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Asymptotically λ-invariant statistical equivalent sequences of fuzzy numbers

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Abstract

This paper presents the following definition which is a natural combination of the definitions for asymptotically equivalent λ-statistical convergence and σ-convergence of fuzzy numbers. Two sequences X and Y of fuzzy numbers are said to be asymptotically λ-invariant statistical equivalent of multiple L provided that for every $\varepsilon > 0$,

$$\lim_n \frac{1}{\lambda_n}\left|\left\{k \in I_n : \bar{d}\left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L\right) \geq \varepsilon\right\}\right| = 0, \text{ uniformly in } m$$

$\left(\text{denoted by } X \overset{S^L_{\sigma,\lambda}(F)}{\sim} Y\right)$ and simply asymptotically λ-invariant statistical equivalent if $L = \bar{1}$.

Keywords: λ-statistical convergence, σ-convergence, Fuzzy numbers

Introduction

Let σ be a one-to-one mapping of the set of positive integers into itself such that $\sigma^k(m) = \sigma\left(\sigma^{k-1}(m)\right)$, $k = 1, 2, 3, \ldots$. The generalized de la Vallee-Pousin mean is defined by

$$t_n(x) = \frac{1}{\lambda_n}\sum_{k \in I_n} x_k,$$

where (λ_n) is a non-decreasing sequence of positive numbers such that $\lambda_{n+1} \leq \lambda_n + 1$, $\lambda_1 = 1$, and $\lambda_n \to \infty$ as $n \to \infty$ and $I_n = [n - \lambda_n + 1, n]$. A sequence $x = (x_k)$ is said to be (V, λ)-summable to a number L if $t_n(x) \to L$ as $n \to \infty$ [1]. (V, λ)-summability reduces to $(C, 1)$-summability when $\lambda_n = n$ for all $n \in N$.

Let D denote the set of all closed and bounded intervals on $\mathbb{R}$, the real line. For $X, Y \in D$, we define

$$d(X, Y) = \max(|a_1 - b_1|, |a_2 - b_2|),$$

where $X = [a_1, a_2]$ and $Y = [b_1, b_2]$. It is known that (D, d) is a complete metric space. A fuzzy real number X is a fuzzy set on $\mathbb{R}$, i.e., a mapping $X : \mathbb{R} \to I (= [0, 1])$ associating each real number t with its grade of membership $X(t)$.

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The set of all upper semicontinuous, normal, and convex fuzzy real numbers is denoted by $\mathbb{R}(I)$. Throughout the paper, by a fuzzy real number X, we mean that $X \in \mathbb{R}(I)$.

The α-cut or α-level set $[X]^\alpha$ of the fuzzy real number X, for $0 < \alpha \leq 1$, is defined by $[X]^\alpha = \{t \in \mathbb{R} : X(t) \geq \alpha\}$; for $\alpha = 0$, it is the closure of the strong 0-cut, i.e., closure of the set $\{t \in R : X(t) > 0\}$. The linear structure of $\mathbb{R}(I)$ induces the addition $X + Y$ and the scalar multiplication μX, $\mu \in \mathbb{R}$, in terms of α-level sets, defined by

$$[X + Y]^\alpha = [X]^\alpha + [Y]^\alpha, \quad [\mu X]^\alpha = \mu [X]^\alpha$$

for each $\alpha \in (0, 1]$.

Let $\bar{d} : \mathbb{R}(I) \times \mathbb{R}(I) \to \mathbb{R}$ be defined by

$$\bar{d}(X, Y) = \sup_{0 \leq \alpha \leq 1} d\left([X]^\alpha, [Y]^\alpha\right).$$

Then, $\bar{d}$ defines a metric on $\mathbb{R}(I)$. It is well known that $\mathbb{R}(I)$ is complete with respect to $\bar{d}$.

A sequence (X_k) of fuzzy real numbers is said to be convergent to the fuzzy real number X_0 if, for every $\varepsilon > 0$, there exists $n_0 \in N$ such that $\bar{d}(X_k, X_0) < \varepsilon$, for all $k \geq n_0$. Let $c(F)$ denote the set of all convergent sequences of fuzzy numbers.

A sequence (X_k) of fuzzy real numbers is said to be bounded if the set $\{X_k : k \in N\}$ of fuzzy numbers is bounded. We denote by $\ell_\infty(F)$ the set of all bounded sequences of fuzzy numbers. In [2], it was shown that $c(F)$ and $\ell_\infty(F)$ are complete metric spaces.

A subset E of N is said to have density (asymptotic or natural) $\delta(E)$ if

$$\delta(E) = \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \varkappa_E(k) \text{ exists,}$$

where $\varkappa_E$ is the characteristic function of E. The definition of statistical convergence was introduced by Fast [2] and studied by several authors [3-9]. The sequence x is statistically convergent to s if for each $\epsilon > 0$,

$$\lim_{n\to\infty} \frac{1}{n} |\{k \le n : |x_k - s| \ge \epsilon\}| = 0,$$

where $|A|$ denotes the number of elements in A. Schoenberg [10] studied statistical convergence as a summability method and listed some of the elementary properties of statistical convergence.

Nuray and Savaş [11] defined the notion of statistical convergence for sequences of fuzzy real numbers and studied some properties. A fuzzy real number (X_k) is said to be statistically convergent to the fuzzy real number X_0 if for every $\varepsilon > 0$,

$$\delta\left(\left\{k \in N : \bar{d}(X_k, X_0) \ge \varepsilon\right\}\right) = 0.$$

Fuzzy sequence are spaces studied by several authors such as [12-19].

In 1993, Marouf [20] presented definitions for asymptotically equivalent sequences of real numbers and asymptotic regular matrices. In 2003, Patterson [21] extended these concepts by presenting an asymptotically statistical equivalent analog of these definitions and natural regularity conditions for nonnegative summability matrices. In 2006, Savaş and Başarir [22] introduced and studied the concept of (σ, λ)-asymptotically statistical equivalent sequences. In 2008, Esi and Esi [23] introduced and studied the concept of asymptotically equivalent difference sequences of fuzzy numbers. In 2009, Esi [24] introduced and studied asymptotically equivalent sequences for double sequences. For sequences of fuzzy numbers, Savaş [25,26] introduced and studied the concepts of strongly λ-summable λ-statistical convergence and asymptotically λ-statistical equivalent sequences, respectively . Recently, Braha [27] defined asymptotically generalized difference lacunary sequences. The goal of this paper is to extend the idea on asymptotically equivalent and λ_{σ^F}-statistical convergence of fuzzy numbers.

Methods

Definitions and notations

Definition 1. Two sequences $X = (X_k)$ and $Y = (Y_k)$ of fuzzy numbers are said to be σ^F-asymptotically equivalent if

$$\lim_k \bar{d}\left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, \bar{1}\right) = 0 \text{, uniformly in } m \left(\text{denoted by } X \overset{\sigma^F}{\sim} Y\right).$$

Definition 2. A sequence of fuzzy numbers, $X = (X_k)$, is said to be $S^L_{\sigma,\lambda}(F)$-statistically convergent or $S^\lambda_{\sigma^F}$-convergent to the fuzzy number L if for every $\varepsilon > 0$,

$$\lim_n \frac{1}{\lambda_n} \left|\left\{k \in I_n : \bar{d}\left(X_{\sigma^k(m)}, L\right) \ge \varepsilon\right\}\right| = 0, \text{ uniformly in } m.$$

In this case, we write $S^\lambda_{\sigma^F} - \lim X = L$ or $X_k \to L\left(S^\lambda_{\sigma^F}\right)$.

Following this result, we introduce two new notions asymptotically $S^L_{\sigma,\lambda}(F)$-statistical equivalent of multiple L and strong $V^L_{\sigma,\lambda}(F)$-asymptotically equivalent of multiple L.

The next definition is a natural combination of Definitions 1 and 2.

Definition 3. Two sequences $X = (X_k)$ and $Y = (Y_k)$ of fuzzy numbers are said to be asymptotically λ-invariant statistical equivalent of multiple L provided that for every $\varepsilon > 0$,

$$\lim_n \frac{1}{\lambda_n} \left|\left\{k \in I_n : \bar{d}\left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L\right) \ge \varepsilon\right\}\right| = 0, \text{ uniformly in } m \left(\text{denoted by } X \overset{S^L_{\sigma,\lambda}(F)}{\sim} Y\right)$$

and simply asymptotically $S_{\sigma,\lambda}(F)$-statistical equivalent if $L = \bar{1}$.

Example 1. Let $\lambda_n = n$ and $\sigma(m) = m+1$ for all $n, m \in \mathbb{N}$. Consider the sequences of fuzzy numbers $X = (X_k)$ and $Y = (Y_k)$ defined by $X_n = \overline{n^{-2}}$ and $Y_n = \overline{n^{-1}}$ for all $n \in \mathbb{N}$. Then,

$$\lim_n \frac{1}{\lambda_n} \left|\left\{k \in I_n : \bar{d}\left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L\right) \ge \varepsilon\right\}\right| = \lim_n \frac{1}{n} \left|\left\{k \in [1, n] : \bar{d}\left(\overline{n^{-1}}, \bar{0}\right) \ge \varepsilon\right\}\right| = 0.$$

If we take $\lambda_n = n$ for all $n \in \mathbb{N}$, the above definition reduces to following definition:

Definition 4. Two sequences $X = (X_k)$ and $Y = (Y_k)$ of fuzzy numbers are said to be asymptotically invariant statistical equivalent of multiple L provided that for every $\varepsilon > 0$,

$$\lim_n \frac{1}{n}\left|\left\{k \le n : \bar{d}\left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L\right) \ge \varepsilon\right\}\right| = 0, \text{ uniformly in } m \left(\text{denoted by } X \overset{S_\sigma^L(F)}{\sim} Y\right)$$

and simply asymptotically $S_\sigma(F)$-statistical equivalent if $L = \bar{1}$.

Definition 5. Two sequences $X = (X_k)$ and $Y = (Y_k)$ of fuzzy numbers are said to be strong $V_{\sigma,\lambda}^L(F)$-asymptotically equivalent of multiple L provided that

$$\lim_n \frac{1}{\lambda_n}\sum_{k\in I_n}\bar{d}\left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L\right) = 0, \text{ uniformly in } m \left(\text{denoted by } X \overset{V_{\sigma,\lambda}^L(F)}{\sim} Y\right)$$

and simply strong $V_{\sigma,\lambda}(F)$-asymptotically statistical equivalent if $L = \bar{1}$.

Example 2. Let $\lambda_n = n$ and $\sigma(m) = m+1$ for all $n, m \in \mathbb{N}$. Consider the sequences of fuzzy numbers $X = (X_k)$ and $Y = (Y_k)$ defined by $X_n = \overline{n^{-2}}$ and $Y_n = \overline{n^{-1}}$ for all $n \in \mathbb{N}$. Then,

$$\lim_n \frac{1}{\lambda_n}\sum_{k\in I_n}\bar{d}\left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, \bar{0}\right) = \lim_n \frac{1}{n}\sum_{k=1}^{n}\bar{d}\left(\frac{\overline{n^{-2k}}}{\overline{n^{-k}}}, \bar{0}\right) = \lim_n \frac{1}{n}\sum_{k=1}^{n}\frac{1}{n^k} < \infty.$$

If we take $\lambda_n = n$ for all $n \in \mathbb{N}$, the above definition reduces to the following definition:

Definition 6. Two sequences X and Y of fuzzy numbers are said to be strong Cesaro $C_\sigma^L(F)$-asymptotically equivalent of multiple L provided that

$$\lim_n \frac{1}{n}\sum_{k=1}^{n}\bar{d}\left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L\right) = 0, \text{ uniformly in } m \left(\text{denoted by } X \overset{C_{\sigma,\lambda}^L(F)}{\sim} Y\right)$$

and simply strong Cesaro $C_\sigma(F)$-asymptotically equivalent if $L = \bar{1}$.

If we take $\sigma(m) = m + 1$, the above definitions reduce the following definitions:

Definition 7. Two sequences $X = (X_k)$ and $Y = (Y_k)$ of fuzzy numbers are said to be asymptotically almost equivalent if

$$\lim_k \bar{d}\left(\frac{X_{k+m}}{Y_{k+m}}, \bar{1}\right) = 0\text{ , uniformly in } m \left(\text{denoted by } X \overset{\widehat{F}}{\sim} Y\right).$$

Definition 8. A sequence of fuzzy numbers $X = (X_k)$ is said to be $\lambda_{\widehat{F}}$-statistically almost convergent or $S_{\widehat{F}}^\lambda$-convergent to the fuzzy number L if for every $\varepsilon > 0$,

$$\lim_n \frac{1}{\lambda_n}\left|\left\{k \in I_n : \bar{d}\left(X_{k+m}, L\right) \ge \varepsilon\right\}\right| = 0 \text{ uniformly in } m.$$

In this case, we write $S_{\widehat{F}}^\lambda - \lim X = L$ or $X_k \to L\left(S_{\widehat{F}}^\lambda\right)$.

Definition 9. Two sequences $X = (X_k)$ and $Y = (Y_k)$ of fuzzy numbers are said to be asymptotically almost λ-statistical equivalent of multiple L provided that for every $\varepsilon > 0$,

$$\lim_n \frac{1}{\lambda_n}\left|\left\{k \in I_n : \bar{d}\left(\frac{X_{k+m}}{Y_{k+m}}, L\right) \ge \varepsilon\right\}\right| = 0\text{, uniformly in } m \left(\text{denoted by } X \overset{S_\lambda^L(\widehat{F})}{\sim} Y\right)$$

and simply asymptotically *almost* λ-statistical equivalent if $L = \bar{1}$.

If we take $\lambda_n = n$ for all $n \in \mathbb{N}$, the above definition reduces to the following definition:

Definition 10. Two sequences $X = (X_k)$ and $Y = (Y_k)$ of fuzzy numbers are said to be asymptotically *almost* statistical equivalent of multiple L provided that for every $\varepsilon > 0$,

$$\lim_n \frac{1}{n}\left|\left\{k \le n : \bar{d}\left(\frac{X_{k+m}}{Y_{k+m}}, L\right) \ge \varepsilon\right\}\right| = 0, \text{ uniformly in } m \left(\text{denoted by } X \overset{S^L(\widehat{F})}{\sim} Y\right)$$

and simply asymptotically *almost* statistical equivalent if $L = \bar{1}$.

Definition 11. Two sequences $X = (X_k)$ and $Y = (Y_k)$ of fuzzy numbers are said to be strong asymptotically almost λ-equivalent of multiple L provided that

$$\lim_n \frac{1}{\lambda_n}\sum_{k\in I_n}\bar{d}\left(\frac{X_{k+m}}{Y_{k+m}}, L\right) = 0, \text{ uniformly in } m \left(\text{denoted by } X \overset{V_\lambda^L(\widehat{F})}{\sim} Y\right)$$

and simply strong asymptotically almost λ-equivalent if $L = \overline{1}$.

If we take $\lambda_n = n$ for all $n \in \mathbb{N}$, the above definition reduces to following definition.

Definition 12. Two sequences $X = (X_k)$ and $Y = (Y_k)$ of fuzzy numbers are said to be strong asymptotically almost equivalent of multiple L provided that

$$\lim_n \frac{1}{n}\sum_{k=1}^{n} \bar{d}\left(\frac{X_{k+m}}{Y_{k+m}}, L\right) = 0, \text{ uniformly in } m \left(\text{denoted by } X \overset{C^L(\widehat{F})}{\sim} Y\right)$$

and simply strong asymptotically almost equivalent if $L = \overline{1}$.

Results and discussion

Theorem 1. *Let $X = (X_k)$ and $Y = (Y_k)$ be two fuzzy real valued sequences. Then, the following conditions are satisfied:*

(i) If $X \overset{V^L_{\sigma,\lambda}(F)}{\sim} Y$, then $X \overset{S^L_{\sigma,\lambda}(F)}{\sim} Y$.

(ii) If $X \in \ell_\infty(F)$ and $X \overset{S^L_{\sigma,\lambda}(F)}{\sim} Y$, then $X \overset{V^L_{\sigma,\lambda}(F)}{\sim} Y$; hence, $X \overset{C^L_{\sigma,\lambda}(F)}{\sim} Y$.

(iii) $X \overset{S^L_{\sigma,\lambda}(F)}{\sim} Y \cap \ell_\infty(F) = X \overset{V^L_{\sigma,\lambda}(F)}{\sim} Y \cap \ell_\infty(F)$.

Proof. **(i)** If $\varepsilon > 0$ and $X \overset{V^L_{\sigma,\lambda}(F)}{\sim} Y$, then

$$\sum_{k\in I_n} \bar{d}\left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L\right) \geq \sum_{\substack{k\in I_n \\ \bar{d}\left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L\right)\geq\varepsilon}} \bar{d}\left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L\right)$$

$$\geq \varepsilon \left|\left\{k \in I_n : \bar{d}\left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L\right) \geq \varepsilon\right\}\right|.$$

Therefore, $X \overset{S^L_{\sigma,\lambda}(F)}{\sim} Y$.

(ii) Suppose that $X = (X_k)$ and $Y = (Y_k)$ are in $\ell_\infty(F)$ and $X \overset{S^L_{\sigma,\lambda}(F)}{\sim} Y$. Then, we can assume that

$$\bar{d}\left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L\right) \leq T, \text{ for all } k \text{ and } m.$$

Given $\varepsilon > 0$,

$$\frac{1}{\lambda_n}\sum_{k\in I_n} \bar{d}\left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L\right) = \frac{1}{\lambda_n} \sum_{\substack{k\in I_n \\ \bar{d}\left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L\right)\geq\varepsilon}} \bar{d}\left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L\right)$$

$$+\frac{1}{\lambda_n} \sum_{\substack{k\in I_n \\ \bar{d}\left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L\right)<\varepsilon}} \bar{d}\left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L\right)$$

$$\leq \frac{T}{\lambda_n}\left|\left\{k \in I_n : \bar{d}\left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L\right) \geq \varepsilon\right\}\right| + \varepsilon.$$

Therefore, $X \overset{V^L_{\sigma,\lambda}(F)}{\sim} Y$. Further, we have

$$\frac{1}{n}\sum_{k=1}^{n} \bar{d}\left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L\right) = \frac{1}{n}\sum_{k=1}^{n-\lambda_n} \bar{d}\left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L\right) + \frac{1}{n}\sum_{k\in I_n} \bar{d}\left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L\right)$$

$$\leq \frac{1}{\lambda_n}\sum_{k=1}^{n-\lambda_n} \bar{d}\left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L\right) + \frac{1}{\lambda_n}\sum_{k\in I_n} \bar{d}\left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L\right)$$

$$\leq \frac{2}{\lambda_n}\sum_{k\in I_n} \bar{d}\left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L\right).$$

Hence, $X \overset{C^L_{\sigma,\lambda}(F)}{\sim} Y$ since $X \overset{V^L_{\sigma,\lambda}(F)}{\sim} Y$.

(iii) Follows from (i) and (ii).

□

In the next theorem, we prove the following relation:

Theorem 2. $X \overset{S^L_{\sigma}(F)}{\sim} Y$ implies $X \overset{S^L_{\sigma,\lambda}(F)}{\sim} Y$ if

$$\liminf \left(\frac{\lambda_n}{n}\right) > 0. \qquad (1)$$

Proof. For a given $\varepsilon > 0$, we have

$$\left\{k \leq n : \bar{d}\left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L\right) \geq \varepsilon\right\} \supset \left\{k \in I_n : \bar{d}\left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L\right) \geq \varepsilon\right\}.$$

Therefore,

$$\frac{1}{n}\left|\left\{k \leq n : \bar{d}\left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L\right) \geq \varepsilon\right\}\right| \geq \frac{1}{n}\left|\left\{k \in I_n : \bar{d}\left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L\right) \geq \varepsilon\right\}\right|$$

$$\geq \frac{\lambda_n}{n}.\frac{1}{\lambda_n}\left|\left\{k \in I_n : \bar{d}\left(\frac{X_{\sigma^k(m)}}{Y_{\sigma^k(m)}}, L\right) \geq \varepsilon\right\}\right|.$$

Taking the limit as $n \to \infty$ and using Equation 1, we get the desired result. This completes the proof. □

Conclusions

The concept of asymptotic equivalence was first suggested by Marouf [20] in 1993. After that, several authors introduced and studied some asymptotically equivalent sequences. The results obtained in this study are much more general than those obtained earlier.

Competing interests

The author declares that he has no competing interests.

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