### **ORIGINAL RESEARCH**

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# Approximation of Jordan homomorphisms in Jordan-Banach algebras

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#### Abstract

Using the direct method based on the Hyers-Ulam-Rassias stability, we investigate and prove the Hyers-Ulam stability of Jordan homomorphisms in Jordan-Banach algebras for the functional equation

$$\sum_{k=2}^{n} \sum_{i_{1}=2}^{k} \sum_{i_{2}=i_{1}+1}^{k+1} \cdots \sum_{i_{n}-k+1=i_{n-k}+1}^{n} f\left(\sum_{i=1, i \neq i_{1}, \cdots, i_{n-k+1}}^{n} x_{i} - \sum_{r=1}^{n-k+1} x_{i_{r}}\right) + f\left(\sum_{i=1}^{n} x_{i}\right) - 2^{n-1}f(x_{1}) = 0,$$

where *n* is an integer greater than 1.

We have proved the Hyers-Ulam stability of Jordan homomorphisms in Jordan-Banach algebras for the above functional equation.

**Keywords:** Hyers-Ulam stability, Jordan homomorphism, Jordan algebra **2010 MSC:** 39B52; 17C65.

#### Introduction

A classical question in the theory of functional equations is that 'when is it true that a function which approximately satisfies a functional equation  $\mathcal{E}$  must be somehow close to an exact solution of  $\mathcal{E}$ . Such a problem was formulated by Ulam [1] in 1940 and solved in the next year for the Cauchy functional equation by Hyers [2]. It gave rise to the stability theory for functional equations. The result of Hyers was generalized by Aoki [3] for approximate additive functions and by Rassias [4] for approximate linear functions. The stability phenomenon that was proved by Rassias is called the Hyers-Ulam-Rassias stability or the generalized Hyers-Ulam stability of functional equations. In 1994, a generalization of the Th.M. Rassias' theorem was obtained by Gåvruta [5] as follows: Suppose that (G, +) is an abelian group and E is a Banach space and that the so-called admissible control function  $\varphi : G \times G \to \mathbb{R}$ satisfies

$$\tilde{\varphi}(x,y) := \sum_{n=0}^{\infty} 2^{-n} \varphi(2^n x, 2^n y) < \infty$$

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$$\|f(x+y) - f(x) - f(y)\| \le \varphi(x,y)$$

for all  $x, y \in G$ , then there exists a unique mapping  $T : G \to E$  such that T(x + y) = T(x) + T(y) and  $||f(x) - T(x)|| \le \tilde{\varphi}(x, x)$  for all  $x, y \in G$ . If, moreover, *G* is a real normed space and f(tx) is continuous in *t* for each fixed *x* in *G*, then *T* is a linear function.

The stability problems of several functional equations have been extensively investigated by a number of authors, and there are many interesting results concerning this problem (see [6-27]).

Recently, Eshaghi Gordji et al. (unpublished work) defined the following *n*-dimensional additive functional equation

$$D_{f}(x_{1}, \cdots, x_{n}) := \sum_{k=2}^{n} \sum_{i_{1}=2}^{k} \sum_{i_{2}=i_{1}+1}^{k+1} \cdots \sum_{i_{n}-k+1=i_{n-k}+1}^{n} \\ \times f\left(\sum_{i=1, i \neq i_{1}^{n}, \cdots, i_{n-k+1}}^{n} x_{i} - \sum_{r=1}^{n-k+1} x_{i_{r}}\right) \\ + f\left(\sum_{i=1}^{n} x_{i}\right) - 2^{n-1}f(x_{1}) = 0, \quad (1.1)$$



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where n is an integer greater than 1, and investigated the functional equation (1.1) in random normed spaces the via the fixed point method.

Note that a unital algebra A, endowed with the Jordan product  $x \circ y = \frac{1}{2}(xy + yx)$  on A, is called a Jordan algebra. A  $\mathbb{C}$ -linear mapping L of a Jordan algebra A into a Jordan algebra B is called a Jordan homomorphism if  $L(x \circ y) = (L(x) \circ L(y))$  holds for all  $x, y \in A$ .

Throughout this paper, let *A* be a Jordan-Banach algebra with norm  $\|\cdot\|$  and unit *e*, and let *B* be a Jordan-Banach algebra with norm  $\|\cdot\|$ .

#### Methods

Using the direct method based on the Hyers-Ulam-Rassias stability, we prove the Hyers-Ulam stability of Jordan homomorphisms in Jordan-Banach algebras for the functional equation (1.1).

#### **Results and discussion**

We need the following lemma in the proof of our main theorem.

**Lemma 2.1.** (Eshaghi Gordji et al., unpublished work) A mapping  $f : \mathcal{A} \to \mathcal{B}$  with f(0) = 0 satisfies (1.1) if and only if  $f : \mathcal{A} \to \mathcal{B}$  is additive.

We are going to prove the main result.

**Theorem 2.2.** Let  $h : A \to B$  be a mapping with h(0) = 0 for which there exists a function  $\varphi : A^{n+2} \to [0, \infty)$  such that

$$\tilde{\varphi}(x_1,\cdots,x_n,z,w) := \sum_{j=0}^{\infty} 2^{-j} \varphi(2^j x_1,\cdots,2^j x_n,2^j z,2^j w) < \infty,$$
(2.1)

$$\left\| \sum_{k=2}^{n} \sum_{i_{1}=2}^{k} \sum_{i_{2}=i_{1}+1}^{k+1} \cdots \sum_{i_{n}-k+1=i_{n-k}+1}^{n} \\ \times h\left( \sum_{i=1,i\neq i_{1},\cdots,i_{n-k+1}}^{n} \mu x_{i} - \sum_{r=1}^{n-k+1} \mu x_{i_{r}} \right) \\ + \mu h\left( \sum_{i=1}^{n} x_{i} \right) - \mu 2^{n-1} h(x_{1}) + h(z \circ w) - h(z) \circ h(w) \right\| \\ \leqslant \varphi(x_{1},\dots,x_{n},z,w)$$
(2.2)

for all  $\mu \in T^1 := \{\lambda \in \mathbb{C} | |\lambda| = 1\}$  and  $x_1, \ldots x_n, z, w \in A$ . Then, there exists a unique Jordan homomorphism  $L : A \rightarrow B$  such that

$$\|h(x) - L(x)\| \le \frac{1}{2^{n-1}}\widetilde{\varphi}(x, x, \underbrace{0\dots 0}_{n-times})$$
(2.3)

for all  $x \in A$ .

*Proof.* Let  $\mu = 1$ . Using the relation

$$1 + \sum_{k=1}^{n-k} \binom{n-k}{k} = \sum_{k=0}^{n-k} \binom{n-k}{k} = 2^{n-k}$$
(2.4)

for all n > k and putting  $x_1 = x_2 = x$  and  $x_i = z = w = 0$ (*i* = 3, ..., *n*) in (2.2), we obtain

$$\left\|2^{n-2}h(2x) - 2^{n-1}h(x)\right\| \le \varphi(x, x, \underbrace{0, \dots, 0}_{n-times})$$
(2.5)

for all  $x \in A$ . So,

$$\left\|\frac{h(2x)}{2} - h(x)\right\| \le \frac{1}{2^{n-1}}\varphi(x, x, \underbrace{0, \dots, 0}_{n-times})$$
(2.6)

for all  $x \in A$ . By induction on *m*, we can show that

$$\left\|\frac{h(2^m x)}{2^m} - h(x)\right\| \le \frac{1}{2^{n-1}} \sum_{j=0}^{m-1} \frac{1}{2^j} \varphi(2^j x, 2^j x, \underbrace{0, \dots, 0}_{n-times})$$
(2.7)

for all  $x \in \mathcal{A}$ . It follows from (2.1) and (2.7) that the sequence  $\left\{\frac{h(2^m x)}{2^m}\right\}$  is a Cauchy sequence for all  $x \in \mathcal{A}$ . Since  $\mathcal{A}$  is complete, the sequence  $\left\{\frac{h(2^m x)}{2^m}\right\}$  converges. Thus, one can define the mapping  $L : \mathcal{A} \to \mathcal{B}$  by

$$L(x) := \lim_{m \to \infty} \frac{h(2^m x)}{2^m}$$

for all  $x \in A$ . Let z = w = 0 and  $\mu = 1$  in (2.2). By (2.1)

$$egin{aligned} \|D_f(x_1,...,x_n)\| &= \lim_{j o \infty} rac{1}{2^j} \left\|D_f\left(2^j x_1,...,2^j x_n
ight)
ight\| \ &\leq \lim_{j o \infty} rac{1}{2^j} arphi\left(2^j x_1,...,2^j x_n,0,0
ight) = 0 \end{aligned}$$

for all  $x_1, \dots, x_n \in A$ . So,  $D_f(x_1, \dots, x_n) = 0$ . By Lemma 2.1, the mapping  $L : A \to B$  is additive. Moreover, passing the limit  $m \to \infty$  in (2.7) we get the inequality (2.3).

Now, let  $L' : \mathcal{A} \to \mathcal{B}$  be another additive mapping satisfying (1.1) and (2.3). Then,

$$\begin{split} \|L(x) - L'(x)\| &= \frac{1}{2^n} \|L(2^n x) - L'(2^n x)\| \\ &\leq \frac{1}{2^m} (\|L(2^n x) - h(2^n x)\| + \|L'(2^n x)\| \\ &- h(2^n x)\|) \\ &\leq \frac{2}{2^m 2^{n-1}} \widetilde{\varphi}(2^m x, 2^m x, \underbrace{0, \dots, 0}_{n-times}) \end{split}$$

which tends to zero as  $m \to \infty$  for all  $x \in A$ . So, we can conclude that L(x) = L'(x) for all  $x \in A$ . This proves the uniqueness of *L*.

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Let  $\mu \in \mathbb{T}^1$ . Set  $x_1 = x$  and  $z = w = x_i = 0$  (i = 2, ..., n) in (2.2). Then, by (2.1), we get

$$\|2^{n-1}h(\mu x) - 2^{n-1}\mu h(x)\| \le \varphi(x, 0, ..., 0, 0, 0) \quad (2.8)$$

for all  $x \in A$ . So,

$$\|2^{-m}(h(2^m\mu x) - \mu h(2^m x))\| \le \frac{2^{-m}}{2^{n-1}}\varphi(2^m x, 0, ..., 0, 0, 0)$$

for all  $x \in A$ . Since the right hand side of the above inequality tends to zero as  $m \to \infty$ , we have

$$L(\mu x) = \lim_{m \to \infty} \frac{h(2^m \mu x)}{2^m} = \lim_{m \to \infty} \frac{\mu h(2^m x)}{2^m} = \mu L(x)$$
(2.9)

for all  $\mu \in \mathbb{T}^1$  and all  $x \in \mathcal{A}$ .

Now let  $\lambda \in \mathbb{C}(\lambda \neq 0)$  and M an integer greater than  $4|\lambda|$ . Then,  $|\lambda/M| < 1/4 < 1 - 2/3 = 1/3$ . By Theorem 1 of [28], there exist three elements  $\mu_1, \mu_2, \mu_3 \in \mathbb{T}^1$  such that  $3\frac{\lambda}{M} = \mu_1 + \mu_2 + \mu_3$ , and  $L(x) = L\left(3 \cdot \frac{1}{3}x\right) = 3L\left(\frac{1}{3}x\right)$  for all  $x \in \mathcal{A}$ . So,  $L\left(\frac{1}{3}x\right) = \frac{1}{3}L(x)$  for all  $x \in \mathcal{A}$ . Thus, by (2.9),

$$L(\lambda x) = L\left(\frac{M}{3} \cdot 3\frac{\lambda}{M}x\right) = M \cdot L\left(\frac{1}{3} \cdot 3\frac{\lambda}{M}x\right)$$
$$= \frac{M}{3}L\left(3\frac{\lambda}{M}x\right)$$
$$= \frac{M}{3}L(\mu_1 x + \mu_2 x + \mu_3 x)$$
$$= \frac{M}{3}(L(\mu_1 x) + L(\mu_2 x) + L(\mu_3 x))$$
$$= \frac{M}{3}(\mu_1 + \mu_2 + \mu_3)L(x)$$
$$= \frac{M}{3} \cdot 3\frac{\lambda}{M}L(x) = \lambda L(x)$$

for all  $x \in \mathcal{A}$ . Hence,

$$L(\zeta x_1 + \eta x_2) = L(\zeta x_1) + L(\eta x_2) = \zeta L(x_1) + \eta L(x_2)$$

for all  $\zeta, \eta \in \mathbb{C}$   $(\zeta, \eta \neq 0)$  and all  $x_1, x_2 \in \mathcal{A}$ , and L(0x) = 0 = 0L(x) for all  $x \in \mathcal{A}$ . So,  $L : \mathcal{A} \to \mathcal{B}$  is  $\mathbb{C}$ -linear. Let  $x_i = 0$   $(i \ge 0)$  in (2.2). Then, we get

$$\|h(z \circ w) - h(z) \circ h(w)\| \le \varphi(\underbrace{0, \cdots, 0}_{n-times}, z, w)$$

for all  $z, w \in A$ . Since

$$\frac{1}{2^{2m}}\varphi(\underbrace{0,\cdots,0}_{n-times},2^mz,2^mw) \leq \frac{1}{2^m}\varphi(\underbrace{0,\cdots,0}_{n-times},2^mz,2^mw),$$

$$\begin{split} \frac{1}{2^{2m}} \left\| h(2^m z \circ 2^m w) - h(2^m z) \circ h(2^m w) \right\| \\ &\leq \frac{1}{2^{2m}} \varphi(\underbrace{0, \dots, 0}_{n-times}, z, w) \\ &\leq \frac{1}{2^m} \varphi(\underbrace{0, \dots, 0}_{n-times}, z, w), \end{split}$$

which tends to zero as  $m \to \infty$  for all  $z, w \in A$ . Hence,

$$L(z \circ w) = \lim_{m \to \infty} \frac{h(2^{2m}(z \circ w))}{2^{2m}}$$
$$= \lim_{m \to \infty} \frac{h(2^m z \circ 2^m w)}{2^{2m}}$$
$$= \lim_{m \to \infty} \frac{1}{2^{2m}} \left(h(2^m z) \circ h(2^m w)\right)$$
$$= \lim_{m \to \infty} \left(\frac{h(2^m z)}{2^m} \circ \frac{h(2^m w)}{2^m}\right)$$
$$= L(z) \circ L(w)$$

for all  $z, w \in A$ . So, the  $\mathbb{C}$ -linear mapping  $L : A \to B$  is a Jordan homomorphism satisfying (2.3).

**Corollary 2.3.** Let  $h : A \to B$  be a mapping with h(0) = 0 for which there exist constants  $\epsilon \ge 0$  and  $p \in [0, 1)$  such that

$$\left\| \sum_{k=2}^{n} \sum_{i_{1}=2}^{k} \sum_{i_{2}=i_{1}+1}^{k+1} \cdots \sum_{i_{n}-k+1=i_{n-k}+1}^{n} \\ \times h\left( \sum_{i=1, i \neq i_{1}, \cdots, i_{n-k+1}}^{n} \mu x_{i} - \sum_{r=1}^{n-k+1} \mu x_{i_{r}} \right) \\ + \mu h\left( \sum_{i=1}^{n} x_{i} \right) - \mu 2^{n-1} h(x_{1}) + h(z \circ w) - h(z) \circ h(w) \\ \leq \epsilon (\|x_{1}\|^{p} + \dots + \|x_{n}\|^{p} + \|z\|^{p} + \|w\|^{p})$$

for all  $\mu \in \mathbb{T}^1$  and all  $x_1, x_2, ..., x_n, z, w \in \mathcal{A}$ . Then, there exists a unique Jordan homomorphism  $L : \mathcal{A} \to \mathcal{B}$  such that

$$||h(x) - L(x)|| \le \frac{\epsilon}{2^n(1-2^{p-1})} ||x||^p$$

for all  $x \in A$ .

*Proof.* Define  $\varphi(x_1, \dots, x_n, z, w) = \epsilon(||x_1||^p + \dots + ||x_n||^p + ||z||^p + ||w||^p)$  and apply Theorem 2.2 Then, we get the desired result.

**Corollary 2.4.** Suppose that  $h : \mathcal{A} \to \mathcal{B}$  is mapping with h(0) = 0 satisfying (2.2) If there exists a function  $\varphi : \mathcal{A}^{n+2} \to [0, \infty)$  such that

$$\tilde{\varphi}(x_1,\cdots,x_n,z,w) := \sum_{j=0}^{\infty} 2^j \varphi(2^{-j}x_1,\cdots,2^{-j}x_n,2^{-j}z,2^{-j}w) < \infty$$
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for all  $z, w, x_i \in A$  (i = 1, ..., n), then there exists a unique Jordan homomorphism  $L : A \to B$  such that

$$\|h(x) - L(x)\| \le \frac{1}{2^{n-1}}\widetilde{\varphi}(x, x, \underbrace{0 \dots 0}_{n-times})$$

for all  $x \in A$ .

*Proof.* By the same method as in the proof of Theorem 2.2 one can obtain that

$$L(x) = \lim_{m \to \infty} \frac{h(2^m x)}{2^m}$$

for all  $x \in A$ .

The rest of the proof is similar to the proof of Theorem 2.2.

**Theorem 2.5.** Let  $h : \mathcal{A} \to \mathcal{B}$  be a mapping with h(0) = 0 for which there exists a function  $\varphi : \mathcal{A}^{n+2} \to [0, \infty)$  satisfying (2.1) such that

$$\left\| \sum_{k=2}^{n} \sum_{i_{1}=2}^{k} \sum_{i_{2}=i_{1}+1}^{k+1} \cdots \sum_{i_{n}-k+1=i_{n-k}+1}^{n} \right. \\ \left. \times h\left( \sum_{i=1, i \neq i_{1}, \cdots, i_{n-k+1}} \mu x_{i} - \sum_{r=1}^{n-k+1} \mu x_{i_{r}} \right) \right. \\ \left. + \mu h\left( \sum_{i=1}^{n} x_{i} \right) - \mu 2^{n-1} h(x_{1}) + h(z \circ w) - h(z) \circ h(w) \right\| \\ \left. \leqslant \varphi(x_{1}, \cdots, x_{n}, z, w) \right.$$
(2.10)

for  $\mu = 1$ , *i* and all  $x_1, \dots, x_n, z, w \in A$ . If h(tx) is continuous in  $t \in \mathbb{R}$  for each fixed  $x \in A$ , then there exists a unique Jordan homomorphism  $L : A \to B$  satisfying (2.3).

*Proof.* Put z = w = 0 in (2.10). By the same reasoning as in the proof of Theorem 2.2, there exists a unique additive mapping  $L : A \to B$  satisfying (2.3). The additive mapping  $L : A \to B$  is given by

$$L(x) = \lim_{m \to \infty} \frac{h(2^m x)}{2^m}$$

for all  $x \in A$ . By the same reasoning as in the proof of Theorem 2.2 the additive mapping  $L : A \to B$  is  $\mathbb{R}$ -linear.

Putting  $x_i = z = w = 0$   $(i = 2, \dots, n)$  and  $\mu = i$  in (2.10), we get

$$\|h(ix) - ih(x)\| \le \varphi(x, \underbrace{0, \cdots, 0}_{(n+1)-times})$$

for all  $x \in A$ . So,

$$\frac{1}{2^n} \|h(2^m ix) - ih(2^m x)\| \le \frac{1}{2^n} \varphi(2^n x, \underbrace{0, \dots, 0}_{(n+1)-times}),$$

which tends to zero as  $m \to \infty$ . Hence,

$$L(ix) = \lim_{m \to \infty} \frac{h(2^m ix)}{2^m} = \lim_{m \to \infty} \frac{ih(2^m x)}{2^m} = iL(x)$$

for all  $x \in A$ .

For each element  $\lambda \in \mathbb{C}$ ,  $\lambda = s + it$ , where  $s, t \in \mathbb{R}$ . So,

$$L(\lambda x) = L(sx + itx) = sL(x) + tL(ix) = sL(x) + itL(x)$$
$$= (s + it)L(x) = \lambda L(x)$$

for all  $x \in \mathcal{A}$ . So,

$$L(\zeta x_1 + \eta x_2) = L(\zeta x_1) + L(\eta x_2) = \zeta L(x_1) + \eta L(x_2)$$

for all  $\zeta, \eta \in \mathbb{C}$ , and all  $x_1, x_2 \in \mathcal{A}$ . Hence, the additive mapping  $L : \mathcal{A} \to \mathcal{B}$  is  $\mathbb{C}$ -linear.

The rest of the proof is the same as in the proof of Theorem 2.2

**Corollary 2.6.** Let  $h : A \to B$  be a mapping with h(0) = 0 for which there exist constants  $\epsilon \ge 0$  and p > 1 such that

$$\begin{aligned} & \left\| \sum_{k=2}^{n} \sum_{i_{1}=2}^{k} \sum_{i_{2}=i_{1}+1}^{k+1} \cdots \sum_{i_{n}-k+1=i_{n-k}+1}^{n} \\ & \times h\left( \sum_{i=1, i \neq i_{1}, \cdots, i_{n-k+1}}^{n} \mu x_{i} - \sum_{r=1}^{n-k+1} \mu x_{i_{r}} \right) \\ & + \mu h\left( \sum_{i=1}^{n} x_{i} \right) - \mu 2^{n-1} h(x_{1}) + h(z \circ w) - (h(z) \circ h(w)) \right\| \\ & \leq \epsilon (\|x_{1}\|^{p} + \ldots + \|x_{n}\|^{p} + \|z\|^{p} + \|w\|^{p}) \end{aligned}$$

for all  $z, w, x_i \in \mathcal{A}$   $(i = 1, 2, \dots, n)$  and all  $\mu \in \mathbb{T}^1$ . Then, there exists a unique Jordan homomorphism  $L : \mathcal{A} \to \mathcal{B}$  such that

$$||h(x) - L(x)|| \le \frac{\epsilon}{2^n(2^{1-p} - 1)} ||x||^p$$

for all  $x \in A$ .

*Proof.* Define  $\varphi(x_1, \dots, x_n, z, w) = \epsilon(||x_1||^p + \dots + ||x_n||^p + ||z||^p + ||w||^p)$  and apply Theorem 2.2 Then, we get the desired result.

#### Conclusions

We have proved the Hyers-Ulam stability of Jordan homomorphisms in Jordan-Banach algebras for the functional equation (1.1).

#### **Competing interests**

The authors declare that they have no competing interests.

#### Authors' contributions

All authors conceived the study, participated in its design and coordination, drafted the manuscript, participated in the sequence alignment, and read and approved the final manuscript.

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