INS Alignment Improvement Using Rest Heading and Zero-Velocity Updates

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In this paper the feasibility of rapid alignment and calibration of a static strapdown inertial navigation system (INS) is evaluated. Resting conditions including zero-velocity update and a known initial heading direction as virtual external measurement data are integrated with INS data. By comparing the virtual external measurements with the estimates of those generated by the aligning INS, estimates of the velocity and heading errors can be obtained and these errors will be propagated in the INS as a result of alignment inaccuracies. An extended Kalman filter based on an augmented process model and a measurement model is designed to estimate alignment attitudes and biases of inertial sensors. Monte Carlo simulation results show that the integration of INS with rest conditions is very effective in rapid and fine leveling and azimuth alignment of INS, but this type of data fusion due to poor acceleration and angular rates of static condition has no chance of valuable calibration of all inertial sensor biases.

Keywords: Aided inertial navigation system, INS, Alignment, Kalman filter, ZUPT

Nomenclature

NED	North-East-Down: navigation frame (n-frame)
L	Latitude
l	Longitude
h	Height above ground
V_N	North velocity
V_E	East velocity
V_D	Down velocity
φ_N	Alignment error in roll attitude
φ_E	Alignment error in pitch attitude
φ_D	Alignment error in heading
C_B^N	Direction cosine matrix from body frame to navigation frame (NED)
δf^B	Accelerometer measurement errors
$\delta \omega_{IB}^B$	Gyroscope measurement errors
r_L	Radius of Earth curvature in the meridian
r_l	Radius of Earth curvature of the ellipsoid in the prime vertical
ω_{IB}^B	Gyroscopes measurements
ω_{IE}^N	Earth's inertial angular velocity in navigation frame
ω_{NB}^B	Turn rate of the body with respect to navigation frame in the body frame
ω_{EN}^N	Turn rate of the local navigation frame with respect the Earth fixed frame in navigation frame
f_B	Accelerometers measurements
Ω_{NB}^B	Skew-symmetric form of ω_{NB}^B
g_l^N	Local gravity vector in navigation frame

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Introduction

The alignment of an strapdown inertial navigation system determines the transformation matrix between a body frame and a navigation frame in the local-level frame [1]. The initial alignment of inertial navigation system is an important process performed prior to normal navigation [2].Since INS is entirely self-contained, it can align itself using the measurements of local gravity and Earth rate. Normally, alignment process is divided into two phases, i.e. the coarse and fine alignment. The purpose of coarse alignment is to provide fairly good initial condition for the fine alignment processing. Typically, the threshold of the attitude errors between the two categories can reach a few degrees [3]. The stationary initial alignment which

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consists of a coarse alignment and a fine alignment is usually performed when a vehicle is at rest. In order to reduce the initial alignment time for some applications, the coarse alignment is only performed or the initial attitudes is directly obtained from other sources such as stored information or a master inertial navigation system [1]. In most ground based applications, gyrocompass sing is known to be a common fine self-alignment method, but it is time-consuming. The basic concept of INS alignment is quite simple and straight forward. However, there are many complications that make alignment both time-consuming and complex. Accurate alignment is crucial; however, if precision navigation is to be achieved over long periods of time without any form of aiding. In many applications, it is essential to achieve an accurate alignment of an inertial navigation system within a very short period of time. This is particularly true in many military applications, in which a very rapid response time is often a prime requirement for obtaining a very short reaction time [4].

As a solution for overcoming these problems, the use of rest constraints, i.e. constant position and attitude, is especially appealing for initial aligning INS or constraining INS errors. In this situation, the motion of the vehicle can be governed by some non-holonomic constraints, [5]. For example, Ref. [6] uses the zero-velocity updates (ZUPT) for initial alignment and calibration of a stationary strapdown INS. ZUPT is applied for constraining foot-mounted INSs, as during ordinary gait, the foot returns to a "stationary" state on a regular base [7, 8]. For details on the benefits of ZUPT, one can refer to [9] and the references there in. Algorithms for fast estimating the azimuth misalignment angle and calibrating gyro drift rates are approached from the point of view of control theory and by introducing the Lyapunov transformation. The equivalence of strapdown INS and gimbaled INS is discussed in [10]. A new solution for the precise azimuth alignment is given in detail in [11] and a new profiler, which consists of an IIR filter and a Kalman filter using hidden Markov model, is designed to attenuate the influence of sensor noise and outer disturbance. Ref. [12] addresses self-alignment of a strapdown INS in near-stationary condition using the east gyro outputs from the inertial measurement unit, along with the velocity outputs in a nonlinear state estimation framework using extended Kalman filter (EKF). In [13], initial alignment and calibration performance of a gimbaled INS is enhanced using the design and implementation of an optimal stochastic close loop control.

All above-mentioned researches are related to the initial alignment of INS using rest conditions a determined and constant heading into the alignment process as an extra constraint in stationary state was not applied. In this work, in order to enhance the alignment and calibration performances, in addition to ZUPT technique, a known direction as new

measurements for updating the Kalman filter is used. By comparing the virtual external velocities and heading with the estimates of those generated by the aligning system, estimates of the velocities and heading errors are obtained and these errors will propagate in the system as a result of alignment inaccuracies. Based on an augmented error model of the system and using an extended Kalman filter, it is possible to deduce the INS alignment and some biases of inertial sensors from the innovation signals. In practical cases, these rest constraints are quite reasonable for vehicles in recharging, though violated due to for example the vibrations of the vehicle engine and buffeting by wave or wind. In fact, in such a situation, the mean attitude of the aligning system with respect to the local geographic frame is fixed, and the specific force and turn rates to which the aligning system is subjected, are nominally fixed.

The paper is organized as follows. Firstly, the process and measurement model for the integration system are provided. Secondly, observability of the system is analyzed. Thirdly, an extended Kalman filter is designed and outlined for the data fusion. Fourthly, some Monte Carlo simulations are carried out to illustrate the functionality and usefulness of the new alignment and calibration method. Finally, the concluding remarks are presented.

Augmented Error Model

The navigation equations for an SDINS in n-frame (north, east and down: NED) are [4]:

$$\dot{L} = \frac{V_N}{r_L + h} \tag{1}$$

$$\dot{l} = \frac{V_E}{(r_l + h)\cos L} \tag{2}$$

$$\dot{h} = -V_D \tag{3}$$

$$\dot{V}^N = C_B^N f_B - (2\omega_{IE}^N + \omega_{EN}^N) \times V^N + g_l^N \tag{4}$$

$$\dot{C}_B^N = C_B^N \Omega_{NB}^B \tag{5}$$

After applying the rest conditions, the linear differential equations of those equations as follows:

$$\dot{\delta L} = \frac{1}{r}\delta V_N \tag{6}$$

$$\dot{\delta l} = \frac{1}{r\cos L}\delta V_E \tag{7}$$

$$\dot{\delta h} = -\delta V_D \tag{8}$$

$$\dot{\delta V}_N = g\varphi_E + 2\omega_D\delta V_E + B_N \tag{9}$$

$$\dot{\delta V}_E = -g\varphi_N - 2\omega_D\delta V_N + B_E \tag{10}$$

$$\dot{\delta V}_D = -2\omega_N\delta V_E + B_D \tag{11}$$

$$\dot{\varphi}_N = \omega_D \varphi_E + \frac{1}{r}\delta V_E + \omega_D \delta L - D_N \qquad (12)$$

$$\dot{\varphi}_E = -\omega_D \varphi_N + \omega_N \varphi_D - \frac{1}{r}\delta V_N - D_E \qquad (13)$$

$$\dot{\varphi}_D = -\omega_N \varphi_E - \frac{\tan L}{r}\delta V_E - \omega_N \delta L - D_D \qquad (14)$$

Where ω_N and ω_D are the Earth rate represented in the n-frame and r is the radius of the Earth with the assumption of:

$$r = r_L + h = r_l + h = r_L = r_l \qquad (15)$$

Also, φ_N, φ_E and φ_D are attitude errors and B and D are fixed biases and drifts of accelerometers and gyros in the n-frame ,respectively.

$$B = [B_N \quad B_E \quad B_D]^T = C_B^N \delta f^B \qquad (16)$$

$$D = [D_N \quad D_E \quad D_D]^T = C_B^N \delta \omega_{IB}^B \qquad (17)$$

Due to short alignment time and using precise inertial sensors, it is suitable to consider these biases as constant variables. Therefore, the dynamic equations for biases are:

$$\dot{B} = 0$$
$$\dot{D} = 0 \qquad (18)$$

The state-space system model can be formed from the differential equations of the navigation errors and the dynamics of inertial sensors. The error state variable x(t) consists of navigation errors and sensor bias errors. In the model, $\omega(t)$ process noise or inertial sensor noise, is white noise with zero mean and covariance Q(t). The linear time-varying stochastic system model is

$$\dot{x}(t) = F(t)x(t) + \omega(t) \qquad (19)$$

Where

$$x = [\delta V_N, \delta V_E, \delta V_D, \varphi_N, \varphi_E, \varphi_D, B_N, B_E, B_D, \dots$$
$$, D_N, D_E, D_D]^T \qquad (20)$$

$$\omega = [\omega_{B_N}, \omega_{B_E}, \omega_{B_D}, \omega_{D_N}, \omega_{D_E}, \omega_{D_D}, 0, 0, 0, \dots$$
$$0,0,0]^T \qquad (21)$$

$$F = \begin{bmatrix} F_{11} & F_{12} & I_{3\times3} & 0_{3\times3} \\ F_{21} & F_{22} & 0_{3\times3} & -I_{3\times3} \\ 0_{6\times3} & 0_{6\times3} & 0_{6\times3} & 0_{6\times3} \end{bmatrix}_{12\times12} \qquad (22)$$

Where

$$F_{11} = \begin{bmatrix} 0 & 2\omega_D & 0 \\ -2\omega_D & 0 & 2\omega_N \\ 0 & -2\omega_N & 0 \end{bmatrix} \qquad (23)$$

$$F_{12} = \begin{bmatrix} 0 & g & 0 \\ -g & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad (24)$$

$$F_{21} = \begin{bmatrix} 0 & 1/r & 0 \\ -1/r & 0 & 0 \\ 0 & -tanL/r & 0 \end{bmatrix} \qquad (25)$$

$$F_{22} = \frac{1}{2}F_{11} \qquad (26)$$

Measurement Model

This paper introduces additional measurements of the velocities and heading to improve the alignment performance of the navigation system. When the navigation unit is assumed to be stationary, it can be inferred that

$$V_{Aid} = [V_N \quad V_E \quad V_D]^T = 0_{3\times1}$$
$$\psi_{Aid} = \psi_0 = Cte \qquad (27)$$

Because the stationary conditions is violated due to for example the vibrations of the vehicle engine and buffering by wave and wind, so the measurement models are formulated by adding white noise, $\gamma(t)$, with a zero mean and covariance R:

$$y_1(t_k) = \hat{V}_{INS} - V_{Aid} = \delta V - \delta V_{Aid} \qquad (28)$$

$$y_2(t_k) = \hat{\psi}_{INS} - \psi_{Aid} = \delta\psi - \delta\psi_{Aid} \qquad (29)$$

Where subscripts INS and AID denote the INS estimates and the aided virtual sensors measurements, respectively. The state space form of measurement is:

$$y(t_k) = H(t_k)x(t_k) + \gamma(t_k) \qquad (30)$$

Two cases are introduced for being studied in this work and their results are compared:
Case study 1: INS and ZUPT will be integrated and the measurement matrix is:

$$H = \begin{bmatrix} 1 & 0 & 0 & 0_{1\times9} \\ 0 & 1 & 0 & 0_{1\times9} \\ 0 & 0 & 1 & 0_{1\times9} \end{bmatrix}_{3\times12} \qquad (31)$$

Case study 2: INS will be aided by ZUPT and a known initial constant heading and the measurement matrix is:

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0_{1\times6} \\ 0 & 1 & 0 & 0 & 0 & 0 & 0_{1\times6} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0_{1\times6} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0_{1\times6} \end{bmatrix}_{4\times12} \qquad (32)$$

Observability Analyses

Observability analysis of a dynamic system indicates the efficiency of a Kalman filter designed to estimate the states of the system. While the observability analysis of a time invariant system is rather simple, the analysis of a time varying system is difficult. If the time varying system is replaced by a piece-wise time invariant system, the observability analysis can be performed simply by the stripped observability matrix suggested by [14]. It was already known that the strap-down INS in a stationary alignment process only using the zero-velocity measurement is not completely observable[15]. The results of analyses using observability matrix, $\mathrm{O}(F, H)$ are as follows (for $\cos L \neq 0$):

For case 1 (ZUPT Aided):

The Rank of O is 9 and these three states are not observable:

North accelerometer bias, B_N

East accelerometer bias, B_E

East gyro bias, D_E

For Case2 (ZUPT and Initial Constant Heading Aided):

The Rank of O is 10and two states are not observable:

North accelerometer bias, B_N

East accelerometer bias, B_E

Kalman Filter

An indirect feedback Kalman filter has been widely used to estimate the errors for a strap-down inertial navigation system alignment employing an EKF in which small nonlinear approximation errors and initial errors are assumed. Probably the most widely used estimator for nonlinear systems is that of Kalman, namely the extended Kalman filter [16]. The EKF applies the Kalman filter to nonlinear systems by simply linearizing all the nonlinear models so that the traditional linear Kalman filter equations (LKF) can be applied.

For applying this filter, it is necessary to use the discrete form of Eq. (19) and to introduce the covariance of the process noises, Q, the covariance of the measurement noises, R, and the covariance of initial estimation error, P_0. The Kalman filter period is 1 second and the initial attitude and heading errors are 1 degree. Some stochastic specifications of the inertial navigation system are as follows:

Table 1. Inertial Navigation System Sensor Specifications

Accelerometers Spec.		Gyros Spec.	
Fixed Bias, (1σ)	100 μg	Fixed Drift, (1σ)	$0.01 \dfrac{\deg}{hr}$
Velocity Random Walk (1σ)	$0.06 \dfrac{m/s}{\sqrt{hr}}$	Angular Random Walk (1σ)	$0.01 \dfrac{\deg}{\sqrt{hr}}$

The process and measurement noise covariance matrix and initial estimation error matrix for case study 1respectively are:

$$Q = diag[(\tfrac{.06}{60}\tfrac{m}{s^2})^2, (\tfrac{.06}{60}\tfrac{m}{s^2})^2, (\tfrac{.06}{60}\tfrac{m}{s^2})^2 \ldots$$
$$, (.01\tfrac{\tfrac{\pi}{180}}{60}\tfrac{rad}{s})^2 , (.01\tfrac{\tfrac{\pi}{180}}{60}\tfrac{rad}{s})^2 \ldots \qquad (33)$$
$$(.01\tfrac{\tfrac{\pi}{180}}{60}\tfrac{rad}{s})^2, (.01\tfrac{\tfrac{\pi}{180}}{60}\tfrac{rad}{s})^2 \; 0_{1\times6}]_{12\times12}$$

$$R = diag\left[(0.01\tfrac{m}{s})^2 \quad (0.01\tfrac{m}{s})^2 \quad (0.01\tfrac{m}{s})^2\right]_{3\times3} \qquad (34)$$

$$P_0 = diag[(.1\tfrac{m}{s})^2, (.1\tfrac{m}{s})^2, (.1\tfrac{m}{s})^2 , \ldots$$
$$(\tfrac{\pi}{180}rad)^2, (\tfrac{\pi}{180}rad)^2, (\tfrac{\pi}{180}rad)^2, (.001\tfrac{m}{s^2})^2 \ldots$$
$$, (.001\tfrac{m}{s^2})^2, (.001\tfrac{m}{s^2})^2, (.01\tfrac{\tfrac{\pi}{180}}{3600}\tfrac{rad}{s})^2 \ldots \qquad (35)$$
$$, (.01\tfrac{\tfrac{\pi}{180}}{3600}\tfrac{rad}{s})^2, (.01\tfrac{\tfrac{\pi}{180}}{3600}\tfrac{rad}{s})^2]_{12\times12}$$

The process and measurement noise covariance matrix and initial estimation error covariance matrix for case study 2 are:
Q and P_0 are equal to those of case1 but R is

$$R = diag\left[\left(\tfrac{.01m}{s}\right)^2, \left(\tfrac{.01m}{s}\right)^2, \left(\tfrac{.01m}{s}\right)^2, \left(\tfrac{0.1\pi}{180}\right)^2\right] \qquad (36)$$

After estimating the bias components in n frame, it is possible to calculate them in b frame:

$$B^B = C_N^B[B_N \quad B_E \quad B_D]^T \qquad (37)$$

$$D^B = C_B^N[D_N \quad D_E \quad D_D]^T \qquad (38)$$

If in some case studies B_N, B_E or D_E are not observable, we can put them equal to zero in the equations to avoid the unobservable states adding an error variance toall the original states that are not measured directly.

Simulation Results

The predictions of system accuracy provided by covariance analysis can be verified by computer simulation of the alignment procedure. Because many of the quantities that are treated only in terms of their statistical parameters in the error analysis must be specified in the simulation, a number of computer runs would be required to provide reliable probability distribution for the estimation errors. Since both the truth model and the filter model are driven by randomly generated noise, each individual Monte Carlo run is expected to be different. Therefore, in order to generate error statistics with a Monte Carlo program, a given case is iterated many times, the iterations differing only in the random number input sequences. The results of the iterations are then averaged to obtain the desired statistics consequently, observing the ensemble statistics of several runs gives an indication of the expected performance of the filter. Naturally, the more runs are made, the more reliable become the statistics. Between 25 and 50 runs is typically used to determine filter performance [17]. Each run produces a different sequence of random numbers to generate the samples of input white-noise processes. The results presented here should be regarded as one run from batch of 25 Monte-Carlo simulations, and as such, only indicate the "potential" accuracy of the alignment technique. Monte-Carlo simulations were carried out for two cases. The first one is the zero velocity updates (case 1) and the second one is the zero velocity updates and initial heading knowledge (case 2), for example the direction of a runway, s fluxgate magnetometer or a compass.

The integration time step for navigation equations is 0.01 sec and True initial attitudes and heading are zero degree. Also, the inertial navigation system is compensated by using the estimation results of velocities, attitudes and heading but because some biases and drifts are not observable, none of these are corrected in the INS. The simulation results are illustrated below:

Fig.1 shows that velocity outputs of the INS are limited. It is obvious that if there are not zero velocity updates, the north and east velocity components oscillate with Schuler period and the down velocity component grows unboundedly.Fig.2-5 present that the attitude alignment results (roll and pitch) are fine and the estimation errors are decreased effectively within few seconds. Also, there is not a significant difference between the results of the two cases. In the estimation of heading, there is a capital difference between two cases. When only zero velocities are measurements although the heading is observable, the estimation error was decreased in a few minutes period. However, in case 2, the heading is limited to the initial heading input as a measurement from the incipient stage of dada fusion.
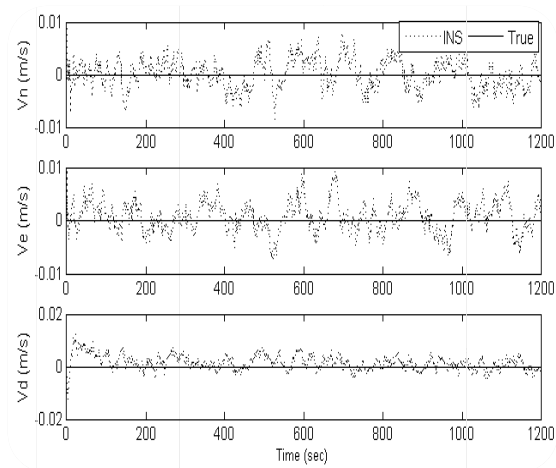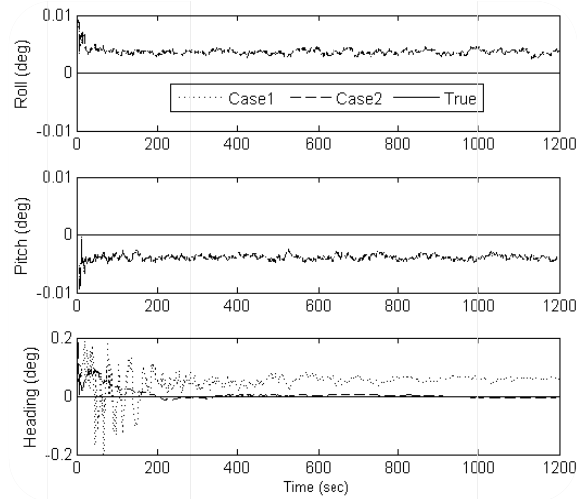

Fig. 1. Velocity components in NED frame
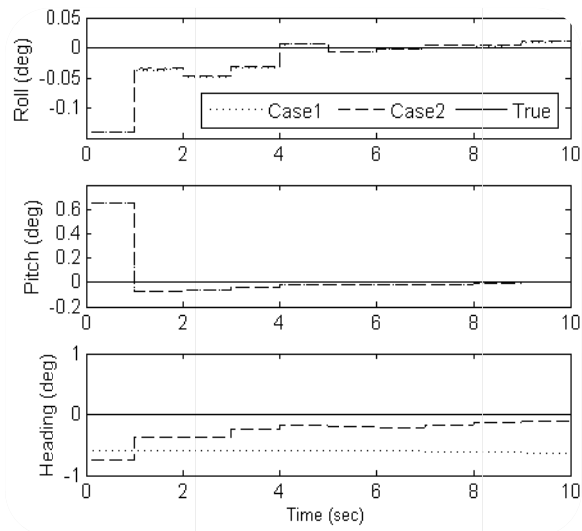

Fig. 2. Roll, pitch and heading history

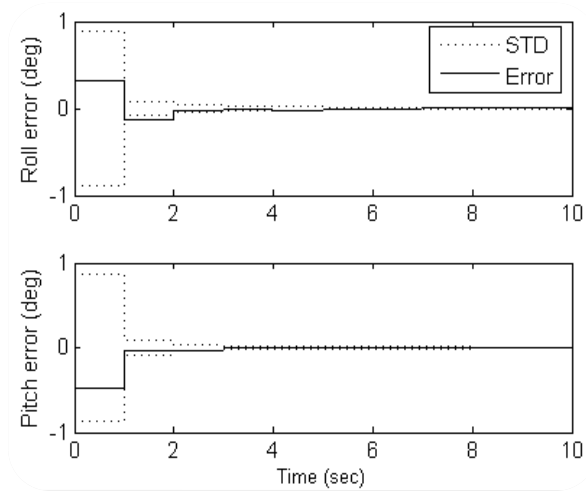**Fig. 3.** Roll, pitch and heading history in short time (zoom window)

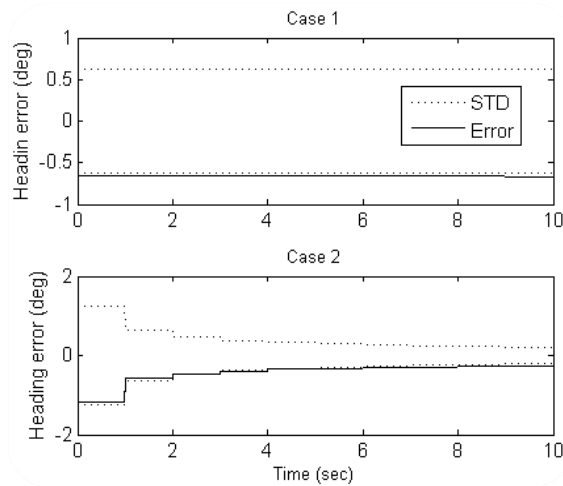**Fig. 4.** Roll and pitch errors and standard deviation ($\pm 1\sigma$)

Fig. 5. Heading error and standard deviation ($\pm 1\sigma$)

Fig. 6. The down component of accelerometers bias estimation

Fig. 7. Estimation error of the down component of accelerometers bias and standard deviation ($\pm 1\sigma$)

Fig. 8. The drifts of gyros in NED frame

As mentioned in the observability analysis paragraph the north and east bias are not observable, hence the filter cannot estimate the biases even after along time and the estimation errors are nearly constant. Nevertheless, the filter estimates the down bias precisely and the convergence time of the estimation is about a few minutes, Figs. 6-7. In the estimation of biases, there are not differences between the filter performances in the two cases.

Fig.8 depicts the drift estimation results of gyros: In case 1, the east drift is not observable and as the results show, the estimation error does not decrease. Also, both the north and down drifts can be estimated but the estimation convergence time of the north drift is better that of the other one. In case 2, the estimation results for the north and down drifts are nearly similar to case1. Nonetheless, due to the observability of the east drift, its estimation error decreases and it can be estimated.
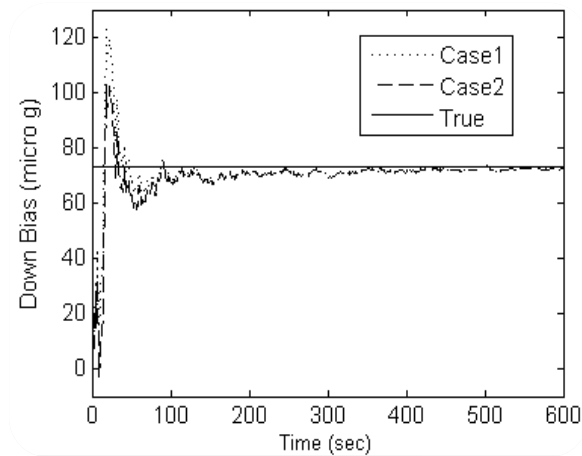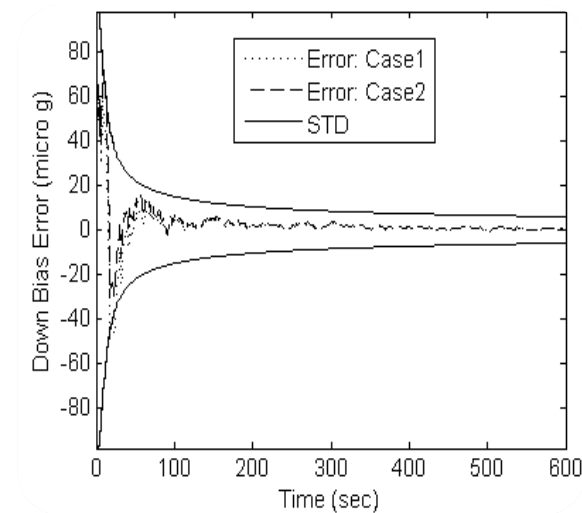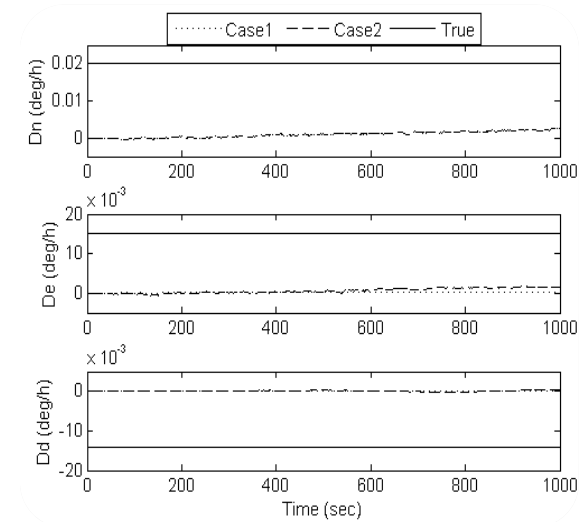
Conclusions and Future Work

Integration of inertial navigation system (INS) with zero-velocity updates (ZUPT) is very effective in rapid and fine leveling of INS but this technique without an extra measurement for heading (e.g. a known direction or heading) has no chance for alignment of heading. Although some biases and drifts of inertial sensors are observable in the fusion of INS data with ZUPT, the estimation convergence speed of nearly all of them are very slow due to poorness of accelerations and angular rates in rest condition. In fact, the fusion cannot help in operation calibration of inertial sensors practically and it is necessary to try new fusions such as INS aided with zero east angular rate of earth condition in future.

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