Gordji et al. Mathematical Sciences 2012, 6:55
http://www.iaumath.com/content/6/1/55

Mathematical Sciences
a SpringerOpen Journal

ORIGINAL RESEARCH Open Access

Approximation of Jordan homomorphisms in Jordan-Banach algebras

Madjid Eshaghi Gordji[1*], Najmeh Karimipour Samani[1] and Choonkil Park[2*]

Abstract

Using the direct method based on the Hyers-Ulam-Rassias stability, we investigate and prove the Hyers-Ulam stability of Jordan homomorphisms in Jordan-Banach algebras for the functional equation

$$\sum_{k=2}^{n}\sum_{i_1=2}^{k}\sum_{i_2=i_1+1}^{k+1}\cdots\sum_{i_{n-k+1}=i_{n-k}+1}^{n} f\left(\sum_{i=1,i\neq i_1,\cdots,i_{n-k+1}}^{n} x_i-\sum_{r=1}^{n-k+1} x_{i_r}\right)+f\left(\sum_{i=1}^{n} x_i\right)-2^{n-1}f(x_1)=0,$$

where n is an integer greater than 1.

We have proved the Hyers-Ulam stability of Jordan homomorphisms in Jordan-Banach algebras for the above functional equation.

Keywords: Hyers-Ulam stability, Jordan homomorphism, Jordan algebra

2010 MSC: 39B52; 17C65.

Introduction

A classical question in the theory of functional equations is that 'when is it true that a function which approximately satisfies a functional equation $\mathcal{E}$ must be somehow close to an exact solution of $\mathcal{E}$.' Such a problem was formulated by Ulam [1] in 1940 and solved in the next year for the Cauchy functional equation by Hyers [2]. It gave rise to the *stability theory* for functional equations. The result of Hyers was generalized by Aoki [3] for approximate additive functions and by Rassias [4] for approximate linear functions. The stability phenomenon that was proved by Rassias is called the *Hyers-Ulam-Rassias stability* or the *generalized Hyers-Ulam stability* of functional equations. In 1994, a generalization of the Th.M. Rassias' theorem was obtained by Găvruta [5] as follows: Suppose that $(G,+)$ is an abelian group and E is a Banach space and that the so-called admissible control function $\varphi : G\times G\to\mathbb{R}$ satisfies

$$\tilde{\varphi}(x,y):=\sum_{n=0}^{\infty}2^{-n}\varphi(2^n x,2^n y)<\infty$$

for all $x,y\in G$. If $f:G\to E$ is a mapping with

$$\|f(x+y)-f(x)-f(y)\|\le\varphi(x,y)$$

for all $x,y\in G$, then there exists a unique mapping $T:G\to E$ such that $T(x+y)=T(x)+T(y)$ and $\|f(x)-T(x)\|\le\tilde{\varphi}(x,x)$ for all $x,y\in G$. If, moreover, G is a real normed space and $f(tx)$ is continuous in t for each fixed x in G, then T is a linear function.

The stability problems of several functional equations have been extensively investigated by a number of authors, and there are many interesting results concerning this problem (see [6-27]).

Recently, Eshaghi Gordji et al. (unpublished work) defined the following n-dimensional additive functional equation

$$D_f(x_1,\cdots,x_n):=\sum_{k=2}^{n}\sum_{i_1=2}^{k}\sum_{i_2=i_1+1}^{k+1}\cdots\sum_{i_{n-k+1}=i_{n-k}+1}^{n}\times f\left(\sum_{i=1,i\neq i_1^n,\cdots,i_{n-k+1}}^{n} x_i-\sum_{r=1}^{n-k+1} x_{i_r}\right)+f\left(\sum_{i=1}^{n} x_i\right)-2^{n-1}f(x_1)=0, \quad (1.1)$$

*Correspondence: meshaghi@semnan.ac.ir; baak@hanyang.ac.kr
[1] Department of Mathematics, Semnan University, Semnan, 35195-363, Iran
Full list of author information is available at the end of the article

where n is an integer greater than 1, and investigated the functional equation (1.1) in random normed spaces the via the fixed point method.

Note that a unital algebra A, endowed with the Jordan product $x \circ y = \frac{1}{2}(xy + yx)$ on A, is called a Jordan algebra. A $\mathbb{C}$-linear mapping L of a Jordan algebra A into a Jordan algebra B is called a Jordan homomorphism if $L(x \circ y) = (L(x) \circ L(y))$ holds for all $x, y \in A$.

Throughout this paper, let A be a Jordan-Banach algebra with norm $\| \cdot \|$ and unit e, and let B be a Jordan-Banach algebra with norm $\| \cdot \|$.

Methods

Using the direct method based on the Hyers-Ulam-Rassias stability, we prove the Hyers-Ulam stability of Jordan homomorphisms in Jordan-Banach algebras for the functional equation (1.1).

Results and discussion

We need the following lemma in the proof of our main theorem.

Lemma 2.1. *(Eshaghi Gordji et al., unpublished work) A mapping $f : \mathcal{A} \to \mathcal{B}$ with $f(0) = 0$ satisfies (1.1) if and only if $f : \mathcal{A} \to \mathcal{B}$ is additive.*

We are going to prove the main result.

Theorem 2.2. *Let $h : \mathcal{A} \to \mathcal{B}$ be a mapping with $h(0) = 0$ for which there exists a function $\varphi : \mathcal{A}^{n+2} \to [0, \infty)$ such that*

$$\tilde{\varphi}(x_1, \cdots x_n, z, w) := \sum_{j=0}^{\infty} 2^{-j} \varphi(2^j x_1, \cdots 2^j x_n, 2^j z, 2^j w) < \infty, \tag{2.1}$$

$$\begin{aligned} &\Bigg\| \sum_{k=2}^{n} \sum_{i_1=2}^{k} \sum_{i_2=i_1+1}^{k+1} \cdots \sum_{i_n-k+1=i_{n-k}+1}^{n} \\ &\quad \times h\left(\sum_{i=1, i \neq i_1, \cdots, i_{n-k+1}}^{n} \mu x_i - \sum_{r=1}^{n-k+1} \mu x_{i_r} \right) \\ &\quad + \mu h\left(\sum_{i=1}^{n} x_i \right) - \mu 2^{n-1} h(x_1) + h(z \circ w) - h(z) \circ h(w) \Bigg\| \\ &\quad \leqslant \varphi(x_1, \ldots x_n, z, w) \end{aligned} \tag{2.2}$$

for all $\mu \in T^1 := \{\lambda \in \mathbb{C} | \ |\lambda| = 1\}$ and $x_1, \ldots x_n, z, w \in \mathcal{A}$. Then, there exists a unique Jordan homomorphism $L : \mathcal{A} \to \mathcal{B}$ such that

$$\|h(x) - L(x)\| \leq \frac{1}{2^{n-1}} \tilde{\varphi}(x, x, \underbrace{0 \ldots 0}_{n-times}) \tag{2.3}$$

for all $x \in \mathcal{A}$.

Proof. Let $\mu = 1$. Using the relation

$$1 + \sum_{k=1}^{n-k} \binom{n-k}{k} = \sum_{k=0}^{n-k} \binom{n-k}{k} = 2^{n-k} \tag{2.4}$$

for all $n > k$ and putting $x_1 = x_2 = x$ and $x_i = z = w = 0$ $(i = 3, \ldots, n)$ in (2.2), we obtain

$$\left\| 2^{n-2} h(2x) - 2^{n-1} h(x) \right\| \leq \varphi(x, x, \underbrace{0, \ldots, 0}_{n-times}) \tag{2.5}$$

for all $x \in \mathcal{A}$. So,

$$\left\| \frac{h(2x)}{2} - h(x) \right\| \leq \frac{1}{2^{n-1}} \varphi(x, x, \underbrace{0, \ldots, 0}_{n-times}) \tag{2.6}$$

for all $x \in \mathcal{A}$. By induction on m, we can show that

$$\left\| \frac{h(2^m x)}{2^m} - h(x) \right\| \leq \frac{1}{2^{n-1}} \sum_{j=0}^{m-1} \frac{1}{2^j} \varphi(2^j x, 2^j x, \underbrace{0, \ldots, 0}_{n-times}) \tag{2.7}$$

for all $x \in \mathcal{A}$. It follows from (2.1) and (2.7) that the sequence $\left\{ \frac{h(2^m x)}{2^m} \right\}$ is a Cauchy sequence for all $x \in \mathcal{A}$. Since $\mathcal{A}$ is complete, the sequence $\left\{ \frac{h(2^m x)}{2^m} \right\}$ converges. Thus, one can define the mapping $L : \mathcal{A} \to \mathcal{B}$ by

$$L(x) := \lim_{m \to \infty} \frac{h(2^m x)}{2^m}$$

for all $x \in \mathcal{A}$. Let $z = w = 0$ and $\mu = 1$ in (2.2). By (2.1)

$$\begin{aligned} \|D_f(x_1, ..., x_n)\| &= \lim_{j \to \infty} \frac{1}{2^j} \left\| D_f\left(2^j x_1, ..., 2^j x_n\right) \right\| \\ &\leq \lim_{j \to \infty} \frac{1}{2^j} \varphi\left(2^j x_1, ..., 2^j x_n, 0, 0\right) = 0 \end{aligned}$$

for all $x_1, \cdots, x_n \in \mathcal{A}$. So, $D_f(x_1, \cdots, x_n) = 0$. By Lemma 2.1, the mapping $L : \mathcal{A} \to \mathcal{B}$ is additive. Moreover, passing the limit $m \to \infty$ in (2.7) we get the inequality (2.3). □

Now, let $L' : \mathcal{A} \to \mathcal{B}$ be another additive mapping satisfying (1.1) and (2.3). Then,

$$\begin{aligned} \|L(x) - L'(x)\| &= \frac{1}{2^n} \|L(2^n x) - L'(2^n x)\| \\ &\leq \frac{1}{2^m} (\|L(2^n x) - h(2^n x)\| + \|L'(2^n x) \\ &\quad - h(2^n x)\|) \\ &\leq \frac{2}{2^m 2^{n-1}} \widetilde{\varphi}(2^m x, 2^m x, \underbrace{0, \ldots, 0}_{n-times}) \end{aligned}$$

which tends to zero as $m \to \infty$ for all $x \in \mathcal{A}$. So, we can conclude that $L(x) = L'(x)$ for all $x \in \mathcal{A}$. This proves the uniqueness of L.

Let $\mu \in \mathbb{T}^1$. Set $x_1 = x$ and $z = w = x_i = 0 \ \ (i = 2, ..., n)$ in (2.2). Then, by (2.1), we get

$$\|2^{n-1}h(\mu x) - 2^{n-1}\mu h(x)\| \le \varphi(x, 0, ..., 0, 0, 0) \quad (2.8)$$

for all $x \in \mathcal{A}$. So,

$$\|2^{-m}(h(2^m\mu x) - \mu h(2^m x))\| \le \frac{2^{-m}}{2^{n-1}}\varphi(2^m x, 0, ..., 0, 0, 0)$$

for all $x \in \mathcal{A}$. Since the right hand side of the above inequality tends to zero as $m \to \infty$, we have

$$L(\mu x) = \lim_{m\to\infty}\frac{h(2^m\mu x)}{2^m} = \lim_{m\to\infty}\frac{\mu h(2^m x)}{2^m} = \mu L(x) \quad (2.9)$$

for all $\mu \in \mathbb{T}^1$ and all $x \in \mathcal{A}$.

Now let $\lambda \in \mathbb{C}(\lambda \neq 0)$ and M an integer greater than $4|\lambda|$. Then, $|\lambda/M| < 1/4 < 1 - 2/3 = 1/3$. By Theorem 1 of [28], there exist three elements $\mu_1, \mu_2, \mu_3 \in \mathbb{T}^1$ such that $3\frac{\lambda}{M} = \mu_1 + \mu_2 + \mu_3$, and $L(x) = L\left(3 \cdot \frac{1}{3}x\right) = 3L\left(\frac{1}{3}x\right)$ for all $x \in \mathcal{A}$. So, $L\left(\frac{1}{3}x\right) = \frac{1}{3}L(x)$ for all $x \in \mathcal{A}$. Thus, by (2.9),

$$\begin{aligned} L(\lambda x) &= L\left(\frac{M}{3}\cdot 3\frac{\lambda}{M}x\right) = M \cdot L\left(\frac{1}{3}\cdot 3\frac{\lambda}{M}x\right) \\ &= \frac{M}{3}L\left(3\frac{\lambda}{M}x\right) \\ &= \frac{M}{3}L(\mu_1 x + \mu_2 x + \mu_3 x) \\ &= \frac{M}{3}(L(\mu_1 x) + L(\mu_2 x) + L(\mu_3 x)) \\ &= \frac{M}{3}(\mu_1 + \mu_2 + \mu_3)L(x) \\ &= \frac{M}{3}\cdot 3\frac{\lambda}{M}L(x) = \lambda L(x) \end{aligned}$$

for all $x \in \mathcal{A}$. Hence,

$$L(\zeta x_1 + \eta x_2) = L(\zeta x_1) + L(\eta x_2) = \zeta L(x_1) + \eta L(x_2)$$

for all $\zeta, \eta \in \mathbb{C} \ \ (\zeta, \eta \neq 0)$ and all $x_1, x_2 \in \mathcal{A}$, and $L(0x) = 0 = 0L(x)$ for all $x \in \mathcal{A}$.

So, $L : \mathcal{A} \to \mathcal{B}$ is $\mathbb{C}$-linear.

Let $x_i = 0 \ \ (i \ge 0)$ in (2.2). Then, we get

$$\|h(z \circ w) - h(z) \circ h(w)\| \le \varphi(\underbrace{0, \cdots, 0}_{n-times}, z, w)$$

for all $z, w \in \mathcal{A}$. Since

$$\frac{1}{2^{2m}}\varphi(\underbrace{0, \cdots, 0}_{n-times}, 2^m z, 2^m w) \le \frac{1}{2^m}\varphi(\underbrace{0, \cdots, 0}_{n-times}, 2^m z, 2^m w),$$

$$\begin{aligned} &\frac{1}{2^{2m}}\left\|h(2^m z \circ 2^m w) - h(2^m z) \circ h(2^m w)\right\| \\ &\quad \le \frac{1}{2^{2m}}\varphi(\underbrace{0, \ldots, 0}_{n-times}, z, w) \\ &\quad \le \frac{1}{2^m}\varphi(\underbrace{0, \ldots, 0}_{n-times}, z, w), \end{aligned}$$

which tends to zero as $m \to \infty$ for all $z, w \in \mathcal{A}$. Hence,

$$\begin{aligned} L(z \circ w) &= \lim_{m\to\infty}\frac{h\left(2^{2m}(z \circ w)\right)}{2^{2m}} \\ &= \lim_{m\to\infty}\frac{h(2^m z \circ 2^m w)}{2^{2m}} \\ &= \lim_{m\to\infty}\frac{1}{2^{2m}}\left(h(2^m z) \circ h(2^m w)\right) \\ &= \lim_{m\to\infty}\left(\frac{h(2^m z)}{2^m} \circ \frac{h(2^m w)}{2^m}\right) \\ &= L(z) \circ L(w) \end{aligned}$$

for all $z, w \in \mathcal{A}$. So, the $\mathbb{C}$-linear mapping $L : \mathcal{A} \to \mathcal{B}$ is a Jordan homomorphism satisfying (2.3).

Corollary 2.3. *Let $h : \mathcal{A} \to \mathcal{B}$ be a mapping with $h(0) = 0$ for which there exist constants $\epsilon \ge 0$ and $p \in [0, 1)$ such that*

$$\begin{aligned} &\left\|\sum_{k=2}^{n}\sum_{i_1=2}^{k}\sum_{i_2=i_1+1}^{k+1}\cdots\sum_{i_{n-k+1}=i_{n-k}+1}^{n}\right. \\ &\times h\left(\sum_{i=1, i\neq i_1,\cdots,i_{n-k+1}}^{n}\mu x_i - \sum_{r=1}^{n-k+1}\mu x_{i_r}\right) \\ &\left.+\mu h\left(\sum_{i=1}^{n}x_i\right) - \mu 2^{n-1}h(x_1) + h(z \circ w) - h(z) \circ h(w)\right\| \\ &\le \epsilon(\|x_1\|^p + \cdots + \|x_n\|^p + \|z\|^p + \|w\|^p) \end{aligned}$$

for all $\mu \in \mathbb{T}^1$ and all $x_1, x_2, ..., x_n, z, w \in \mathcal{A}$. Then, there exists a unique Jordan homomorphism $L : \mathcal{A} \to \mathcal{B}$ such that

$$\|h(x) - L(x)\| \le \frac{\epsilon}{2^n(1 - 2^{p-1})}\|x\|^p$$

for all $x \in \mathcal{A}$.

Proof. Define $\varphi(x_1, \cdots x_n, z, w) = \epsilon(\|x_1\|^p + \cdots + \|x_n\|^p + \|z\|^p + \|w\|^p)$ and apply Theorem 2.2 Then, we get the desired result. □

Corollary 2.4. *Suppose that $h : \mathcal{A} \to \mathcal{B}$ is mapping with $h(0) = 0$ satisfying (2.2) If there exists a function $\varphi : \mathcal{A}^{n+2} \to [0, \infty)$ such that*

$$\tilde{\varphi}(x_1, \cdots x_n, z, w) := \sum_{j=0}^{\infty}2^j\varphi(2^{-j}x_1, \cdots 2^{-j}x_n, 2^{-j}z, 2^{-j}w) < \infty$$

for all $z, w, x_i \in \mathcal{A}$ $(i = 1, ..., n)$, then there exists a unique Jordan homomorphism $L : \mathcal{A} \to \mathcal{B}$ such that

$$\|h(x) - L(x)\| \leq \frac{1}{2^{n-1}} \widetilde{\varphi}(x, x, \underbrace{0 \ldots 0}_{n-times})$$

for all $x \in \mathcal{A}$.

Proof. By the same method as in the proof of Theorem 2.2 one can obtain that

$$L(x) = \lim_{m \to \infty} \frac{h(2^m x)}{2^m}$$

for all $x \in \mathcal{A}$. □

The rest of the proof is similar to the proof of Theorem 2.2.

Theorem 2.5. *Let $h : \mathcal{A} \to \mathcal{B}$ be a mapping with $h(0) = 0$ for which there exists a function $\varphi : \mathcal{A}^{n+2} \to [0, \infty)$ satisfying (2.1) such that*

$$\Bigg\| \sum_{k=2}^{n} \sum_{i_1=2}^{k} \sum_{i_2=i_1+1}^{k+1} \cdots \sum_{i_n-k+1=i_{n-k}+1}^{n} \times h \left(\sum_{i=1, i \neq i_1, \cdots, i_{n-k+1}} \mu x_i - \sum_{r=1}^{n-k+1} \mu x_{i_r} \right) + \mu h \left(\sum_{i=1}^{n} x_i \right) - \mu 2^{n-1} h(x_1) + h(z \circ w) - h(z) \circ h(w) \Bigg\| \leqslant \varphi(x_1, \cdots x_n, z, w) \quad (2.10)$$

for $\mu = 1, i$ and all $x_1, \cdots, x_n, z, w \in \mathcal{A}$. If $h(tx)$ is continuous in $t \in \mathbb{R}$ for each fixed $x \in \mathcal{A}$, then there exists a unique Jordan homomorphism $L : \mathcal{A} \to \mathcal{B}$ satisfying (2.3).

Proof. Put $z = w = 0$ in (2.10). By the same reasoning as in the proof of Theorem 2.2, there exists a unique additive mapping $L : \mathcal{A} \to \mathcal{B}$ satisfying (2.3). The additive mapping $L : \mathcal{A} \to \mathcal{B}$ is given by

$$L(x) = \lim_{m \to \infty} \frac{h(2^m x)}{2^m}$$

for all $x \in \mathcal{A}$. By the same reasoning as in the proof of Theorem 2.2 the additive mapping $L : \mathcal{A} \to \mathcal{B}$ is $\mathbb{R}$-linear. □

Putting $x_i = z = w = 0$ $(i = 2, \cdots, n)$ and $\mu = i$ in (2.10), we get

$$\|h(ix) - ih(x)\| \leq \varphi(x, \underbrace{0, \cdots, 0}_{(n+1)-times})$$

for all $x \in \mathcal{A}$. So,

$$\frac{1}{2^n} \left\| h(2^m ix) - ih(2^m x) \right\| \leq \frac{1}{2^n} \varphi(2^n x, \underbrace{0, \ldots, 0}_{(n+1)-times}),$$

which tends to zero as $m \to \infty$. Hence,

$$L(ix) = \lim_{m \to \infty} \frac{h(2^m ix)}{2^m} = \lim_{m \to \infty} \frac{ih(2^m x)}{2^m} = iL(x)$$

for all $x \in \mathcal{A}$.

For each element $\lambda \in \mathbb{C}$, $\lambda = s + it$, where $s, t \in \mathbb{R}$. So,

$$L(\lambda x) = L(sx + itx) = sL(x) + tL(ix) = sL(x) + itL(x) = (s + it)L(x) = \lambda L(x)$$

for all $x \in \mathcal{A}$. So,

$$L(\zeta x_1 + \eta x_2) = L(\zeta x_1) + L(\eta x_2) = \zeta L(x_1) + \eta L(x_2)$$

for all $\zeta, \eta \in \mathbb{C}$, and all $x_1, x_2 \in \mathcal{A}$. Hence, the additive mapping $L : \mathcal{A} \to \mathcal{B}$ is $\mathbb{C}$-linear.

The rest of the proof is the same as in the proof of Theorem 2.2

Corollary 2.6. *Let $h : \mathcal{A} \to \mathcal{B}$ be a mapping with $h(0) = 0$ for which there exist constants $\epsilon \geq 0$ and $p > 1$ such that*

$$\Bigg\| \sum_{k=2}^{n} \sum_{i_1=2}^{k} \sum_{i_2=i_1+1}^{k+1} \cdots \sum_{i_n-k+1=i_{n-k}+1}^{n} \times h \left(\sum_{i=1, i \neq i_1, \cdots, i_{n-k+1}}^{n} \mu x_i - \sum_{r=1}^{n-k+1} \mu x_{i_r} \right) + \mu h \left(\sum_{i=1}^{n} x_i \right) - \mu 2^{n-1} h(x_1) + h(z \circ w) - (h(z) \circ h(w)) \Bigg\| \leq \epsilon (\|x_1\|^p + \ldots + \|x_n\|^p + \|z\|^p + \|w\|^p)$$

for all $z, w, x_i \in \mathcal{A}$ $(i = 1, 2, \cdots, n)$ and all $\mu \in \mathbb{T}^1$. Then, there exists a unique Jordan homomorphism $L : \mathcal{A} \to \mathcal{B}$ such that

$$\|h(x) - L(x)\| \leq \frac{\epsilon}{2^n (2^{1-p} - 1)} \|x\|^p$$

for all $x \in \mathcal{A}$.

Proof. Define $\varphi(x_1, \cdots x_n, z, w) = \epsilon(\|x_1\|^p + \cdots + \|x_n\|^p + \|z\|^p + \|w\|^p)$ and apply Theorem 2.2 Then, we get the desired result. □

Conclusions

We have proved the Hyers-Ulam stability of Jordan homomorphisms in Jordan-Banach algebras for the functional equation (1.1).

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors conceived the study, participated in its design and coordination, drafted the manuscript, participated in the sequence alignment, and read and approved the final manuscript.

Author details
[1]Department of Mathematics, Semnan University, Semnan, 35195-363, Iran. [2]Research Institute for Natural Sciences, Hanyang University, Seoul, 133-791, Korea.

Received: 7 July 2012 Accepted: 12 August 2012
Published: 24 October 2012

References
1. Ulam, SM: A Collection of the Mathematical Problems. Interscience Publishers, New York (1960)
2. Hyers, DH: On the stability of the linear functional equation. Proc. Nat. Acad. Sci. **27**, 222–224 (1941)
3. Aoki, T: On the stability of the linear transformation in Banach spaces. J. Math. Soc. Japan. **2**, 64–66 (1950)
4. Rassias, TM: On the stability of the linear mapping in Banach spaces Proc. Amer. Math. Soc. **72**, 297–300 (1978)
5. Găvruta, P: A generalization of the Hyers-Ulam-Rassias stability of approximately additive mappings. J. Math. Anal. Appl. **184**, 431–436 (1994)
6. Badora, R: On approximate ring homomorphisms. J. Math. Anal. Appl. **276**, 589–597 (2002)
7. Badora, R: On approximate derivations. Math. Inequal. Appl. **9**, 167–173 (2006)
8. Czerwik, S: Stability of Functional Equations of Ulam-Hyers-Rassias Type. Hadronic Press, Palm Harbor (2003)
9. Gajda, Z: On stability of additive mappings. Internat. J. Math. Math. Sci. **14**, 431–434 (1991)
10. Găvruta, P, Găvruta, L: A new method for the generalized Hyers-Ulam-Rassias stability. Int. J. Nonlinear Anal. Appl. **1**(2), 11–18 (2010)
11. Jun, K, Lee, Y: A generalization of the Hyers-Ulam-Rassias stability of Jensen's equation. J. Math. Anal. Appl. **238**, 305–315 (1999)
12. Khodaei, H, Kamyar, M: Fuzzy approximately additive mappings. Int. J. Nonlinear Anal. Appl. **1**(2), 44–53 (2010)
13. Khodaei, H, Rassias, TM: Approximately generalized additive functions in several variables. Int. J. Nonlinear Anal. Appl. **1**(1), 22–41 (2010)
14. Park, C: On the stability of the linear mapping in Banach modules. J. Math. Anal. Appl. **275**, 711–720 (2002)
15. Park, C: Modified Trif's functional equations in Banach modules over a C^*-algebra and approximate algebra homomorphisms. J. Math. Anal. Appl. **278**, 93–108 (2003)
16. Park, C: Lie $*$-homomorphisms between Lie C^*-algebras and Lie $*$-derivations on Lie C^*-algebras. J. Math. Anal. Appl. **293**, 419–434 (2004)
17. Park, C, Park, W: On the Jensen's equation in Banach modules. Taiwanese J. Math. **6**, 523–531 (2002)
18. Park, C, Eshaghi Gordji, M: Comment on "Approximate ternary Jordan derivations on Banach ternary algebras" [Bavand Savadkouhi et al. J. Math. Phys. 50, 042303 (2009)]. J. Math. Phys. **51**, 044102 (2010). doi:10.1063/1.3299295
19. Park, C, Najati, A: Generalized additive functional inequalities in Banach algebras. Int. J. Nonlinear Anal. Appl. **1**(2), 54–62 (2010)
20. Park, C, Rassias, TM: Isomorphisms in unital C^*-algebras. Int. J. Nonlinear Anal. Appl. **1**(2), 1–10 (2010)
21. Rassias, JM: On approximation of approximately linear mappings by linear mappings. J. Funct. Anal. **46**, 126–130 (1982)
22. Rassias, JM: On approximation of approximately linear mappings by linear mappings. Bull. Sci. Math. **108**, 445–446 (1984)
23. Rassias, JM: On a new approximation of approximately linear mappings by linear mappings. Discuss. Math. **7**, 193–196 (1985)
24. Rassias, JM: Solution of a problem of Ulam. J. Approx. Theory. **57**, 268–273 (1989)
25. Rassias, TM: Problem 16; 2, Report of the 27th international symposium on functional equations. Aequationes Math. **39**, 292–293 (1990)
26. Rassias, TM: Functional Equations, Inequalities and Applications. Kluwer, Boston (2003)
27. Shakeri, S, Saadati, R, Park, C: Stability of the quadratic functional equation in non-Archimedean $\mathcal{L}$-fuzzy normed spaces. Int. J. Nonlinear Anal. Appl. **1**(2), 72–83 (2010)
28. Kadison, RV, Pedersen, G: Means and convex combinations of unitary operators. Math. Scand. **57**, 249–266 (1985)

doi:10.1186/2251-7456-6-55
Cite this article as: Gordji *et al.*: **Approximation of Jordan homomorphisms in Jordan-Banach algebras.** *Mathematical Sciences* 2012 **6**:55.