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Product (N, p_n) $(C, 1)$ summability of a sequence of Fourier coefficients

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Abstract

Purpose: The purpose of the present paper is to study the product (N, p_n) $(C, 1)$ summability of a sequence of Fourier coefficients which extends a theorem of Prasad.

Methods: We use $N_p. C^{\ 1}$ summability methods with dropping monotonicity on the generating sequence $\{p_{n-k}\}$ (that is, by weakening the conditions on the filter, we improve the quality of digital filter).

Results: Let $B_n(x)$ denote the nth term of conjugate series of a Fourier series. Mohanty and Nanda were the first to establish a result for C_1 summability of the sequence $\{n\ B_n(x)\}$. Varshney improved the result for $H_1. C_1$ summability which was generalized by various investigators using different summability methods with different sets of conditions. In this paper, we extend a result of Prasad by dropping the monotonicity on the sequence $\{p_{n-k}\}$.

Conclusions: Various results pertaining to the C_1 and $H_1. C_1$ summabilities of the sequence $\{n\ B_n(x)\}$ have been reviewed and the condition of monotonicity on the means generating the sequence $\{p_{n-k}\}$ has been relaxed. Moreover, a proper set of conditions have been discussed to rectify the errors pointed out in Remark 3.2 (1) and (2).

Keywords: Conjugate Fourier series, $(C, 1)$ summability, (N, p_n) summability and product, (N, p_n) $(C, 1)$ summability

Introduction

Let $\sum_{n=0}^{\infty} u_n$ be a given infinite series with sequence of its nth partial sums $\{s_n\}$. If the $\{p_n\}$ be a nonnegative and nondecreasing, which generates sequences of constants, real or complex, let us write

$$P_n = \sum_{k=0}^{n} P_k \neq 0 \ \forall n \geq 0, \ P_{-1} = 0 = P_{-1} \ \text{and}$$

$$P_n \to \infty \ \text{as}\ n \to \infty.$$

The condition for regularity of Nörlund summability are easily seen to be

$$\lim_{n\to\infty} \frac{P_n}{P_n} \to 0 \ \ and \qquad (1.)$$

$$\sum_{k=0}^{\infty} |p_k| = O(P_n), \ \text{as}\ n\to\infty. \qquad (2.)$$

The sequence-to-sequence transformation

$$t_n^N = \frac{1}{P_n} \sum_{k=0}^{n} p_{n-k}\, s_k. \qquad (1.1)$$

defines the sequence $\{t_n^N\}$ of Nörlund means of the sequence $\{s_n\}$, as generated by the sequence of coefficients $\{p_n\}$. The series $\sum_{n=0}^{\infty} u_n$ is said to be summable (N, p_n) to the sum s if $lim_{n\to\infty}\, t_n^N$ exists and equal to s.

In the special case in which

$$p_n = \binom{n+\alpha-1}{\alpha-1} = \frac{\Gamma(n+\alpha)}{\Gamma(n+1)\Gamma(\alpha)}; \ (\alpha > -1)$$

the Nörlund summability (N, p_n) reduces to the familiar (C, α) summability.

The product of $N_p$summability with a C^1summability defines $N_p. C^{\ 1}$ summability. Thus the $N_p. C^{\ 1}$ mean is given by $t^{NC}{}_n(x) = P_n^{-1} \sum_{k=1}^{n} p_{n-k}\, C_k(x)$.

If $t_n^{NC} \to s \ as\ n \to \infty$, then the infinite series $\sum_{n=0}^{\infty} u_n$ is said to be the summable $N_p.C^1$ to the sum s if $lim_{n\to\infty}\, t_n{}^{NC}$ exists and is equal to s.

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Let $f(x)$ be a 2π-periodic function and Lebesgue integrable. The Fourier series of $f(x)$ at any point x is given by

$$f(x) \sim \frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos kx + b_k \sin kx) \equiv \sum_{k=0}^{\infty} A_k(x). \tag{1.2}$$

With nth partial sum, $s_n(f;x)$ is called trigonometric polynomial of degree (order) n of the first $(n + 1)$ terms of the Fourier series of f.

The conjugate series of Fourier series (1.2) is given by

$$\sum_{k=1}^{\infty} (b_k \cos kx - a_k \sin kx) \equiv \sum_{k=1}^{\infty} B_k(x) \tag{1.3}$$

The regularity conditions of $N_p . C^1$ are as follows: $nB_n \to s \Rightarrow C^1(nB_n) = t_n^C = n^{-1} \sum_{k=1}^{n} k B_k(x) \to s$, as $n \to \infty$, C^1 method is regular, $\Rightarrow N_p \{C^1(nB_n)\} = t_n^{NC} = P_n^{-1} \sum_{k=1}^{n} P_{n-k} (k^{-1} \sum_{r=1}^{k} r B_r(x)) \to s$, as $n \to \infty$, N_p method is also regular, and $\Rightarrow C^1 . N_p$ method is regular. We note that t_n^N and t_n^{NC} are also trigonometric polynomials of degree (order) n.

Abel's transformation

The formula

$$\sum_{k=m}^{n} u_k v_k = \sum_{k=m}^{n-1} U_k (v_k - v_{k+1}) - U_{m-1} v_m + U_n v_n, \tag{1.4}$$

where $0 \le m \le n$, $U_k = u_0 + u_1 + u_2 + ... + u_k$, if $k \ge 0$, $U_{-1} = 0$, which can be verified, is known as Abel's transformation and will be used extensively in the succeeding discussion.

If $v_m, v_{m+1}, \ldots, v_n$ are nonnegative and nonincreasing, the left hand side of (1.4) does not exceed $2 v_m \max_{m-1 \le k \le n} |U_k|$ in the absolute value. In fact,

$$\left| \sum_{k=m}^{n} u_k v_k \right| \le \max |U_k| \left\{ \sum_{k=m}^{n-1} (v_k - v_{k+1}) + v_m + v_n \right\} = 2 v_m \max |U_k|.$$

Throughout in this paper, we use the following notations

$$\psi(t) = \psi_x(t) = f(x+t) - f(x-t) - l,$$

$$\Psi(t) = \int_0^t |\psi(u)| du,$$

$$\mathrm{Q}(n,t) = \frac{1}{\pi P_n} \sum_{k=1}^{n} p_{n-k} \left\{ \frac{\sin kt}{kt^2} - \frac{\cos kt}{t} \right\},$$

$$\Delta_k p_{n-k} = p_{n-k} - p_{n-k-1}, 0 \le k \le n,$$

and $\tau = [1/t]$ is the largest integer contained in $1/t$, where l is a constant.

The $(C, 1)$ and $(H, 1)$ denotes the Cesàro and harmonic summabilities respectively of order one. The product summability (N, p_n) $(C, 1)$ is obtained by superimposing (N, p_n) summability on $(C, 1)$ summability, and the product summability (N, p_n) $(C, 1)$ plays an important role in signal theory as a double digital filter in finite impulse response in particular [1].

Methods

Known theorems

The theory of summability is a very extensive field. Mohanty and Nanda [2] proved the following theorem on C_1 summability of the sequence $\{n\, B_n(x)\}$.

Theorem 2.1 [2] *If*

$$\Psi(t) = o(t/\log(1/t)), \text{ as } t \to +0 \tag{2.1}$$

and

$$a_n = O(n^{-\delta}); b_n = O(n^{-\delta}), \text{ as } t \to +0, \tag{2.2}$$

then the sequence $\{n\, B_n(x)\}$ *is the summable* C_1 *to the value of* l/π.

Varshney [3] improved Theorem 2.1 by extending it to product $H_1 . C_1$ summability. He has proved that

Theorem 2.2 [3] *if*

$$\Psi(t) = o(t/\log(1/t)), \text{ as } t \to +0, \tag{2.3}$$

then the sequence $\{n\, B_n(x)\}$ *is the summable* $H_1 . C_1$ *to the value of* l/π.

Various investigators such as Sharma [4], Rhoades [5] (cor. 19, p. 533), Pandey [6], Rai [7], Dwivedi [8], Mittal and Prasad [9], Prasad [10], Mittal [11], Chandra [12], Mittal et al. [13,14], and Mittal and Singh [1] used different summability methods with different sets of conditions. In particular, Prasad [10] has proved the following:

Theorem 2.3 [6] *Let* $p(u)$ *be monotonically decreasing and strictly positive value with* $u \ge 0$. *Let* $p_n = p(n)$ *and*

$$P(u) = \int_0^u p(x) dx \to \infty, \text{ as } u \to \infty. \tag{2.4}$$

Let $\alpha(t)$ *be a positive and nondecreasing function of t. If*

$$\Psi(t) = o(t/\alpha(1/t)), \text{ as } t \to +0, \tag{2.5}$$

then a sufficient condition that the sequence $\{n\, B_n(x)\}$ *be a summable* $N_P . C_1$ *to the value of* l/π *is that*

$$\int_1^n \frac{P(x)}{x\,\alpha(x)} dx = O(P(n)), \text{ as } n \to \infty. \tag{2.6}$$

Results and discussion

Main theorem

In the present paper, we extend Theorem 2.3 by dropping the monotonicity on the generating sequence $\{P_{n-k}\}$ (that is, by weakening the conditions on the filter, we improve the quality of the digital filter). More precisely, we prove in Theorem 3.1:

Theorem 3.1 *Let* $\{p_k\}$ *be a nonnegative value such that*

$$\text{(i.)} \sum_{k=r}^{n} |\Delta_k p_{n-k}| = O(p_{n-r}),\ \text{(ii.)}\ n\, p_n = O(p_n). \tag{3.1}$$

Let α(t) be a positive and increasing function of t such that

$$\Psi(t) = o(t/\alpha(1/t)),\ \text{as}\ t \to +0 \tag{3.2}$$

and

$$\alpha(n) \to \infty,\ \text{as}\ n \to \infty \tag{3.3}$$

then a sufficient condition for the sequence $\{nB_n(x)\}$ *to be as the summable* (N, p_n) *(C, 1) to the value of* l/π *is*

$$\int_{1/\delta}^{n} \frac{P(x)}{x\,\alpha(x)}\,dx = O(P(n)),\ \text{as}\ n \to \infty. \tag{3.4}$$

Remark 3.2 (1) If $p_{n-k} \le p_{n-k-1}, \forall\, 0 \le k < n$, as used in Theorem 2.3, then both the conditions (3.1) holds. Thus Theorem 3.1 extends Theorem 2.3. (2) In the proof of Theorem 2.3, author in [10] has used the condition (3.3) but did not mention in his statement.

Lemmas. For the proof of our Theorem 3.1, we require the following lemmas.

Lemma 4.1 [10] *If* $0 \le t \le 1/n$, *then*

$$|Q(n,t)| = 0(n)$$

Lemma 4.2 [15] *For all values of n and t*

$$\left|\sum_{k=0}^{n} \frac{\sin(k+1)t}{k+1}\right| \le 1 + \frac{\pi}{2}. \tag{4.2}$$

Lemma 4.3 *Under the regularity conditions of matrix* (N, p_n) *in satisfying (3.1), we get* $Q(n,t) = O(t^{-1} P(\tau)/P_n) + O(t^{-2} p_1/P_n)$, *for*

$$1/n \le t \le \delta. \tag{4.3}$$

Proof We have $Q(n,t) = \sum_{k=0}^{n-1} p_{n-k}/\pi P_n \left\{\frac{\sin(n-k)t}{(n-k)t^2} - \frac{\cos(n-k)t}{t}\right\} = Q_1(n,t) + Q_2(n,t)$, as we say.

By using Abel's transformation, Lemma 4.2, and condition (3.1), we have

$$\begin{aligned}
|Q_1(n,t)| &= \left|\sum_{k=0}^{n-1} p_{n-k}/\pi P_n \frac{\sin(n-k)t}{(n-k)t^2}\right| \\
&\le \left|\sum_{k=0}^{\tau-1} p_{n-k}/\pi P_n \frac{\sin(n-k)t}{(n-k)t^2}\right| + \left|\sum_{k=\tau}^{n-1} p_{n-k}/\pi P_n \frac{\sin(n-k)t}{(n-k)t^2}\right| \\
&\le \left[t^{-1}\sum_{k=0}^{\tau-1} p_{n-k}\left|\frac{\sin(n-k)t}{(n-k)t}\right| + t^{-2}\left|\sum_{k=\tau}^{n-1} p_{n-k}\frac{\sin(n-k)t}{n-k}\right|\right]/\pi P_n \\
&\le \left[t^{-1}\sum_{k=0}^{\tau-1} p_{n-k} + t^{-2}\left|\sum_{k=\tau}^{n-2}\left(\Delta_k p_{n-k}\sum_{r=0}^{k}\frac{\sin(n-r)t}{n-r}\right)\right|\right]/\pi P_n \\
&\quad + t^{-2}\left|p_{n-\tau}/\pi P_n \sum_{k=0}^{\tau-1}\frac{\sin(n-k)t}{n-k}\right| + t^{-2}\left|p_1/\pi P_n \sum_{k=0}^{n-1}\frac{\sin(n-k)t}{n-k}\right| \\
&\le \left[t^{-1}\sum_{k=0}^{\tau} p_{n-k} + t^{-2}\left(1+\frac{\pi}{2}\right) \times \left(\sum_{k=\tau}^{n-2}|\Delta_k p_{n-k}| + p_{n-\tau} + p_1\right)\right]/\pi P_n \\
&= O(t^{-1})((P(\tau) + (\tau+1)p_{n-\tau})/P_n) + O(t^{-2} p_1/P_n) \\
&= O(t^{-1}P(\tau)/P_n) + O(t^{-2} p_1/P_n).
\end{aligned}$$

Again by using Abel's transformation and condition (3.1), we have

$$\begin{aligned}
Q_2(n,t) &= \sum_{k=0}^{n-1} p_{n-k}/\pi P_n \frac{\cos(n-k)t}{t} \\
&= O(t^{-1})\left[P(\tau) + \sum_{k=0}^{n-2}(\Delta_k p_{n-k})\sum_{r=0}^{k}\cos(n-r)t - p_{n-\tau}\sum_{r=0}^{\tau-1}\cos(n-r)t\right]P_n^{-1} \\
&\quad + O(t^{-1})P_n^{-1} p_1 \sum_{r=0}^{n}\cos(n-r)t
\end{aligned}$$

$$\begin{aligned}
|Q_2(n,t)| &= O(t^{-1})\left[P(\tau) + t^{-1}\sum_{k=\tau}^{n-2}|\Delta_k p_{n-k}| + t^{-1}p_{n-\tau} + t^{-1}p_1\right]/P_n \\
&= O(t^{-1}P(\tau)/P_n) + O(t^{-2}p_1/P_n).
\end{aligned}$$

By collecting $Q_1(n,t)$, $Q_2(n,t)$ and $Q(n,t)$, we get

$$Q(n,t) = O\left(t^{-1}P(\tau)/P_n\right) + O\left(t^{-2}p_1/P_n\right).$$

This completes the proof of Lemma 4.3.

Proof of Theorem 3.1 The C_1 transform of the sequence $\{n\, B_n(x)\}$ denoted by $C_n(x)$ is defined by

$$C_n(x) = \frac{1}{n}\sum\nolimits_{k=1}^{n} k B_k(x).$$

The $N_p. C_1$ transform of the sequence $\{n\ B_n(x)\}$, which is denoted by $t_n^{N\ C}(x)$, is given by

$$t^{N\,C}{}_n(x) = P_n^{-1}\sum\nolimits_{k=1}^{n} p_{n-k}\, C_k(x)$$
$$= P_n^{-1}\sum\nolimits_{k=1}^{n} p_{n-k}\left(\frac{1}{k}\sum\nolimits_{r=1}^{k} rB_r(x)\right).$$

Therefore, following Mohanty and Nanda [2], we obtain

$$t^{N\,C}{}_n(x) - l/\pi = P_n^{-1}\sum\nolimits_{k=1}^{n}\left\{p_{n-k}\left(\frac{1}{k}\sum\nolimits_{r=1}^{k} r\,B_r(x) - l/\pi\right)\right\}$$

$$= P_n^{-1}\sum_{k=1}^{n} p_{n-k}\left\{\frac{1}{\pi}\int_0^{\pi}\psi(t)\left(\frac{\sin kt}{kt^2} - \frac{\cos kt}{t}\right)dt + o(1)\right\}$$

$$= \frac{1}{\pi}\int_0^{\pi}\psi(t)\,P_n^{-1}\sum\nolimits_{k=1}^{n} p_{n-k}\left(\frac{\sin kt}{kt^2} - \frac{\cos kt}{t}\right)dt + o(1)$$

$$= \frac{1}{\pi}\left(\int_0^{1/n} + \int_{1/n}^{\delta} + \int_{\delta}^{\pi}\right)\psi(t)Q(n,t)dt + o(1), \quad \text{where } 0 < \delta < \pi$$

$$= \frac{1}{\pi}(I_1 + I_2 + I_3) + o(1), \tag{5.1}$$

where

$$I_1 = \int_0^{1/n}\psi(t)Q(n,t)dt = O(n)\int_0^{1/n}|\psi(t)|dt$$
$$= O(n)\Psi(1/n) = O(n)o(1/n\alpha(n))$$
$$= o(1/\alpha(n)) = o(1), \text{as } n\to\infty, \tag{5.2}$$

in view of Lemma 4.1, conditions (3.2), and (3.3).

Using Lemma 4.3, we have

$$I_2 = \int_{1/n}^{\delta}\psi(t)Q(n,t)dt$$

$$= \int_{1/n}^{\delta}|\psi(t)|P_n^{-1}\left\{O\left(t^{-2}p_1\right) + O\left(t^{-1}P(\tau)\right)\right\}dt$$

$$= I_{2,1} + I_{2,2} \text{ as we say.} \tag{5.3}$$

Now, using conditions (3.1-ii), (3.2), (3.3), and second mean value theorem for integrals, we have

$$I_{2,1} = O(1)\int_{1/n}^{\delta} t^{-2}P_n^{-1}p_1|\psi(t)|\,dt = O\left(P_n^{-1}p_1\right)\int_{1/n}^{\delta} t^{-2}|\psi(t)|\,dt$$
$$= O\left(\frac{1}{n}\right)\left\{\left(t^{-2}\,\Psi(t)\right)_{1/n}^{\delta} + \int_{1/n}^{\delta} t^{-3}\,\Psi(t)dt\right\}$$
$$= o\left(\frac{1}{n}\right)\left(\frac{1}{t\,\alpha(1/t)}\right)_{1/n}^{\delta} + o\left(\frac{1}{n}\right)\int_{1/n}^{\delta}\frac{dt}{t^2\,\alpha(1/t)}$$
$$= o\left(\frac{1}{n}\right) + o\left(\frac{1}{\alpha(n)}\right) + o\left(\frac{1}{n\,\alpha(1/\delta)}\right)\int_{1/n}^{\delta}\frac{dt}{t^2}$$
$$= o\left(\frac{1}{n}\right) + o\left(\frac{1}{\alpha(n)}\right) + o\left(\frac{1}{n\,\alpha(1/\delta)}\right)(\delta - 1/n)$$
$$= o(1), \text{as } n\to\infty. \tag{5.4}$$

Using conditions (3.1-ii), (3.2), (3.3), and (3.4), we have

$$I_{2,2} = O(1)\int_{1/n}^{\delta}\frac{|\psi(t)|}{t}P_n^{-1}P(\tau)\,dt = O(1)\left(\Psi(t)P_n^{-1}\frac{P(\tau)}{t}\right)_{1/n}^{\delta}$$
$$+ O(1)\int_{1/n}^{\delta}\Psi(t)P_n^{-1}\frac{P(\tau)}{t^2}\,dt + O(1)\int_{1/n}^{\delta}\frac{\Psi(t)}{t}\,d\left(P_n^{-1}P(\tau)\right)$$
$$= o(1) + o\left(\frac{P_n^{-1}P_n}{\alpha(n)}\right) + o(1)\int_{1/n}^{\delta}\frac{P_n^{-1}P(\tau)}{t\,\alpha(1/t)}\,dt$$
$$+ O\left(P_n^{-1}\right)\int_{1/n}^{\delta}\Psi(t)\,d\left(\frac{P(1/t)\alpha(1/t)}{t\,\alpha(1/t)}\right)$$
$$= o(1) + o\left(\frac{P_n^{-1}P_n}{\alpha(n)}\right) + o(1)\int_{1/n}^{\delta}\frac{P_n^{-1}P(\tau)}{t\,\alpha(1/t)}\,dt$$
$$+ O\left(P_n^{-1}\right)\int_{1/n}^{\delta} o\left(\frac{t}{\alpha(1/t)}\right)d\left(\frac{P(1/t)}{t\,\alpha(1/t)}\right)\alpha(1/t)$$
$$+ O\left(P_n^{-1}\right)\int_{1/n}^{\delta} o\left(\frac{t}{\alpha(1/t)}\right)\frac{P(1/t)}{t\,\alpha(1/t)}\;d(\alpha(1/t))$$
$$= o(1) + o\left(\frac{1}{\alpha(n)}\right) + o(1)\int_{1/n}^{\delta}\frac{P_n^{-1}P(\tau)}{t\,\alpha(1/t)}\,dt$$
$$+ o\left(P_n^{-1}\right)\int_{1/n}^{\delta} t\,d\left(\frac{P(1/t)}{t\,\alpha(1/t)}\right) + o(1)\int_{1/n}^{\delta}\frac{d(\alpha(1/t))}{\{\alpha(1/t)\}^2}$$
$$= o(1) + o\left(\frac{1}{\alpha(n)}\right) + o(1)\int_{1/n}^{\delta}\frac{P_n^{-1}P(\tau)}{t\,\alpha(1/t)}\,dt$$
$$+ o\left(P_n^{-1}\right)\left\{\left[\frac{t\,P(1/t)}{t\,\alpha(1/t)}\right]_{1/n}^{\delta} - \int_{1/n}^{\delta}\frac{P(1/t)}{t\,\alpha(1/t)}dt\right\}$$
$$+ o(1)\left[-\frac{1}{\alpha(1/t)}\right]_{1/n}^{\delta}$$
$$= o(1) + o\left(\frac{1}{\alpha(n)}\right) + o(1)\int_{1/n}^{\delta}\frac{P_n^{-1}P(\tau)}{t\,\alpha(1/t)}\,dt$$
$$= o(1) + o(1)\int_{1/\delta}^{n}\frac{P_n^{-1}P(x)}{x\,\alpha(x)}\,dx$$
$$= o(1) + o(1)O\left(P_n^{-1}P(n)\right) = o(1), \text{as } n\to\infty. \tag{5.5}$$

On combining (5.3), (5.4) and (5.5), we get

$$I_2 = o(1), \text{as } n \to \infty \tag{5.6}$$

Finally, by Riemann-Lebesgue Theorem, we have

$$I_3 = \int_\delta^\pi \psi(t) Q(n,t) dt = o(1), \text{as } n \to \infty \tag{5.7}$$

By collecting (5.2), (5.6), and (5.7), we get

$$t^{NC}{}_n(x) - l/\pi = o(1), \text{as } n \to \infty.$$

This completes the proof of Theorem 3.1.

Conclusions

Various results pertaining to the C_1 and $H_1 C_1$ summabilities of the sequence $\{n\ B_n(x)\}$ have been reviewed, and the condition of monotonicity on the means of generating the sequence $\{p_{n-k}\}$ has been relaxed. Moreover, a proper set of conditions have been discussed to rectify the errors pointed out in Remark 3.2 (1) and (2).

Competing interest

The authors declare that they have no competing interests.

Authors' contributions

VNM, KK, and LNM contributed equally to this work. All the authors read and approved the final manuscript.

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References

1. Mittal, ML, Singh, U: $T \cdot C_1$ summability of a sequence of Fourier coefficients. Appl. Math. Comp. **204**, 702–706 (2008)
2. Mohanty, R, Nanda, M: On the behavior of Fourier coefficients. Proc Am Math Soc **5**, 79–84 (1954)
3. Varshney, OP: On a sequence of Fourier coefficients. Proc. Amer. Math. Soc. **10**, 790–795 (1959)
4. Sharma, RM: On $(N, p_n).C_1$ summability of the sequence$\{nB_n(x)\}$. Rend. Circ. Mat. Palermo **2**(19), 217–224 (1970)
5. Rhoades, BE: Corrigendum to: generalized harmonic summability of a sequence of Fourier coefficients. Analysis **10**, 97–98 (1990)
6. Pandey, GS: On a sequence of Fourier coefficients. Math. Student **45**, 80–83 (1977)
7. Rai, OP: Harmonic summability of a sequence of Fourier coefficients. Proc. Japan Acad. **41**, 123–127 (1965)
8. Dwivedi, GK: On a sequence of Fourier coefficients. Annal. Soc. Math. Polon Series-I, Commen. Math **15**, 61–66 (1971)
9. Mittal, ML, Prasad, G: On a sequence of Fourier coefficients. Indian J. Pure Appl. Math. **25**(3), 235–241 (1992)
10. Prasad, K: On the (N, p_n). C_1 summability of a sequence of Fourier coefficients. Indian J. Pure Appl. Math. **12**(7), 874–881 (1981)
11. Mittal, ML: On the $\|T\|.C_1$ summability of a sequence of Fourier coefficients. Bull. Cal. Math. Soc **81**, 25–31 (1989)
12. Chandra, P: Trigonometric approximation of functions in *Lp*-norm. J Math Anal Appl **275**(1), 13–26 (2002)
13. Mittal, ML, Rhoades, B.E., Mishra, V.N., Singh, U.: Using infinite matrices to approximate functions of class using trigonometric polynomials. J Math Anal Appl **326**, 667–676 (2007)
14. Mittal, ML, Singh, U., Mishra, V.N., Priti, S., Mittal, S.S.: Approximation of functions (signals) belonging to Lip(ξ(t), p)-class by means of conjugate Fourier series using linear operators. Indian J. Math. **47**, 217–229 (2005)
15. Gronwall, TH: Über die Gibbssche Erscheinung und die trigonometrische summen sin x + (1/2)sin $2x$ + (1/3)sin $3x$ + _ _ _ + (1/n)sin nx. Math Ann **72**, 228–243 (1912)

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