied. Also, the fundamental homomorphism theorem in the soft topological soft group setting is established. Throughout this section, X and Y are assumed to be groups.

Definition 17. Let (F, A) be a soft group over X and τ be a soft topology on X, and then (F, A, τ) is called a *soft topological soft group* over X if for each $\alpha \in A$, $[F(\alpha), \tau^{\alpha}_{F(\alpha)}]$ is a topological group on $F(\alpha)$ where $\tau^{\alpha}_{F(\alpha)}$ is the relativized topology on $F(\alpha)$ induced from τ^{α}.

Example 1. Let $X = S_3 = \{e, (12), (13), (23), (123), (132)\}$, $A = \{\alpha_1, \alpha_2\}$, (F, A) be a soft set defined by $F(\alpha_1) = \{e, (12)\}$, $F(\alpha_2) = \{e, (13)\}$, and $\tau = \{(\widetilde{\phi}, A), (\widetilde{X}, A), (F_1, A), (F_2, A), (F_3, A)\}$ where $F_1(\alpha_1) = \{e\}$, $F_2(\alpha_1) = \{(12)\}$, $F_3(\alpha_1) = \{e, (12)\}$, $F_1(\alpha_2) = \{e\}$, $F_2(\alpha_2) = \{(13)\}$, $F_3(\alpha_2) = \{e, (13)\}$, and then clearly, (F, A) is a soft group over X. Now, $\tau^{\alpha_1} = \{\phi, X, \{e\}, \{(12)\} \{e, (12)\}\}$ and $\tau^{\alpha_1}_{F(\alpha_1)} = \{\phi, \{e\}, \{(12)\} \{e, (12)\}\}$.

It can be easily shown that $\tau^{\alpha_1}_{F(\alpha_1)}$ is a group topology on $F(\alpha_1)$, and similarly, $\tau^{\alpha_2}_{F(\alpha_2)}$ is a group topology on $F(\alpha_2)$. Therefore, (F, A, τ) is a soft topological soft group over X.

Definition 18. Let (F_1, A, τ_1) and (F_2, A, τ_2) be two soft topological soft groups over X, and then their *intersection* defined by $(F_1, A, \tau_1) \widetilde{\cap} (F_2, A, \tau_2) = (F_1 \widetilde{\cap} F_2, A, \tau_1 \widetilde{\cap} \tau_2)$, where $(F_1 \widetilde{\cap} F_2)(\alpha) = F_1(\alpha) \cap F_2(\alpha)$, *for all* $\alpha \in A$, and $\tau_1 \widetilde{\cap} \tau_2$ is defined in as Proposition 6.

Remark 1. If (F_1, A, τ_1) and (F_2, A, τ_2) be two soft topological soft groups on X, then $(F_1 \widetilde{\cap} F_2, A)$ is a soft set, and $\tau_1 \widetilde{\cap} \tau_2$ is a soft topology on X, but in general, $(F_1 \widetilde{\cap} F_2, A, \tau_1 \widetilde{\cap} \tau_2)$ is not necessarily a soft topological soft group, as shown by the following example. However, if $\tau_1 = \tau_2 = \tau$ (say) then by Theorem 16, the intersection of such soft topological soft groups is a soft topological soft group.

Example 2. Let $X = S_3 = \{e, (12), (13), (23), (123), (132)\}$, $A = \{\alpha\}$, (F, A) be a soft set defined by $F(\alpha) = \{e, (123), (132)\}$. Therefore, (F, A) is a soft group over X. Also, let $(F_1, A) = \{\alpha/\{e\}\}$, $(F_2, A) = \{\alpha/\{(123)\}\}$, $(F_3, A) = \{\alpha/\{(132)\}\}$, $(F_4, A) = \{\alpha/\{e, (123)\}\}$, $(F_5, A) = \{\alpha/\{e, (132)\}\}$, $(F_6, A) = \{\alpha/\{(123), (132)\}\}$, $(F_7, A) = \{\alpha/\{e, (123), (132)\}\}$, $(F_8, A) = \{\alpha/\{(12), (132)\}\}$, $(F_9, A) = \{\alpha/\{(12), (123), (132)\}\}$, $(F_{10}, A) = \{\alpha/\{e, (12), (132)\}\}$, and $(F_{11}, A) = \{\alpha/\{e, (12), (123), (132)\}\}$ are the soft sets over X. If $\tau = \{(\widetilde{\phi}, A), (\widetilde{X}, A), (F_1, A), (F_2, A), (F_3, A), (F_4, A), (F_5, A), (F_6, A), (F_7, A)\}$, then τ is a soft topology on X. Thus, $\tau^{\alpha} = \{\phi, X, \{e\}, \{(123)\}, \{(132)\}, \{e, (123)\}, \{e, (132)\}, \{(123), (132)\}, \{e, (123), (132)\}\}$ and $\tau^{\alpha}_{F(\alpha)} = \{\phi, \{e\}, \{(123)\}, \{(132)\}, \{e, (123)\}, \{e, (132)\}, \{(123), (132)\}, \{e, (123), (132)\}\}$ is a group topology on $F(\alpha)$. Again, if $\nu = \{(\widetilde{\phi}, A), (\widetilde{X}, A), (F_1, A), (F_2, A), (F_4, A), (F_8, A), (F_9, A), (F_10, A), (F_11, A)\}$, then ν is a soft topology on X, and $\nu^{\alpha} = \{\phi, X, \{e\}, \{(123)\}, \{e, (123)\}, \{(12), (132)\}, \{(12), (123), (132)\}, \{e, (12), (132)\}, \{e, (12), (123), (132)\}\}$ and $\nu^{\alpha}_{F(\alpha)} = \{\phi, \{e\}, \{(123)\}, \{(132)\}, \{e, (123)\}, \{e, (132)\}, \{(123), (132)\}, \{e, (123), (132)\}\}$ is a group topology on $F(\alpha)$. Therefore, (F, A, τ) and (F, A, ν) are two soft topological soft groups over X. Now, $\tau \widetilde{\cap} \nu = \{(\widetilde{\phi}, A), (\widetilde{X}, A), (F_1, A), (F_2, A), (F_4, A)\}$. Hence, $(\tau \widetilde{\cap} \nu)^{\alpha} = \{\phi, X, \{e\}, \{(123)\}, \{e, (123)\}\}$ and $(\tau \widetilde{\cap} \nu)^{\alpha}_{F(\alpha)} = \{\phi, \{e\}, \{(123)\}, \{e, (123)\}, \{e, (123), (132)\}\}$. Here, $(123), (132) \in F(\alpha)$ and $(123)(132) = (e) \in \{e\}$, but the only one open set containing (132) is $\{e, (123), (132)\}$ and $(123)\{e, (123), (132)\} = \{e, (123), (132)\} \not\subseteq \{e\}$. Hence, $(\tau \widetilde{\cap} \nu)^{\alpha}_{F(\alpha)}$ is not a group topology on $F(\alpha)$. Therefore, $(F, A, \tau \widetilde{\cap} \nu)$ is not a soft topological soft group over X.

Theorem 14. *Let* H *and* G *be two subgroups of* X *and* τ *be a soft topology on* X. *If* $u \in \tau^{\alpha}_{H}$ *and* $v \in \tau^{\alpha}_{G}$*, then* $u \cap v \in \tau^{\alpha}_{H \cap G}$.

Proof. Since $u \in \tau^{\alpha}_{H}$,, it follows that there exists $u_1 \in \tau^{\alpha}$ such that $u = u_1 \cap H$. Now, $u_1 \in \tau^{\alpha}$ implies that $\exists\, (F_1, A) \in \tau$ such that $F_1(\alpha) = u_1$. Again, since $v \in \tau^{\alpha}_{G}$, it follows that there exists $v_1 \in \tau^{\alpha}$ such that $v = v_1 \cap G$. Now, $v_1 \in \tau^{\alpha}$ implies that $\exists\, (F_2, A) \in \tau$ such that $F_2(\alpha) = v_1$. Hence, $(F_1 \widetilde{\cap} F_2, A) \in \tau$ and $(F_1 \widetilde{\cap} F_2)(\alpha) = F_1(\alpha) \cap F_2(\alpha) = u_1 \cap v_1$. Thus, $u_1 \cap v_1 \in \tau^{\alpha} \Rightarrow H \cap G \cap u_1 \cap v_1 \in \tau^{\alpha}_{H \cap G} \Rightarrow (H \cap u_1) \cap (G \cap v_1) = u \cap v \in \tau^{\alpha}_{H \cap G}$. □

Theorem 15. *Let* H *and* G *be two subgroups of* X *and* τ *be a soft topology on* X. *If* (H, τ^{α}_{H}), (G, τ^{α}_{G}) *be two topological groups on* H, G*, respectively, then* $[H \cap G, \tau^{\alpha}_{H \cap G}]$ *is a topological group on* $H \cap G$.

Proof. Let $x, y \in H \cap G$ and $W \in \tau^{\alpha}_{H \cap G}$ such that $xy^{-1} \in W. \Rightarrow\ \exists\, w \in \tau^{\alpha}$ such that $W = w \cap H \cap G. \Rightarrow\ \exists\, (F, A) \in \tau$

such that $F(\alpha) = w \in \tau^{\alpha}$. Now, $F(\alpha) \cap H \in \tau_H^{\alpha}$ and $xy^{-1} \in F(\alpha) \cap H$. Since τ_H^{α} is a topological group, $\exists\ u_1, v_1 \in \tau_H^{\alpha}$ and $x \in u_1,\ y \in v_1$ such that $u_1 v_1^{-1} \subseteq F(\alpha) \cap H$. Similarly $\exists\ u_2, v_2 \in \tau_G^{\alpha}$ and $x \in u_2,\ y \in v_2$ such that $u_2 v_2^{-1} \subseteq F(\alpha) \cap G$. Hence, by Theorem 14, we have $u_1 \cap u_2 \in \tau_{H\cap G}^{\alpha}$ and $v_1 \cap v_2 \in \tau_{H\cap G}^{\alpha}$. Also, $x \in u_1 \cap u_2,\ y \in v_1 \cap v_2$ such that $(u_1 \cap u_2)(v_1 \cap v_2)^{-1} \subseteq u_1 v_1^{-1} \cap u_2 v_2^{-1} \subseteq F(\alpha) \cap G \cap H = W$. Therefore, $[H \cap G, \tau_{H\cap G}^{\alpha}]$ is a topological group on $H \cap G$. □

Theorem 16. *Let (F, A, τ) and (G, A, τ) be two soft topological soft groups over X, and then $(F, A, \tau)\widetilde{\cap}(G, A, \tau) = (F\widetilde{\cap}G, A, \tau)$ is a soft topological soft group over X.*

Proof. (F, A, τ) and (G, A, τ) be two soft topological soft groups over X.
$\Rightarrow$ (F, A) and (G, A) be two soft groups over X, and hence, $(F\widetilde{\cap}G, A)$ is a soft group over X.
Also, for each $\alpha \in A$, $[F(\alpha), \tau_{F(\alpha)}^{\alpha}]$ and $[G(\alpha), \tau_{G(\alpha)}^{\alpha}]$ are two topological groups on $F(\alpha)$ and $G(\alpha)$, respectively. Therefore, by Theorem 15, $[F(\alpha) \cap G(\alpha), \tau_{F(\alpha)\cap G(\alpha)}^{\alpha}] = [(F\widetilde{\cap}G)(\alpha), \tau_{(F\widetilde{\cap}G)(\alpha)}^{\alpha}]$ is a topological group on $F(\alpha) \cap G(\alpha)$, *for all* $\alpha \in A$. Thus, (F, A, τ) $\widetilde{\cap}(G, A, \tau)$ is a soft topological soft group over X. □

Note 1. *Let $\{(H_i, A, \tau);\ i \in I\}$ be a nonempty family of soft topological soft subgroups over X where I is an index set, and then $\widetilde{\bigcap}_{i\in I}(H_i, A, \tau)$ $(= (\widetilde{\cap}_{i\in I} H_i, A, \tau))$ is a soft topological soft subgroup over X.*

Theorem 17. *Let (F, A, τ) be a soft topological soft group over X and $f : X \to Y$ be an open homomorphism, and then $(f(F), A, f(\tau))$ is a soft topological soft group over Y.*

Proof. It is clear that $(f(F), A)$ is a soft group over Y and $f(\tau)$ is a soft topology on Y. We show that $[f(\tau)]_{f(F(\alpha))}^{\alpha}$ is a group topology on $f(F(\alpha))$, *for all* $\alpha \in A$. Let $y_1,\ y_2 \in f(F(\alpha))$ and $W \in [f(\tau)]_{f(F(\alpha))}^{\alpha}$ be such that $y_1 y_2^{-1} \in W$. Now, $y_1, y_2 \in f(F(\alpha)) \Rightarrow \exists\ x_1, x_2 \in F(\alpha)$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$. Again, $W \in [f(\tau)]_{f(F(\alpha))}^{\alpha} \Rightarrow \exists\ (G, A) \in f(\tau)$ such that $W = G(\alpha) \cap f(F(\alpha))$. Therefore, $(f^{-1}(G), A) \in \tau$. Hence, $\Rightarrow f^{-1}(G(\alpha)) \cap F(\alpha) \in [\tau]_{F(\alpha)}^{\alpha}$, *for all* $\alpha \in A$. Again, $f(x_1 x_2^{-1}) = f(x_1)(f(x_2))^{-1} = y_1 y_2^{-1} \in W = G(\alpha) \cap f(F(\alpha)) \subset G(\alpha)$. Therefore, $x_1 x_2^{-1} \in f^{-1}(G(\alpha))$. Also, since $x_1, x_2 \in F(\alpha)$ and $F(\alpha)$ is a group, $x_1 x_2^{-1} \in F(\alpha)$. Hence, $x_1 x_2^{-1} \in f^{-1}(G(\alpha)) \cap F(\alpha)$. Since $[\tau]_{F(\alpha)}^{\alpha}$ is a group topology on $F(\alpha)$ and $x_1 x_2^{-1} \in f^{-1}(G(\alpha)) \cap F(\alpha) \in [\tau]_{F(\alpha)}^{\alpha}$, there exists $u, v \in [\tau]_{F(\alpha)}^{\alpha}$ such that $x_1 \in u,\ x_2 \in v$ and $uv^{-1} \subseteq f^{-1}(G(\alpha)) \cap F(\alpha)$. Hence, $\exists\ (H, A) \in \tau$ such that $u = H(\alpha) \cap F(\alpha)$. Since f is open, we have $[f(H), A] \in f(\tau)$ and $y_1 \in f(u) = f[H(\alpha) \cap F(\alpha)] \subseteq f(H(\alpha)) \cap f(F(\alpha)) \in [f(\tau)]_{f(F(\alpha))}^{\alpha}$. Similarly, $y_2 \in f(v) \in [f(\tau)]_{f(F(\alpha))}^{\alpha}$. Thus, $f(u)[f(v)]^{-1} = f(uv^{-1}) \subseteq f[f^{-1}(G(\alpha)) \cap F(\alpha)] \subseteq ff^{-1}(G(\alpha)) \cap f(F(\alpha)) \subseteq G(\alpha) \cap f(F(\alpha)) = W$. Therefore, $(f(F), A, f(\tau))$ is a soft topological soft group over Y. □

Theorem 18. *Let (G, A, ν) be a soft topological soft group over Y and $f : X \to Y$ be an onto homomorphism, and then $(f^{-1}(G), A, f^{-1}(\nu))$ is a soft topological soft group over X.*

Proof. It is clear that $(f^{-1}(G), A)$ is a soft group over X and $f^{-1}(\nu)$ is a soft topology on X. We show that $[f^{-1}(\nu)]_{f^{-1}(G(\alpha))}^{\alpha}$ is a group topology on $f^{-1}(G(\alpha))$, *for all* $\alpha \in A$. Let $x_1,\ x_2 \in f^{-1}(G(\alpha))$ and $W \in [f^{-1}(\nu)]_{f^{-1}(G(\alpha))}^{\alpha}$ be such that $x_1 x_2^{-1} \in W$. Let $f(x_1) = y_1$ and $f(x_2) = y_2$. Thus, $y_1, y_2 \in G(\alpha)$. Again, $W \in [f^{-1}(\nu)]_{f^{-1}(G(\alpha))}^{\alpha} \Rightarrow \exists\ (F, A) \in f^{-1}(\nu)$ such that $W = F(\alpha) \cap f^{-1}(G(\alpha))$ Now, $(F, A) \in f^{-1}(\nu) n \Rightarrow \exists\ (H_1, A) \in \nu$ such that $f^{-1}(H_1, A) = (F, A)$. Hence, $W = f^{-1}(H_1(\alpha) \cap f^{-1}(G(\alpha)) = f^{-1}[G(\alpha) \cap H_1(\alpha)]$. Thus, $y_1 y_2^{-1} = f(x_1)[f(x_2)]^{-1} = f(x_1 x_2^{-1}) \in f(W)$ and $f(W) = ff^{-1}[H_1(\alpha) \cap G(\alpha)] = H_1(\alpha) \cap G(\alpha) \in [\nu]_{G(\alpha)}^{\alpha}$ (since f is onto). Since $[\nu]_{G(\alpha)}^{\alpha}$ is a group topology on $G(\alpha)$, it follows that there exists $u,\ v \in [\nu]_{G(\alpha)}^{\alpha}$ with $y_1 \in u$ and $y_2 \in v$ such that $uv^{-1} \subseteq H_1(\alpha) \cap G(\alpha)$. Also, $u \in [\nu]_{G(\alpha)}^{\alpha} \Rightarrow \exists (H_2, A) \in \nu$ such that $u = H_2(\alpha) \cap G(\alpha)$. Therefore, $[f^{-1}(H_2), A] \in f^{-1}(\nu)$ and $x_1 \in f^{-1}(u) = f^{-1}[H_2(\alpha) \cap G(\alpha)] = f^{-1}(H_2(\alpha)) \cap f^{-1}(G(\alpha)) \in [f^{-1}(\nu)]_{f^{-1}(G(\alpha))}^{\alpha}$. Similarly, $x_2 \in f^{-1}(v) \in [f^{-1}(\nu)]_{f^{-1}(G(\alpha))}^{\alpha}$. Thus, $f^{-1}(u)[f^{-1}(v)]^{-1} = f^{-1}(uv^{-1}) \subseteq f^{-1}[H_1(\alpha) \cap G(\alpha)] = W$. Therefore, $(f^{-1}(G), A, f^{-1}(\nu))$ is a soft topological soft group over X. □

Definition 19. Let (F, A, τ) be a soft topological soft group over X, and then

(i) (F, A, τ) is said to be *an identity soft topological soft group* if $F(\alpha) = \{e\}$, *for all* $\alpha \in A$, where e is the identity element of X.
(ii) (F, A, τ) is said to be *an absolute soft topological soft group* if $F(\alpha) = X$, *for all* $\alpha \in A$.

Theorem 19. *Let τ be a soft topology on X and $f : X \to Y$ be an open homomorphism:*

(i) If (F, A, τ) be an identity soft topological soft group over X, then $(f(F), A, f(\tau))$ is an identity soft topological soft group over Y.

(ii) *If (F, A, τ) be a soft topological soft group over X and $F(\alpha) = Kerf$, for all $\alpha \in A$, then $(f(F), A, f(\tau))$ is an identity soft topological soft group over Y.*
(iii) *If f is onto and (F, A, τ) be an absolute soft topological soft group over X, then $(f(F), A, f(\tau))$ is an absolute soft topological soft group over Y.*

Proof. We give the proof for (i). Proofs of other results are similar:

(i) From Theorem 17, $(f(F), A, f(\tau))$ is a soft topological soft group over Y. Again, (F, A, τ) is an identity soft topological soft group. Therefore, $F(\alpha) = \{e\}$, *for all* $\alpha \in A$, and hence, $[f(F)](\alpha) = f[F(\alpha)] = \{e'\}$, *for all* $\alpha \in A$, where e' is the identity element of Y. Therefore, $(f(F), A, f(\tau))$ is an identity soft topological soft group over Y. □

Theorem 20. *Let ν be a soft topology on Y and $f : X \to Y$ be a homomorphism:*

(i) *If (G, A, ν) be an identity soft topological soft group over Y and $Kerf = \{e\}$, then $(f^{-1}(G), A, f^{-1}(\nu))$ is an identity soft topological soft group over X.*
(ii) *If (G, A, ν) be an absolute soft topological soft group over Y, then $(f^{-1}(G), A, f^{-1}(\nu))$ is an absolute soft topological soft group over X.*

Proof. It follows from Theorem 18 and Theorem 19. □

Definition 20. Let (F_1, A, τ_1) and (F_2, A, τ_2) be two soft topological soft groups over X, and then (F_1, A, τ_1) is said to be a *soft topological soft subgroup (soft topological soft normal subgroup)* of (F_2, A, τ_2) if the following conditions are met:

(i) (F_1, A) is a soft subgroup (*soft normal subgroup*) of (F_2, A), and
(ii) $[\tau_1^{\alpha}]_{F_1(\alpha)} = [[\tau_2^{\alpha}]_{F_2(\alpha)}]_{F_1(\alpha)} = [\tau_2^{\alpha}]_{F_2(\alpha)/F_1(\alpha)}$, *for all* $\alpha \in A$.

This is denoted by $(F_1, A, \tau_1) \tilde{\leq} (F_2, A, \tau_2)$ $((F_1, A, \tau_1) \tilde{\triangleleft} (F_2, A, \tau_2))$.

Example 3. Let $X = S_3 = \{e, (12), (13), (23), (123), (132)\}$ and $A = \{\alpha_1, \alpha_2\}$. Also, let $(F_1, A) = \{\{e\}, \{e, (12)\}\}$, $(F_2, A) = \{\{e, (123), (132)\}, \{e, (12)\}\}$, $(H_1, A) = \{\{e\}, \{e\}\}$, $(H_2, A) = \{\{(123)\}, \{(12)\}\}$, $(H_3, A) = \{\{(132)\}, \{(13)\}\}$, $(H_4, A) = \{\{e, (123)\}, \{e, (12)\}\}$, $(H_5, A) = \{\{e, (132)\}, \{e, (13)\}\}$, $(H_6, A) = \{\{(123), (132)\}, \{(12), (13)\}\}$, $(H_7, A) = \{\{e, (123), x\ (132)\}, \{e, (12)\ (13)\}\}$ and $\tau_1 = \{(\tilde{\phi}, A), (\tilde{X}, A), (H_1, A), (H_2, A), (H_4, A)\}$, $\tau_2 = \{(\tilde{\phi}, A), (\tilde{X}, A), (H_1, A), (H_2, A), (H_3, A), (H_4, A), (H_5, A), (H_6, A), (H_7, A)\}$, then clearly, (F_1, A) is a soft subgroup (soft normal subgroup) of (F_2, A). Now, $\tau_1^{\alpha_1} = \{\phi, X, \{e\}, \{(123)\}, \{e, (123)\}\}$. Hence, $[\tau_1^{\alpha_1}]_{F_1(\alpha_1)} = \{\phi, \{e\}\}$ is a discrete topology on $F_1(\alpha_1)$, and hence, $(F_1(\alpha_1), [\tau_1^{\alpha_1}]_{F_1(\alpha_1)})$ is a topological group on $F_1(\alpha_1)$. Similarly, $[\tau_1^{\alpha_2}]_{F_1(\alpha_2)} = \{\phi, \{e\}, \{(12)\}, \{e, (12)\}\}$, $[\tau_2^{\alpha_1}]_{F_2(\alpha_1)} = \{\phi, \{e\}, \{(123)\}, \{(132)\}, \{e, (123)\}, \{e, (132)\}, \{(123), (132)\}, \{e, (123), (132)\}\}$ and $[\tau_2^{\alpha_2}]_{F_2(\alpha_2)} = \{\phi, \{e\}, \{(12)\}, \{e, (12)\}\}$ are discrete topologies on $F_1(\alpha_2)$, $F_2(\alpha_1)$ and $F_2(\alpha_2)$, respectively, and hence, $(F_1(\alpha_2), [\tau_1^{\alpha_2}]_{F_1(\alpha_2)})$, $(F_2(\alpha_1), [\tau_2^{\alpha_1}]_{F_2(\alpha_1)})$ and $(F_2(\alpha_2), [\tau_2^{\alpha_2}]_{F_2(\alpha_2)})$ are topological groups on $F_1(\alpha_2)$, $F_2(\alpha_1)$ and $F_2(\alpha_2)$, respectively. Thus, (F_1, A, τ_1) and (F_2, A, τ_2) are two soft topological soft groups over X. Again, $[\tau_2^{\alpha_1}]_{F_2(\alpha_1)/F_1(\alpha_1)} = \{\phi, \{e\}\} = [\tau_1^{\alpha_1}]_{F_1(\alpha_1)}$ and $[\tau_2^{\alpha_2}]_{F_2(\alpha_2)/F_1(\alpha_2)} = \{\phi, \{e\}, \{(12)\}, \{e, (12)\}\} = [\tau_1^{\alpha_2}]_{F_1(\alpha_2)}$. Therefore, (F_1, A, τ_1) is a soft topological soft subgroup (soft topological soft normal subgroup) of (F_2, A, τ_2).

Theorem 21. *Let (F, A, τ) and (G, A, τ) be two soft topological soft groups over X. If $F(\alpha) \subseteq G(\alpha)$, for all $\alpha \in A$, then $(F, A, \tau) \tilde{\leq} (G, A, \tau)$.*

Theorem 22. *Let (F, A, τ) be a soft topological soft group over X and $\{(H_i, A, \tau);\ i \in I\}$ be a nonempty family of soft topological soft subgroups (soft topological soft normal subgroups) of (F, A, τ) where I is an index set, and then $\tilde{\bigcap}_{i \in I}(H_i, A, \tau)$ $(= (\tilde{\cap}_{i \in I} H_i, A, \tau))$ is a soft topological soft subgroup (soft topological soft normal subgroup) of (F, A, τ).*

Proof. From Theorem 14, we get $\tilde{\bigcap}_{i \in I}(H_i, A, \tau)$ is a soft topological soft groups over X. Also, $(\cap_{i \in I} H_i)(\alpha)$ is a subgroup (normal subgroup) of $F(\alpha)$, *for all* $\alpha \in A$. Hence, $(\cap_{i \in I} H_i, A)$ is a soft subgroup (soft normal subgroup) of (F, A). Again, $[\tau^{\alpha}]_{(\cap_{i \in I} H_i)(\alpha)} = [\tau^{\alpha}]_{F(\alpha)/(\cap_{i \in I} H_i)(\alpha)}$. Therefore, $\tilde{\bigcap}_{i \in I}(H_i, A, \tau)$ is a soft topological soft subgroup (soft topological soft normal subgroup) of (F, A, τ). □

Theorem 23. *Let (F, A, τ) be a soft topological soft group over X. If (N, A) be a soft subgroup of (F, A), then (N, A, τ) is a soft topological soft group over X and (N, A, τ) is a soft topological soft subgroup of (F, A, τ).*

Proof. Since (F, A, τ) be a soft topological soft group over X, then for each $\alpha \in A$, $[F(\alpha), \tau^{\alpha}_{F(\alpha)}]$ is a topological group on $F(\alpha)$. Also, $N(\alpha)$ is a subgroup of $F(\alpha)$, *for all* $\alpha \in A$. Thus, $[\tau^{\alpha}_{F(\alpha)}]_{N(\alpha)} = [\tau^{\alpha}_{N(\alpha)}]$ is a group topology on $N(\alpha)$, *for all* $\alpha \in A$. Therefore, (N, A, τ) is a soft topological soft group over X. Also, from Definition 20, (N, A, τ) is a soft topological soft subgroup of (F, A, τ). □

Theorem 24. *Let τ be a soft topology over X. Let (F_1, A, τ) and (F_2, A, τ) be two soft topological soft groups over X. If $f : X \to Y$ be a soft open homomorphism, then $(f(F_1), A, f(\tau))$ and $(f(F_2), A, f(\tau))$ are both soft topo-*

logical soft groups over Y. Also, if $(F_1, A, \tau) \widetilde{\leq} (\widetilde{\lhd}) (F_2, A, \tau)$, then $(f(F_1), A, f(\tau)) \widetilde{\leq} (\widetilde{\lhd}) (f(F_2), A, f(\tau))$.

Proof. From Theorem 17, we get $(f(F_1), A, f(\tau))$ and $(f(F_2), A, f(\tau))$ are both soft topological soft groups over Y. Also, since $(F_1, A, \tau) \widetilde{\leq} (\widetilde{\lhd}) (F_2, A, \tau)$, then $(F_1, A) \widetilde{\leq} (\widetilde{\lhd}) (F_2, A)$ and $\tau^{\alpha}_{F_1(\alpha)} = \tau^{\alpha}_{F_2(\alpha)/F_1(\alpha)}$. Thus, $f(F_1), A) \widetilde{\leq} (\widetilde{\lhd}) (f(F_2), A)$ and $[f(\tau)]^{\alpha}_{F_1(\alpha)} = [f(\tau)]^{\alpha}_{f(F_1(\alpha))/f(F_2(\alpha))}$, *for all* $\alpha \in A$. Therefore, $(f(F_1), A, f(\tau)) \widetilde{\leq} (\widetilde{\lhd}) (f(F_2), A, f(\tau))$. □

Theorem 25. *Let ν be a soft topology over Y. Let (G_1, A, ν) and (G_2, A, ν) be two soft topological soft groups over Y. If $f : X \to Y$ be an onto homomorphism, then $(f^{-1}(G_1), A, f^{-1}(\nu))$ and $(f^{-1}(G_2), A, f^{-1}(\nu))$ are both soft topological soft groups over X. Also, if $(G_1, A, \nu) \widetilde{\leq} (\widetilde{\lhd}) (G_2, A, \nu)$, then $(f^{-1}(G_1), A, f^{-1}(\nu)) \widetilde{\leq} (\widetilde{\lhd}) (f^{-1}(G_2), A, f^{-1}(\nu))$.*

Proof. From Theorem 18, we get $(f^{-1}(G_1), A, f^{-1}(\nu))$ and $(f^{-1}(G_2), A, f^{-1}(\nu))$ are both soft topological soft groups over X. Also, since $(G_1, A, \nu) \widetilde{\leq} (\widetilde{\lhd}) (G_2, A, \nu)$, then $(G_1, A) \widetilde{\leq} (\widetilde{\lhd}) (G_2, A)$ and $\nu^{\alpha}_{G_1(\alpha)} = \nu^{\alpha}_{G_2(\alpha)/G_1(\alpha)}$. $\Rightarrow (f^{-1}(G_1), A) \widetilde{\leq} (\widetilde{\lhd}) (f^{-1}(G_2), A)$ and $[f^{-1}(\nu)]^{\alpha}_{G_1(\alpha)} = [f^{-1}(\nu)]^{\alpha}_{f^{-1}(G_1(\alpha))/f^{-1}(G_2(\alpha))}$, *for all* $\alpha \in A$. Therefore, $(f^{-1}(G_1), A, f^{-1}(\nu)) \widetilde{\leq} (\widetilde{\lhd}) (f^{-1}(G_2), A, f^{-1}(\nu))$. □

Theorem 26. *Let (F, A, τ) be a soft topological soft group over X and $f : X \to Y$ be a homomorphism. Define the set $K_f(\alpha)$ by $K_f(\alpha) = [Ker(f)]_{F(\alpha)} = (Kerf) \cap F(\alpha) = \{g \in F(\alpha); f(g) = e_Y\}$, for all $\alpha \in A$, then*

(i) (K_f, A, τ) is a soft topological soft group over X.
(ii) (K_f, A, τ) is a soft topological normal soft subgroup of (F, A, τ).

Proof. It follows from Definition 7 and Theorem 23. □

Remark 2. Let (N, A, τ) and (F, A, τ) be two soft topological soft groups over X such that (N, A, τ) is a soft topological soft normal subgroup of (F, A, τ). Define a mapping $\frac{F}{N}$ over A by $\frac{F}{N}(\alpha)$ = the factor group $\frac{F(\alpha)}{N(\alpha)}$, *for all* $\alpha \in A$. Therefore, $(\frac{F}{N}, A)$ is a factor soft group. Again, *for all* $\alpha \in A$, $\tau^{\alpha}_{F(\alpha)}$ is a group topology on $F(\alpha)$. If $\psi_{\alpha} : F(\alpha) \to \frac{F(\alpha)}{N(\alpha)}$ be a canonical mapping and define $\tau^{\alpha}_{\frac{F(\alpha)}{N(\alpha)}} = \{U \subseteq \frac{F(\alpha)}{N(\alpha)} : \psi_{\alpha}^{-1}(U) \in \tau^{\alpha}_{F(\alpha)}\}$. Therefore, for each $\alpha \in A$, $(\frac{F(\alpha)}{N(\alpha)}, \tau^{\alpha}_{\frac{F(\alpha)}{N(\alpha)}})$ is a topological group on $\frac{F(\alpha)}{N(\alpha)}$ (from part b Theorem 8).

Definition 21. Let (F_1, A, τ) and (F_2, A, ν) be two soft topological soft groups over X and Y, respectively, and then (F_1, A, τ) is said to be *soft topological soft homomorphic* to (onto) (F_2, A, ν), denoted by $(F_1, A, \tau) \sim (F_2, A, \nu)$, if for each $\alpha \in A$, $\exists\, \phi_{\alpha} : (F_1(\alpha), \tau^{\alpha}_{F_1(\alpha)}) \to (F_2(\alpha), \nu^{\alpha}_{F_2(\alpha)})$ such that

(i) $\phi_{\alpha} : F_1(\alpha) \to F_2(\alpha)$ is a homomorphism (onto homomorphism).
(ii) $\phi_{\alpha} : (F_1(\alpha), \tau^{\alpha}_{F_1(\alpha)}) \to (F_2(\alpha), \nu^{\alpha}_{F_2(\alpha)})$ is continuous.

Definition 22. Let (F_1, A, τ_1) and (F_2, A, τ_2) be two soft topological soft groups over X and Y, respectively. Also, let (F_1, A, τ_1) be soft topological soft homomorphic to (F_2, A, τ_2). Define $\phi F_1 : A \to P(Y)$ by $(\phi F_1)(\alpha) = (\phi_{\alpha}(F_1(\alpha)))$, *for all* $\alpha \in A$ where ϕ_{α} satisfies relations (i) and (ii) of the Definition 21.

Definition 23. Let (F_1, A, τ_1) and (F_2, A, τ_2) be two soft topological soft groups over X and Y, respectively. Also, let (F_1, A, τ_1) be soft topological soft homomorphic to (F_2, A, τ_2). Define $\phi^{-1} F_2 : A \to P(X)$ by $(\phi^{-1} F_2)(\alpha) = (\phi_{\alpha}^{-1}(F_2(\alpha)))$, *for all* $\alpha \in A$ where ϕ_{α} satisfies relations (i) and (ii) of the Definition 21.

Theorem 27. *Let (F_1, A, τ) and (F_2, A, ν) be two soft topological soft groups over X and Y, respectively. Also, let (F_1, A, τ) be soft topological soft homomorphic to (F_2, A, ν). If $\phi_{\alpha} : (F_1(\alpha), \tau^{\alpha}_{F_1(\alpha)}) \to (F_2(\alpha), \nu^{\alpha}_{F_2(\alpha)})$ be the corresponding homomorphism for each $\alpha \in A$, then*

(i) $(\phi F_1, A, \nu)$ is a soft topological soft group over Y and $(\phi F_1, A, \nu) \widetilde{\leq} (F_2, A, \nu)$.
(ii) $(\phi^{-1} F_2, A, \tau)$ is a soft topological soft group over X and $(\phi^{-1} F_2, A, \tau) \widetilde{\leq} (F_1, A, \tau)$.

Proof. (i) For each $\alpha \in A$, $(\phi F_1)(\alpha) = \phi_{\alpha}(F_1(\alpha))$ is a subgroup of $F_2(\alpha)$. Hence, $(\phi F_1, A)$ is a soft subgroup of (F_2, A). By Theorem 22, $(\phi F_1, A, \nu)$ is a soft topological soft group over Y and $(\phi F_1, A, \nu) \widetilde{\leq} (F_2, A, \nu)$.
(ii) For each $\alpha \in A$, $(\phi^{-1} F_2)(\alpha) = \phi_{\alpha}^{-1}(F_2(\alpha))$ is a subgroup of $F_1(\alpha)$. Hence, $(\phi^{-1} F_2, A)$ is a soft subgroup of (F_1, A), and then by Theorem 22, $(\phi^{-1} F_2, A, \tau)$ is a soft topological soft group over X and $(\phi^{-1} F_2, A, \tau) \widetilde{\leq} (F_1, A, \tau)$.

□

Theorem 28. *Let (F_1, A, τ_1) and (F_2, A, τ_2) be two soft topological soft groups over X and Y, respectively. Also, let (F_1, A, τ_1) be soft topological soft homomorphic onto (F_2, A, τ_2).*

(i) If (F_3, A, τ_1) be a soft topological soft normal subgroup of (F_1, A, τ_1), then $(\phi F_3, A, \tau_2)$ is a soft topological soft normal subgroup of $(\phi F_1, A, \tau_2)$, where ϕF is as defined in Definition 22.

(ii) If (F_4, A, τ_2) be a soft topological soft normal subgroup of (F_2, A, τ_2), then $(\phi^{-1}F_4, A, \tau_1)$ is a soft topological soft normal subgroup of $(\phi^{-1}F_2, A, \tau_1)$, where $\phi^{-1}F$ is as in Definition 23.

Proof. (i) Since (F_1, A, τ_1) be soft topological soft homomorphic onto (F_2, A, τ_2), then it is clear that $(\phi F_3, A, \tau_2)$ and $(\phi F_1, A, \tau_2)$ are soft topological soft subgroups over Y. Also, *for all* $\alpha \in A$, $\phi_\alpha(F_1(\alpha))$ and $\phi_\alpha(F_3(\alpha))$ are both subgroups of $F_2(\alpha)$ and $\phi_\alpha(F_3(\alpha))$ is a normal subgroup of $\phi_\alpha(F_1(\alpha))$. Thus, *for all* $\alpha \in A$, $\phi_\alpha(F_3(\alpha))$ is a normal subgroup of $\phi_\alpha(F_1(\alpha))$. Thus, $(\phi F_3, A)$ is a soft normal subgroup of $(\phi F_1, A)$. Therefore, $(\phi F_3, A, \tau_2)$ is a soft topological soft normal subgroup of $(\phi F_1, A, \tau_2)$.

(ii) Since (F_1, A, τ_1) be soft topological soft homomorphic onto (F_2, A, τ_2), then it is clear that $(\phi^{-1}F_3, A, \tau_1)$ and $(\phi^{-1}F_1, A, \tau_1)$ are soft topological soft subgroups over X. Also, *for all* $\alpha \in A$, $\phi_\alpha^{-1}(F_4(\alpha))$ and $\phi_\alpha^{-1}(F_2(\alpha))$ are both subgroups of $F_1(\alpha)$ and $\phi_\alpha^{-1}(F_4(\alpha))$ is a normal subgroup of $\phi_\alpha^{-1}(F_2(\alpha))$. Hence, *for all* $\alpha \in A$, $\phi_\alpha^{-1}(F_3(\alpha))$ is a normal subgroup of $\phi_\alpha^{-1}(F_2(\alpha))$. Thus, $(\phi^{-1}F_3, A)$ is a soft normal subgroup of $(\phi^{-1}F_2, A)$. Therefore, $(\phi^{-1}F_3, A, \tau_1)$ is a soft topological soft normal subgroup of $(\phi^{-1}F_2, A, \tau_1)$. □

Definition 24. Let (F_1, A, τ) and (F_2, A, ν) be two soft topological soft groups over X and Y, respectively, and then (F_1, A, τ) is said to be *soft topological soft isomorphic* to (F_2, A, ν), denoted by $(F_1, A, \tau) \simeq (F_2, A, \nu)$, if for each $\alpha \in A, \exists\, \phi_\alpha : (F_1(\alpha), \tau^\alpha_{F_1(\alpha)}) \to (F_2(\alpha), \nu^\alpha_{F_2(\alpha)})$ such that

(i) $\phi_\alpha : F_1(\alpha) \to F_2(\alpha)$ is an isomorphism.
(ii) $\phi_\alpha : (F_1(\alpha), \tau^\alpha_{F_1(\alpha)}) \to (F_2(\alpha), \nu^\alpha_{F_2(\alpha)})$ is homeomorphism.

Theorem 29. *Let (N, A, τ) be a soft topological soft normal subgroup of (F, A, τ), and then for each $\alpha \in A$, the canonical mapping $\psi_\alpha : (F(\alpha), \tau^\alpha_{F(\alpha)}) \to \left(\frac{F(\alpha)}{N(\alpha)}, \tau^\alpha_{\frac{F(\alpha)}{N(\alpha)}}\right)$, given by $\psi_\alpha(\xi) = \xi N(\alpha)$, $\xi \in F(\alpha)$, is an open homomorphism.*

Proof. (N, A, τ) is a soft topological soft normal subgroup of (F, A, τ). $\Rightarrow$ $N(\alpha)$ is normal subgroup of $F(\alpha)$, *for all* $\alpha \in A$. $\Rightarrow$ $(N(\alpha), \tau^\alpha_{N(\alpha)})$ is a topological normal subgroup of $(F(\alpha), \tau^\alpha_{F(\alpha)})$. Therefore, from part c of Theorem 8, the canonical mapping $\psi_\alpha : (F(\alpha), \tau^\alpha_{F(\alpha)}) \to \left(\frac{F(\alpha)}{N(\alpha)}, \tau^\alpha_{\frac{F(\alpha)}{N(\alpha)}}\right)$ is an open homomorphism. □

Theorem 30. *Let (F_1, A, τ) and (F_2, A, ν) be two soft topological groups over X and Y, respectively. Also, let (F_1, A, τ) be soft topological soft homomorphic to (F_2, A, ν). If $\phi_\alpha : (F_1(\alpha), \tau^\alpha_{F_1(\alpha)}) \to (F_2(\alpha), \nu^\alpha_{F_2(\alpha)})$ be the corresponding homomorphism for each $\alpha \in A$ and $K(\alpha)$ be the kernel of ϕ_α, then $\phi_\alpha : (F_1(\alpha), \tau^\alpha_{F_1(\alpha)}) \to (F_2(\alpha), \nu^\alpha_{F_2(\alpha)})$ is continuous (open) if $\phi^0_\alpha : \left(\frac{F_1(\alpha)}{K(\alpha)}, \tau^\alpha_{\frac{F_1(\alpha)}{K(\alpha)}}\right) \to \left(F_2(\alpha), \nu^\alpha_{F_2(\alpha)}\right)$ is continuous (open), where $\phi^0_\alpha(\xi K(\alpha)) = \phi_\alpha(\xi)$.*

Proof. Since $\phi_\alpha : (F_1(\alpha), \tau^\alpha_{F_1(\alpha)}) \to (F_2(\alpha), \nu^\alpha_{F_2(\alpha)})$ is the corresponding homomorphism and $K(\alpha)$ be the kernel of ϕ_α. Let $\psi_\alpha : F_1(\alpha) \to \frac{F_1(\alpha)}{K(\alpha)}$ be the canonical mapping defined by $\psi_\alpha(\xi) = \xi K(\alpha), \xi \in F_1(\alpha)$. Again, define $\phi^0_\alpha : \left(\frac{F_1(\alpha)}{K(\alpha)}, \tau^\alpha_{\frac{F_1(\alpha)}{K(\alpha)}}\right) \to \left(F_2(\alpha), \nu^\alpha_{F_2(\alpha)}\right)$ by $\phi^0_\alpha[\xi K(\alpha)] = \phi_\alpha(\xi), \xi \in F_1(\alpha)$, and then $\phi_\alpha(\xi) = \phi^0_\alpha[\xi K(\alpha)] = \phi^0_\alpha(\psi_\alpha(\xi)) = (\phi^0_\alpha \psi_\alpha)(\xi)$, *for all* $\xi \in F_1(\alpha)$. Therefore, $\phi_\alpha = \phi^0_\alpha \psi_\alpha$, and hence, by Theorem 9, $\phi_\alpha : (F_1(\alpha), \tau^\alpha_{F_1(\alpha)}) \to (F_2(\alpha), \nu^\alpha_{F_2(\alpha)})$ is continuous (open) if $\phi^0_\alpha : \left(\frac{F_1(\alpha)}{K(\alpha)}, \tau^\alpha_{\frac{F_1(\alpha)}{K(\alpha)}}\right) \to \left(F_2(\alpha), \nu^\alpha_{F_2(\alpha)}\right)$ is continuous (open). □

Theorem 31. ***(Fundamental homomorphism theorem)*** *Let (F_1, A, τ) and (F_2, A, ν) be two soft topological soft groups over X and Y, respectively, and $(F_1, A, \tau) \sim (F_2, A, \nu)$. Also, let $\phi_\alpha : (F_1(\alpha), \tau^\alpha_{F_1(\alpha)}) \to (F_2(\alpha), \nu^\alpha_{F_2(\alpha)})$ be the corresponding homomorphism and $K(\alpha)$ be the kernel of ϕ_α, for all $\alpha \in A$. If ϕ_α is open and $\psi_\alpha : F_1(\alpha) \to \frac{F_1(\alpha)}{K(\alpha)}$ be the canonical mapping, then $(F_1/K, A) \simeq (F_2, A)$ such that for all $\alpha \in A$, $\phi^0_\alpha : \left(\frac{F_1(\alpha)}{K(\alpha)}, \tau^\alpha_{\frac{F_1(\alpha)}{K(\alpha)}}\right) \to \left(F_2(\alpha), \nu^\alpha_{F_2(\alpha)}\right)$ is a homeomorphism.*

Proof. Since $(F_1, A, \tau) \sim (F_2, A, \nu)$ and $\phi_\alpha : (F_1(\alpha), \tau^\alpha_{F_1(\alpha)}) \to (F_2(\alpha), \nu^\alpha_{F_2(\alpha)})$ be the corresponding homomorphism, then the mapping $\phi_\alpha : F_1(\alpha) \to F_2(\alpha)$ is an algebraic homomorphism, and $\phi_\alpha : (F_1(\alpha), \tau^\alpha_{F_1(\alpha)}) \to (F_2(\alpha), \nu^\alpha_{F_2(\alpha)})$ is continuous. Also, $K(\alpha)$ be the kernel of ϕ_α, then by Theorem 7, $(F_1/K, A) \simeq (F_2, A)$. Again, the mapping $\phi^0_\alpha : \left(\frac{F_1(\alpha)}{K(\alpha)}, \tau^\alpha_{\frac{F_1(\alpha)}{K(\alpha)}}\right) \to \left(F_2(\alpha), \nu^\alpha_{F_2(\alpha)}\right)$, defined by $\phi^0_\alpha[\xi K(\alpha)] = \phi_\alpha(\xi)$, *for all* $\xi \in F_1(\alpha)$, is an algebraic isomorphism from the group $\frac{F_1(\alpha)}{K(\alpha)}$ onto the group $F_2(\alpha)$ and $\phi_\alpha = \phi^0_\alpha \psi_\alpha$, *for all* $\alpha \in A$. Since ϕ_α is continuous and open, then by Theorem 30, $\phi^0_\alpha : \left(\frac{F_1(\alpha)}{K(\alpha)}, \tau^\alpha_{\frac{F_1(\alpha)}{K(\alpha)}}\right) \to \left(F_2(\alpha), \nu^\alpha_{F_2(\alpha)}\right)$, *for all* $\alpha \in A$ is also continuous and open. Since ϕ^0_α is bijective and open, $[\phi^0_\alpha]^{-1}$ is continuous. Therefore, $\phi^0_\alpha : \left(\frac{F_1(\alpha)}{K(\alpha)}, \tau^\alpha_{\frac{F_1(\alpha)}{K(\alpha)}}\right) \to \left(F_2(\alpha), \nu^\alpha_{F_2(\alpha)}\right)$, *for all* $\alpha \in A$ is a homeomorphism. □

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
SN and SKS contributed to the paper equally. Both authors read and approved the final manuscript for publication.

Authors' information
SN is a research scholar at the Department of Mathematics, Visva-Bharati and an assistant professor of Mathematics at Govt. College of Education, Burdwan, Kazirhat, Lakurdi, Burdwan, West Bengal, 713102, India. SKS is a professor of Mathematics at the Department of Mathematics, Visva-Bharati, Santiniketan, West Bengal, 731235, India.

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Author details
[1]Gov't. College of Education, Burdwan, Kazirhat, Lakurdi, Burdwan, West Bengal, 713102, India. [2]Department of Mathematics, Visva-Bharati, Santiniketan, West Bengal, 731235, India.

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