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Dynamic Terminal Sliding Mode Control for an Aerospace Launch Vehicle

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Tracking guidance commands for a time-varying aerospace launch vehicle during the atmospheric flight is considered in this paper. Hence, the dynamic terminal sliding mode control law is constructed for this purpose and dynamic sliding mode control is utilized. The terminal sliding manifold causes the dynamic sliding mode to converge asymptotically to zero in finite-time. The actuator and rate gyro dynamics are included in the model of launch vehicle. Dynamic sliding mode control accommodates unmatched disturbances, while the terminal sliding mode control is used to accelerate the system to reach the dynamic sliding manifold. Finally, the effectiveness of the proposed control is demonstrated in the presence of unmatched disturbances and is compared with the dynamic sliding mode.

Keywords: Terminal sliding mode, Dynamic sliding mode, Unmatched disturbance, Finite-time convergence

Nomenclature

e	tracking error
h_0	Initial height
$\mathfrak{I}$	DSM surface
q	pitch rate
S	sliding mode surface
TC	thrust due to control engines
U	speed of launch vehicle
V	Lyapunov function
V_x	velocity in x direction
V_z	velocity in z direction
ρ	air density, positive constant
ε	width of boundary layer
μ	gravitational parameter
θ	pitch angle

Introduction

The main purpose of the flight control systems or autopilots in an Aerospace Launch Vehicle (ALV) is to maintain the vehicle attitude commanded by the guidance section. The autopilot determines the ALV attitude via an inertial measurement unit (IMU) and commands the appropriate change in the engine thrust vector to achieve the commanded attitude. Design of the launch vehicle autopilot must satisfy three main, often conflicting requirements: stabilizing the vehicle, ensuring adequate responses to guidance commands while minimizing trajectory deviation, and minimizing angle of attack in the region of high dynamic pressure to ensure the structural integrity of the vehicle [1].

Flight control of aeronautical and space vehicles involves attitude maneuvering through a wide range of flight conditions, wind disturbances, and plant uncertainties, including aerodynamic surfaces and engine failures. Application of differential geometric methods to the system state-flow analysis and feedback synthesis has led to the development of many powerful control design methods for flight control systems. Such a controller would drastically decrease the amount of time spent in the pre-flight analysis, and at the same time improve flight vehicle reliability and robustness in the face of failure and damages. Sliding mode control (SMC) methods have already found their places in the arsenal of control design tools for aeronautical and space vehicles [2]. The sliding mode control has been studied for many decades and it is now one of the most active areas of research on nonlinear system theory. The sliding mode control is characterized by the choice of a sliding surface

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describing the desired performance and by the determination of a control law in a way that drives the system states to reach and remain on this surface. An asymptotic convergence to the final state will be achieved in the sliding mode [3].

Dynamic sliding mode (DSM) is used by many authors and in some of the works it has been combined with other sliding mode methods such as terminal sliding mode ([2, 4-7]). Feng et al. employ terminal sliding mode and conventional sliding mode to construct second-order sliding mode for a class of uncertain input-delay systems. A linear sliding mode manifold is predesigned to represent the ideal dynamics of the system. Another terminal sliding mode manifold surface is presented to derive the linear sliding mode to reach zero in finite-time [8].

In sliding mode, the purpose of control is reaching the manifold in finite-time and remaining on this surface for all of the time. Terminal sliding mode is used to accelerate the system to reach the linear sliding mode so as to increase the response rate of system ([8, 10]).

In this paper, a new sliding mode by using two sliding mode methods is presented. Dynamic sliding mode for a lunch vehicle, which has non-minimum phase and time-variable model, is designed. Another terminal sliding manifold is presented in order to converge the dynamic sliding surface to zero in finite-time. Saturation function is used to eliminate or reduce chattering, but in the saturation function employed here a new sliding surface is presented inside the control structure. The combination of the saturation function with this terminal dynamic sliding surface is in a way that the variation of the Lyapunov function is hastened in the proximity of zero. The saturation function, which considers the terminal sliding mode surface as its variable, forces Lyapunov function derivative to hasten when reaching the approximation of zero; it causes control law to have more potent asymptotically stability for the equilibrium point in the presence of disturbances and parameter uncertainties. This difference in the dynamic sliding surface exists because of the use of terminal nonlinear manifold to converge the tracking error to zero in finite-time.

The paper is organized as follows: in section II the model formulation is presented. In section III sliding mode concepts are explained. Terminal sliding mode and dynamic sliding mode control theory are separately presented in section III. In the other words, this section explains the theoretical base of control design. In section IV, after the choice of sliding surface, the control structure is determined. Section V illustrates simulation and results for the proposed control in comparison with dynamic sliding mode, and finally concluding remarks are given in section VI.

Model Formulation

For describing the dynamics of the model, obtaining equation of motion for the launch vehicle is necessary. The equation of motion for the launch vehicle can be derived from Newton's second law of motion, which states that the summation of all external forces acting on a body must be equal to the time rate of change of its momentum and the summation of the external moments must be equal to the time rate of change of its moment of momentum (angular momentum). The time rates of change are all taken with respect to inertial space [10]. These laws can be expressed as follows:

$$F_x = m.(\dot{V}_x + q.V_z)$$
$$F_z = m.(\dot{V}_z - q.V_x) \quad (1)$$
$$M_y = I_y.\dot{q}$$

This equation is simplified to 3-degree-of-freedom equations. Because the goal is longitudinal control, planar equations are used. By substituting aerodynamic, gravitational and control forces in the above equations, the 3DOF equation of motion is simplified as follows:

$$\dot{V}_x = \frac{F_x}{m} - q.V_z \quad (2)$$

$$\dot{V}_z = \left(\frac{1}{2m}\rho USC_{z\alpha}\right)V_z + \left(\frac{1}{4m}\rho USDC_{zq} + U\right)q - g\cos\theta - \left(\frac{2}{m}.TC\right)\sin(\delta_e) \quad (3)$$

$$\dot{q} = \left(\frac{1}{2I_y}\rho USx_{ac}C_{z\alpha}\right)V_z + \left(\frac{1}{4I_y}\rho USD^2C_{mq}\right)q - \left(\frac{2}{I_y}.TC.dx\right)\sin(\delta_e) \quad (4)$$

Where F_x is the resultant of all longitudinal forceses thrust, control thrust vectors (that decomposed in total longitudinal) and aerodynamic drag. x_{ac} is the static margin and δ_e is the angle of control engines deflection. Above equations are used for the simulation of the model.

The control design for a lunch vehicle by some linearization is due to the strength of sliding mode controller that lies in its ability to handle nonlinearities in the control dynamics [11]. The model from which it can be exploited for control is as follows:

$$\dot{v}_z = Z_v v_z + Z_q q - \frac{\mu\sin\theta}{(\int u\sin\theta + h_0)^2}\theta + Z_{\delta_e}\delta_e$$
$$\dot{q} = M_{VZ}v_z + M_q q + M_{\delta_e}\delta_e \quad (5)$$

In the above equations, being purposed for control, Z and M indicate the aerodynamics and force coefficients for ALV that vary with time, and μ is the gravitational parameter (i.e. μ=GM). The δ_e shows the deflection of control thrusters that are used as control actuators for supply control commands.

The servomechanism's and Rate gyro's Transfer functions are shown in (6), (7) as follows:

$$[TF]_{servo} = \frac{\delta}{\delta_c} = \frac{1}{0.1s+1} \quad (6)$$

$$[TF]_{gyro} = \frac{(80\pi)^2}{s^2 + (40\pi)s + (80\pi)^2} \quad (7)$$

The rate gyro is implemented for measuring the pitch rate of ALV and feedbacking data to the controller to be compared with the reference pitch rate and to calculate the tracking error.

Terminal and Dynamic Sliding Mode Control

In this section fundamentals of conventional and terminal sliding mode are expressed. Then terminal sliding mode is used to introduce a new saturation function that is utilized to implement the terminal dynamic sliding mode control. Finally, the dynamic sliding theory is presented.

Sliding Mode Control Theory

The tracking error is defined as$e = X - X_d$ in the variable X, which must track the desired trajectory. The time-variable surface S(t) in the state-space by the scalar equation s(x, t) = 0 is defined as:

$$S(X,t) = \left(\frac{d}{dt} + \lambda\right)^{n-1} e \tag{8}$$

In the other words, S is simply a weighted sum of the position error and the velocity error. The λ coefficient is a strictly positive constant. More precisely, the problem of tracking the n-dimensional vector can be replaced by a 1st-order stabilization problem in s [12].

The simplified 1st-order stabilization problem can be satisfied by sufficiently choosing the control law so that outside of S (t) [12] we have:

$$\frac{1}{2}\frac{d}{dt}S^2 \le -\eta|S| \tag{9}$$

Where η in the sliding condition is a positive constant. This equation implies that the squared distance to the surface, as measured by S^2, decreases along all system trajectories. Satisfying (9) makes the surfaces an invariant set [12].

Lyapunov direct method can also be used to obtain the control law. A candidate function is selected as:

$$V = \frac{1}{2} s s^T \tag{10}$$

That must guarantee the Lyapunov conditions that equal the sliding condition expressed in (9) and make the surface invariant set. The condition guaranteeing an ideal sliding motion is the sliding condition [13]. Therefore, a control input can be chosen so that $\dot{V}(s) < 0$ for all time for every nonzero S. Indeed, feedback linearization is used for the first term of input to make S derivative zero and the second term is designed in a way that $\dot{V}(s) < 0$ is satisfied. These conditions cause the asymptotical convergence to the surface (s=0). Thus, the input can be determined in a way that the last condition is satisfied.

In the first step, the continuous part is designed so that the derivation of s becomes zero (for reaching the surface). In the second step, discontinuous part is designed to retain variables in manifold and slide on this for all times. To this end, a sign function is considered. However, because of the discontinuity across the sliding surfaces, the preceding control law may result in control chattering. Chattering is an undesirable phenomenon, since it involves high frequency action and may excite high-frequency dynamics neglected in the modeling. The discontinuity in the control law can be dealt with by defining a thin layer of ε width around the sliding surface [14]. Hence, we use a saturation function that lets variables not be exactly in the surface but be in the boundary layer about S=0. In this way, the chattering is eliminated and system has adequate time for switching.

Choosing the boundary layer thickness is a tradeoff between precision and chattering. Greater thickness results in low chattering and less precision. Due to this choice, the effect of noise is degraded. Therefore, designer must choose a sufficient ε in saturation function by knowing about noises and uncertainty in plant.

Terminal Sliding Mode Control and its Use in Saturation Function:

Recently, the terminal sliding mode control has been developed to achieve the finite-time convergence of the system dynamics in the terminal sliding mode. Instead of using hyper planes as the sliding surfaces, this method adopts nonlinear sliding surfaces [15]. The terminal sliding mode design is based on a particular choice of the sliding surface and good determination of a control law permitting to derive the system states that remain on this surface. When the representative point of the system movement slides on the surface, a terminal sliding mode is established and a fast finite convergence is guaranteed. To reach this goal, one defines a nonlinear sliding surface:

$$S = x_2 + \beta x_1^{\frac{q}{p}} \tag{11}$$

Where β>0, p and q are positive odd integers verifying q>p [3], and 1<q/p<2 [16]. Equation (11) satisfies Lyapunov conditions for Lyapunov function (10) when:

$$\dot{V} = S\dot{S} = S\left(\dot{x}_2 + \beta\frac{q}{p}x_1^{\frac{q}{p}-1}\dot{x}_1\right) \le 0 \tag{12}$$

That for asymptotical stability, except equilibrium point, Lyapunov function must be negative (i.e., just when states are zero, this function can be zero). The fast response in this method can be found in its difference with conventional sliding mode. The sliding manifold is nonlinear by adding power. In the vicinity of zero point (equilibrium point) rate of error elimination is more than conventional sliding mode, because of the existence of $\frac{q}{p}x_1^{\frac{q}{p}-1}$ in the sliding manifold derivative as seen in (12).

In both second-order and terminal sliding mode for satisfying the last Lyapunov condition (if we use saturation function) the following term should finally be obtained:

$$\dot{\mathrm{V}} = -\rho \mathrm{S}\,\mathrm{sat}\left(\frac{\mathrm{S}}{\varepsilon}\right) < 0 \quad (13)$$

ρ is a positive constant. Terminal sliding mode idea led us to a new approach in the second part of input that appeared in (13).

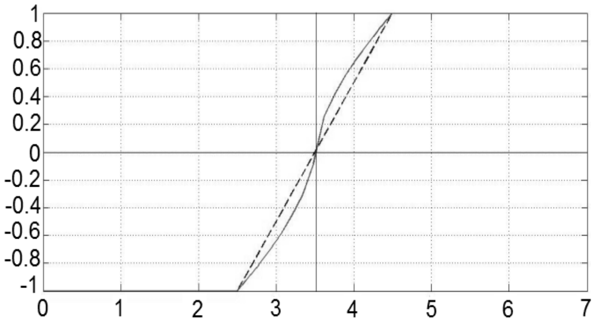


Fig. 1. Sat(s) ---- in comparison with sat(S^0.64)____

If instead of using S in saturation function, S to power of p/q-1 is used, boundary layer thickness (i.e. ε) is not changed but like the terminal sliding mode in the vicinity of zero the saturation function becomes greater than before. This causes more negative Lyapunov function. This concept can be seen in figure1.This means that control is more robust against disturbances and uncertainties. Furthermore, the finite-time convergence and asymptotical stability is more guaranteed.

Dynamic Sliding Mode Theory

Non-minimum phase nature of a plant restricts the application of the powerful nonlinear control techniques such as sliding mode control. Accordingly, the existence condition of conventional sliding modes cannot be completely met for a bounded control and the system experiences instability due to unstable internal dynamics. As a remedy to this problem, a dynamic sliding-mode controller was proposed that could be applied to non-minimum phase output tracking [7].

The main characteristic of the dynamic sliding mode is being a compensator [13]. In order to reduce the effect of the unmatched disturbance to the output steady-state tracking error, the dynamic sliding manifold was designed as a linear dynamic operator acting on some states and on tracking errors as follows[17]:

$$\mathfrak{I}(x_2, e) = x_2 + W(s)e \quad (14)$$

Where $s = \frac{d}{dt}$, $W(s) = \frac{P(s)}{Q(s)}$ and P(s), Q(s) are polynomials of s. A candidate for the Lyapunov function [13]:

$$\mathrm{V} = 0.5\,(\mathfrak{I})^{\mathrm{T}}\mathfrak{I} > 0 \quad (15)$$

Is built, and its derivative must satisfy the Lyapunov condition.

In the next section we combine the dynamic and terminal sliding to construct a new control law and finally we use this control law to make controller more robust and with higher tracking capability.

Control Design

In this section, the control structure is designed by using the previous section information. The purpose of designing the control structure is tracking the pitch program command that is pre-programmed for atmosphere flight as shown in Figure 2.

For designing the sliding manifold, dynamic terminal sliding surface is considered. Since the goal is to track non-minimum phase system in the presence of unmatched disturbance, the dynamic sliding manifold is chosen as following:

$$\mathfrak{I} = \delta + W(s)\dot{\theta}_e \quad (16)$$

That $\theta_e = \theta_c - \theta$ is tracking error for pitch angle and the compensator of dynamic surface is as follows:

$$\mathrm{W(s)} = \frac{\mathrm{s}^2 + 280\mathrm{s} + 2880}{\mathrm{s}^2 + 25.8\mathrm{s}} \quad (17)$$

The above compensator has a positive steady state error. We considered (15) as Lyapunov function and for satisfying Lyapunov condition for asymptotic stability, the following equation must be satisfied:

$$\dot{\mathrm{V}} = \mathfrak{I}\dot{\mathfrak{I}} < 0 \quad (18)$$

Input for thruster deflection command will be obtained as follows like what is proposed in [13]:

$$\delta_{\mathrm{c}} = -\rho\,\mathrm{sat}\left(\frac{\mathfrak{I}}{\varepsilon}\right) \quad (19)$$

In [13], this control is achieved by considering the integral of time multiplied by absolute tracking error criterion for dynamic sliding design.

Terminal sliding mode idea is used for this dynamic sliding manifold and is compared with dynamic sliding mode without terminal sliding mode. The dynamic sliding manifold is used as sliding variables that must converge to zero in finite-time. The terminal nonlinear manifold is assumed as:

$$\mathrm{S} = \delta^{0.72} + W(s)\dot{\theta}_e^{0.72} \quad (20)$$

In (20), the power is according to what proposed in section III and with *q/p* magnitude that satisfies the mentioned condition for terminal sliding mode. Thus input command is as follow:

$$\delta_c = -\rho \mathrm{sat}\left(\frac{S}{\varepsilon}\right) \quad (21)$$

In this command, the terminal sliding mode idea in saturation function can be seen, because in the sliding surface $\frac{q}{p} - 1$ power is considered as 0.72.

In the next section, these two sliding mode control laws are compared in simulation.

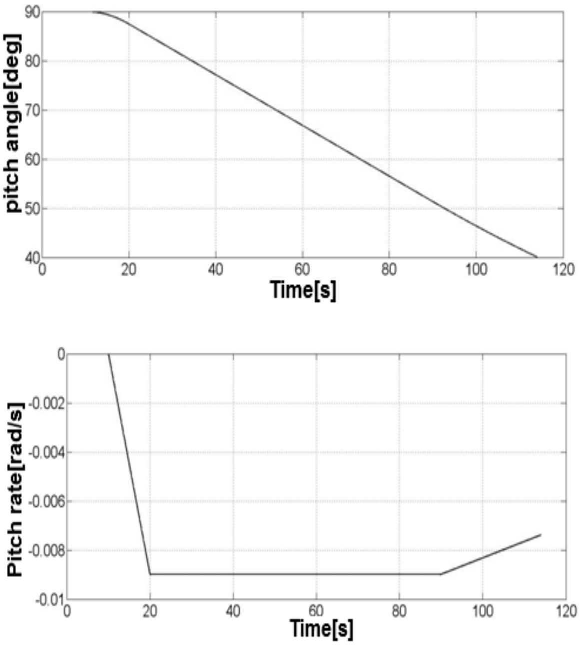


Fig. 2. The desired profile for ALV

Simulation Results

A simulation is performed to illustrate the validity and performance of the proposed control for tracking the desired trajectory of ALV. The model of this paper is like [1, 11]. The coefficients of (5) are depicted in Figure 3 with respect to time from [1, 11].

Remark: the gravity term in longitudinal axes of ALV is shown as Z-θ for illustrating ALV coefficient variation, but is computed online while other coefficients are considered offline.

In order to show the robustness and accuracy of the constructed control, the unmatched disturbance depicted in Figure 4is exerted. Simulation results in comparison with dynamic sliding mode designed in the previous section are shown. Hence, the dynamic sliding manifold is as (16) for both controls. But in the proposed control this manifold is used for terminal nonlinear sliding manifold.

The tracking error is depicted for both dynamic and proposed dynamic terminal sliding control in Figure 5 and Figure 6. For simulation purpose, $\rho = 8.8, \varepsilon = 1.25$ is considered ε.

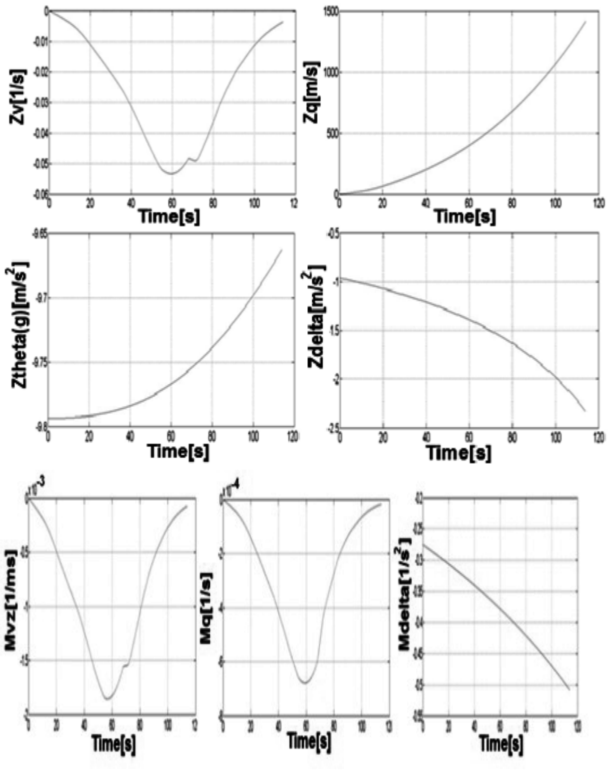
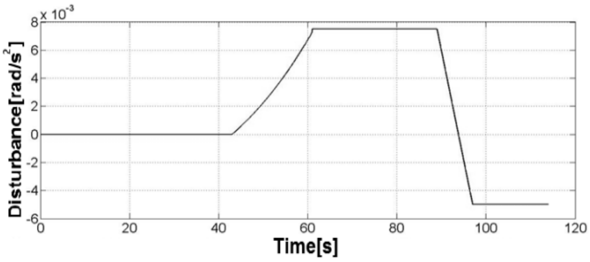
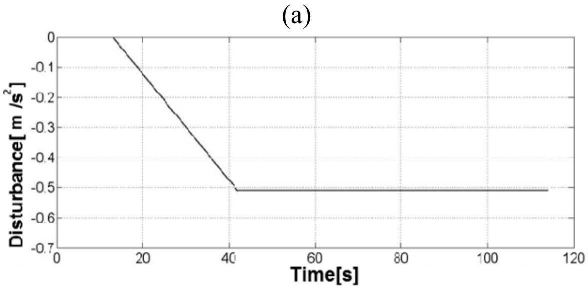


Fig. 3. Longitudinal dynamics coefficients for ALV



(a)



(b)

Fig. 4. Unmatched disturbances profiles (a) exerted on pitch equation, and (b) exerted on vertical velocity equation

As shown in Figure 5 and Figure 6 the error in the proposed control is a lot lower than the dynamic sliding mode control. The proposed dynamic terminal sliding mode has better results and can adapted with the exerted unmatched disturbances. The proposed control has a fast response and a more precise tracking.

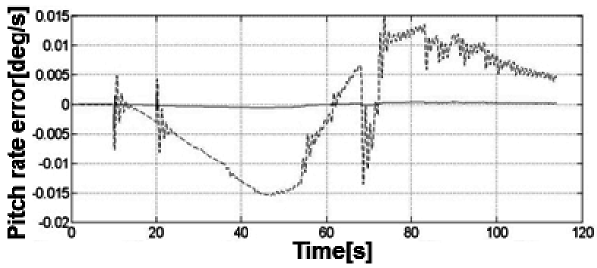


Fig. 5. Pitch rate error (---Dynamic SMC, _Proposed DSMC)

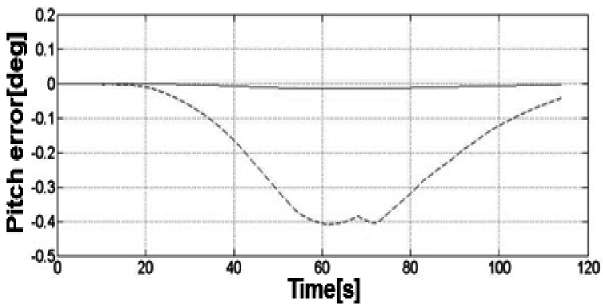


Fig. 6. Pitch angle error (---Dynamic SMC, _ Proposed DSMC)

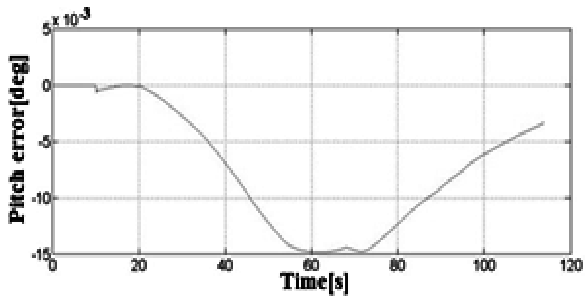


Fig. 7. Pitch angle error for proposed DSMC

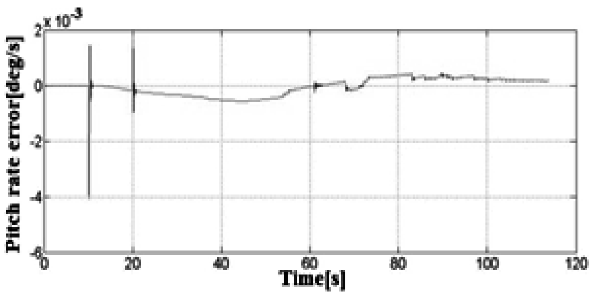


Fig. 8. Pitch rate error for proposed DSMC

For higher resolution of the performance of proposed control and its behavior, results are shown separately in Figure 7 and Figure 8. It is done as the scale of error in the second-order sliding mode is less than the ordinary sliding mode. Figure 9 shows the control effort or the control thrusters deflections in time.

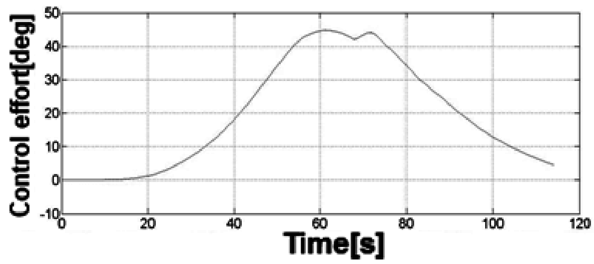


Fig. 9. Control effort for proposed DSMC

Conclusion

In this paper, a novel sliding mode approach to achieve the asymptotic stability and disturbance rejection for tracking guidance commands profile in the presence of unmatched disturbances is introduced. The innovative idea of this paper is focused on more robustness and asymptotic stability by choosing terminal nonlinear sliding manifold for converging the sliding variables in the dynamic sliding surface to zero. This idea can be seen in the saturation function that is given for thruster commands. The simulation results show the effectiveness and improvement in the performance of the proposed control in comparison with the introduced dynamic sliding mode control for ALV. This control can reject the effects of exerted unmatched disturbances which may have been caused by atmosphere condition or gusts. Furthermore, this control has adequate robustness and efficient performance.

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