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The evaluation of educational systems: an application study

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Abstract

It is more appropriate that many industrial products be evaluated and qualified by an imprecise (fuzzy) quality. By this idea the products could be evaluated using two membership functions for specification limits rather than two real numbers used in classical quality control. This idea leads the researchers to be able to deal with the vague process capability indices modeled as triangular fuzzy numbers. In this paper, we discuss on such fuzzy qualities and review some fuzzy process capability indices. Then we will bring them up to analyze several educational systems, such as comparing capability indices of two or more teachers, schools, and so on. The idea of this paper could be applied in other similar evaluation schemes as well.

Keywords: Fuzzy process capability indices, Triangular fuzzy number, Interval estimation

Introduction

The process capability index (PCI) compares the output of a process to the specification limits (SLs) by using capability indices. Frequently, this comparison is made by forming the ratio of the width between the process SLs to the width of the natural tolerance limits which is measured by 6 process standard deviation units. This method leads to make a statement about how well the process meets the specifications [1]. A process is said to be capable if with high probability, the real valued quality characteristic of the produced items lies between the lower and upper specification limits [2].

There are several statistics such as Cp, Cpk, Cpm, Cpmk, and so on, which are used to estimate the capability of a manufacturing process where in most cases, the normal distribution and a large sample size is assumed for population of data [3,4].

After the inception of the notion of fuzzy sets by Zadeh [5], there are efforts by many authors to apply this notion in statistics. For these trends one can see [6]. In quality control, we may confront imprecise concepts. One case is a situation in which the upper and lower specification limits (SLs) are imprecise. If we introduce vagueness into SLs and express SLs by fuzzy terms, we face quite new, reasonable and interesting processes, where ordinary capability indices are not appropriate for measuring the capability of these processes. Recently, several PCIs are developed for such situation by authors [7-17] and are applied in real cases [18-22]. In this paper which is an extended version of [22], some fuzzy PCIs are applied in comparing educational systems, where the point estimates and confidence region estimates for fuzzy PCIs are used in this analysis. The idea used in this paper could be applied in similar real world problems as well.

The organization of this paper is as follows. In the section Preliminaries, some preliminaries are discussed about fuzzy set and extension principle. In section Classical process capability indices, the traditional process capability indices are reviewed. In section Fuzzy process capability indices, we review the new extended process capability indices from Parchami et al. [9], also we present a method based on a binary relation for the comparison of fuzzy processes. In Section An application in educational systems, this method is clarified by an application on real data at Shahid Bahonar University of Kerman (SBUK) on the educational systems comparison. The final section is the Conclusions.

Methods

Preliminaries

Let R be the set of real numbers. Let $F(R) = \{A | A : R \to [0, 1],$ A is a continuous function$\}$, and $F_T(R) = \{T_{a,b,c} | a, b, c \in R,$ $a \le b \le c\}$, where

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$$T_{a,b,c}(x) = \begin{cases} (x-a)/(b-a) & if \quad a \le x < b \\ (c-x)/(c-b) & if \quad b \le x < c \\ 0 & \text{elsewhere.} \end{cases}$$

Any $A \in F(R)$ is called a fuzzy quantity on R and any $T_{a,b,c} \in F_T(R)$ is called a fuzzy triangular number, which we sometimes write as $T(a,b,c)$.

Now, we present a definition from [5,23].

Definition 2.1 (Extension Principle). Let $g: X \times Y \to Z$ be a function. Then g induces a function $G: F(X) \times F(Y) \to F(Z)$ as defined by

$$G(A,B)(z) = \underset{z=g(x,y)}{Sup} \min(A(x), B(y)); A \in F(X), B \in F(Y) \quad (1)$$

where the supremum over the empty set is taken to be 0.

The following definition could be given by using Definition 2.1.

Definition 2.2 Let $T(a,b,c), T(a',b',c') \in F_T(R)$, $k \in R$, $k > 0$ and $a \ge c'$. Define the operations **A** and $\oslash$ on F_T (R) as the following:

$$T(a,b,c) \ominus T(a',b',c') = T(a-c', b-b', c-a'), \quad (2)$$

which is called the width (difference) between T(a,b,c) and T′(a′,b′,c′), and

$$T(a,b,c)_{\oslash} k = T\left(\frac{a}{k}, \frac{b}{k}, \frac{c}{k}\right), \quad (3)$$

which is called the division of T(a,b,c) by k.

In order to clarify the above mentioned arithmetic operations, the following example is presented.

Example 4.1 Let T(8,12,14) ∈ FT (R) and T(17,19,20)∈ FT (R), which indicate 'approximately 12' and 'approximately 19', respectively (see Figure 1).

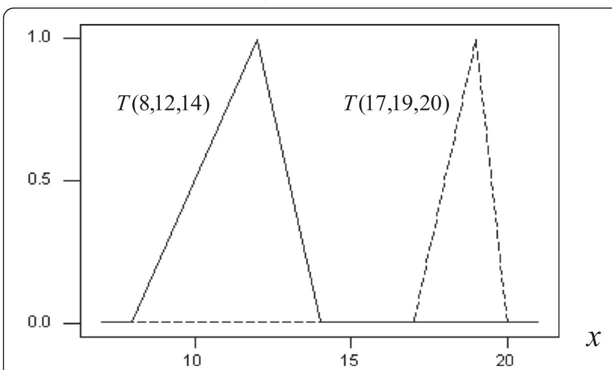

Figure 1 The membership function of two triangular fuzzy numbers in Example 4.1.

By (2) we can compute the width (difference) between two triangular fuzzy numbers as T(3,7,12), which is indicated in Figure 2.

Results and discussion

Classical process capability indices

In process improvement efforts, the process capability index or process capability ratio is a statistical measure of process capability: the ability of a process to produce output within specification limits. Several PCIs are introduced in the literature such as Cp, Cpk, Cpm and so on [2-4,24,25]. For convenience we will denote the upper and lower specification limits by U and L, respectively, rather than the more customary USL and LSL notations. When univariate measurements are concerned, we will denote the corresponding random variable by X. The expected value and standard deviation of X will be denoted by μ and σ, respectively. The commonly recognized PCIs are the following:

$$C_p = \frac{U-L}{6\sigma} = \frac{w}{6\sigma}, \quad (4)$$

where $w = U - L$. This Cp is used when μ = M with $M = (U + L)/2$.

$$C_{pk} = \frac{w - 2|\mu - M|}{6\sigma} = \frac{\min\{U-\mu, \mu-L\}}{3\sigma} \quad (5)$$

and

$$C_{pm} = \frac{w}{6\sqrt{\sigma^2 + (\mu - T)^2}} = \frac{w}{6\sqrt{E\left[(X-T)^2\right]}}, \quad (6)$$

Where T is target value and E[.] denotes the expected value.

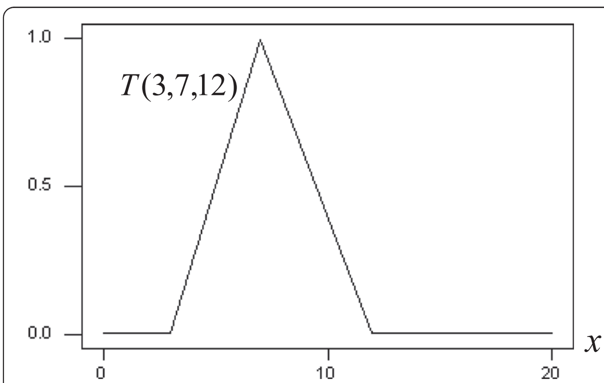

Figure 2 The membership function of difference between two triangular fuzzy numbers in Example 4.1.

There is also the hybrid index of

$$C_{pmk} = \frac{w - 2|\mu - M|}{6\sqrt{\sigma^2 + (\mu - T)^2}} = \frac{w - 2|\mu - M|}{6\sqrt{E\left[(X - T)^2\right]}}. \quad (7)$$

Usually, $T = M$. If $T \neq M$, the situation is sometimes described as 'asymmetric tolerances' [26-28]. Introduction of Cp is ascribed to Juran [29]; Cpk, to Kane [30]; Cpm, for the most part to Hsiang and Taguchi [31]; and Cpmk, to Pearn et al. [24]. Substituting the sample mean and standard deviation provides a point estimate for any of indices. For more details about this discussion, see [3,4].

Fuzzy process capability indices

Point estimations

It is natural to use fuzzy numbers such as $U(a_u, b_u, c_u) = T(a_u, b_u, c_u) \in F_T(R)$ and $L(a_l, b_l, c_l) = T(a_l, b_l, c_l) \in F_T(R)$ for the upper and lower engineering specification limits, respectively, if the process specification limits are fuzzy rather than real numbers. In this section we quote some definitions from [9].

Definition 4.1 A process with fuzzy specification limits, which we call a fuzzy process for short, is one which approximately satisfies the normal distribution condition and its specification limits are fuzzy [9].

Definition 4.2 Suppose we have a fuzzy process with fixed σ, for which the upper and lower specification limits are the fuzzy sets $U(a_u, b_u, c_u), L(a_l, b_l, c_l) \in F_T(R)$, where $a_u \geq c_l$. Then, by (2) and (3) the fuzzy process capability indices are triangular fuzzy numbers, which are defined as follows [9]:

$$\tilde{C}_p = T\left(\frac{a_u - c_l}{6\sigma}, \frac{b_u - b_l}{6\sigma}, \frac{c_u - a_l}{6\sigma}\right) \quad (8)$$

$$\tilde{C}_{pk} = T\left(\frac{a_u - c_l - 2|\mu - m|}{6\sigma}, \frac{b_u - b_l - 2|\mu - m|}{6\sigma}, \frac{c_u - a_l - 2|\mu - m|}{6\sigma}\right) \quad (9)$$

$$\tilde{C}_{pm} = T\left(\frac{a_u - c_l}{6\sqrt{\sigma^2 + (\mu - t)^2}}, \frac{b_u - b_l}{6\sqrt{\sigma^2 + (\mu - t)^2}}, \frac{c_u - a_l}{6\sqrt{\sigma^2 + (\mu - t)^2}}\right) \quad (10)$$

and

$$\tilde{C}_{pmk} = T\left(\frac{a_u - c_l - 2|\mu - m|}{6\sqrt{\sigma^2 + (\mu - t)^2}}, \frac{b_u - b_l - 2|\mu - m|}{6\sqrt{\sigma^2 + (\mu - t)^2}}, \frac{c_u - a_l - 2|\mu - m|}{6\sqrt{\sigma^2 + (\mu - t)^2}}\right) \quad (11)$$

where $m = (b_u + b_l)/2$ and t is target value.

When we have several fuzzy processes, a criterion for comparing of two fuzzy subsets is needed. There are many ways to do this comparison [32]. We use Yuan's approach, since it is a reasonable approach and it has appropriate properties such as distinguish ability and robustness. For the following definitions see [33].

Definition 4.3 Let Ci, $C_j \in F(R)$ be normal and convex fuzzy quantities. Then, the degree of bigness of Ci relative to Cj, is defined as [33]:

$$\mu(C_i, C_j) = \frac{\Delta_{ij}}{\Delta_{ij} + \Delta_{ji}}, \quad (12)$$

in which $\Delta_{ij} = \int_{c^+_{i\alpha} > c^-_{j\alpha}} \left(c^+_{i\alpha} - c^-_{j\alpha}\right) d\alpha + \int_{c^-_{i\alpha} > c^+_{j\alpha}} \left(c^-_{i\alpha} - c^+_{j\alpha}\right) d\alpha$ and $c^+_{i\alpha} = sup\{x : x \in C_{i\alpha}\}$, $c^-_{i\alpha} = inf\{x : x \in C_{i\alpha}\}$ where ciα is the α-cut of Ci for $\alpha \in (0, 1]$.

Definition 4.4 Let Ci, $C_j \in F(R)$ then [33]:

(i) Ci is bigger than Cj if and only if $\mu(C_i, C_j) > 0.5$ and

(ii) Ci and Cj are equal if and only if $\mu(C_i, C_j) = 0.5$.

Using the preference relation defined for each ordered pair, it is easy to rank n alternatives {C1,C1,. . .,Cn}. The procedure is as follows: calculate μ(Ci,Cj) for i = 1,. . .,n and j = 1,. . .,n, which consists of an n×n matrix. By using the fact that $\mu(C_i, C_j) = 1 - \mu(C_j, C_i)$, we only need to calculate $n(n - 1)/2$ membership values. Then sort $\{C_1, C_2, \ldots, C_n\}$ into $\{C_{k_1}, C_{k_2}, \ldots, C_{k_n}\}$ so that for any $i < j$, $\mu\left(C_{k_i}, C_{k_j}\right) \geq 0.5$. Based on the sorting, we can conclude that C_{k_1} is the most preferred choice and C_{k_2} is the second [9].

Confidence regions

Substituting the sample mean and the sample standard deviation in (8–11) provides fuzzy point estimates for $\tilde{C}_p$, $\tilde{C}_{pk}$, $\tilde{C}_{pm}$, and $\tilde{C}_{pmk}$. Since the point estimate of fuzzy PCIs, e.g., $\hat{\tilde{C}}_p$, like other statistics is subject to the sampling variation, it is critical to compute a confidence interval to provide a range which includes the true PCI $\tilde{C}_p$ with high and certain probability. In the following two theorems, we are going to provide two 100 (1–α)% fuzzy confidence intervals, named 100 (1–α)% fuzzy confidence regions, from [12] and [15]. Also, for more details about constructing the fuzzy confidence region for fuzzy capability indices see [13].

Theorem 4.1 Suppose that $X_1, X_2, \cdots, X_n$ are independent, identically distributed random variables with N(μ, σ2) and let $U(a_u, b_u, c_u) \in F_T(R)$ then let $L(a_l, b_l, c_l) \in F_T(R)$ be the engineering fuzzy specification limits, where au ≥ cl. Then the following interval is a 100(1−α)% fuzzy confidence region for $\tilde{C}_p$

$$\left[\hat{\tilde{C}}_p \otimes \sqrt{\frac{\chi^2_{n-1,\alpha/2}}{n-1}}, \hat{\tilde{C}}_p \otimes \sqrt{\frac{\chi^2_{n-1,1-\alpha/2}}{n-1}}\right] \qquad (13)$$

where $\chi^2_{n,\alpha}$ is the α-quantile of Chi-square distribution with n degrees of freedom, and $\hat{\tilde{C}}_p = T\left(\frac{a_u-c_l}{6S_{n-1}}, \frac{b_u-b_l}{6S_{n-1}}, \frac{c_u-a_l}{6S_{n-1}}\right)$ is the point estimate of $\tilde{C}_p$ where $S^2_{n-1} = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$ [12].

Theorem 4.2 Under the same assumptions as in Theorem 4.1, an approximate 100(1−α)% confidence region for $\tilde{C}_p$ is

$$\left[\hat{\tilde{C}}_{pm(n)} \otimes \sqrt{\frac{\chi^2_{\nu_n,\alpha/2}}{\nu_n}}, \hat{\tilde{C}}_{pm(n)} \otimes \sqrt{\frac{\chi^2_{\nu_n,1-\alpha/2}}{\nu_n}}\right], \qquad (14)$$

where $\hat{\tilde{C}}_{pm} = T\left(\frac{a_u-c_l}{6S_{T(n)}}, \frac{b_u-b_l}{6S_{T(n)}}, \frac{c_u-a_l}{6S_{T(n)}}\right)$ is the point estimate of $\tilde{C}_{pm}$, and $S^2_{T(n)} = \frac{1}{n}\sum_{i=1}^{n}(X_i - t)^2$, $\nu_n = n(1+\delta_n^2)^2/(1+2\delta_n^2)$, $\delta_n = (\bar{X} - t)/S_n$, and $S_n^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2$ [15].

An application in the educational systems

In this section, we are going to compare several educational systems using the introduced fuzzy PCIs in the following two real applications. The data are real and are collected from the grades of some of classes at SBUK. The grading scale is the interval [0, 20].

Example 5.1 We are going to perform a comparison between two classes, where their students are taking the Quality Control course with one specific teacher; the grades in two classes A and B at SBUK are as follows:

Class A
15.25, 12, 12, 13.75, 17, 14.75, 14.5, 19.5, 13.25, 13.5, 15.75, 15.5, 16.5, 15.75, 18.25, 16.5, 13.25.

Class B
14.5, 14.25, 18.75, 16.5, 16.5, 12.25, 12, 13.25, 18, 12.5, 15.25, 14, 18, 13.75, 15.5, 16.25, 14.25, 12, 18.75, 13, 14.25, 12, 14, 15.75, 13.25, 9.5, 16, 14.75, 16.25.

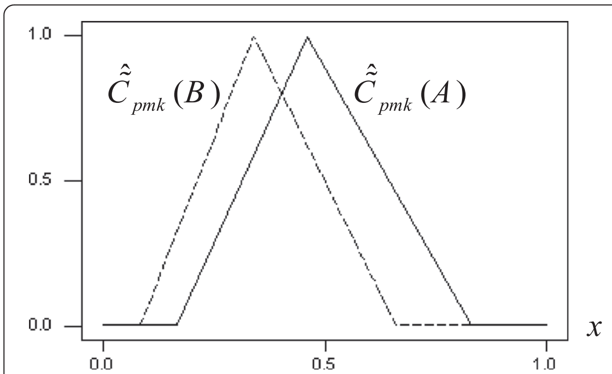

Figure 3 The membership functions of the estimated fuzzy capability indices in Example 5.1.

From the data, the following sufficient statistics are computed:

$$\bar{x}_A = 15.12, s_A = 2.07, \bar{x}_B = 14.66 \text{ and } s_B = 2.23.$$

We are going to evaluate and compare the students of class A with the students of class B. If the processes are evaluated by taking the fuzzy set theory into account, the result will be more sensitive and informative [21]. Therefore, for this comparison, using fuzzy SLs is more suitable than using real SLs. According to situations and our goals, we decide to use $\tilde{C}_{pmk}$ index for this comparison. By an expert personal opinion, we decide to employ $L(8, 12, 14) = T(8, 12, 14) \in F_T(R)$, $U(17, 19, 20) = T(17, 19, 20) \in F_T(R)$ as fuzzy SLs and target value t = 16 for the recorded grades of Quality Control course depicted in Figure 1.

By Anderson-Darling normality test, our observations approximately satisfy the normal distribution condition, and SLs are fuzzy quantity. Hence, by Definition 4.1, we have two educational fuzzy processes. Substituting the mean and the standard deviation of grades in (11) provides a point estimate for $\tilde{C}_{pmk}$ index in each class, which is shown in Figure 3. For example for class A, we have

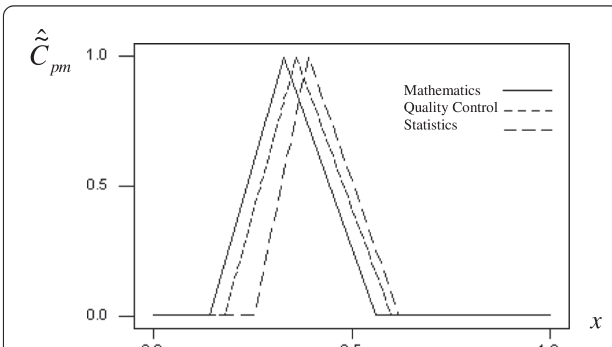

Figure 4 The membership functions of the estimated fuzzy capability indices in Example 5.2.

Table 1 The results of calculations for Example 5.2

Comparison between	Δ_{ji} value	Δ_{ji} value	Better teacher	Degree of bigness
Math and Stat	0.306	0.479	Stat is better than Math	0.61
Math and Qual	0.380	0.459	Qual is better than Math	0.55
Stat and Qual	0.440	0.346	Stat is better than Qual	0.56

$$\hat{\tilde{C}}_{pmk}(\mathrm{A}) = T\left(\frac{a_u - c_l - 2|\bar{x}_\mathrm{A} - m|}{6\sqrt{s_\mathrm{A}^2 + (\bar{x}_\mathrm{A} - t)^2}}, \frac{b_u - b_l - 2|\bar{x}_\mathrm{A} - m|}{6\sqrt{s_\mathrm{A}^2 + (\bar{x}_\mathrm{A} - t)^2}}, \frac{c_u - a_l - 2|\bar{x}_\mathrm{A} - m|}{6\sqrt{s_\mathrm{A}^2 + (\bar{x}_\mathrm{A} - t)^2}}\right) = T(0.166, 0.46, 0.83)$$

Similarly, one can compute $\hat{\tilde{C}}_{pmk}(\mathrm{B}) = T(0.084, 0.34, 0.66)$.

The comparison between two classes leads to the comparison between their students, since these two classes have the same teacher with a similar teaching method. By (14) and by the aid of Maple software, we can calculate Δ_{AB} = 0.746 and Δ_{BA} = 0.494. Hence, by Definition 4.4, $\hat{\tilde{C}}_{pmk}(\mathrm{A})$ is bigger than $\hat{\tilde{C}}_{pmk}(\mathrm{B})$ with 0.602 degree of bigness. Therefore, we can conclude that the students in class A are more active and capable than the students in class B.

Example 5.2 We wish to perform a comparison between three teachers; Mathematics (Math), Statistics (Stat), and Quality Control (Qual) at SBUK. Among the students we choose all those who have taken the three mentioned courses with three specified teachers; the size of given sample is n=19. In this example we just present the estimation of the sufficient statistics rather than all observed grades. From the data the following statistics are computed:

(1) Mathematics
$\bar{x}_\mathrm{M} = 13.3, s_\mathrm{M} = 2.31$;
(2) Statistics
$\bar{x}_\mathrm{S} = 14.27, s_\mathrm{S} = 1.94$;
(3) Quality control
$\bar{x}_\mathrm{Q} = 15.12, s_\mathrm{Q} = 2.07$.

According to the situations and our goals, we use $\tilde{C}_{pm}$ index for this comparison. An expert person introduces the following fuzzy SLs and target values for each course.

Mathematics: $L_\mathrm{M}(8, 12, 14) = T(8, 12, 14)$, $U_\mathrm{M}(17, 19, 20) = T(17, 19, 20)$ and $t_\mathrm{M} = 16$;
Statistics: $L_\mathrm{S}(9, 12, 13.5) = T(9, 12, 13.5)$, $U_\mathrm{S}(18, 19, 20) = T(18, 19, 20)$ and $t_\mathrm{S} = 16.5$;
Quality control: $L_\mathrm{QC}(10, 13, 15) = T(10, 13, 15)$, $U_\mathrm{QC}(18, 19, 20) = T(18, 19, 20)$ and $t_\mathrm{QC} = 17$.

According to the Anderson-Darling normality test, our observations approximately satisfy the normal distribution condition. Substituting the sample mean and the sample standard deviation of grades in (10) provides a point estimate for the $\tilde{C}_{pm}$ index of each class, which are shown in Figure 4. For instance, we have for Mathematics class:

$$\hat{\tilde{C}}_{pm}(\mathrm{M}) = T\left(\frac{a_u - c_l}{6\sqrt{s_\mathrm{M}^2 + (\bar{x}_\mathrm{M} - t_\mathrm{M})^2}}, \frac{b_u - b_l}{6\sqrt{s_\mathrm{M}^2 + (\bar{x}_\mathrm{M} - t_\mathrm{M})^2}}, \frac{c_u - a_l}{6\sqrt{s_\mathrm{M}^2 + (\bar{x}_\mathrm{M} - t_\mathrm{M})^2}}\right) = T(0.141, 0.33, 0.56).$$

Similarly, one can calculate $\hat{\tilde{C}}_{pm}(\mathrm{S}) = T(0.254, 0.39, 0.62)$ and $\hat{\tilde{C}}_{pm}(\mathrm{QC}) = T(0.18, 0.36, 0.6)$.

The comparison between three classes leads to the comparison between three teachers, since students are identical in the three classes and similar conditions in three exams are applied. In Table 1, we presented the results of the pairwise comparison for three fuzzy process capability indices by aid of Maple software. For example, $\hat{\tilde{C}}_{pm}(\mathrm{S})$ is bigger than $\hat{\tilde{C}}_{pm}(\mathrm{M})$ with 0.61 *degree of bigness*; therefore, in the teaching process we can assert that the Statistics teacher is more capable than the Mathematics one. Actually, we have the sorting

Table 2 Fuzzy confidence regions for $\tilde{C}_p$ and $\tilde{C}_{pm}$

Course	Fuzzy PCI	95% confidence region	99% confidence region
Mathematics	$\tilde{C}_p$	[T(0.146, 0.342, 0.585), T(0.286, 0.668, 1.146)]	[T(0.128, 0.298, 0.511), T(0.311, 0.726, 1.244)]
	$\tilde{C}_{pm}$	[T(0.106, 0.247, 0.424), T(0.179, 0.417, 0.714)]	[T(0.096, 0.224, 0.383), T(0.191, 0.446, 0.764)]
Statistics	$\tilde{C}_p$	[T(0.261, 0.407, 0.639), T(0.512, 0.796, 1.251)]	[T(0.228, 0.355, 0.558), T(0.555, 0.864, 1.358)]
	$\tilde{C}_{pm}$	[T(0.191, 0.297, 0.466), T(0.323, 0.502, 0.788)]	[T(0.172, 0.268, 0.421), T(0.345, 0.537, 0.844)]
Quality control	$\tilde{C}_p$	[T(0.163, 0.327, 0.544), T(0.320, 0.639, 1.066)]	[T(0.143, 0.285, 0.475), T(0.347, 0.694, 1.157)]
	$\tilde{C}_{pm}$	[T(0.131, 0.261, 0.439), T(0.232, 0.464, 0.774)]	[T(0.117, 0.234, 0.389), T(0.250, 0.499, 0.832)]

$\left\{\hat{\tilde{C}}_{pm}(\mathrm{S}), \hat{\tilde{C}}_{pm}(\mathrm{QC}), \hat{\tilde{C}}_{pm}(\mathrm{M})\right\}$. Based on the given sorting method in Subsection Point estimations, we can conclude that the statistics teacher has the most capability among the three teachers.

As mentioned earlier, one can provide the point estimates for fuzzy PCIs and then compares them for obtaining the best educational system or the most capable procedure. But, since the point estimates of any fuzzy PCI like other statistics is subject to sampling variation, it is critical to compute a confidence region to provide a range which includes the true PCI with a certain and high probability. Also, it must be mentioned that the most evaluations on process capability indices focus on only point estimates, which may result in unreliable assessments of process potential, but using the results of Theorem 4.1 and Theorem 4.2, we obtain two confidence regions at levels 95% and 99% for fuzzy PCIs $\tilde{C}_p$ and $\tilde{C}_{pm}$ in Table 2. For instance by Theorem 4.1 one can claim for the Mathematics course that the fuzzy region $[T(0.106, 0.247, 0.424),\ T(0.179, 0.417, 0.714)]$ is an approximate 95% fuzzy confidence region for the unknown capability index $\tilde{C}_{pm}$, and for the upper confidence in estimation the fuzzy region $[T(0.128, 0.298, 0.511),\ T(0.311, 0.726, 1.244)]$ is an approximate 99% fuzzy confidence region.

Conclusions

In this paper the fuzzy process capability indices (PCIs), when the engineering specification limits are considered as triangular fuzzy numbers, are reviewed. If the SLs are defined by fuzzy quantities, it is more appropriate to define the PCIs as fuzzy numbers. In this situation, the point estimates and the confidence regions are presented for fuzzy PCIs. A meaningful application of these new PCIs on the evaluation of the educational systems at Shahid Bahonar University of Kerman is discussed, and the performance of the proposed method is shown. The results reported here may be applied in similar situation that one needs comparison and analysis in systems where specification limits are expressed by linguistics terms.

Endnotes

This paper is an extended version of [22].

Competing interest

The authors declare that they have no competing interests.

Authors' contributions

Both authors read and approved the final manuscript.

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