Journal of Mathematical Extension
Vol. 5, No. 2 (2), (2011), 91-104

Exact Solutions of the Couple Boiti-Leon-Pempinelli System by the Generalized $(\frac{G'}{G})$-expansion Method

M. Fazli Aghdaei
Payame Noor university of Tehran

J. Manafianheris*
Islamic Azad University, Ahar Branch

Abstract. In this work, the generalized $(\frac{G'}{G})$–expansion method is applied to solve the coupled Boiti-Leon-Pempinelli system (BLP system). Generalized $(\frac{G'}{G})$-expansion method was used to construct solitary wave solutions of the nonlinear evolution equations. This method is developed for searching the exact travelling wave solutions of nonlinear partial differential equations. It is shown that the $(\frac{G'}{G})$-expansion method, with the help of symbolic computation, provides a straightforward and powerful mathematical tool for solving nonlinear partial differential equations.

AMS Subject Classification: 2015
Keywords and Phrases: Generalized $(\frac{G'}{G})$-expansion method; Coupled Boiti-Leon-Pempinelli system; Solitons;

1. Introduction

In this article, we consider the generalized $(\frac{G'}{G})$-expansion method to investigate the coupling Boiti-Leon-Pempinelli system ([3,11,12,14,15,17,19,24,25]) as follows

$$u_{ty} = \left(u^2 - u_x\right)_{xy} + 2v_{xxx}, \tag{1}$$

Received November 2010; Final Revised August 2011
*Corresponding author

$v_t = v_{xx} + 2uv_x$,

and obtained some new exact travelling solutions. Recently, the investigation of exact travelling wave solutions to nonlinear partial differential equations plays an important role in the study of nonlinear in modelling physical phenomena. A variety of powerful methods has been presented including the inverse scattering transform ([1]), homotopy perturbation method ([4,5]), variational iteration method ([5]), homotopy analysis method ([6,7]), Exp-function method ([8,9,13,18]), tanh-function method ([10]), Hirota's bilinear method ([14]), B$\ddot{a}$cklund transformation ([17]), sine-cosine method ([21]) and so on. Here, we use an effective method for constructing a range of exact solutions for the following nonlinear partial differential equations which was first proposed by Wang ([20]). A new method called the $(\frac{G'}{G})$-expansion method is presented to look for travelling wave solutions of nonlinear evolution equations. Bekir [2] is applied the $(\frac{G'}{G})$-expansion method for the nonlinear evolution equations. Zhang et al. ([22]) have examined the generalized $(\frac{G'}{G})$-expansion method. In ([23]) Zhang, Tong and Wang have solved the mKdV equation with variable coefficients using the $(\frac{G'}{G})$-expansion method. By using the aforementioned method, we will obtain the exact solutions of the coupling Boiti-Leon-Pempinelli system. The aim of this paper is to obtain the analytical solutions of the coupling Boiti-Leon-Pempinelli system and to determine the accuracy of this method in solving this equation. The remainder of the paper is organized as follows: In Section 2, first we briefly give the steps of this method and apply the method to solve the nonlinear partial differential equations. In Section 3, the application of the $(\frac{G'}{G})$-expansion method to the coupled Boiti-Leon-Pempinelli system will be introduced briefly. Also a conclusion is given in Section 4.

2. Basic Idea of $(\frac{G'}{G})$-Expansion Method

We consider the detailed description of this method which first presented by Wang ([20]).

Step 1. We consider the nonlinear partial differential equation (NLPDE) with independent variables $X = (x, y, t)$ and dependent variable u as follows:

$$\mathcal{P}(u, u_t, u_x, u_y, u_{xx}, u_{yy}, u_{xy}, u_{tt}, u_{tx}, u_{ty}, ...) = 0, \quad (2)$$

in which $\mathcal{P}$ can be converted to an ordinary differential equation (ODE)

$$\mathcal{M}(u, -cu', u', u', u'', ...) = 0, \quad (3)$$

which transformation

$$\xi = x + y - ct, \quad (4)$$

is wave variable. Also, c is constant to be determined later.

Step 2. We seek its solutions in the more general polynomial form as follows

$$u(\xi) = a_0 + \sum_{k=1}^{m} a_k \left(\frac{G'(\xi)}{G(\xi)}\right)^k, \quad (5)$$

where $G(\xi)$ satisfies the second order linear ordinary differential equation (LODE) in the form

$$G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0, \quad (6)$$

where a_0, $a_k(k = 1, 2, ..., m)$, λ and μ are constants to be determined later, $a_m = 0$, but the degree of which is generally equal to or less than $m - 1$, the positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in Eq. (3).

Step 3. Substituting (5) and Eq. (6) into Eq. (3) with the value of m obtained in Step 1. Collecting the coefficients of $\left(\frac{G'(\xi)}{G(\xi)}\right)^k$ $(k = 0, 1, 2, ...)$, then setting each coefficient to zero, we can get a set of over-determined partial differential equations for $a_0, a_i(i = 1, 2, ..., n), \lambda, c$ and μ with the aid of symbolic computation Maple 12.

Step 4. Solving the algebraic equations in Step 3, then substituting $a_i, ..., a_m, c$ and general solutions of Eq. (6) into (5) we can obtain a series of fundamental solutions of Eq. (2) depending of the solution $G(\xi)$ of Eq. (6).

3. The Couple Boiti-Leon-Pempinelli System

We employ the generalized $(\frac{G'}{G})$-expansion method to the Boiti-Leon-Pempinelli system as follows

$$u_{ty} = \left(u^2 - u_x\right)_{xy} + 2v_{xxx}, \qquad (7)$$
$$v_t = v_{xx} + 2uv_x.$$

Introducing a complex variation ξ defined as (4) and then system (7) becomes an ODE, in the form of

$$-cu'' = \left(u^2 - u'\right)'' + 2v''', \qquad (8)$$
$$-cv' = v'' + 2uv',$$

where by integrating twice the first equation we obtain

$$v' = \frac{1}{2}u' - \frac{1}{2}cu - \frac{1}{2}u^2. \qquad (9)$$

Integrating Eq. (9) with respect to ξ and considering the zero constants for integration, we obtain

$$v = \frac{1}{2}u - \frac{1}{2}\int(cu + u^2)d\xi. \qquad (10)$$

Substituting Eq. (9) into the second equation of Eq. (8), we get

$$u'' - 2u^3 - 3cu^2 - c^2u = 0. \qquad (11)$$

In order to determine value of m, we balance the linear term of the highest order u'' with the highest order nonlinear term u^3 in Eq. (11) and using Eq. (5) we have

$$u^3(\xi) = a_m^3\left(\frac{G'(\xi)}{G(\xi)}\right)^{3m} + ..., \qquad (12)$$

$$u_{\xi\xi}(\xi) = m(m+1)a_m \left(\frac{G'(\xi)}{G(\xi)}\right)^{m+2} + ...$$

Balancing u'' with u^3 in Eq. (11), we conclude that $m + 2 = 3m \Rightarrow m = 1$. We can suppose that the solutions of Eq. (7) is in the form

$$u(\xi) = a_0 + a_1 \left(\frac{G'(\xi)}{G(\xi)}\right), \qquad a_1 \neq 0, \tag{13}$$

therefore

$$u^3(\xi) = a_1^3 \left(\frac{G'(\xi)}{G(\xi)}\right)^3 + 3a_0a_1^2 \left(\frac{G'(\xi)}{G(\xi)}\right)^2 + 3a_0^2a_1 \left(\frac{G'(\xi)}{G(\xi)}\right) + a_0^3, \tag{14}$$

and

$$u_{\xi\xi}(\xi) = 2a_1 \left(\frac{G'(\xi)}{G(\xi)}\right)^3 + 3a_1\lambda \left(\frac{G'(\xi)}{G(\xi)}\right)^2 + (\lambda^2 a_1 + 2a_1\mu) \left(\frac{G'(\xi)}{G(\xi)}\right) + \lambda a_1 \mu. \tag{15}$$

Substituting (13)–(15), and using the well-known Maple 12 software, we will have

$$a_0 = \frac{\lambda \pm \sqrt{\lambda^2 - 4\mu}}{2}, \quad a_1 = 1, \quad c = \mp\sqrt{\lambda^2 - 4\mu}, \quad \lambda = \lambda, \tag{16}$$

or

$$a_0 = \frac{-\lambda \pm \sqrt{\lambda^2 - 4\mu}}{2}, \quad a_1 = -1, \quad c = \mp\sqrt{\lambda^2 - 4\mu}, \quad \lambda = \lambda, \tag{17}$$

where λ and μ are arbitrary constants. Substituting (16) and (17) into expression (13), can be written as

$$u(\xi) = \frac{\lambda \pm \sqrt{\lambda^2 - 4\mu}}{2} + \left(\frac{G'(\xi)}{G(\xi)}\right), \quad \xi = x + y \pm \sqrt{\lambda^2 - 4\mu}t, \tag{18}$$

or

$$u(\xi) = \frac{-\lambda \pm \sqrt{\lambda^2 - 4\mu}}{2} - \left(\frac{G'(\xi)}{G(\xi)}\right), \quad \xi = x + y \pm \sqrt{\lambda^2 - 4\mu}t, \tag{19}$$

Substituting the general solutions of Eq. (6) into (18) and (19) we have three types of the exact solutions of Eq. (7) as follows:

Case 1:

When $\lambda^2 - 4\mu > 0$, we obtain hyperbolic function solution

$$u_1(\xi) = \frac{\lambda \pm \sqrt{\lambda^2 - 4\mu}}{2} + \frac{\sqrt{\lambda^2 - 4\mu}}{2}$$

$$\left(\frac{C_1 \sinh\left(\frac{\sqrt{\lambda^2-4\mu}\xi}{2}\right) + C_2 \cosh\left(\frac{\sqrt{\lambda^2-4\mu}\xi}{2}\right)}{C_1 \cosh\left(\frac{\sqrt{\lambda^2-4\mu}\xi}{2}\right) + C_2 \sinh\left(\frac{\sqrt{\lambda^2-4\mu}\xi}{2}\right)} \right), \tag{20}$$

if $\mathrm{C}_1 > 0$, $\mathrm{C}_1^2 > \mathrm{C}_2^2$, then $\mathrm{v}_1(\xi)$ can be written as

$$\mathrm{v}_1(\xi) = \mp \frac{1}{8\sqrt{\lambda^2 - 4\mu}} \left\{ \mp 2\mathrm{c}\sqrt{\lambda^2 - 4\mu} \ln \left[\tanh^2\left(\frac{\sqrt{\lambda^2 - 4\mu}\xi}{2} + \xi_0 \right) - 1 \right] \right.$$

$$\pm 2\mathrm{c}\lambda\sqrt{\lambda^2 - 4\mu}\xi + 2\mathrm{c}(\lambda^2 - 4\mu)\xi \mp 4(\lambda^2 - 4\mu) \tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}\xi}{2} + \xi_0 \right)$$

$$+ \left(-5\lambda^2 + 16\mu - 4\lambda\sqrt{\lambda^2 - 4\mu} \right) \ln \left[\tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}\xi}{2} + \xi_0 \right) - 1 \right]$$

$$\left. +\lambda^2 \ln \left[\tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}\xi}{2} + \xi_0 \right) + 1 \right] \mp 2\lambda\sqrt{\lambda^2 - 4\mu} - 2(\lambda^2 - 4\mu) \right\},$$

where $\xi_0 = \tanh^{-1}\left(\frac{\mathrm{C}_2}{\mathrm{C}_1}\right)$ and $\xi = \mathrm{x} + \mathrm{y} \pm \sqrt{\lambda^2 - 4\mu}\mathrm{t}$. When $\lambda^2 - 4\mu < 0$, we have trigonometric function solution

$$u_2(\xi) = \frac{\lambda \pm \sqrt{\lambda^2 - 4\mu}}{2} + \frac{\sqrt{4\mu - \lambda^2}}{2} \left(\frac{-C_1 \sin\left(\frac{\sqrt{4\mu-\lambda^2}\xi}{2}\right) + C_2 \cos\left(\frac{\sqrt{4\mu-\lambda^2}\xi}{2}\right)}{C_1 \cos\left(\frac{\sqrt{4\mu-\lambda^2}\xi}{2}\right) + C_2 \sin\left(\frac{\sqrt{4\mu-\lambda^2}\xi}{2}\right)} \right), \tag{21}$$

$$\mathrm{v}_2(\xi) = \pm \frac{1}{4\sqrt{4\mu - \lambda^2}} \left\{ \pm 2(4\mu - \lambda^2) \tan\left(\frac{-\sqrt{4\mu - \lambda^2}\xi}{2} + \xi_0 \right) + \right.$$

$$\sqrt{4\mu - \lambda^2} \left(\pm\lambda + \sqrt{\lambda^2 - 4\mu} \right) (1 - \mathrm{c}\xi) + \ln \left[1 + \tan^2\left(\frac{-\sqrt{4\mu - \lambda^2}\xi}{2} + \xi_0 \right) \right]$$

$$\times\sqrt{4\mu-\lambda^2}\left(\pm \mathrm{c}\pm\lambda+\sqrt{\lambda^2-4\mu}\right)+(\pm 3\lambda^2+2\lambda\sqrt{\lambda^2-4\mu}\mp 8\mu)$$
$$\times\arctan\left[\tan\left(\frac{-\sqrt{4\mu-\lambda^2}\xi}{2}+\xi_0\right)\right]\Bigg\},$$

where $\xi_0=\tan^{-1}\left(\frac{C_2}{C_1}\right)$ and $\xi=x+y\pm\sqrt{\lambda^2-4\mu}t$. When $\lambda^2-4\mu=0$, we get rational solution

$$u_3(\xi)=\frac{\lambda}{2}+\frac{C_2}{(C_1+C_2\xi)},\qquad \xi=x+y\pm\sqrt{\lambda^2-4\mu}t, \tag{22}$$

$$v_3(\xi)=\frac{-1}{8(C_1+C_2\xi)}\left[-2\lambda C_1-8C_2+(\lambda^2C_1-2\lambda C_2+2c\lambda C_1)\xi+\right.$$
$$\left.(\lambda^2C_2+2c\lambda C_2)\xi^2+4(\lambda+c)(C_1+C_2\xi)\ln(C_1+C_2\xi)\right].$$

If $C_1\neq 0, C_2=0, \lambda>0, \mu=0$, then (20) give

$$u_4(\xi)=\lambda\left(1+\frac{1}{2}\tanh\frac{\lambda}{2}\xi\right),\qquad \xi=x+y+\lambda t, \tag{23}$$

$$v_4(\xi)=\frac{-\lambda}{4}\ln\left[\tanh^2\frac{\lambda}{2}\xi-1\right]+\frac{\lambda}{8}\left[\ln\frac{(\tanh\frac{\lambda}{2}\xi-1)^9}{(\tanh\frac{\lambda}{2}\xi+1)}\right]+\frac{\lambda}{2}\tanh\frac{\lambda\xi}{2}+\frac{\lambda}{2}(1+\lambda\xi),$$

$$u_5(\xi)=\frac{\lambda}{2}\tanh\frac{\lambda}{2}\xi,\qquad \xi=x+y-\lambda t, \tag{24}$$

$$v_5(\xi)=\frac{c}{4}\ln\left[\tanh^2\frac{\lambda}{2}\xi-1\right]+\frac{\lambda}{8}\left[\ln\frac{(\tanh\frac{\lambda}{2}\xi-1)}{(\tanh\frac{\lambda}{2}\xi+1)}\right]+\frac{\lambda}{2}\tanh\frac{\lambda\xi}{2}.$$

If $C_2\neq 0, C_1=0, \lambda>0, \mu=0$, then (20) become

$$u_6(\xi)=\lambda\left(1+\frac{1}{2}\coth\frac{\lambda}{2}\xi\right),\qquad \xi=x+y+\lambda t, \tag{25}$$

$$v_6(\xi)=\frac{-\lambda}{4}\ln\left[\coth^2\frac{\lambda}{2}\xi-1\right]+\frac{\lambda}{8}\left[\ln\frac{(\coth\frac{\lambda}{2}\xi-1)^9}{(\coth\frac{\lambda}{2}\xi+1)}\right]+\frac{\lambda}{2}\coth\frac{\lambda\xi}{2}+\frac{\lambda}{2}(1+\lambda\xi),$$

$$u_7(\xi)=\frac{\lambda}{2}\coth\frac{\lambda}{2}\xi,\qquad \xi=x+y-\lambda t, \tag{26}$$

$$v_7(\xi)=\frac{c}{4}\ln\left[\coth^2\frac{\lambda}{2}\xi-1\right]+\frac{\lambda}{8}\left[\ln\frac{(\coth\frac{\lambda}{2}\xi-1)}{(\coth\frac{\lambda}{2}\xi+1)}\right]+\frac{\lambda}{2}\coth\frac{\lambda\xi}{2}.$$

In particular, if $C_1 \neq 0, C_2 = 0, \lambda = 0, \mu > 0$, then (21) give

$$u_8(\xi) = \sqrt{-\mu} - \sqrt{\mu}\,\tan\sqrt{\mu}\xi, \quad \xi = x + y + 2\sqrt{-\mu}t, \tag{27}$$

$$v_8(\xi) = \frac{1}{2}\sqrt{-\mu} - \mu\xi - \sqrt{\mu}\ \tan\sqrt{\mu}\xi + \sqrt{\mu}\ \arctan\left(\tan\sqrt{\mu}\xi\right),$$

$$u_9(\xi) = -\sqrt{-\mu} - \sqrt{\mu}\,\tan\sqrt{\mu}\xi, \quad \xi = x + y - 2\sqrt{-\mu}t, \tag{28}$$

$$v_9(\xi) = -\frac{1}{2}\sqrt{-\mu} - \mu\xi - \sqrt{\mu}\ \tan\sqrt{\mu}\xi + \sqrt{\mu}\ \arctan\left(\tan\sqrt{\mu}\xi\right).$$

If $C_2 \neq 0, C_1 = 0, \lambda = 0, \mu > 0$, then (21) become

$$u_{10}(\xi) = \sqrt{-\mu} - \sqrt{\mu}\,\cot\sqrt{\mu}\xi, \quad \xi = x + y + 2\sqrt{-\mu}t, \tag{29}$$

$$v_{10}(\xi) = \frac{1}{2}\sqrt{-\mu} - \mu\xi - \sqrt{\mu}\ \arctan\left(\cot\sqrt{\mu}\xi\right),$$

$$u_{11}(\xi) = -\sqrt{-\mu} - \sqrt{\mu}\,\cot\sqrt{\mu}\xi, \quad \xi = x + y - 2\sqrt{-\mu}t, \tag{30}$$

$$v_{11}(\xi) = -\frac{1}{2}\sqrt{-\mu} - \mu\xi - \sqrt{\mu}\ \arctan\left(\cot\sqrt{\mu}\xi\right).$$

Case 2:

When $\lambda^2 - 4\mu > 0$, we obtain hyperbolic function solution

$$u_{12}(\xi) = \frac{-\lambda \pm \sqrt{\lambda^2 - 4\mu}}{2} - \frac{\sqrt{\lambda^2 - 4\mu}}{2}\left(\frac{C_1 \sinh\left(\frac{\sqrt{\lambda^2-4\mu}\xi}{2}\right) + C_2 \cosh\left(\frac{\sqrt{\lambda^2-4\mu}\xi}{2}\right)}{C_1 \cosh\left(\frac{\sqrt{\lambda^2-4\mu}\xi}{2}\right) + C_2 \sinh\left(\frac{\sqrt{\lambda^2-4\mu}\xi}{2}\right)}\right), \tag{31}$$

if $C_1 > 0$, $C_1^2 > C_2^2$, then $v_{12}(\xi)$ can be written as

$$v_{12}(\xi) = \frac{-1}{8\sqrt{\lambda^2 - 4\mu}}\left\{2c\sqrt{\lambda^2 - 4\mu}\ln\left[\tanh^2\left(\frac{\sqrt{\lambda^2 - 4\mu}\xi}{2} + \xi_0\right) - 1\right] + \right.$$
$$2(\lambda \mp \sqrt{\lambda^2 - 4\mu})\sqrt{\lambda^2 - 4\mu}(1 - c\xi) \mp \lambda^2 \ln\left[\tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}\xi}{2} + \xi_0\right) - 1\right]$$
$$\left.\pm\left(5\lambda^2 - 16\mu - 4\lambda\sqrt{\lambda^2 - 4\mu}\right)\ln\left[\tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}\xi}{2} + \xi_0\right) + 1\right]\right\},$$

where $\xi_0 = \tanh^{-1}\left(\frac{C_2}{C_1}\right)$ and $\xi = x + y \pm \sqrt{\lambda^2 - 4\mu}t$.

When $\lambda^2 - 4\mu < 0$, we have trigonometric function solution

$$u_{13}(\xi)=\frac{-\lambda\pm\sqrt{\lambda^2-4\mu}}{2}-\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin\left(\frac{\sqrt{4\mu-\lambda^2}\xi}{2}\right)+C_2\cos\left(\frac{\sqrt{4\mu-\lambda^2}\xi}{2}\right)}{C_1\cos\left(\frac{\sqrt{4\mu-\lambda^2}\xi}{2}\right)+C_2\sin\left(\frac{\sqrt{4\mu-\lambda^2}\xi}{2}\right)}\right), \tag{32}$$

$$v_{13}(\xi)=\frac{\mp 1}{4\sqrt{4\mu-\lambda^2}}\left\{\sqrt{4\mu-\lambda^2}\left(\mp c\mp\lambda+\sqrt{\lambda^2-4\mu}\right)\right.$$

$$\ln\left[1+\tan^2\left(\frac{-\sqrt{4\mu-\lambda^2}\xi}{2}+\xi_0\right)\right]$$

$$+(\mp 3\lambda^2+2\lambda\sqrt{\lambda^2-4\mu}\pm 8\mu)\times\arctan\left[\tan\left(\frac{-\sqrt{4\mu-\lambda^2}\xi}{2}+\xi_0\right)\right]$$

$$\left.\pm\sqrt{4\mu-\lambda^2}\left(\lambda-\sqrt{\lambda^2-4\mu}\right)(1-c\xi)\right\},$$

where $\xi_0=\tan^{-1}\left(\frac{C_2}{C_1}\right)$ and $\xi=x+y\pm\sqrt{\lambda^2-4\mu}\,t$. When $\lambda^2-4\mu=0$, we get rational solution

$$u_{14}(\xi)=\frac{-\lambda}{2}+\frac{C_2}{(C_1+C_2\xi)},\qquad \xi=x+y\pm\sqrt{\lambda^2-4\mu}\,t, \tag{33}$$

$$v_{14}(\xi)=\frac{-1}{8(C_1+C_2\xi)}\left[2\lambda C_1-8C_2+(\lambda^2C_1+2\lambda C_2-2c\lambda C_1)\xi+\right.$$
$$\left.(\lambda^2C_2-2c\lambda C_2)\xi^2+4(c-\lambda)(C_1+C_2\xi)\ln(C_1+C_2\xi)\right].$$

If $C_1\neq 0, C_2=0, \lambda>0, \mu=0$, then (31) give

$$u_{15}(\xi)=\frac{\lambda}{2}\tanh\frac{\lambda}{2}\xi,\qquad \xi=x+y+\lambda t, \tag{34}$$

$$v_{15}(\xi)=\frac{\lambda}{4}\ln\left[\tanh^2\frac{\lambda}{2}\xi-1\right]+\frac{\lambda}{8}\left[\ln\frac{(\tanh\frac{\lambda}{2}\xi-1)}{(\tanh\frac{\lambda}{2}\xi+1)}\right]+\frac{\lambda}{2}\tanh\frac{\lambda\xi}{2},$$

$$u_{16}(\xi)=-\lambda\left(1-\frac{1}{2}\tanh\frac{\lambda}{2}\xi\right),\qquad \xi=x+y-\lambda t, \tag{35}$$

$$v_{16}(\xi) = \frac{c}{4}\ln\left[\tanh^2\frac{\lambda}{2}\xi - 1\right] + \frac{\lambda}{8}\left[\ln\frac{(\tanh\frac{\lambda}{2}\xi - 1)}{(\tanh\frac{\lambda}{2}\xi + 1)^9}\right] + \frac{\lambda}{2}\tanh\frac{\lambda\xi}{2} - \frac{\lambda}{2}(1 - c\xi).$$

But, if $C_2 \neq 0, C_1 = 0, \lambda > 0, \mu = 0$, then (31) become

$$u_{17}(\xi) = \frac{\lambda}{2}\coth\frac{\lambda}{2}\xi, \quad \xi = x + y + \lambda t, \tag{36}$$

$$v_{17}(x,t) = \frac{\lambda}{4}\ln\left[\coth^2\frac{\lambda}{2}\xi - 1\right] + \frac{\lambda}{8}\left[\ln\frac{(\coth\frac{\lambda}{2}\xi - 1)}{(\coth\frac{\lambda}{2}\xi + 1)}\right] + \frac{\lambda}{2}\coth\frac{\lambda\xi}{2},$$

$$u_{18}(\xi) = -\lambda\left(1 - \frac{1}{2}\coth\frac{\lambda}{2}\xi\right), \quad \xi = x + y - \lambda t, \tag{37}$$

$$v_{18}(\xi) = \frac{c}{4}\ln\left[\coth^2\frac{\lambda}{2}\xi - 1\right] + \frac{\lambda}{8}\left[\ln\frac{(\coth\frac{\lambda}{2}\xi - 1)}{(\coth\frac{\lambda}{2}\xi + 1)^9}\right] + \frac{\lambda}{2}\coth\frac{\lambda\xi}{2} - \frac{\lambda}{2}(1 - c\xi).$$

In particular, if $C_1 \neq 0, C_2 = 0, \lambda = 0, \mu < 0$, then (31) give

$$u_{19}(\xi) = \sqrt{-\mu}(1 + \tanh\sqrt{-\mu}\xi), \quad \xi = x + y + 2\sqrt{-\mu}t, \tag{38}$$

$$v_{19}(\xi) = \frac{1}{2\sqrt{-\mu}}\left(-\mu - 2\mu\sqrt{-\mu}\xi + \mu\ln\left[\tanh^2\sqrt{-\mu}\xi - 1\right] - 2\mu\ln\left[\tanh\sqrt{-\mu}\xi - 1\right]\right),$$

$$u_{20}(\xi) = -\sqrt{-\mu}(1 - \tanh\sqrt{-\mu}\xi), \quad \xi = x + y - 2\sqrt{-\mu}t, \tag{39}$$

$$v_{20}(\xi) = -\frac{1}{2\sqrt{-\mu}}\left(-\mu + 2\mu\sqrt{-\mu} + \mu\ln\left[\tanh^2\sqrt{-\mu}\xi - 1\right] - 2\mu\ln\left[\tanh\sqrt{-\mu}\xi + 1\right]\right).$$

If $C_2 \neq 0, C_1 = 0, \lambda = 0, \mu < 0$, then (31) become

$$u_{21}(\xi) = \sqrt{-\mu}(1 + \coth\sqrt{-\mu}\xi), \quad \xi = x + y + 2\sqrt{-\mu}t, \tag{40}$$

$$v_{21}(\xi) = \frac{1}{2\sqrt{-\mu}}\left(-\mu - \mu\sqrt{-\mu}\xi + \mu\ln\left[\coth^2\sqrt{-\mu}\xi - 1\right] - 2\mu\ln\left[\coth\sqrt{-\mu}\xi - 1\right]\right),$$

$$u_{22}(\xi) = -\sqrt{-\mu}(1 - \coth\sqrt{-\mu}\xi), \quad \xi = x + y - 2\sqrt{-\mu}t, \tag{41}$$

$$v_{22}(\xi) = \frac{1}{2\sqrt{-\mu}}\left(-\mu + 2\mu\sqrt{-\mu} + \mu\ln\left[\coth^2\sqrt{-\mu}\xi - 1\right] - 2\mu\ln\left[\coth\sqrt{-\mu}\xi + 1\right]\right),$$

which are the exact solutions of the discussed equation.

4. Conclusion

In this paper, we investigated the coupled Boiti-Leon-Pempinelli system. The generalized $(\frac{G'}{G})$-expansion method is a useful method for finding travelling wave solutions of nonlinear evolution equations. This method has been successfully applied to obtain some new generalized solitonary solutions to the coupled BLP system. These exact solutions include hyperbolic function solution, trigonometric function solution, and rational solution. The generalized $(\frac{G'}{G})$-expansion method is more powerful in searching for the exact solutions of nonlinear partial differential equations (NLPDEs). Also, new results are formally developed in this paper. It can be concluded that this method is a very powerful and efficient technique in finding exact solutions for wide classes of problems. In our work, we made use of the well-known symbolic software Maple 12 Package to calculate the algebraic equations obtained from the analytical scheme that were employed.

Acknowledgment:
The authors would like to thank the referees for their valuable suggestions and comments.

References

[1] M. J. Ablowitz and P.A. Clarkson, Solitons, *Nonlinear evolution equations and inverse scattering,* Cambridge: Cambridge University Press., 1991.

[2] A. Bekir, Application of the $(\frac{G'}{G})$-expansion method for nonlinear evolution equations, *Phys. Lett. A.,* 372 (2008), 3400-3406.

[3] M. Boiti, J. J. P. Leon, and F. Pempinelli, Integrable two-dimensional generalisation of the sine- and sinh-Gordon equations, *Inverse Prob.,* 3 (1987), 37-49.

[4] M. Dehghan and J. Manafian, The solution of the variable coefficients fourth–order parabolic partial differential equations by homotopy perturbation method, *Z. Naturforsch,* 64 (2009), 420-430.

[5] M. Dehghan, J. Manafian, and A. Saadatmandi, Application of semi-analytic methods for the Fitzhugh-Nagumo equation, which models the

transmission of nerve impulses, *Math. Meth. Appl. Sci.,* 33 (2010), 1384-1398.

[6] M. Dehghan, J. Manafian, and A. Saadatmandi, Solving nonlinear fractional partial differential equations using the homotopy analysis method, *Num. Meth. Partial Differential Eq. J.,* 26 (2010), 448-479.

[7] M. Dehghan, J. Manafian, and A. Saadatmandi, The solution of the linear fractional partial differential equations using the homotopy analysis method, *Z. Naturforsch,* 65a (2010), 935-949.

[8] M. Dehghan, J. Manafian Heris, and A. Saadatmandi, Application of the Exp-function method for solving a partial dierential equation arising in biology and population genetics, *Int. J. Num. Methods for Heat Fluid Flow,* 21 (2011), 736-753.

[9] M. Dehghan, J. Manafian Heris, and A. Saadatmandi, Analytical treatment of some partial differential equations arising in mathematical physics by using the Exp-function method, *Int. J. Modern Physics,* B., 25 (2011), 2965-2981.

[10] E. Fan, Extended tanh-function method and its applications to nonlinear equations, *Phys. Lett. A.,* 277 (2000), 212-218.

[11] J. P. Fang, C. L. Zheng, and J. M. Zhu, New variable separation excitations, rectangle-like solutions and fractal solutions in the Boiti-Leon-Pempinelli system, *Acta. Phys. Sin.,* 54 (2005), 2990-3006.

[12] W. G. Feng, K. M. Li, Y. Z. Li, and C. Lin, Explicit exact solutions for $(2+1)$-dimensional BoitiLeonPempinelli equation, *Commun. Nonlinear Sci. Num. Simulat,* 14 (2009), 2013-2017.

[13] J. H. He and X. H. Wu, Exp-function method for nonlinear wave equations, *Chaos Solitons Fractals,* 30 (2006), 700-708.

[14] R. Hirota, *The Direct Method in Soliton Theory,* Cambridge Univ. Press., 2004.

[15] D. J. Huang and H. Q. Zhang, Exact travelling wave solutions for the Boiti-Leon-Pempinelli equation, *Chaos Soliton Fract,* 22 (2004), 243-247.

[16] X. Lü, H. W. Zhu, X. H. Meng, Z. C. Yang, and B. Tian, Soliton solutions and a Bäcklund transformation for a generalized nonlinear Schrödinger equation with variable coefficients from optical fiber communications, *J. Math. Anal. Appl.,* 336 (2007), 1305-1315.

[17] Z. S. Lü and Q. H. Zhang, Multi-soliton like solutions for the Boiti-Leon-Pempinelli equation, *Chaos Soliton Fract,* 19 (2004), 527-531.

[18] J. Manafian Heris and M. Bagheri, Exact solutions for the modified KdV and the generalized KdV equations via Exp-function method, *J. Math. Extension,* 4 (2010), 77-98.

[19] Y. J. Ren, S. T. Liu, and H. Q. Zhang, A new generalized algebra method and its application in the $(2+1)$ dimensional Boiti-Leon-Pempinelli equation, *Chaos Soliton Fract,* 32 (2007), 1655-1665.

[20] M. Wang, X. Li, and J. Zhang, The $(\frac{G'}{G})$-expansion method and travelling wave solutions of nonlinear evolution equations in mathematical physics, *Phys. Lett. A.,* 372 (2008), 417-423.

[21] E. Yusufoğlu, A. Bekir, and M. Alp, Periodic and solitary wave solutions of Kawahara and modified Kawahara equations by using Sine-Cosine method, *Chaos Solitons Fractals,* 37 (2008), 1193-1197.

[22] J. Zhang, X. Wei, and Y. Lu, A generalized $(\frac{G'}{G})$-expansion method and its applications evolution equations in mathematical physics, *Phys. Lett. A.,* 372 (2008), 3653-3658.

[23] S. Zhang, J. L. Tong, and W. Wang, A generalized $(\frac{G'}{G})$-expansion method for the mKdV equation with variable coefficients, *Phys. Lett. A.,* 372 (2008). 3653-3658.

[24] C. L. Zheng, J. P. Fang, and L. Q. Chen, Bell-like and peak-like loop solitons in $(2+1)$-dimensional Boiti-Leon-Pempinelli system, *Acta. Phys. Sin.,* 54 (2005), 1468-1508.

[25] S. D. Zhu, The generalizing Riccati equation mapping method in nonlinear evolution equation: application to $(2+1)$-dimensional Boiti-Leon-Pempinelle equation, *Chaos Soliton Fract,* 37 (2008), 1335-1342.

Mehdi Fazli Aghdaei
Department of Mathematics
Instructor of Mathematics
Payame noor university
P.O. Box 19395-3697
Tehran, Iran
E-mail: Fazli-112@yahoo.com

Jalil Manafian Heris
Department of Mathematics
Ph.D of student Applied Mathematics
Islamic Azad University-Ahar Branch
Ahar, Iran
E-mail: manafian@aut.ac.ir