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Quasi a-ideals in BCI-algebras

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Abstract

Non-classical logic has become a considerable formal tool for computer science on computational intelligence to deal with fuzzy information and uncertain information. BCI-algebras and BCK-algebras are two classes of non-classical logic algebras that are introduced by Iseki in 1966 and they are algebraic formulations of BCK and BCI-system in logic algebras. Lele used the notion of fuzzy point to study some properties of BCI-algebras. Jun and Lele used the notion of fuzzy points for establishing quasi q-ideal in the set of all fuzzy points of a fixed BCI-algebras and give some characterizations of these ideals.

Keywords: BCI-algebras, Quasi ideals, Quasi a-ideals

Introduction

Lele et al. [1] used the notion of fuzzy point to study some properties of BCK-algebras. Jun and Lele [2] used the notion of fuzzy points for establishing quasi ideals. As a continuation of [2], in this chapter, we introduce the notion of a quasi a-ideal and a quasi q-ideal in the set of all fuzzy points of a fixed BCI-algebra.

Definition 1.1. [3] A BCI-algebra is an algebra $(X, *, 0)$ satisfying the following conditions:

(I) $((x*y)*(x*z))*(z*y) = 0$,
(II) $(x*(x*y))*y = 0$,
(III) $x*x = 0$,
(IV) $x*y = 0$ and $y*x = 0$ implied as $x = y$; for all $x, y, z, \in X$, we can define a partial ordering '$\leq$' on X by $x \leq y$ if and only if $x*y = 0$.

Definition 1.2. If ξ is the family of all fuzzy sets on X, $x_\alpha \in \xi$ is called a *fuzzy point* if and only if there exists $\alpha > 0$ such that for all $y \in X$,

$$x_\alpha(y) = \begin{cases} \alpha & \text{if } \ x = y, \\ 0 & \text{otherwise.} \end{cases}$$

Definition 1.3. [1] We denote by $\mathrm{FP}(X) = \{x_\alpha \mid x \in X, \alpha \in (0,1]\}$, the set of all fuzzy points on X. Define a binary operation $\odot$ on $\mathrm{FP}(X)$ by

$$x_\alpha \odot x_\beta = (x*y)_{\min\{\alpha,\beta\}},$$

where $*$ is a binary operation on X. If $(X, *, 0)$ is a BCI-algebra, then

(i) $((x_\alpha \odot y_\beta) \odot (x_\alpha \odot z_\gamma)) \odot (z_\gamma \odot y_\beta) = 0_{\min\{\alpha,\beta,\gamma\}}$,
(ii) $((x_\alpha \odot (x_\alpha \odot y_\beta)) \odot y_\beta = 0_{\min\{\alpha,\beta\}}$,
(iii) $x_\alpha \odot x_\alpha = 0_\alpha$,

for all $x_\alpha, y_\beta, z_\gamma \in \mathrm{FP}(X)$.

Corollary 1.1. *[1] For all $x_\alpha, y_\beta, z_\gamma \in FP(X)$, the following hold:*

(1) $x_\alpha \odot 0_\beta = x_{\min\{\alpha,\beta\}}$.
(2) $0_\alpha \odot (x_\beta \odot y_\gamma) = (0_{\alpha\beta}) \odot (0_\alpha \odot y_\gamma)$.
(3) *If* $x_\alpha \odot y_\beta = 0_{\min\{\alpha,\beta\}}$, *then* $(x_{\alpha\gamma}) \odot (y_{\beta\gamma} = 0_{\min\{\alpha,\beta,\gamma\}}, (z_\gamma \odot y_\beta) \odot (z_\gamma \odot x_\alpha) = 0_{\min\{\alpha,\beta,\gamma\}}$.
(4) $(x_\alpha \odot y_\beta) \odot z_\gamma = (x_\alpha \odot z_\gamma) \odot y_\beta$.
(5) $x_\alpha \odot (x_\alpha \odot (x_\alpha \odot y_\beta)) = x_\alpha \odot y_\beta$.

Definition 1.4. [1] For a fuzzy set μ in a BCI-algebra X, we define the set $\mathrm{FP}(\mu)$ of all fuzzy points in X covered by μ to be the set

$$\mathrm{FP}(\mu) = \{x_\alpha \in \mathrm{FP}(X) \mid \mu(x) \geq \alpha, \quad 0 < \alpha \leq 1\}.$$

Definition 1.5. [1] A subset $\mathrm{FP}(\mu)$ of $\mathrm{FP}(X)$ is called a *quasi ideal* of $\mathrm{FP}(X)$ if $0_\alpha \in \mathrm{FP}(\mu)$ for all $\alpha \in \mathrm{Im}(\mu)$ and $(QI1) x_\alpha \odot y_\beta,\ y_\beta \in \mathrm{FP}(\mu) \Rightarrow x_{\min\{\alpha,\beta\}} \in \mathrm{FP}(\mu)$ for all $x_\alpha, y_\beta \in FP(X)$.

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A quasi ideal $FP(\mu)$ is said to be *closed* if for any $\theta \in Im(\mu)$,

$$0_\theta \odot x_\alpha \in FP(\mu) \text{ when } x_\alpha \in FP(\mu).$$

Definition 1.6. [2] For a fuzzy set μ in a BCI-algebra X, the set $FP(\mu)$ of all fuzzy points in X covered by μ is called a *quasi a-ideal* of $FP(X)$ if for all $\theta \in Im(\mu)$ and $x_\alpha, y_\beta, z_\gamma \in FP(X)$, $(QA1)$ $0_\theta \in FP(\mu)$, $(QA2)(x_\alpha \odot z_\gamma) \odot (0_\theta \odot y_\beta)$, $z_\gamma \in FP(\mu) \Rightarrow y_\beta \odot x_{\min\{\alpha,\theta,\gamma\}} \in FP(\mu)$.

Main results

Theorem 2.1. *Every quasi a-ideal of FP(X) is also a quasi ideal.*

Proof. Let $FP(\mu)$ be a quasi a-ideal of $FP(X)$, then by $(QA1)$ for all $\theta \in Im(\mu)$, $0_\theta \in FP(\mu)$. Let $x_\alpha, y_\beta \in FP(X)$ be such that $x_\alpha \odot y_\beta \in FP(\mu)$ and $y_\beta \in FP(\mu)$. we prove $x_{\min\{\alpha,\beta\}} \in FP(\mu)$.

As $(x_\alpha \odot y_\beta) \odot (0_\alpha \odot 0_\alpha) \in FP(\mu)$ and $y_\beta \in FP(\mu)$ by $(QA2)$, we obtain

$$0_\alpha \odot x_{\min\{\alpha,\beta\}} \in FP(\mu). \tag{1}$$

If $z_\alpha \in FP(\mu)$ then $z_\alpha = (z_\alpha \odot 0_\alpha) \odot (0_\alpha \odot 0_\alpha) \in FP(\mu), 0_\alpha \in FP(\mu)$ by $(QA2)$ it follows that $0_\alpha \odot z_\alpha \in FP(\mu)$. That is

$$z_\alpha \in FP(\mu) \Rightarrow 0_\alpha \odot z_\alpha \in FP(\mu). \tag{2}$$

Combining (1) and (2) we have $0_\alpha \odot (0_\alpha \odot x_\beta) \in FP(\mu)$.

As $0_\alpha \odot (0_\alpha \odot x_\beta) = (0_\alpha \odot 0_\alpha) \odot (0_\alpha \odot x_\beta) \in FP(\mu)$, $0_\alpha \in FP(\mu)$ by $(QA2)$ we obtain that $x_\beta \odot 0_\alpha = x_{min\{\alpha,\beta\}} \in FP(\mu)$, as required. □

The converse of Theorem 2.1 may not be true as seen in the following example.

Example 2.1. Let $X = \{0, a, b, c, d\}$ be a BCI-algebra with the following table:

$*$	0	a	b	c	d
0	0	0	0	0	0
a	a	0	a	0	0
b	b	b	0	0	0
c	c	c	c	0	0
d	d	c	d	a	

Let μ be a fuzzy set in X defined by $\mu(0) = \mu(c) = 0.8$, $\mu(a) = \mu(b) = \mu(d) = 0.3$.

Consider the set $FP(\mu) = \{0_\theta, a_\alpha, b_\beta, c_\gamma, d_t \mid \theta, \gamma \in (0, 0.8],\ \alpha, \beta, t \in (0, 0.3]\}$. Then $FP(\mu)$ is a quasi ideal of $FP(X)$. Note that $(a_{0.6} \odot c_{0.7}) \odot (0_{0.6} \odot b_{0.5}) = (a * c)_{0.6} \odot (0 * b)_{0.5} = 0_{0.6} \odot 0_{0.5} = 0_{0.5} \in FP(\mu)$ and $c_{0.7} \in FP(\mu)$. But $b_{0.5} \odot a_{0.6} = (b * a)_{\min\{0.5,0.6\}} = b_{0.5} \notin FP(\mu)$. This shows that $FP(\mu)$ is not a quasi a-ideal of $FP(X)$.

Therefore, the converse of Theorem 2.1 is true only in a special case.

Theorem 2.2. *Let μ be a fuzzy set in a BCI-algebra X. If $FP(\mu)$ is a quasi ideal of $FP(X)$ such that for all $x_\alpha, y_\beta, z_\gamma \in FP(X)$, and $\theta \in Im(\mu)$*

$$(x_\alpha \odot z_\gamma) \odot (0_\theta \odot y_\beta) \in FP(\mu) \Rightarrow (y_\beta \odot x_\alpha) \odot z_\gamma \in FP(\mu),$$

then $FP(\mu)$ is a quasi a-ideal of $FP(X)$.

Proof. Let μ be a fuzzy ideal of X. Let $\theta \in Im(\mu)$ and suppose that $\theta = \mu(x)$. Since μ is a fuzzy ideal, we have $\mu(0) \geq \mu(x) = \theta$. Hence, $0_\theta \in FP(\mu)$. Let $x_\alpha, y_\beta, z_\gamma \in FP(X)$ and $\theta \in Im(\mu)$ be such that $(x_\alpha \odot z_\gamma) \odot (0_\theta \odot y_\beta) \in FP(\mu)$ *and* $z_\gamma \in FP(\mu)$. Then by hypothesis, we have $(y_\beta \odot x_\alpha) \odot z_\gamma \in FP(\mu)$ *and* $z_\gamma \in FP(\mu)$. Since $FP(\mu)$ is a quasi ideal of $FP(X)$ then $y_\beta \odot x_{min\{\alpha,\gamma\}} \in FP(\mu)$. But $\min\{\alpha, \beta, \theta, \gamma\} \leq min\{\alpha, \beta, \gamma\}$ *and* $y_\beta \odot x_{\min\{\alpha,\gamma\}} \in FP(\mu)$, imply that $y_\beta \odot x_{\min\{\alpha,\gamma,\theta\}} \in FP(\mu)$. Hence, $FP(\mu)$ is a quasi a-ideal of $FP(X)$. □

Competing interests
The author declares that he has no competing interest.

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