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Hybrid generalized contractions

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Abstract

Sintunavarat and Kumam introduced the notion of hybrid generalized multi-valued contraction mapping and established a common fixed point theorem. We extend their result for four mappings and prove common coincidence and common fixed point theorem.

Keywords: Common fixed points, Coincidence points, R -weakly commuting

MSC 2000: 47H10, 54H25

Introduction and preliminaries

Alber and Guerre-Delabriere [1] introduced the concept of weak contraction in Hilbert spaces. Rhoades [2] has shown that the result concluded by Alber and Guerre-Delabriere in [1] is also valid in complete metric spaces. Berinde and Berinde [3] extended weak contraction for multi-valued mappings and introduced the notion of multi-valued (θ, L) -weak contraction and multi-valued (α, L) -weak contraction. Kamran [4] extended these contractions for hybrid pair of mappings and introduced multi-valued (f, θ, L) -weak contraction and multi-valued (f, α, L) -weak contraction. Sintunavarat and Kumam [5] introduced the notion of generalized (f, α, β) -weak contraction to extend the notion of multi-valued (f, α, L) -weak contraction. They further extended the notion of generalized (f, α, β) -weak contraction by introducing hybrid generalized multi-valued contraction mappings and established a common fixed point theorem [6]. The purpose of this paper is to extend the notion of hybrid generalized multi-valued contraction and to prove common coincidence and common fixed point theorems.

Let (X, d) be a metric space. For $x \in X$ and $A \subseteq X$, $d(x, A) = \inf\{d(x, y) : y \in A\}$. We denote by $CB(X)$ the class of all nonempty closed and bounded subsets of X . For every $A, B \in CB(X)$

$$H(A, B) = \max\{\sup_{x \in A} d(x, B), \sup_{y \in B} d(y, A)\}.$$

Such a map H is called Hausdorff metric induced by d . A point $x \in X$ is said to be a common fixed point of $f : X \rightarrow X$ and $T : X \rightarrow CB(X)$ if $x = fx \in Tx$. The point $x \in X$ is said to be a coincidence point of $f : X \rightarrow X$ and $T : X \rightarrow CB(X)$ if $fx \in Tx$. Some works for hybrid pair are available in [7-10].

The mappings $f : X \rightarrow X$ and $T : X \rightarrow CB(X)$ are called R -weakly commuting [11,12] if for all $x \in X$, $fTx \in CB(X)$, and there exists a positive real number R such that $H(fTx, Tf x) \leq Rd(fx, Tx)$.

Lemma 1. [4] Let (X, d) be a metric space, $\{A_k\}$ be a sequence in $CB(X)$, and $\{x_k\}$ be a sequence in X such that $x_k \in A_{k-1}$. Let $\phi : [0, \infty) \rightarrow [0, 1)$ be a function satisfying $\limsup_{r \rightarrow t^+} \phi(r) < 1$ for every $t \in [0, \infty)$. Suppose $d(x_{k-1}, x_k)$ to be a nonincreasing sequence such that

$$H(A_{k-1}, A_k) \leq \phi(d(x_{k-1}, x_k))d(x_{k-1}, x_k),$$

$$d(x_{k+1}, x_k) \leq H(A_{k-1}, A_k) + \phi^{n_k}(d(x_{k-1}, x_k)),$$

where $n_1 < n_2 < \dots, k, n_k \in \mathbb{N}$. Then, $\{x_k\}$ is a Cauchy sequence in X .

Lemma 2. [13] If $A, B \in CB(X)$ and $a \in A$, then for each $\epsilon > 0$, there exists $b \in B$ such that

$$d(a, b) \leq H(A, B) + \epsilon.$$

Definition 1. [6] Let (X, d) be a metric space, $f : X \rightarrow X$ be a single-valued mapping, and $T : X \rightarrow CB(X)$ be a multi-valued mapping. T is said to be a hybrid generalized multi-valued contraction mapping if and only if there exist two functions $\phi : [0, \infty) \rightarrow [0, 1)$ satisfying

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$\limsup_{r \rightarrow t^+} \phi(r) < 1$ for every $t \in [0, \infty)$ and $\varphi : [0, \infty) \rightarrow [0, \infty)$ such that

$$H(Tx, Ty) \leq \phi(M(x, y))M(x, y) + \varphi(N(x, y))N(x, y), \quad (1)$$

for each $x, y \in X$,

where

$$M(x, y) = \max\{d(fx, fy), d(fy, Tx)\},$$

and

$$N(x, y) = \min\{d(fx, fy), d(fx, Tx), d(fy, Ty), d(fx, Ty), d(fy, Tx)\}.$$

Main results

Definition 2. Let (X, d) be a metric space, $f, g : X \rightarrow X$ and $T : X \rightarrow CB(X)$. A mapping $S : X \rightarrow CB(X)$ is said to be an extended hybrid generalized multi-valued f contraction if and only if there exist two functions $\phi : [0, \infty) \rightarrow [0, 1)$ satisfying $\limsup_{r \rightarrow t^+} \phi(r) < 1$ for every $t \in [0, \infty)$ and $\varphi : [0, \infty) \rightarrow [0, \infty)$ such that

$$H(Sx, Ty) \leq \phi(M(x, y))M(x, y) + \varphi(N(x, y))N(x, y), \quad (2)$$

for each $x, y \in X$,

where

$$M(x, y) = \max\{d(fx, gy), \min\{d(fx, Ty), d(gy, Sx)\}\},$$

and

$$N(x, y) = \min\{d(fx, gy), d(fx, Tx), d(gy, Sy), d(fx, Ty), d(gy, Sx)\}.$$

Remark 1. If $f = g$ and $T = S$, then Definition 2 reduces to Definition 1.

Lemma 3. Let (X, d) be a metric space, $f, g : X \rightarrow X$ and $T : X \rightarrow CB(X)$. Let $S : X \rightarrow CB(X)$ be the extended hybrid generalized multi-valued f contraction. Let $\{gx_{2k+1}\}$ be a g -orbit of S at x_0 and $\{fx_{2k+2}\}$ be an f -orbit of T at x_1 such that

$$d(y_{k+1}, y_{k+2}) \leq H(A_k, A_{k+1}) + \phi^{n_{k+1}}(M(x_k, x_{k+1})), \quad (3)$$

for each $k \in \{0, 2, 4, 6, \dots\}$, and

$$d(y_{k+2}, y_{k+1}) \leq H(A_{k+1}, A_k) + \phi^{n_{k+1}}(M(x_{k+1}, x_k)), \quad (4)$$

for each $k \in \{1, 3, 5, \dots\}$, where $y_{2k+1} = gx_{2k+1} \in Sx_{2k} = A_{2k}$, $y_{2k+2} = fx_{2k+2} \in Tx_{2k+1} = A_{2k+1}$, for each $k \geq 0$. Further, $n_1 < n_2 < \dots$ and $\{d(y_k, y_{k+1})\}$ is a nonincreasing sequence. Then, $\{y_k\}$ is a Cauchy sequence in X .

Proof. Let $y_0 = x_0$. Then, we construct a sequence $\{y_k\}$ in X , A_k in $CB(X)$ such that $y_{2k+1} = gx_{2k+1} \in Sx_{2k} = A_{2k}$ and $y_{2k+2} = fx_{2k+2} \in Tx_{2k+1} = A_{2k+1}$.

For $k \in \{0, 2, 4, 6, \dots\}$, it follows from the extended hybrid generalized multi-valued f contraction that

$$\begin{aligned} H(A_k, A_{k+1}) &= H(Sx_k, Tx_{k+1}); \\ &\leq \phi(M(x_k, x_{k+1}))M(x_k, x_{k+1}) \\ &\quad + \varphi(N(x_k, x_{k+1}))N(x_k, x_{k+1}); \\ &= \phi(\max\{d(fx_k, gx_{k+1}), \min\{d(fx_k, Tx_{k+1}), d(gx_{k+1}, Sx_k)\}\}) \\ &\quad \times \max\{d(fx_k, gx_{k+1}), \min\{d(fx_k, Tx_{k+1}), d(gx_{k+1}, Sx_k)\}\} \\ &\quad + \varphi(\min\{d(fx_k, gx_{k+1}), d(fx_k, Tx_k), d(gx_{k+1}, Sx_{k+1}), d(fx_k, Tx_{k+1}), d(gx_{k+1}, Sx_k)\}) \min\{d(fx_k, gx_{k+1}), d(fx_k, Tx_k), d(gx_{k+1}, Sx_{k+1}), d(fx_k, Tx_{k+1}), d(gx_{k+1}, Sx_k)\}); \\ &= \phi(d(fx_k, gx_{k+1}))d(fx_k, gx_{k+1}); \\ &= \phi(d(y_k, y_{k+1}))d(y_k, y_{k+1}). \end{aligned}$$

Similarly, we show that for $k = \{1, 3, 5, \dots\}$, we have

$$H(A_{k+1}, A_k) \leq \phi(d(y_{k+1}, y_k))d(y_{k+1}, y_k).$$

By (3), for $k \in \{0, 2, 4, 6, \dots\}$, we have

$$\begin{aligned} d(y_{k+1}, y_{k+2}) &= d(gx_{k+1}, fx_{k+2}); \\ &\leq H(Sx_k, Tx_{k+1}) + \phi^{n_{k+1}}(M(x_k, x_{k+1})); \\ &= H(A_k, A_{k+1}) + \phi^{n_{k+1}}(\max\{d(fx_k, gx_{k+1}), \min\{d(fx_k, Tx_{k+1}), d(gx_{k+1}, Sx_k)\}\}); \\ &= H(A_k, A_{k+1}) + \phi^{n_{k+1}}(d(fx_k, gx_{k+1})); \\ &= H(A_k, A_{k+1}) + \phi^{n_{k+1}}(d(y_k, y_{k+1})). \end{aligned}$$

Similarly, for $k = \{1, 3, 5, \dots\}$, we have

$$d(y_{k+2}, y_{k+1}) \leq H(A_{k+1}, A_k) + \phi^{n_{k+1}}(d(y_{k+1}, y_k)).$$

Given that $\{d(y_k, y_{k+1})\}$ is a nonincreasing sequence, thus, all the conditions of Lemma 1 are satisfied. Hence, $\{y_k\}$ is a Cauchy sequence in X . \square

Theorem 1. Let (X, d) be a complete metric space, $f, g : X \rightarrow X$, $T : X \rightarrow CB(X)$ are continuous mappings, and $S : X \rightarrow CB(X)$ is a continuous extended hybrid generalized multi-valued f contraction such that $SX \subseteq gX$ and $TX \subseteq fX$. Then,

- (i) if g, T and f, S are R -weakly commuting, then g, T and f, S have a common coincidence point say z .
- (ii) Moreover, if $ggz = gz, fgz = gz$, then f, g, T , and S have a common fixed point.

Proof. Let x_0 be an arbitrary point in X and $y_0 = fx_0$. Then, we construct a sequence $\{y_k\}$ in X , A_k in $CB(X)$ respectively as follows. Since $SX \subseteq gX$, there exists a point

$x_1 \in X$ such that $y_1 = gx_1 \in Sx_0 = A_0$. We can choose a positive integer n_1 such that

$$\phi^{n_1}(d(y_0, y_1)) \leq [1 - \phi(M(x_0, x_1))]M(x_0, x_1). \quad (5)$$

Since $TX \subseteq fX$, there exists $y_2 = fx_2 \in Tx_1 = A_1$ such that

$$d(y_1, y_2) \leq H(Sx_0, Tx_1) + \phi^{n_1}(d(y_0, y_1)). \quad (6)$$

Using (5) and the notion of extended hybrid generalized multi-valued f contraction in the above inequality, we have

$$\begin{aligned} d(y_1, y_2) &\leq H(Sx_0, Tx_1) + \phi^{n_1}(d(y_0, y_1)); \\ &\leq \phi(M(x_0, x_1))M(x_0, x_1) \\ &\quad + \varphi(N(x_0, x_1))N(x_0, x_1) \\ &\quad + [1 - \phi(M(x_0, x_1))]M(x_0, x_1); \\ &= M(x_0, x_1); \\ &= \min\{d(fx_k, Tx_{k+1}), d(gx_{k+1}, Sx_k)\}; \\ &= d(fx_0, gx_1); \\ &= d(y_0, y_1). \end{aligned}$$

Now, we can choose a positive integer $n_2 > n_1$ such that

$$\phi^{n_2}(d(y_2, y_1)) \leq [1 - \phi(M(x_2, x_1))]M(x_2, x_1). \quad (7)$$

There exists $y_3 = gx_3 \in Sx_2 = A_2$ such that

$$d(y_3, y_2) \leq H(Sx_2, Tx_1) + \phi^{n_2}(d(y_2, y_1)). \quad (8)$$

Using (7) and the notion of extended hybrid generalized multi-valued f contraction in the above inequality, we have

$$\begin{aligned} d(y_3, y_2) &\leq H(Sx_2, Tx_1) + \phi^{n_2}(d(y_2, y_1)); \\ &\leq \phi(M(x_2, x_1))M(x_2, x_1) \\ &\quad + \varphi(N(x_2, x_1))N(x_2, x_1) \\ &\quad + [1 - \phi(M(x_2, x_1))]M(x_2, x_1); \\ &= M(x_2, x_1); \\ &= \max\{d(fx_2, gx_1), \min\{d(fx_2, Tx_1), \\ &\quad d(gx_1, Sx_2)\}\}; \\ &= d(fx_2, gx_1); \\ &= d(y_2, y_1). \end{aligned}$$

Now, we can choose a positive integer $n_3 > n_2$ such that

$$\phi^{n_3}(d(y_2, y_3)) \leq [1 - \phi(M(x_2, x_3))]M(x_2, x_3). \quad (9)$$

There exists $y_4 = fx_4 \in Tx_3 = A_3$ such that

$$d(y_3, y_4) \leq H(Sx_2, Tx_3) + \phi^{n_3}(d(y_2, y_3)). \quad (10)$$

Using (9) and the notion of extended hybrid generalized multi-valued f contraction in the above inequality, we have

$$\begin{aligned} d(y_3, y_4) &\leq H(Sx_2, Tx_3) + \phi^{n_3}(d(y_2, y_3)); \\ &\leq \phi(M(x_2, x_3))M(x_2, x_3) \\ &\quad + \varphi(N(x_2, x_3))N(x_2, x_3) \\ &\quad + [1 - \phi(M(x_2, x_3))]M(x_2, x_3); \\ &= M(x_2, x_3); \\ &= \max\{d(fx_2, gx_3), \min\{d(fx_2, Tx_3), \\ &\quad d(gx_3, Sx_2)\}\}; \\ &= d(fx_2, gx_3); \\ &= d(y_2, y_3). \end{aligned}$$

Now, we can choose a positive integer $n_4 > n_3$ such that

$$\phi^{n_4}(d(y_4, y_3)) \leq [1 - \phi(M(x_4, x_3))]M(x_4, x_3). \quad (11)$$

There exists $y_5 = gx_5 \in Sx_4 = A_4$ such that

$$d(y_5, y_4) \leq H(Sx_4, Tx_3) + \phi^{n_4}(d(y_4, y_3)). \quad (12)$$

Using (11) and the notion of extended hybrid generalized multi-valued f contraction in the above inequality, we have

$$\begin{aligned} d(y_5, y_4) &\leq H(Sx_4, Tx_3) + \phi^{n_4}(d(y_4, y_3)); \\ &\leq \phi(M(x_4, x_3))M(x_4, x_3) \\ &\quad + \varphi(N(x_4, x_3))N(x_4, x_3) \\ &\quad + [1 - \phi(M(x_4, x_3))]M(x_4, x_3); \\ &= M(x_4, x_3); \\ &= \max\{d(fx_4, gx_3), \min\{d(fx_4, Tx_3), \\ &\quad d(gx_3, Sx_4)\}\}; \\ &= d(fx_4, gx_3); \\ &= d(y_4, y_3). \end{aligned}$$

By repeating this process for all $k \in \mathbb{W}$, we have the following:

Case (i). For $k \in \{0, 2, 4, 6, \dots\}$, we can choose a positive integer n_{k+1} such that

$$\phi^{n_{k+1}}(d(y_k, y_{k+1})) \leq [1 - \phi(M(x_k, x_{k+1}))]M(x_k, x_{k+1}). \quad (13)$$

There exists $y_{k+2} = fx_{k+2} \in Tx_{k+1} = A_{k+1}$ such that

$$d(y_{k+1}, y_{k+2}) \leq H(Sx_k, Tx_{k+1}) + \phi^{n_{k+1}}(d(y_k, y_{k+1})). \quad (14)$$

Using (13) and the notion of extended generalized multi-valued f contraction in the above inequality, we have

$$\begin{aligned} d(y_{k+1}, y_{k+2}) &\leq H(Sx_k, Tx_{k+1}) + \phi^{n_{k+1}}(d(y_k, y_{k+1})); \\ &\leq \phi(M(x_k, x_{k+1}))M(x_k, x_{k+1}) \\ &\quad + \varphi(N(x_k, x_{k+1}))N(x_k, x_{k+1}) \\ &\quad + [1 - \phi(M(x_k, x_{k+1}))]M(x_k, x_{k+1}); \\ &= M(x_k, x_{k+1}); \\ &= \max\{d(fx_k, gx_{k+1}), \min\{d(fx_k, Tx_{k+1}), \\ &\quad d(gx_{k+1}, Sx_k)\}\}; \\ &= d(fx_k, gx_{k+1}); \\ &= d(y_k, y_{k+1}). \end{aligned}$$

Case (ii). For $k \in \{1, 3, 5, 7, \dots\}$, we can choose a positive integer n_k such that

$$\phi^{n_{k+1}}(d(y_{k+1}, y_k)) \leq [1 - \phi(M(x_{k+1}, x_k))]M(x_{k+1}, x_k). \tag{15}$$

There exists $y_{k+2} = gx_{k+2} \in Sx_{k+1} = A_{k+1}$ such that

$$d(y_{k+2}, y_{k+1}) \leq H(Sx_{k+1}, Tx_k) + \phi^{n_{k+1}}(d(y_{k+1}, y_k)). \tag{16}$$

Using (15) and the notion of extended generalized multi-valued f contraction in the above inequality, we have

$$\begin{aligned} d(y_{k+2}, y_{k+1}) &\leq H(Sx_{k+1}, Tx_k) + \phi^{n_{k+1}}(d(y_{k+1}, y_k)); \\ &\leq \phi(M(x_{k+1}, x_k))M(x_{k+1}, x_k) \\ &\quad + \varphi(N(x_{k+1}, x_k))N(x_{k+1}, x_k) \\ &\quad + [1 - \phi(M(x_{k+1}, x_k))]M(x_{k+1}, x_k); \\ &= M(x_{k+1}, x_k); \\ &= \max\{d(fx_{k+1}, gx_k), \min\{d(fx_{k+1}, Tx_k), \\ &\quad d(gx_k, Sx_{k+1})\}\}; \\ &= d(fx_{k+1}, gx_k); \\ &= d(y_{k+1}, y_k). \end{aligned}$$

Hence, $\{d(y_k, y_{k+1})\}$ is a nonincreasing sequence for each $k \geq 0$. Thus, by Lemma 3, $\{y_k\}$ is a Cauchy sequence in X . Then, (2) ensures that $\{A_k\}$ is a Cauchy sequence in $CB(X)$. As we know that if X is complete, then $CB(X)$ is also complete. Therefore, there exist $z \in X$ and $A \in CB(X)$ such that $y_k \rightarrow z$ and $A_k \rightarrow A$. Moreover, $gx_{2k+1} \rightarrow z$ and $f_{2k+2} \rightarrow z$, since

$$d(z, A) = \lim_{k \rightarrow \infty} d(y_k, A_k) \leq \lim_{k \rightarrow \infty} H(A_{k-1}, A_k) = 0. \tag{17}$$

It follows that $z \in A$, since A is closed. Thus, we have

$$\begin{aligned} \lim_{k \rightarrow \infty} gx_{2k+1} &= z \in A = \lim_{k \rightarrow \infty} Sx_{2k} \text{ and} \\ \lim_{k \rightarrow \infty} fx_{2k+2} &= z \in A = \lim_{k \rightarrow \infty} Tx_{2k+1} \end{aligned}$$

As g, T and f, S are R -weakly commuting, we have

$$d(gfx_{2k+2}, Tgx_{2k+1}) \leq H(gTx_{2k+1}, Tgx_{2k+1}) \tag{18} \\ \leq Rd(gx_{2k+1}, Tx_{2k+1}).$$

$$d(gx_{2k+1}, Sfx_{2k}) \leq H(fSx_{2k}, Sfx_{2k}) \leq Rd(fx_{2k}, Sx_{2k}). \tag{19}$$

Letting $k \rightarrow \infty$ in (18) and (19) and using (17) and the continuity of f, g, T , and S , we get

$$gz \in Tz \text{ and } fz \in Sz.$$

By condition (ii) of Theorem 1, we have $ggz = gz, fgz = gz$. Let $v = gz$ and then we have $gv = v = fv$. From (2), we have

$$\begin{aligned} H(Sv, Tz) &\leq \phi(\max\{d(fv, gz), \\ &\quad \min\{d(fv, Tz), d(gz, Sv)\}\}) \\ &\quad \times \max\{d(fv, gz), \\ &\quad \min\{d(fv, Tz), d(gz, Sv)\}\} \\ &\quad + \varphi(\min\{d(fv, gz), d(fv, Tv), d(gz, Sz), \\ &\quad d(fv, Tz), d(gz, Sv)\}) \\ &\quad \times \min\{d(fv, gz), d(fv, Tv), d(gz, Sz), \\ &\quad d(fv, Tz), d(gz, Sv)\}. \end{aligned}$$

Note that $fv = gz$ and $fv \in Tz$. Hence, we have $H(Sv, Tz) = 0$, i.e., $Sv = Tz$. Again from (2), we have

$$\begin{aligned} H(Sv, Tv) &\leq \phi(\max\{d(fv, gv), \\ &\quad \min\{d(fv, Tv), d(gv, Sv)\}\}) \\ &\quad \times \max\{d(fv, gv), \\ &\quad \min\{d(fv, Tv), d(gv, Sv)\}\} \\ &\quad + \varphi(\min\{d(fv, gv), d(fv, Tv), d(gv, Sv), \\ &\quad d(fv, Tv), d(gv, Sv)\}) \\ &\quad \times \min\{d(fv, gv), d(fv, Tv), d(gv, Sv), \\ &\quad d(fv, Tv), d(gv, Sv)\}. \end{aligned}$$

Note that $fv = gv$ and $gv \in Sv$. Hence, we have $H(Sv, Tv) = 0$, i.e., $Sv = Tv$. Therefore, we have $v = fv = gv \in Sv = Tv$. \square

Remark 2. Theorem 1 improves and extends some known results of Kamran [12], Nadler [13], Hu [14], Kaneko [15], and Mizoguchi and Takahashi [16].

Example 1. Let $X = [0, \infty)$ be endowed with the metric

$$d(x, y) = \begin{cases} x + y & \text{if } x \neq y, \\ 0 & \text{if } x = y. \end{cases} \tag{20}$$

Define $T, S : X \rightarrow CB(X)$ and $f, g : X \rightarrow X$ by $Tx = [0, \frac{x}{3}]$, $Sx = \{0\}$, $fx = \frac{3x}{2}$, and $gx = \frac{x}{2}$ for all $x \in X$. Let $\phi(t) = \frac{2t}{3}$ and $\varphi(t) = t$ for all $t \geq 0$. It is easy to show that S is the

extended hybrid generalized multi-valued f contraction. It is easy to check that all the conditions of Theorem 1 hold, and O is a common fixed point of f, g, T , and S .

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

MUA and TK contributed equally in this article. Both authors read and approved the final manuscript.

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