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Survey of B-spline functions to approximate the solution of mathematical problems

Jalil Rashidinia[1,2*] and Sh Sharifi[2]

*Correspondence:
Rashidinia@iust.ac.ir
[1]School of Mathematics, Iran University of Science and Technology, Tehran, Iran
[2]Department of Mathematics and Statistics, Islamic Azad University - Central Tehran Branch, Tehran, Iran

Abstract

Purpose: In the present paper, we describe a survey of B-spline techniques which have been used for numerical solutions of mathematical problems recently.

Methods: Here, we discussed the definition of B-splines of various degrees by two different approaches to generate the recurrence relation to drive the formulation of B-splines.

Results: Cubic B-spline applied on two test equations and absolute errors in interpolation are compared with cubic and quintic splines. Some remarks have been included.

Conclusions: Numerical results are tabulated in tables; these tables show that the results obtained by cubic B-spline are considerable and accurate with respect to the cubic spline and more or less similar to the quintic spline.

Keywords: Spline functions, Derivation of formula, Alternative approach, Numerical illustration

Introduction

The theory of spline functions is a very attractive field of approximation theory. Usually, a spline is a piecewise polynomial function defined in region D, such that there exists a decomposition of D into subregions in each of which the function is a polynomial of some degree k. Also, the function, as a rule, is continuous in D, together with its derivatives of order up to $(k-1)$ [1-7]. Generally, the piecewise polynomial is considered, and $[a, b] \subset R$ is a finite interval. We introduce a set of partition $\Delta_n = \{x_0, x_1, \cdots, x_n\}$ of $[a, b]$, where x_i $(i = 0(1)n)$ are called nodes of the partition. The set of piecewise polynomial of degree k defined on a partition Δ_n is denoted by $S_k(\Delta_n)$ in each subinterval; $I_i = [x_{i-1}, x_i]$ is a kth degree polynomial. Specifically, the type of bases B-spline for our purpose is considered, for which we only use the equidistance partition. Moreover, we extend the set of nodes by taking $h = \frac{b-a}{n}$, $x_0 = a$, and $x_i = x_0 + ih$ where $i = \pm1, \pm2, \pm3, \cdots$.

Let $\{\Delta_n\}$ be a partition of $[a, b] \subset R$. A B-spline of degree k is a spline from $S_k(\Delta_n)$ with minimal support and the partition of unity holding.

The B-spline of degree k is denoted by $B_i^k(x)$, where $i \in Z$, and then we have the following properties:

1. Supp $(B_i^k) = [x_i, x_{i+k+1}]$
2. $B_i^k(x) \geq 0$, $\forall x \in R$ (non-negativity)

3. $\sum_{i=-\infty}^{\infty} B_i^k(x) = 1$, $\forall x \in R$ (partition of unity)

The next section explains the explicit definition of B-splines.

Methods

Derivation of B-spline functions

In this section we give an introduction of B-splines. The B-splines were so named because they formed a basis for the set of all splines [4]. Through out this section, we suppose that an infinite set of knots $\{x_i\}$ has been prescribed in such a way that

$$\cdots < x_{-2} < x_{-1} < x_0 < x_1 < x_2 < \cdots$$
$$\lim_{i \to \infty} x_i = \infty = - \lim_{i \to \infty} x_{-i}$$

The B-spline to be defined now depends on this set of knots.

Definition 1. Support of function f is defined as the set of points x when $f(x) \neq 0$.

B-spline of degree 0

The B-spline of degree 0 is defined by $B_i^0(x) = \begin{cases} 1 & x_i \le x < x_{i+1} \\ 0 & \text{otherwise.} \end{cases}$

B-spline of degree 0 is characterized by the following:

1. The support of B_i^0 is a half-open interval $[x_i, x_{i+1}]$.
2. $B_i^0(x) \ge 0$ for all x and for all i.
3. B_i^0 is continuous.
4. $\sum_{i=-\infty}^{\infty} B_i^0(x) = 1$ for all x.
5. Any spline of degree 0 can be expressed as a linear combination of the B-splines B_i^0.

We generate all the higher degree B-splines by a simple recursive definition [3,4]:

$$B_i^k(x) = \left(\frac{x - x_i}{x_{i+k} - x_i}\right) B_i^{k-1}(x) + \left(\frac{x_{i+k+1} - x}{x_{i+k+1} - x_{i+1}}\right) B_{i+1}^{k-1}(x); \qquad k \ge 1, \tag{1}$$

The B_i^k functions as defined by Equation 1 are called B-splines of degree k. Since each B_i^k is obtained by applying a linear factor to B_i^{k-1} and B_{i+1}^{k-1}, we see that degrees actually increased by 1 at each step. Therefore, B_i^1 is piecewise linear, B_i^2 is piecewise quadratic, and so on.

With the function B_i^0 as a starting point and Equation 1, we obtain the higher degree B-splines.

B-spline of degree 1

To illustrate Equation 1, let us determine B_i^1 in an alternative form:

$$B_i^1(x) = \left(\frac{x - x_i}{x_{i+1} - x_i}\right) B_i^0(x) + \left(\frac{x_{i+2} - x}{x_{i+2} - x_{i+1}}\right) B_{i+1}^0(x),$$
$$= \begin{cases} \frac{x - x_i}{x_{i+1} - x_i} & x_i \le x < x_{i+1} \\ \frac{x_{i+2} - x}{x_{i+2} - x_{i+1}} & x_{i+1} \le x < x_{i+2} \\ 0 & \text{otherwise.} \end{cases}$$

B-spline of degree 1 is characterized by the following:

1. The support of B_i^1 is a half-open interval $[x_i, x_{i+2}]$.
2. $B_i^1(x) \geq 0$ for all x and for all i.
3. $B_i^1(x)$ is continuous at all points.
4. $\sum_{i=-\infty}^{\infty} B_i^1(x) = 1$, for all x.

Quadratic B-spline

We determine B_i^2 in an alternative form:

$$B_i^2(x) = \left(\frac{x - x_i}{x_{i+2} - x_i}\right) B_i^1(x) + \left(\frac{x_{i+3} - x}{x_{i+3} - x_{i+1}}\right) B_{i+1}^1(x),$$

At first, we determine $B_{i+1}^1(x)$ in an alternative form:

$$B_{i+1}^1(x) = \left(\frac{x - x_{i+1}}{x_{i+2} - x_{i+1}}\right) B_{i+1}^0(x) + \left(\frac{x_{i+3} - x}{x_{i+3} - x_{i+2}}\right) B_{i+2}^0(x),$$

$$= \begin{cases} \frac{x - x_{i+1}}{x_{i+2} - x_{i+1}} & x \in [x_{i+1}, x_{i+2}) \\ \frac{x_{i+3} - x}{x_{i+3} - x_{i+2}} & x \in [x_{i+2}, x_{i+3}) \\ 0 & \text{otherwise}, \end{cases}$$

Thus,

$$B_i^2(x) = \begin{cases} \frac{(x - x_i)^2}{(x_{i+2} - x_i)(x_{i+1} - x_i)} & x \in [x_i, x_{i+1}) \\ \frac{(x - x_i)(x_{i+2} - x)}{(x_{i+2} - x_i)(x_{i+2} - x_{i+1})} & \\ \quad + \frac{(x_{i+3} - x)(x - x_{i+1})}{(x_{i+3} - x_{i+1})(x_{i+2} - x_{i+1})} & x \in [x_{i+1}, x_{i+2}) \\ \frac{(x_{i+3} - x)^2}{(x_{i+3} - x_{i+1})(x_{i+3} - x_{i+2})} & x \in [x_{i+2}, x_{i+3}) \\ 0 & \text{otherwise}. \end{cases}$$

Alternative approach to drive the B-spline relations

In this section, we give another approach for driving the B-splines which are more applicable with respect to the recurrence relation for the formulations of B-splines of higher degrees [2]. At first, we recall that the kth forward difference $f(x_0)$ of a given function $f(x)$ at x_0, which is defined recursively by the following:

$$\Delta f(x_0) = f(x_1) - f(x_0),$$
$$\Delta^{k+1} f(x_0) = \Delta^k f(x_1) - \Delta^k f(x_0).$$

In particular,

$$\begin{aligned}
\Delta^2 f(x_0) &= \Delta f(x_1) - \Delta f(x_0) = f(x_2) - 2f(x_1) + f(x_0),\\
\Delta^3 f(x_0) &= \Delta^2 f(x_1) - \Delta^2 f(x_0) = f(x_3) - 3f(x_2) + 3f(x_1) - f(x_0),\\
\Delta^4 f(x_0) &= \Delta^3 f(x_1) - \Delta^3 f(x_0) = f(x_4) - 4f(x_3) + 6f(x_2) - 4f(x_1) + f(x_0),\\
\Delta^5 f(x_0) &= \Delta^4 f(x_1) - \Delta^4 f(x_0) = f(x_5) - 5f(x_4) + 10f(x_3) - 10f(x_2) + 5f(x_1) - f(x_0),\\
\Delta^6 f(x_0) &= \Delta^5 f(x_1) - \Delta^5 f(x_0) = f(x_6) - 6f(x_5) + 15f(x_4) - 20f(x_3) + 15f(x_2)\\
&\quad - 6f(x_1) + f(x_0),\\
\Delta^7 f(x_0) &= \Delta^6 f(x_1) - \Delta^6 f(x_0) = f(x_7) - 7f(x_6) + 21f(x_5) - 35f(x_4) + 35f(x_3)\\
&\quad - 21f(x_2) + 7f(x_1) - f(x_0).
\end{aligned} \tag{2}$$

Remark 1. The coefficient of $f(x_k)$ in $\Delta^n f(x_0)$ is simply the binomial coefficient $(-1)^{n-k}\binom{n}{k}$. It is well known that with evenly spaced knots $x_i = x_0 + ih$ and $x_{i+j} = x_{i+j}h$, Δ^n annihilates all polynomials of degree $(n-1)$.

Theorem 1. *The nth forward difference $\Delta^n f(x)$ with evenly spaced knots of any polynomial of degree $n-1$ is identically equal to zero.*

Theorem 2. *The function $(x-t)_+^m$ is defined as follows:*

$$(x-t)_+^m = \begin{cases} (x-t)^m & x \geq t \\ 0 & x < t \end{cases}$$

It is clear that $(x-t)_+^m$ is $(m-1)$ times continuously differentiable both with respect to t and x.

The B-spline of order m is defined as follows:

$$B_i^m(t) = \frac{1}{h^m}\sum_{j=0}^{m+1}\binom{m+1}{j}(-1)^{m+1-j}(x_{i-2+j} - t)_+^m = \frac{1}{h^m}\Delta^{m+1}(x_{i-2} - t)_+^m.$$

Hence, we apply this approach to obtain the B-spline of order one. Let $m = 1$; thus,

$$B_i^1(t) = \frac{1}{h}\Delta^2(x_{i-2} - t)_+ = \frac{1}{h}[\,(x_{i-2} - t)_+ - 2(x_{i-1} - t)_+ + (x_i - t)_+\,],$$

$$B_i^1(t) = \frac{1}{h}\begin{cases} (x_i - t) - 2(x_{i-1} - t) & x_{i-2} < t \leq x_{i-1} \\ (x_i - t) & x_{i-1} < t \leq x_i \\ 0 & \text{otherwise,} \end{cases}$$

$$B_i^1(t) = \frac{1}{h}\begin{cases} t - x_{i-2} & x_{i-2} < t \leq x_{i-1} \\ x_i - t & x_{i-1} < t \leq x_i \\ 0 & \text{otherwise.} \end{cases}$$

In order to obtain the quadratic B-spline, let $m = 2$. Thus,

$$\begin{aligned} B_i^2(t) &= \frac{1}{h^2}\Delta^3(x_{i-2}-t)_+^2 \\ &= \frac{1}{h_2}[\,(x_{i+1}-t)_+^2 - 3(x_i-t)_+^2 + 3(x_{i-1}-t)_+^2 - (x_{i-2}-t)_+^2\,], \end{aligned}$$

$$B_i^2(t) = \frac{1}{h^2}\begin{cases} (x_{i+1}-t)^2 - 3(x_i-t)^2 + 3(x_{i-1}-t)^2 & x_{i-2} < t \le x_{i-1} \\ (x_{i+1}-t)^2 - 3(x_i-t)^2 & x_{i-1} < t \le x_i \\ (x_{i+1}-t)^2 & x_i \;\; < t \le x_{i+1} \\ 0 & \text{otherwise.} \end{cases}$$

In order to obtain the cubic B-spline, let $m = 3$. Thus,

$$\begin{aligned} B_i^3(t) &= \frac{1}{h^3}\Delta^4(x_{i-2}-t)_+^3 \\ &= \frac{1}{h^3}[\,(x_{i+2}-t)_+^3 - 4(x_{i+1}-t)_+^3 + 6(x_i-t)_+^3 - 4(x_{i-1}-t)_+^3 + (x_{i-2}-t)_+^3\,], \end{aligned}$$

$$B_i^3(t) = \frac{1}{h^3}\begin{cases} (x_{i+2}-t)^3 - 4(x_{i+1}-t)^3 + 6(x_i-t)^3 & \\ \qquad - 4(x_{i-1}-t)^3 & x_{i-2} < t \le x_{i-1} \\ (x_{i+2}-t)^3 - 4(x_{i+1}-t)^3 + 6(x_i-t)^3 & x_{i-1} < t \le x_i \\ (x_{i+2}-t)^3 - 4(x_{i+1}-t)^3 & x_i \;\; < t \le x_{i+1} \\ (x_{i+2}-t)^3 & x_{i+1} < t \le x_{i+2} \\ 0 & \text{otherwise.} \end{cases}$$

In order to obtain the quartic B-spline which is used by [8], let $m = 4$:

$$\begin{aligned} B_i^4(t) &= \frac{1}{h^4}\Delta^5(x_{i-2}-t)_+^4 \\ &= \frac{1}{h^4}[\,(x_{i+3}-t)_+^4 - 5(x_{i+2}-t)_+^4 + 10(x_{i+1}-t)_+^4 - 10(x_i-t)_+^4 \\ &\quad + 5(x_{i-1}-t)_+^4 - (x_{i-2}-t)_+^4\,], \end{aligned}$$

$$B_i^4(t) = \frac{1}{h^4}\begin{cases} (x_{i+3}-t)^4 - 5(x_{i+2}-t)^4 + 10(x_{i+1}-t)^4 & \\ \qquad - 10(x_i-t)^4 + 5(x_{i-1}-t)^4 & x_{i-2} < t \le x_{i-1} \\ (x_{i+3}-t)^4 - 5(x_{i+2}-t)^4 + 10(x_{i+1}-t)^4 & \\ \qquad - 10(x_i-t)^4 & x_{i-1} < t \le x_i \\ (x_{i+3}-t)^4 - 5(x_{i+2}-t)^4 + 10(x_{i+1}-t)^4 & x_i \;\; < t \le x_{i+1} \\ (x_{i+3}-t)^4 - 5(x_{i+2}-t)^4 & x_{i+1} < t \le x_{i+2} \\ (x_{i+3}-t)^4 & x_{i+2} < t \le x_{i+3} \\ 0 & \text{otherwise.} \end{cases}$$

In order to obtain the quintic B-spline which is used by [9], let $m = 5$:

$$B_i^5(t) = \frac{1}{h^5}\Delta^6(x_{i-2}-t)_+^5$$
$$= \frac{1}{h^5}[\,(x_{i+4}-t)_+^5 - 6(x_{i+3}-t)_+^5 + 15(x_{i+2}-t)_+^5 - 20(x_{i+1}-t)_+^5$$
$$+ 15(x_i-t)_+^5 - 6(x_{i-1}-t)_+^5 + (x_{i-2}-t)_+^5\,],$$

$$B_i^5(t) = \frac{1}{h^5}\begin{cases} (x_{i+4}-t)^5 - 6(x_{i+3}-t)^5 + 15(x_{i+2}-t)^5 \\ \quad - 20(x_{i+1}-t)^5 + 15(x_i-t)^5 \\ \quad - 6(x_{i-1}-t)^5 & x_{i-2} < t \le x_{i-1} \\ (x_{i+4}-t)^5 - 6(x_{i+3}-t)^5 + 15(x_{i+2}-t)^5 \\ \quad - 20(x_{i+1}-t)^5 + 15(x_i-t)^5 & x_{i-1} < t \le x_i \\ (x_{i+4}-t)^5 - 6(x_{i+3}-t)^5 + 15(x_{i+2}-t)^5 \\ \quad - 20(x_{i+1}-t)^5 & x_i < t \le x_{i+1} \\ (x_{i+4}-t)^5 - 6(x_{i+3}-t)^5 + 15(x_{i+2}-t)^5 & x_{i+1} < t \le x_{i+2} \\ (x_{i+4}-t)^5 - 6(x_{i+3}-t)^5 & x_{i+2} < t \le x_{i+3} \\ (x_{i+4}-t)^5 & x_{i+3} < t \le x_{i+4} \\ 0 & \text{otherwise.} \end{cases}$$

At last in order to obtain Sextic B-spline which is used by [10], let $m = 6$

$$B_i^6(t) = \frac{1}{h^6}\Delta^7(x_{i-2}-t)_+^6$$
$$= \frac{1}{h^6}[\,(x_{i+5}-t)_+^6 - 7(x_{i+4}-t)_+^6 + 21(x_{i+3}-t)_+^6$$
$$- 35(x_{i+2}-t)_+^6 + 35(x_{i+1}-t)_+^6 - 21(x_i-t)_+^6 + 7(x_{i-1}-t)_+^6 - (x_{i-2}-t)_+^6\,],$$

$$B_i^6(t) = \frac{1}{h^6}\begin{cases} (x_{i+5}-t)^6 - 7(x_{i+4}-t)^6 + 21(x_{i+3}-t)^6 \\ \quad - 35(x_{i+2}-t)^6 + 35(x_{i+1}-t)^6 \\ \quad - 21(x_i-t)^6 + 7(x_{i-1}-t)^6 & x_{i-2} < t \le x_{i-1} \\ (x_{i+5}-t)^6 - 7(x_{i+4}-t)^6 + 21(x_{i+3}-t)^6 \\ \quad - 35(x_{i+2}-t)^6 + 35(x_{i+1}-t)^6 \\ \quad - 21(x_i-t)^6 & x_{i-1} < t \le x_i \\ (x_{i+5}-t)^6 - 7(x_{i+4}-t)^6 + 21(x_{i+3}-t)^6 \\ \quad - 35(x_{i+2}-t)^6 + 35(x_{i+1}-t)^6 & x_i < t \le x_{i+1} \\ (x_{i+5}-t)^6 - 7(x_{i+4}-t)^6 + 21(x_{i+3}-t)^6 \\ \quad - 35(x_{i+2}-t)^6 & x_{i+1} < t \le x_{i+2} \\ (x_{i+5}-t)^6 - 7(x_{i+4}-t)^6 + 21(x_{i+3}-t)^6 & x_{i+2} < t \le x_{i+3} \\ (x_{i+5}-t)^6 - 7(x_{i+4}-t)^6 & x_{i+3} < t \le x_{i+4} \\ (x_{i+5}-t)^6 & x_{i+4} < t \le x_{i+5} \\ 0 & \text{otherwise.} \end{cases}$$

Table 1 The absolute errors at different points

x	Cubic B-spline	Cubic spline [19]	Quintic spline [19]
$\frac{1}{12}$	-1.84×10^{-5}	-2.51×10^{-4}	-1.05×10^{-5}
$\frac{3}{12}$	-2.97×10^{-4}	-6.87×10^{-4}	-2.86×10^{-5}
$\frac{5}{12}$	-3.98×10^{-4}	-9.39×10^{-4}	-3.91×10^{-5}
$\frac{7}{12}$	-3.98×10^{-4}	-9.39×10^{-4}	-3.91×10^{-5}
$\frac{9}{12}$	-2.97×10^{-4}	-6.87×10^{-4}	-2.86×10^{-5}
$\frac{11}{12}$	-1.84×10^{-5}	-2.51×10^{-4}	-1.04×10^{-5}

Remark 2. The above B-splines have been used to approximate the solution of recent differential equations [11-13] and integral equations [14], as well as for interpolation [15-18].

Results and discussion

Numerical illustration

We applied the cubic B-spline to interpolate the following test problems. The maximum errors in the interpolation are tabulated in Tables 1 and 2. Results obtained by cubic B-spline are compared with the absolute error in the interpolation by cubic and quintic splines given in [19].

1. **Example 1**

$$f(x) = sin(\pi x), \qquad x \in [0, 1],$$

We interpolate this problem with the step size, $h = \frac{1}{6}$.

2. **Example 2**

$$f(x) = \frac{e^{kx} - 1}{e^{k} - 1}, \qquad x \in [0, 0.8], k = 12,$$

We interpolate this problem with the step size, $h = \frac{2}{15}$.

Conclusions

These tables show that the results obtained by cubic B-spline are considerable and accurate with respect to the cubic spline and are more or less similar to the quintic spline.

Table 2 The absolute errors at different points

x	Cubic B-spline	Cubic spline [19]	Quintic spline [19]
$\frac{1}{15}$	5.55×10^{-7}	-1.23×10^{-6}	1.54×10^{-6}
$\frac{3}{15}$	-8.71×10^{-6}	-6.13×10^{-6}	2.99×10^{-6}
$\frac{5}{15}$	9.14×10^{-6}	-3.03×10^{-5}	1.48×10^{-5}
$\frac{7}{15}$	-8.81×10^{-5}	-1.50×10^{-4}	7.34×10^{-5}
$\frac{9}{15}$	9.81×10^{-5}	-7.45×10^{-4}	3.64×10^{-4}
$\frac{11}{15}$	9.55×10^{-4}	-3.69×10^{-3}	1.56×10^{-2}

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
JR and SS contributed equally to this work. Both authors read and approved the final manuscript.

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