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Nonlinear fuzzy approximation of a mixed type ACQ functional equation via fixed point alternative

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Abstract

Using the fixed point method, we prove the generalized Hyers-Ulam stability of the following additive-cubic-quartic functional equation:

$$\begin{aligned} 11f(x+2y)+11f(x-2y) &= 44f(x+y)+44f(x-y)+12f(3y) \\ &\quad -48f(2y)+60f(y)-66f(x) \end{aligned}$$

in fuzzy Banach spaces.

Keywords: Hyers-Ulam stability, Fuzzy Banach space, Fixed point method

2010 Mathematics subject classification: 39B52; 46S40; 34K36; 47S40; 26E50; 47H10; 39B82.

Introduction

Katsaras [1] defined a fuzzy norm on a vector space to construct a fuzzy vector topological structure on the space. Some mathematicians have defined fuzzy norms as a vector space from various points of view (see [2-4]). In particular, Bag and Samanta [5], following Cheng and Mordeson [6], gave an idea of fuzzy norm in such a manner that the corresponding fuzzy metric is of Karmosil and Michalek type [7]. They established a decomposition theorem of a fuzzy norm into a family of crisp norms and investigated some properties of fuzzy normed spaces [8].

Definition 1. *Let X be a real vector space. A function $N : X \times \mathbb{R} \to [0,1]$ is called a fuzzy norm on X if for all $x, y \in X$ and all $s, t \in \mathbb{R}$ (Bag and Samanta [5]):*
$(N1)$ $N(x,t) = 0$ *for* $t \leq 0$;
$(N2)$ $x = 0$ *if and only if* $N(x,t) = 1$ *for all* $t > 0$;
$(N3)$ $N(cx,t) = N\left(x, \frac{t}{|c|}\right)$ *if* $c \neq 0$;
$(N4)$ $N(x+y, c+t) \geq min\{N(x,s), N(y,t)\}$;
$(N5)$ $N(x,.)$ *is a non-decreasing function of* $\mathbb{R}$ *and* $\lim_{t\to\infty} N(x,t) = 1$;
$(N6)$ *for* $x \neq 0$, $N(x,.)$ *is continuous on* $\mathbb{R}$.

Example 1. *Let $(X, \|.\|)$ be a normed linear space and $\alpha, \beta > 0$. Then*

$$N(x,t) = \begin{cases} \frac{\alpha t}{\alpha t + \beta\|x\|} & t > 0, x \in X \\ 0 & t \leq 0, x \in X \end{cases}$$

is a fuzzy norm on X.

Definition 2. *Let (X, N) be a fuzzy normed vector space. A sequence $\{x_n\}$ in X is said to be convergent or converges if there exists an $x \in X$ such that $\lim_{t\to\infty} N(x_n - x, t) = 1$ for all $t > 0$. In this case, x is called the limit of the sequence $\{x_n\}$ in X, and we denote it by $N - \lim_{t\to\infty} x_n = x$ (Bag and Samanta [5]).*

Definition 3. *Let (X, N) be a fuzzy normed vector space. A sequence $\{x_n\}$ in X is called Cauchy if for each $\epsilon > 0$ and each $t > 0$ there exists an $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$ and all $p > 0$, we have $N(x_{n+p} - x_n, t) > 1 - \epsilon$* (Bag and Samanta [5]).

It is well known that every convergent sequence in a fuzzy normed vector space is Cauchy. If each Cauchy sequence is convergent, then the fuzzy norm is said to be

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complete, and the fuzzy normed vector space is called a fuzzy Banach space.

We say that a mapping $f : X \to Y$ between fuzzy normed vector spaces X and Y is continuous at a point $x \in X$ if for each sequence $\{x_n\}$ converging to $x_0 \in X$, then the sequence $\{f(x_n)\}$ converges to $f(x_0)$. If $f : X \to Y$ is continuous at each $x \in X$, then $f : X \to Y$ is said to be continuous on X (see [8]).

Definition 4. *Let X be a set. A function $d : X \times X \to [0, \infty]$ is called a generalized metric on X if d satisfies the following conditions:*

(1) $d(x, y) = 0$ if and only if $x = y$ for all $x, y \in X$;

(2) $d(x, y) = d(y, x)$ for all $x, y \in X$;

(3) $d(x, z) \leq d(x, y) + d(y, z)$ for all $x, y, z \in X$.

Theorem 1. *Let (X, d) be a complete generalized metric space and $J : X \to X$ be a strictly contractive mapping with Lipschitz constant $L < 1$ [9,10]. Then, for all $x \in X$, either*

$$d(J^n x, J^{n+1} x) = \infty,$$

for all nonnegative integers n or there exists a positive integer n_0 such that

(1) $d(J^n x, J^{n+1} x) < \infty$ for all $n_0 \geq n_0$;

(2) the sequence $\{J^n x\}$ converges to a fixed point y^* of J;

(3) y^* is the unique fixed point of J in the set $Y = \{y \in X : d(J^{n_0} x, y) < \infty\}$;

(4) $d(y, y^*) \leq \frac{1}{1-L} d(y, Jy)$ for all $y \in Y$.

The stability problem of functional equations originated from a question of Ulam [11] concerning the stability of group homomorphisms. Hyers [12] gave a first affirmative partial answer to the question of Ulam for Banach spaces. Hyers' theorem was generalized by Themistocles M Rassias [13] for linear mappings by considering an unbounded Cauchy difference.

The functional equation $f(x + y) + f(x - y) = 2f(x) + 2f(y)$ is called a *quadratic functional equation*. In particular, every solution of the quadratic functional equation is said to be a *quadratic mapping*. The Hyers-Ulam stability of the quadratic functional equation was proved by Skof [14] for mappings $f : X \to Y$, where X is a normed space and Y is a Banach space. Cholewa [15] noticed that the theorem of Skof is still true if the relevant domain X is replaced by an Abelian group. Czerwik [16] proved the Hyers-Ulam stability of the quadratic functional equation.

In the study of Eshaghi Gordji et. al [17], they proved that the following functional equation is an *additive-cubic-quartic* functional equation:

$$\begin{aligned} 11f(x + 2y) + 11f(x - 2y) = & \ 44f(x + y) + 44f(x - y) \\ & +12f(3y) - 48f(2y) \\ & +60f(y) - 66f(x). \end{aligned} \quad (1)$$

In this paper, we prove the generalized Hyers-Ulam stability of the functional equation (Equation 1) in fuzzy Banach spaces.

The stability problems of several functional equations have been extensively investigated by a number of authors, and there are many interesting results concerning this problem (see [18]–[43]).

Methods

Fuzzy stability of the functional equation (Equation 1): an odd case

In this section, using the fixed point alternative approach, we prove the generalized Hyers-Ulam stability of the functional equation (Equation 1) in fuzzy Banach spaces: an odd case. Throughout this paper, assume that X is a vector space and that (Y, N) is a fuzzy Banach space.

In the work of Lee et al. [32], they considered the following quartic functional equation:

$$f(2x+y)+f(2x-y) = 4\{f(x+y)+f(x-y)\}+24f(x)-6f(y). \quad (2)$$

It is easy to show that the function $f(x) = x^4$ satisfies the functional equation (Equation 2), which is called a quartic functional equation, and every solution of the quartic functional equation is said to be a quartic mapping.

One can easily show that an even mapping $f : X \to Y$ satisfies Equation 1 if and only if the even mapping $f : X \to Y$ is a quartic mapping, that is,

$$f(2x+y)+f(2x-y) = 4\{f(x+y)+f(x-y)\}+24f(x)-6f(y), \quad (3)$$

and an odd mapping $f : X \to Y$ satisfies Equation 1 if and only if the odd mapping $f : X \to Y$ is a additive-cubic mapping, that is,

$$f(2x+y)+f(2x-y) = 4\{f(x+y)+f(x-y)\}-6f(x). \quad (4)$$

It was shown in Lemma 2.2 in the study of Eshaghi Gordji et. al [17] that $g(x) = f(2x) - 2f(x)$ and $h(x) = f(2x) - 8f(x)$ are cubic and additive, respectively, and that $f(x) := \frac{1}{6}g(x) - \frac{1}{6}h(x)$.

For a given mapping $f : X \to Y$, we define the following:

$$\begin{aligned} \Phi_f(x, y) = & \ 11f(x + 2y) + 11f(x - 2y) \\ & -44\{f(x + y) + f(x - y)\} \\ & -12f(3y) + 48f(2y) - 60f(y) + 66f(x), \end{aligned}$$

for all $x, y \in X$.

Theorem 2. *Let $\varphi : X^2 \to [0, \infty)$ be a function such that there exists an $\alpha < 1$ with*

$$\varphi\left(\frac{x}{2}, \frac{y}{2}\right) \leq \frac{\alpha}{8} \varphi(x, y), \quad (5)$$

for all $x, y \in X$. Let $f : X \to Y$ be an odd mapping, satisfying

$$N\left(\Phi_f(x,y), t\right) \geq \frac{t}{t+\varphi(x,y)}, \tag{6}$$

for all $x, y \in X$ and all $t > 0$, and then the limit

$$C(x) := N\text{-}\lim_{n\to\infty} 8^n \left(f\left(\frac{x}{2^{n-1}}\right) - 2f\left(\frac{x}{2^n}\right)\right)$$

exists for each $x \in X$ and defines a unique cubic mapping $C : X \to Y$ such that

$$N(f(2x) - 2f(x) - C(x), t) \geq \frac{(264-264\alpha)t}{(264-264\alpha)t + 17\alpha\varphi(2x,x) + 17\alpha\varphi(0,x)}. \tag{7}$$

Proof. Putting $x = 0$ in Equation 6, we have the following:

$$N\left(12f(3y) - 48f(2y) + 60f(y), t\right) \geq \frac{t}{t+\varphi(0,y)}, \tag{8}$$

for all $y \in X$ and $t > 0$.

Replacing x by $2y$ in Equation 6, we obtain the following:

$$N\left(11f(4y) - 56f(3y) + 114f(2y) - 104f(y), t\right) \geq \frac{t}{t+\varphi(2y,y)}, \tag{9}$$

for all $y \in X$ and $t > 0$.

By Equations 8 and 9, we have the following:

$$\begin{aligned} &N\left(f(4y) - 10f(2y) + 16f(y), \frac{17t}{33}\right) \\ &\geq \min\Bigg(N\left(\frac{11f(4y) - 56f(3y) + 114f(2y) - 104f(y)}{11}, \frac{t}{11}\right), \\ &N\left(\frac{14(12f(3y) - 48f(2y) + 60f(y))}{33}, \frac{14t}{33}\right)\Bigg) \\ &\geq \frac{t}{t+\varphi(2y,y)+\varphi(0,y)}, \end{aligned} \tag{10}$$

for all $y \in X$ and all $t > 0$. Letting $y := \frac{x}{2}$ and $g(x) = f(2x) - 2f(x)$ for all $x \in X$, we get the following:

$$\begin{aligned} N\left(g(x) - 8g\left(\frac{x}{2}\right), \frac{17t}{33}\right) &\geq \frac{t}{t+\varphi\left(x,\frac{x}{2}\right)+\varphi\left(0,\frac{x}{2}\right)} \\ &\geq \frac{\frac{8t}{\alpha}}{\frac{8t}{\alpha}+\varphi(2x,x)+\varphi(0,x)}. \end{aligned} \tag{11}$$

Consider the set $S := \{g : X \to Y\}$ and the generalized metric d in S defined by the following:

$$d(f,g) = \inf_{\mu\in\mathbb{R}^+}\left\{N(g(x)-h(x),\mu t) \geq \frac{t}{t+\varphi(2x,x)+\varphi(0,x)}, \forall x \in X,\ t > 0\right\},$$

where $\inf\emptyset = +\infty$. It is easy to show that (S, d) is complete (see Lemma 2.1 of [33]).

Now, we consider a linear mapping $J : S \to S$ such that

$$Jg(x) := 8g\left(\frac{x}{2}\right),$$

for all $x \in X$. Let $g, h \in S$ satisfy $d(g, h) = \epsilon$ and then

$$N(g(x) - h(x), \epsilon t) \geq \frac{t}{t+\varphi(2x,x)+\varphi(0,x)},$$

for all $x \in X$ and $t > 0$. Hence,

$$\begin{aligned} N(Jg(x) - Jh(x), \alpha\epsilon t) &= N\left(8g\left(\frac{x}{2}\right) - 8h\left(\frac{x}{2}\right), \alpha\epsilon t\right) \\ &= N\left(g\left(\frac{x}{2}\right) - h\left(\frac{x}{2}\right), \frac{\alpha\epsilon t}{8}\right) \\ &\geq \frac{\frac{\alpha t}{8}}{\frac{\alpha t}{8}+\varphi\left(x,\frac{x}{2}\right)+\varphi\left(0,\frac{x}{2}\right)} \\ &\geq \frac{\frac{\alpha t}{8}}{\frac{\alpha t}{8}+\frac{\alpha}{8}\varphi(2x,x)+\frac{\alpha}{8}\varphi(0,x)} \\ &= \frac{t}{t+\varphi(2x,x)+\varphi(0,x)}, \end{aligned}$$

for all $x \in X$ and $t > 0$. Thus, $d(g, h) = \epsilon$ implies that $d(Jg, Jh) \leq \alpha\epsilon$. This means that

$$d(Jg, Jh) \leq \alpha d(g, h),$$

for all $g, h \in S$. It follows from Equation 11 that

$$N\left(g(x) - 8g\left(\frac{x}{2}\right), \frac{17\alpha t}{264}\right) \geq \frac{t}{t+\varphi(2x,x)+\varphi(0,x)}.$$

Thus,

$$d(g, Jg) \leq \frac{17\alpha}{264}.$$

By Theorem 1, there exists a mapping $C : X \to Y$, satisfying the following:

(1) C is a fixed point of J, that is,

$$C\left(\frac{x}{2}\right) = \frac{1}{8}C(x), \tag{12}$$

for all $x \in X$. The mapping C is a unique fixed point of J in the following set: $\Omega = \{h \in S : d(g, h) < \infty\}$.

This implies that C is a unique mapping, satisfying Equation 12, such that there exists $\mu \in (0, \infty)$, satisfying the following:

$$\begin{aligned} N(g(x) - C(x), \mu t) &= N(f(2x) - 2f(x) - C(x), \mu t) \\ &\geq \frac{t}{t+\varphi(2x,x)+\varphi(0,x)}, \end{aligned}$$

for all $x \in X$ and $t > 0$.

(2) $d(J^n g, C) \to 0$ as $n \to \infty$. This implies the following equality:

$$N\text{-}\lim_{n\to\infty} 8^n g\left(\frac{x}{2^n}\right) = N\text{-}\lim_{n\to\infty} 8^n \left(f\left(\frac{x}{2^{n-1}}\right) - 2f\left(\frac{x}{2^n}\right)\right) = C(x),$$

for all $x \in X$.

(3) $d(g, C) \leq \frac{d(g,Jg)}{1-\alpha}$ with $f \in \Omega$, which implies the following inequality:

$$d(g, C) \leq \frac{17\alpha}{264 - 264\alpha}.$$

This implies that the inequality (Equation 7) holds.

Since $\Phi_g(x, y) = \Phi_f(2x, 2y) - 2\Phi_f(x, y)$, using Equation 6, we obtain the following:

$$\begin{aligned} N\left(8^n \Phi_g\left(\frac{x}{2^n}, \frac{y}{2^n}\right), 8^n t\right) &= N\left(8^n \Phi_f\left(\frac{x}{2^{n-1}}, \frac{y}{2^{n-1}}\right) - 2 \cdot 8^n \Phi_f\left(\frac{x}{2^n}, \frac{y}{2^n}\right), 8^n t\right) \\ &\geq \frac{t}{t + \varphi\left(\frac{x}{2^n}, \frac{y}{2^n}\right)}, \end{aligned} \tag{13}$$

for all $x, y \in X$, $t > 0$ and all $n \in \mathbb{N}$. Thus, by Equation 5, we have the following:

$$N\left(8^n \Phi_g\left(\frac{x}{2^n}, \frac{y}{2^n}\right), t\right) \geq \frac{\frac{t}{8^n}}{\frac{t}{8^n} + \frac{\alpha^n}{8^n}\varphi(x, y)},$$

for all $x, y \in X$, $t > 0$ and all $n \in \mathbb{N}$. Since $\lim_{n\to\infty} \frac{\frac{t}{8^n}}{\frac{t}{8^n} + \frac{\alpha^n}{8^n}\varphi(x,y)} = 1$ for all $x, y \in X$ and all $t > 0$, we deduce that $N\left(\Phi_C(x, y), t\right) = 1$ for all $x, y \in X$ and all $t > 0$. Thus, the mapping $C : X \to Y$, satisfying Equation 1, as desired. This completes the proof. □

Corollary 1. *Let $\theta \geq 0$ and let r be a real number with $r > 1$. Let X be a normed vector space with norm $\|\cdot\|$. Let $f : X \to Y$ be an odd mapping, satisfying the following:*

$$N\left(\Phi_f(x, y), t\right) \geq \frac{t}{t + \theta\left(\|x\|^r + \|y\|^r\right)}, \tag{14}$$

for all $x, y \in X$ and all $t > 0$, and then,

$$C(x) := N\text{-}\lim_{n\to\infty} 8^n \left(f\left(\frac{x}{2^{n-1}}\right) - 2f\left(\frac{x}{2^n}\right)\right)$$

exists for each $x \in X$ and defines a unique cubic mapping $C : X \to Y$ such that

$$N(f(2x) - 2f(x) - C(x), t) \geq \frac{33(8^r - 8)t}{33(8^r - 8)t + 17(2^r + 2)\theta\|x\|^r},$$

for all $x \in X$ and all $t > 0$.

Proof. The proof follows from Theorem 2 by taking $\varphi(x, y) := \theta\left(\|x\|^r + \|y\|^r\right)$ for all $x, y \in X$, and then we can choose $\alpha = 8^{1-r}$ and get the desired result. □

Theorem 3. *Let $\varphi : X^2 \to [0, \infty)$ be a function such that there exists an $\alpha < 1$ with the following:*

$$\varphi\left(2x, 2y\right) \leq 8\alpha\varphi(x, y), \tag{15}$$

for all $x, y \in X$. Let $f : X \to Y$ be an odd mapping, satisfying Equation 6, and then the limit

$$C(x) := N\text{-}\lim_{n\to\infty} \frac{f(2^{n+1}x) - 2f(2^n x)}{8^n}$$

exists for each $x \in X$ and defines a unique cubic mapping $C : X \to Y$ such that

$$N(f(2x) - 2f(x) - C(x), t) \geq \frac{(264 - 264\alpha)t}{(264 - 264\alpha)t + 17\varphi(2x, x) + 17\varphi(0, x)} \tag{16}$$

Proof. Let (S, d) be the generalized metric space defined as in the proof of Theorem 2. Consider the linear mapping $J : S \to S$ such that $Jg(x) := \frac{1}{8}g(2x)$, for all $x \in X$. Let $g, h \in S$ be such that $d(g, h) = \epsilon$, and

$$N(g(x) - h(x), \epsilon t) \geq \frac{t}{t + \varphi(2x, x) + \varphi(0, x)},$$

for all $x \in X$ and $t > 0$. Hence,

$$\begin{aligned} N(Jg(x) - Jh(x), \alpha\epsilon t) &= N\left(\frac{g(2x)}{8} - \frac{h(2x)}{8}, \alpha\epsilon t\right) \\ &= N\left(g(2x) - h(2x), 8\alpha\epsilon t\right) \\ &\geq \frac{8\alpha t}{8\alpha t + 8\alpha\varphi(2x, x) + 8\alpha\varphi(0, x)} \\ &= \frac{t}{t + \varphi(2x, x) + \varphi(0, x)}, \end{aligned}$$

for all $x \in X$ and $t > 0$. Thus, $d(g, h) = \epsilon$ implies that $d(Jg, Jh) \leq \alpha\epsilon$. This means that $d(Jg, Jh) \leq \alpha d(g, h)$, for all $g, h \in S$. It follows from Equation 10 that

$$N\left(\frac{g(2x)}{8} - g(x), \frac{17t}{264}\right) \geq \frac{t}{t + \varphi(2x, x) + \varphi(0, x)},$$

for all $x \in X$ and $t > 0$. Thus, $d(g, Jg) \leq \frac{17}{264}$.

By Theorem 1, there exists a mapping $C : X \to Y$, satisfying the following:

(1) C is a fixed point of J, that is,

$$8C(x) = C(2x), \tag{17}$$

for all $x \in X$. The mapping C is a unique fixed point of J in the set $\Omega = \{h \in S : d(g, h) < \infty\}$.

This implies that C is a unique mapping, satisfying Equation 17, such that there exists $\mu \in (0,\infty)$, satisfying the following:

$$N(g(x)-C(x),\mu t)=N(f(2x)-2f(x)-C(x),\mu t) \geq \frac{t}{t+\varphi(2x,x)+\varphi(0,x)},$$

for all $x\in X$ and $t>0$.

(2) $d(J^n g, C)\to 0$ as $n\to\infty$. This implies the following equality:

$$N\text{-}\lim_{n\to\infty}\frac{g(2^n x)}{8^n}=N\text{-}\lim_{n\to\infty}\frac{f(2^{n+1}x)-2f(2^n x)}{8^n}=C(x),$$

for all $x\in X$.

(3) $d(g,C)\leq \frac{d(g,Jg)}{1-\alpha}$ with $f\in\Omega$, which implies the following inequality: $d(g,C)\leq\frac{17}{264-264\alpha}$. This implies that the inequality (Equation 16) holds.

The rest of the proof is similar to that of the proof of Theorem 2. □

Corollary 2. *Let $\theta\geq 0$ and let r be a real number with $0<r<1$. Let X be a normed vector space with norm $\|\cdot\|$. Let $f: X\to Y$ be an odd mapping, satisfying Equation 14, and the limit*

$$C(x):=N\text{-}\lim_{n\to\infty}\frac{f(2^{n+1}x)-2f(2^n x)}{8^n}$$

exists for each $x\in X$ and defines a unique cubic mapping $C: X\to Y$ such that

$$N(f(2x)-2f(x)-C(x),t) \geq \frac{132(1-8^{-r})t}{132(1-8^{-r})t+17(2^{r-1}+1)\theta\|x\|^r},$$

for all $x\in X$ and all $t>0$.

Proof. The proof follows from Theorem 3 by taking $\varphi(x,y):=\theta(\|x\|^r+\|y\|^r)$ for all $x,y\in X$, and then we can choose $\alpha=8^{-r}$ and get the desired result. □

Theorem 4. *Let $\varphi: X^2\to[0,\infty)$ be a function such that there exists an $\alpha<1$ with the following:*

$$\varphi\left(\frac{x}{2},\frac{y}{2}\right)\leq\frac{\alpha}{2}\varphi(x,y), \tag{18}$$

for all $x,y\in X$. Let $f: X\to Y$ be an odd mapping, satisfying Equation 6, and then the limit

$$A(x):=N\text{-}\lim_{n\to\infty}2^n\left(f\left(\frac{x}{2^{n-1}}\right)-8f\left(\frac{x}{2^n}\right)\right)$$

exists for each $x\in X$ and defines a unique additive mapping $A: X\to Y$ such that

$$N(f(2x)-8f(x)-A(x),t) \geq \frac{(66-66\alpha)t}{(66-66\alpha)t+17\alpha\varphi(2x,x)+17\alpha\varphi(0,x)}. \tag{19}$$

Proof. Let (S,d) be the generalized metric space defined as in the proof of Theorem 2.

Letting $y:=\frac{x}{2}$ and $h(x): f(2x)-8f(x)$ for all $x\in X$ in Equation 10, we obtain the following:

$$N\left(h(x)-2h\left(\frac{x}{2}\right),\frac{17t}{33}\right)\geq\frac{t}{t+\varphi\left(x,\frac{x}{2}\right)+\varphi\left(0,\frac{x}{2}\right)}. \tag{20}$$

Consider the linear mapping $J: S\to S$ such that

$$Jh(x):=2h\left(\frac{x}{2}\right),$$

for all $x\in X$. Let $g,h\in S$ be such that $d(g,h)=\epsilon$, and then

$$N(g(x)-h(x),\epsilon t)\geq\frac{t}{t+\varphi(2x,x)+\varphi(0,x)},$$

for all $x\in X$ and $t>0$. Hence,

$$N(Jg(x)-Jh(x),\alpha\epsilon t)=N\left(2g\left(\frac{x}{2}\right)-2h\left(\frac{x}{2}\right),\alpha\epsilon t\right) \geq\frac{t}{t+\varphi(2x,x)+\varphi(0,x)},$$

for all $x\in X$ and $t>0$. Thus, $d(g,h)=\epsilon$ implies that $d(Jg,Jh)\leq\alpha\epsilon$. This means that $d(Jg,Jh)\leq\alpha d(g,h)$, for all $g,h\in S$. It follows from Equation 20 that

$$N\left(2h\left(\frac{x}{2}\right)-h(x),\frac{17\alpha t}{66}\right)\geq\frac{t}{t+\varphi(2x,x)+\varphi(0,x)},$$

for all $x\in X$ and $t>0$. Thus, $d(g,Jg)\leq\frac{17\alpha}{66}$.

By Theorem 1, there exists a mapping $A: X\to Y$, satisfying the following:

(1) A is a fixed point of J, that is,

$$\frac{1}{2}A(x)=A\left(\frac{x}{2}\right), \tag{21}$$

for all $x\in X$. The mapping A is a unique fixed point of J in the set $\Omega=\{h\in S: d(g,h)<\infty\}$.

This implies that A is a unique mapping, satisfying Equation 21, such that there exists $\mu\in(0,\infty)$, satisfying the following:

$$N(h(x)-A(x),\mu t)=N(f(2x)-8f(x)-A(x),\mu t) \geq\frac{t}{t+\varphi(2x,x)+\varphi(0,x)},$$

for all $x\in X$ and $t>0$.

(2) $d(J^n h, A) \to 0$ as $n \to \infty$. This implies the following equality:

$$N\text{-}\lim_{n\to\infty} 2^n h\left(\frac{x}{2^n}\right) = N\text{-}\lim_{n\to\infty} 2^n\left(f\left(\frac{x}{2^{n-1}}\right) - 8f\left(\frac{x}{2^n}\right)\right) = A(x)$$

for all $x \in X$.

(3) $d(h, A) \le \frac{d(h,Jh)}{1-\alpha}$ with $f \in \Omega$, which implies the following inequality: $d(h, A) \le \frac{17\alpha}{66-66\alpha}$. This implies that the inequality (Equation 19) holds. The rest of the proof is similar to that of the proof of Theorem 2. □

Corollary 3. *Let $\theta \ge 0$ and let r be a real number with $r > 1$. Let X be a normed vector space with norm $\|\cdot\|$. Let $f : X \to Y$ be an odd mapping, satisfying Equation 14, and then*

$$A(x) := N\text{-}\lim_{n\to\infty} 2^n\left(f\left(\frac{x}{2^{n-1}}\right) - 8f\left(\frac{x}{2^n}\right)\right)$$

exists for each $x \in X$ and defines a unique additive mapping $A : X \to Y$ such that

$$N(f(2x) - 8f(x) - A(x), t) \ge \frac{33(2^r-2)t}{33(2^r-2)t + 17(2^r+2)\theta\|x\|^r},$$

for all $x \in X$ and all $t > 0$.

Proof. The proof follows from Theorem 4 by taking $\varphi(x, y) := \theta\left(\|x\|^r + \|y\|^r\right)$ for all $x, y \in X$, and then we can choose $\alpha = 2^{1-r}$ and get the desired result. □

Theorem 5. *Let $\varphi : X^2 \to [0, \infty)$ be a function such that there exists an $\alpha < 1$ with the following:*

$$\varphi(2x, 2y) \le 2\alpha\varphi(x, y), \tag{22}$$

for all $x, y \in X$. Let $f : X \to Y$ be an odd mapping, satisfying Equation 6, and then the limit

$$A(x) := N\text{-}\lim_{n\to\infty} \frac{f(2^{n+1}x) - 8f(2^n x)}{2^n}$$

exists for each $x \in X$ and defines a unique additive mapping $A : X \to Y$ such that

$$N(f(2x) - 8f(x) - A(x), t) \ge \frac{(66-66\alpha)t}{(66-66\alpha)t + 17\varphi(2x, x) + 17\varphi(0, x)}. \tag{23}$$

Proof. Let (S, d) be the generalized metric space defined as in the proof of Theorem 2. Consider the linear mapping $J : S \to S$ such that $Jh(x) := \frac{1}{2}h(2x)$, for all $x \in X$. Let $g, h \in S$ be such that $d(g, h) = \epsilon$. Then

$$N(g(x) - h(x), \epsilon t) \ge \frac{t}{t + \varphi(2x, x) + \varphi(0, x)},$$

for all $x \in X$ and $t > 0$. Hence,

$$\begin{aligned} N(Jg(x) - Jh(x), \alpha\epsilon t) &= N\left(\frac{g(2x)}{2} - \frac{h(2x)}{2}, \alpha\epsilon t\right) \\ &= N\left(g(2x) - h(2x), 2\alpha\epsilon t\right) \\ &\ge \frac{2\alpha t}{2\alpha t + \varphi(4x, 2x) + \varphi(0, 2x)} \\ &= \frac{t}{t + \varphi(2x, x) + \varphi(0, x)}, \end{aligned}$$

for all $x \in X$ and $t > 0$. Thus, $d(g, h) = \epsilon$ implies that $d(Jg, Jh) \le \alpha\epsilon$. This means that $d(Jg, Jh) \le \alpha d(g, h)$, for all $g, h \in S$. It follows from Equation 10 that

$$N\left(\frac{h(2x)}{2} - h(x), \frac{17t}{66}\right) \ge \frac{t}{t + \varphi(2x, x) + \varphi(0, x)},$$

for all $x \in X$ and $t > 0$. Thus,

$$d(g, Jg) \le \frac{17}{66}.$$

By Theorem 1, there exists a mapping $A : X \to Y$, satisfying the following:

(1) A is a fixed point of J, that is,

$$2A(x) = A(2x), \tag{24}$$

for all $x \in X$. The mapping A is a unique fixed point of J in the set $\Omega = \{h \in S : d(g, h) < \infty\}$.

This implies that A is a unique mapping, satisfying Equation 24, such that there exists $\mu \in (0, \infty)$, satisfying the following:

$$\begin{aligned} N(h(x) - A(x), \mu t) &= N(f(2x) - 8f(x) - A(x), \mu t) \\ &\ge \frac{t}{t + \varphi(2x, x) + \varphi(0, x)}, \end{aligned}$$

for all $x \in X$ and $t > 0$.

(2) $d(J^n h, A) \to 0$ as $n \to \infty$. This implies the following equality:

$$N\text{-}\lim_{n\to\infty} \frac{h(2^n x)}{2^n} = N\text{-}\lim_{n\to\infty} \frac{f(2^{n+1}x) - 8f(2^n x)}{2^n} = A(x),$$

for all $x \in X$.

(3) $d(h, A) \le \frac{d(h,Jh)}{1-\alpha}$ with $f \in \Omega$, which implies the following inequality: $d(h, A) \le \frac{17}{66-66\alpha}$.

This implies that the inequality (Equation 23) holds. The rest of the proof is similar to that of the proof of Theorem 2. □

Corollary 4. *Let $\theta \geq 0$ and let r be a real number with $0 < r < 1$. Let X be a normed vector space with norm $\| \cdot \|$. Let $f : X \to Y$ be an odd mapping, satisfying Equation 14, and then the limit*

$$A(x) := N\text{-}\lim_{n\to\infty} \frac{f(2^{n+1}x) - 8f(2^n x)}{2^n}$$

exists for each $x \in X$ and defines a unique additive mapping $A : X \to Y$ such that

$$N(f(2x) - 8f(x) - A(x), t) \geq \frac{33(2^r - 1)t}{33(2^r - 1)t + 17 \cdot 2^r(2^{r-1} + 1)\theta\|x\|^r},$$

for all $x \in X$ and all $t > 0$.

Proof. The proof follows from Theorem 5 by taking $\varphi(x, y) := \theta(\|x\|^r + \|y\|^r)$, for all $x, y \in X$, and then we can choose $\alpha = 2^{-r}$ and get the desired result. □

Fuzzy stability of the functional equation (Equation 1): an even case

Throughout this section, using the fixed point alternative approach, we prove the generalized Hyers-Ulam stability of the functional equation (Equation 1) in fuzzy Banach spaces: an even case.

Theorem 6. *Let $\varphi : X^2 \to [0, \infty)$ be a function such that there exists an $\alpha < 1$ with the following:*

$$\varphi\left(\frac{x}{2}, \frac{y}{2}\right) \leq \frac{\alpha}{16}\varphi(x, y), \tag{25}$$

for all $x, y \in X$. Let $f : X \to Y$ be an even mapping, satisfying the following:

$$N\left(\Phi_f(x, y), t\right) \geq \frac{t}{t + \varphi(x, y)}, \tag{26}$$

for all $x, y \in X$ and all $t > 0$, and then the limit

$$Q(x) := N\text{-}\lim_{n\to\infty} 16^n f\left(\frac{x}{2^n}\right)$$

exists for each $x \in X$ and defines a unique quartic mapping $Q : X \to Y$ such that

$$N(f(x) - Q(x), t) \geq \frac{(352 - 352\alpha)t}{(352 - 352\alpha)t + 13\alpha\varphi(x, x) + 13\alpha\varphi(0, x)}. \tag{27}$$

Proof. Putting $x = 0$ in Equation 26, we have the following:

$$N\left(12f(3y) - 70f(2y) + 148f(y), t\right) \geq \frac{t}{t + \varphi(0, y)}, \tag{28}$$

for all $y \in X$ and $t > 0$.

Substituting $x = y$ in Equation 26, we obtain the following:

$$N\left(f(3y) - 4f(2y) - 17f(y), t\right) \geq \frac{t}{t + \varphi(y, y)}, \tag{29}$$

for all $y \in X$ and $t > 0$.

By Equations 28 and 29, we have the following:

$$\begin{aligned} N\left(f(2y) - 16f(y), \frac{13t}{22}\right) &\geq \min\left(N\left(\frac{12f(3y) - 70f(2y) + 148f(y)}{22}, \frac{t}{22}\right),\right. \\ &\quad \left. N\left(\frac{6(f(3y) - 4f(2y) - 17f(y))}{22}, \frac{6t}{11}\right)\right) \\ &\geq \frac{t}{t + \varphi(y, y) + \varphi(0, y)}, \end{aligned} \tag{30}$$

for all $y \in X$ and all $t > 0$. By replacing $y := \frac{x}{2}$ for all $x \in X$, we get the following:

$$\begin{aligned} N\left(f(x) - 16\left(\frac{x}{2}\right), \frac{11t}{22}\right) &\geq \frac{t}{t + \varphi\left(0, \frac{x}{2}\right) + \varphi\left(\frac{x}{2}, \frac{x}{2}\right)} \\ &\geq \frac{\frac{16t}{\alpha}}{\frac{16t}{\alpha} + \varphi(x, x) + \varphi(0, x)}. \end{aligned} \tag{31}$$

Consider the set $S := \{g : X \to Y\}$, and the generalized metric d in S defined by

$$d^*(f, g) = \inf_{\mu\in\mathbb{R}^+}\left\{N(g(x) - h(x), \mu t) \geq \frac{t}{t + \varphi(x, x) + \varphi(0, x)}, \forall x \in X,\ t > 0\right\},$$

where $\inf \emptyset = +\infty$. It is easy to show that (S, d^*) is complete (see Lemma 2.1 in [33]).

Now, we consider a linear mapping $J : S \to S$ such that $Jg(x) := 16g\left(\frac{x}{2}\right)$, for all $x \in X$. Let $g, h \in S$ satisfy $d^*(g, h) = \epsilon$, and then

$$N(g(x) - h(x), \epsilon t) \geq \frac{t}{t + \varphi(x, x) + \varphi(0, x)},$$

for all $x \in X$ and $t > 0$. Hence,

$$\begin{aligned} N(Jg(x) - Jh(x), \alpha\epsilon t) &= N\left(16g\left(\frac{x}{2}\right) - 16h\left(\frac{x}{2}\right), \alpha\epsilon t\right) \\ &= N\left(g\left(\frac{x}{2}\right) - h\left(\frac{x}{2}\right), \frac{\alpha\epsilon t}{16}\right) \\ &\geq \frac{\frac{\alpha t}{16}}{\frac{\alpha t}{16} + \varphi\left(\frac{x}{2}, \frac{x}{2}\right) + \varphi\left(0, \frac{x}{2}\right)} \\ &\geq \frac{\frac{\alpha t}{16}}{\frac{\alpha t}{16} + \frac{\alpha}{16}\varphi(x, x) + \frac{\alpha}{8}\varphi(0, x)} \\ &= \frac{t}{t + \varphi(x, x) + \varphi(0, x)}, \end{aligned}$$

for all $x \in X$ and $t > 0$. Thus, $d^*(g,h) = \epsilon$ implies that $d^*(Jg, Jh) \leq \alpha\epsilon$. This means that $d^*(Jg, Jh) \leq \alpha d^*(g,h)$, for all $g, h \in S$. It follows from Equation 31 that

$$N\left(f(x) - 16\left(\frac{x}{2}\right), \frac{13\alpha t}{352}\right) \geq \frac{t}{t + \varphi(x,x) + \varphi(0,x)}.$$

Thus, $d^*(f, Jf) \leq \frac{13\alpha}{352}$. By Theorem 1, there exists a mapping $Q : X \to Y$, satisfying the following:

(1) Q is a fixed point of J, that is,

$$Q\left(\frac{x}{2}\right) = \frac{1}{16}Q(x), \tag{32}$$

for all $x \in X$. The mapping Q is a unique fixed point of J in the following set: $\Omega = \{h \in S : d^*(g,h) < \infty\}$. This implies that Q is a unique mapping, satisfying Equation 32, such that there exists $\mu \in (0, \infty)$, satisfying the following:

$$N(f(x) - Q(x), \mu t) \geq \frac{t}{t + \varphi(x,x) + \varphi(0,x)},$$

for all $x \in X$ and $t > 0$.

(2) $d^*(J^n f, Q) \to 0$ as $n \to \infty$. This implies the following equality: $N\text{-}\lim_{n\to\infty} 16^n f\left(\frac{x}{2^n}\right) = Q(x)$, for all $x \in X$.

(3) $d^*(f, Q) \leq \frac{d^*(f,Jf)}{1-\alpha}$ with $f \in \Omega$, which implies the following inequality: $d^*(f, Q) \leq \frac{13\alpha}{352-352\alpha}$. This implies that the inequality (Equation 27) holds.

On the other hand, by Equation 26, we obtain the following:

$$N\left(16^n \Phi_f\left(\frac{x}{2^n}, \frac{y}{2^n}\right), 16^n t\right) \geq \frac{t}{t + \varphi\left(\frac{x}{2^n}, \frac{y}{2^n}\right)},$$

for all $x, y \in X$, $t > 0$ and all $n \in \mathbb{N}$. Thus,

$$N\left(16^n \Phi_f\left(\frac{x}{2^n}, \frac{y}{2^n}\right), t\right) \geq \frac{\frac{t}{16^n}}{\frac{t}{16^n} + \frac{\alpha^n}{16^n}\varphi(x,y)},$$

for all $x, y \in X$, $t > 0$ and all $n \in \mathbb{N}$. Since $\lim_{n\to\infty} \frac{\frac{t}{16^n}}{\frac{t}{16^n} + \frac{\alpha^n}{16^n}\varphi(x,y)} = 1$ for all $x, y \in X$ and all $t > 0$, we deduce that $N\left(\Phi_Q(x,y), t\right) = 1$ for all $x, y \in X$ and all $t > 0$. Thus, the mapping $Q : X \to Y$, satisfying Equation 1, as desired. This completes the proof. □

Corollary 5. *Let $\theta \geq 0$ and let r be a real number with $r > 1$. Let X be a normed vector space with norm $\|\cdot\|$. Let $f : X \to Y$ be an even mapping, satisfying Equation 14, and then the limit*

$$Q(x) := N\text{-}\lim_{n\to\infty} 16^n f\left(\frac{x}{2^n}\right)$$

exists for each $x \in X$ and defines a unique quartic mapping $Q : X \to Y$ such that

$$N(f(x) - Q(x), t) \geq \frac{352(16^r - 1)t}{352(16^r - 1)t + 39\theta\|x\|^r},$$

for all $x \in X$ and all $t > 0$.

Proof. The proof follows from Theorem 6 by taking $\varphi(x,y) := \theta\left(\|x\|^r + \|y\|^r\right)$, for all $x, y \in X$, and then we can choose $\alpha = 16^{-r}$ and get the desired result. □

Theorem 7. *Let $\varphi : X^2 \to [0, \infty)$ be a function such that there exists an $\alpha < 1$ with the following:*

$$\varphi\left(2x, 2y\right) \leq 16\alpha\varphi(x,y), \tag{33}$$

for all $x, y \in X$. Let $f : X \to Y$ be an even mapping, satisfying Equation 26, and then the limit

$$Q(x) := N\text{-}\lim_{n\to\infty} \frac{f(2^n x)}{16^n}$$

exists for each $x \in X$ and defines a unique quartic mapping $Q : X \to Y$ such that

$$N(f(x) - Q(x), t) \geq \frac{(352-352\alpha)t}{(352-352\alpha)t + 13\varphi(x,x) + 13\varphi(0,x)}. \tag{34}$$

Proof. Let (S, d^*) be the generalized metric space defined as in the proof of Theorem 6. Consider the linear mapping $J : S \to S$ such that

$$Jg(x) := \frac{1}{16}g(2x),$$

for all $x \in X$. Let $g, h \in S$ be such that $d^*(g,h) = \epsilon$, and then

$$N(g(x) - h(x), \epsilon t) \geq \frac{t}{t + \varphi(x,x) + \varphi(0,x)},$$

for all $x \in X$ and $t > 0$. Hence,

$$\begin{aligned} N(Jg(x) - Jh(x), \alpha\epsilon t) &= N\left(\frac{g(2x)}{16} - \frac{h(2x)}{16}, \alpha\epsilon t\right) \\ &= N\left(g(2x) - h(2x), 16\alpha\epsilon t\right) \\ &\geq \frac{16\alpha t}{16\alpha t + 16\alpha\varphi(x,x) + 16\alpha\varphi(0,x)} \\ &= \frac{t}{t + \varphi(x,x) + \varphi(0,x)}, \end{aligned}$$

for all $x \in X$ and $t > 0$. Thus, $d^*(g,h) = \epsilon$ implies that $d^*(Jg, Jh) \leq \alpha\epsilon$. This means that $d^*(Jg, Jh) \leq \alpha d^*(g,h)$ for all $g, h \in S$. It follows from Equation 30 that

$$N\left(\frac{f(2x)}{16} - f(x), \frac{13t}{352}\right) \geq \frac{t}{t + \varphi(x,x) + \varphi(0,x)},$$

for all $x \in X$ and $t > 0$. Thus, $d^*(g, Jg) \leq \frac{13}{352}$.

By Theorem 1, there exists a mapping $Q : X \to Y$ satisfying the following:

(1) Q is a fixed point of J, that is,

$$16Q(x) = Q(2x), \tag{35}$$

for all $x \in X$. The mapping Q is a unique fixed point of J in the set $\Omega = \{h \in S : d^*(g,h) < \infty\}$.

This implies that Q is a unique mapping, satisfying Equation 35, such that there exists $\mu \in (0,\infty)$, satisfying the following:

$$N(f(x) - Q(x), \mu t) \geq \frac{t}{t + \varphi(x,x) + \varphi(0,x)},$$

for all $x \in X$ and $t > 0$.

(2) $d^*(J^n f, Q) \to 0$ as $n \to \infty$. This implies the following equality: $N\text{-}\lim_{n\to\infty} \frac{f(2^n x)}{16^n} = Q(x)$, for all $x \in X$.

(3) $d^*(f,Q) \leq \frac{d^*(f,Jf)}{1-\alpha}$ with $f \in \Omega$, which implies the following inequality: $d(f,Q) \leq \frac{13}{352-352\alpha}$. This implies that the inequality (Equation 34) holds. The rest of the proof is similar to that of the proof of Theorem 2. □

Corollary 6. *Let $\theta \geq 0$ and let r be a real number with $0 < r < 1$. Let X be a normed vector space with norm $\|\cdot\|$. Let $f : X \to Y$ be an even mapping, satisfying Equation 14, and then the limit*

$$Q(x) := N\text{-}\lim_{n\to\infty} \frac{f(2^n x)}{16^n}$$

exists for each $x \in X$ and defines a unique quartic mapping $Q : X \to Y$ such that

$$N(f(x) - Q(x), t) \geq \frac{352(16 - 16^r)t}{352(16 - 16^r)t + 624\theta\|x\|^r},$$

for all $x \in X$ and all $t > 0$.

Proof. The proof follows from Theorem 7 by taking the following: $\varphi(x,y) := \theta\left(\|x\|^r + \|y\|^r\right)$, for all $x, y \in X$, and then we can choose $\alpha = 16^{r-1}$ and get the desired result. □

Results and discussion

We linked here three different disciplines, namely fuzzy Banach spaces, functional equations, and fixed point theory. We established the Hyers-Ulam-Rassias stability of functional Equation 1 in fuzzy Banach spaces by fixed point method.

Conclusions

Throughout this paper, using the fixed point method, we proved the Hyers-Ulam-Rassias stability of a mixed type ACQ functional equation in fuzzy Banach spaces.

Competing interests

The author declares that there are no competing interests.

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