Design of Integral Augmented Sliding Mode Control for Pitch Angle of a 3-DOF Bench-top Helicopter

Amir Hossein Zaeri[1], Samsul Bahari Mohd Noor[1], Maryam Mohd Isa[1], Farah Saleena Taip[2]
1- Department of Electrical and Electronic Engineering, University Putra Malaysia,
43400 UPM Serdang, Selangor Darul Ehsan, Malaysia,
Email: amzaeri@gmail.com, samsul@eng.upm.edu.my, misa@eng.upm.edu.my
2- Department of Process and Food Engineering Engineering, University Putra Malaysia,
43400 UPM Serdang, Selangor Darul Ehsan, Malaysia,
Email: saleena@eng.upm.edu.my

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ABSTRACT:
Sliding mode control (SMC) is a nonlinear controller that is used to achieve desired performance in the presence of unstructured uncertainty as a result of carelessness of parameter specification of the system. The main aim for sliding control, or also called Variable Structure Control, is to control the nonlinear plant by introducing a sliding surface. The sliding surface should be reached by making the state of system approach this level. In this paper, an integral augmented sliding mode control (SMC+I) is proposed to improve the control performance of a plant with uncertainty giving the example of bench-top helicopter and the results are compared with the results obtained from conventional sliding mode control with and without a boundary layer.

KEYWORDS: Sliding Mode Control, Chattering Phenomenon, Boundary Layer, Pitch Angel of a 3-DOF Helicopter.

1. INTRODUCTION

Variable-Structure systems (VSSs) with a sliding mode were first proposed in early 1950s. However, due to the implementation difficulties of high-speed switching, it was not until the 1970s that the approach received the attention it deserved [1]. Sliding mode control is a robust controller that can be used to control linear and non-linear plants to achieve desired performance in the presence of unstructured uncertainty as a result of carelessness of parameter specification of the system. Because of these properties, diverse applications of the sliding mode control methodology can be found in the areas of electric motors, manipulators, power systems, mobile robots, spacecraft, and automotive control [2]. Sliding mode control or also called Variable Structure Control is suitable for highly nonlinear plants which have extensive domain where linear approximation cannot be done. In this specific problem, sliding mode controller is operated in linear plants with high uncertainty. The concept of sliding mode control is to bring the state of system to a desired surface, called sliding surface and remain. There are two important rules in the sliding mode control law; first is to bring the system towards the sliding surface and second is to ensure the state of the system remains on the sliding surface. However, chattering always occurs in sliding mode design because of the changes in structure of the sliding mode controller [3]. In order to remove this unwanted chattering effect, a number of methods are available [3-6]. One of them is by introducing a boundary layer around the sliding surface [4-5]. Error within some decided beforehand boundary layer is considered in these functions [5, 7]. For smoothness (to reduce chattering) of the control input signal, larger width of the boundary layer is preferred. However, for better tracking accuracy, a boundary layer with smaller width is needed [8].

Conventional sliding mode control can be improved such that an integral dynamic with an autonomous tuning parameter as a constant coefficient is added into conventional sliding surface to improve transient performance and steady-state accuracy, and to overcome disadvantages of the conventional SMC method [7].

This paper focuses on this and considers the design of a single-input single-output, SISO, laboratory scale bench-top helicopter. Finally, simulation tests are carried out for the plant with uncertainties.

2. A REVIEW OF CONVENTIONAL SLIDING MODE CONTROL FUNDAMENTALS

In sliding mode control, there are a number of laws to be followed.

We consider this dynamic equation [4-5]

$$\mathbf{X}^{(n)} = f(\mathbf{X}) + b(\mathbf{X})\,u \quad (1)$$

Where X =[x □ ... $x(n\text{-}1)$]^T is the state vector. The functions f(X) and b(X) are not exactly known but they are bounded. At first, we define a time-varying surface, $s(t)$ in the state space $R^{(n)}$ where

$$s = (D + \lambda)^{(n-1)}\tilde{x} = 0 \quad \lambda > 0 \quad (2)$$

$\tilde{x} = x - x_d$ is the tracking error. The control law contains two parts

$$u = u_{eq} + u_{sw} \quad (3)$$

In Eqn.(3), the equivalent control (u_{eq}) is the part of u, which can be interpreted as the continuous control law would maintain □ = 0 if the dynamics were exactly known and so the switching control (u_{sw}) is the part of the input which can transpose the system state to surface s = 0 if the dynamics were not known.

If n = 2 we have

$$u_{eq} = \frac{1}{\hat{b}}\left[-\hat{f}(X) - \lambda\,\dot{\tilde{x}} + \ddot{x}_d\right] = \hat{b}^{-1}\hat{u} \quad (4)$$

Where

$$\hat{b} = (b_{\min}.b_{\max})^{\frac{1}{2}},\ \hat{f} = \frac{f_{\min} + f_{\max}}{2} \quad (5)$$

For choosing u_1, we need to obtain a sliding condition for crossing a sliding surface (s = 0). Usually, this condition is

$$\frac{1}{2}\frac{ds^2}{dt} \le -\eta\,|s|,\ (\eta > 0)$$

If $b(x)$ is known then u_1 is

$$u_{sw} = -\hat{b}^{-1}\{\beta(F(X) + \eta) + (\beta - 1)\hat{u}\}Sgn(s) \quad (6)$$

where

$$F(X) = \max(f - \hat{f}),\ \beta = \sqrt{\frac{b_{\max}}{b_{\min}}} \quad (7)$$

$$\mathrm{sgn}(s) = \begin{cases} 1 & s > 0 \\ 0 & s = 0 \\ -1 & s < 0 \end{cases} \quad (8)$$

Because of sgn(s), we have some unwanted oscillation around the sliding surface called the chattering phenomenon. In order to eliminate the chattering, normally the saturation function is used [4-5]:

$$u_{sw} = -\hat{b}^{-1}\{\beta(F(X) + \eta) + (\beta - 1)\,\hat{u}\,\}sat(\frac{s}{\varphi}) \quad (9)$$

that

$$sat(\frac{s}{\varphi}) = \begin{cases} 1 & s > \varphi \\ \frac{s}{\varphi} & -\varphi \le s \le \varphi \\ -1 & s < -\varphi \end{cases} \quad (10)$$

The saturation function approximates the sign function term in the boundary layer of the sliding surface (Figure 1). Basically, the boundary layer approach is designed to avoid chattering by replacing the discontinuous switching action with a continuous function depending on the width of the boundary layer.

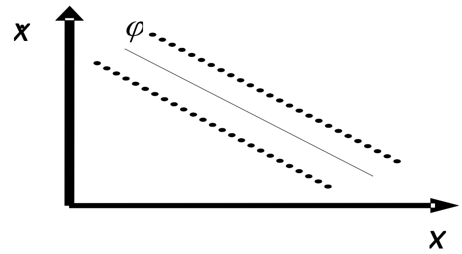

Fig. 1. Sliding surface with boundary layer

3. INTEGRAL AUGMENTED SLIDING MODE CONTROL (SMC + I)

The sliding surface can be improved by introducing an integral action into the sliding surface for steady-state accuracy defined as [7, 9-10]:

$$s(t) = (D + \lambda)^{(n-1)}\tilde{x}(t) + k_i\int_0^t \tilde{x}(\tau)d\tau \quad (11)$$

where k_i is the integral gain, $k_i \in R^+$. The phase plane ally with the proposed switching surface, Eq.(11), is three-dimensional and the switching surface is a plane going through the origin, if the order of uncontrolled system is assumed to be two (n=2). The importance of the proposed control approach is that the solution is obtained on a plane, while in conventional SMC the solution is obtained on a line.

Taking derivative of the sliding surface given in Eq.(11) with respect to time, for n = 2 one has:

$$\dot{s}(t) = \ddot{\tilde{x}}(t) + \lambda\dot{\tilde{x}}(t) + k_i\tilde{x}(t) \quad (12)$$

A necessary condition for the tracking error to remain on the sliding surface s(t) is $\dot{s}$ (t) = 0 [2, 4, 7]:

$$\ddot{\tilde{x}}(t) + \lambda\dot{\tilde{x}}(t) + k_i\tilde{x}(t) = 0 \quad (13)$$

If the control gains, λ and k_i , are properly chosen so that the characteristic polynomial in Eq.(13) is strictly stable, that is, the roots of the polynomial are in the open left-half of the complex plane, it infers that $\lim_{t\to\infty}\tilde{x} = 0$ means the closed-loop system is globally asymptotically stable [7, 11-12].

The equivalent control law is obtained when □(t)=0, therefore:

$$u_{eq} = \frac{1}{\hat{b}}\left[-\hat{f}(X) - \lambda\,\dot{\tilde{x}} - k_i\tilde{x} + \ddot{x}_d\right] = \hat{b}^{-1}\hat{u} \quad (14)$$

and:

$$u_{sw} = -\hat{b}^{-1}\{\beta(F(X) + \eta) + (\beta - 1)\,\hat{u}\,\}Sat(\frac{s}{\varphi}) \quad (15)$$

4. BENCH-TOP HELICOPTER MODEL

The bench-top helicopter is shown in Figure 2 [13-16]. It is a laboratory scale plant with 3 Degrees of Freedom (3DOF), roll angle φ, pitch angle θ, and yaw angle ψ, each one measured by an absolute encoder. Two electrical DC motors are attached to the helicopter body, forcing the two propellers to turn. The total force F caused by aerodynamic makes the total system turn around an angle measured by an encoder. A counter weight of mass M helps the propellers lift the body weight due to the mass m. The dynamics of the pitch angle is obtained by applying Lagrange's equations to the mathematical scheme, so that:

$$Fl_1 - mg[(h+d)\sin\theta + \cos\theta] + Mg(l_2 + l_2\cos\alpha)\cos\theta + Mg(l_3\sin\theta - h)\sin\theta - b_e\frac{d\theta}{dt} = J_e\frac{d^2\theta}{dt^2} \quad (16)$$

Where h, d, l_1, l_2 and l_3 are lengths, m is the sum of both motors' mass and M, the counterweight mass; b_e is the dynamic coefficient, g the gravity acceleration, J_e the initial moment of the whole system around the pitch angle θ, and α a fixed construction angle. The total non-linear model obtained from the previous equation can be simplified by linearizing around the operational point $\theta_0 = 0$. It yields a second order transfer function between the pitch angle θ and the motor signal U

$$P(s) = \frac{\theta(s)}{U(s)} = \frac{k\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (17)$$

Using system identification techniques from experimental data the parameter variations obtained are $k \in [0.01, 0.099]$, $\zeta \in [0.1\ 0.16]$, and $\omega_n \in [0.55, 0.58]$ [5].

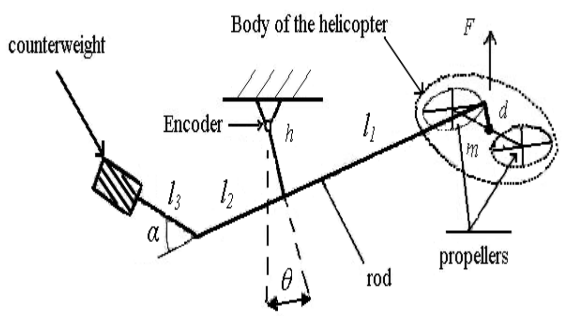

Fig. 2. A laboratory scale bench-top helicopter

5. RESULTS AND DISCUSSION

The sliding mode controllers design procedures were applied to the laboratory scale bench-top helicopter. A step input has been applied to the plant where the set point of the pitch angle, θ is set at 4°.

The system reached the desired input by using sliding mode controller without a boundary layer (Figure 3) in a very short time but s (Figure 4) and the control signal (Figure 5) have very unfavorable chattering. This chattering is undesirable in practice because it involves high control activity and may excite high frequency dynamics neglected in the course of modeling [4-5].

Using sliding mode controller with boundary layer, the system reached the desired input and becomes stable in a very short time. Figure 6 shows the responses for φ=0.1 and φ=1. Both responses become stable after 6 seconds but the response with φ=1 is better because the chattering effect in the control signal does not exist (Figure 8). In addition, with φ=1, the sliding surface is reached faster than φ=0.1 although its response has a steady state error.

Responses of the close-loop control system to 4 degree set-point changes are illustrated in Figure 9 for SMC and SMC+I (with φ=1 for both of them) control system, respectively. The performance of the system with the proposed sliding mode controller (SMC+I) is much better than the system with the conventional sliding mode controller (SMC), where smaller rise time, settling time and steady-state error in magnitude were obtained from SMC+I.

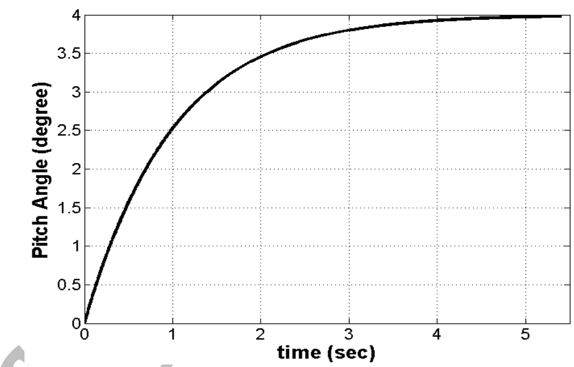

Fig. 3. Step response of 4° at t=0s using sliding mode controller without boundary layer.

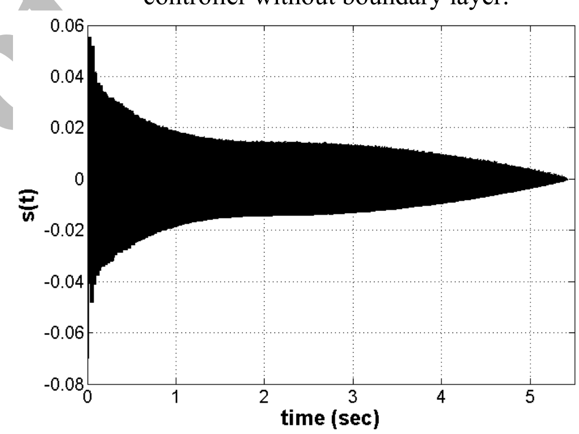

Fig. 4. $s(t)$ in sliding mode controller without boundary layer.

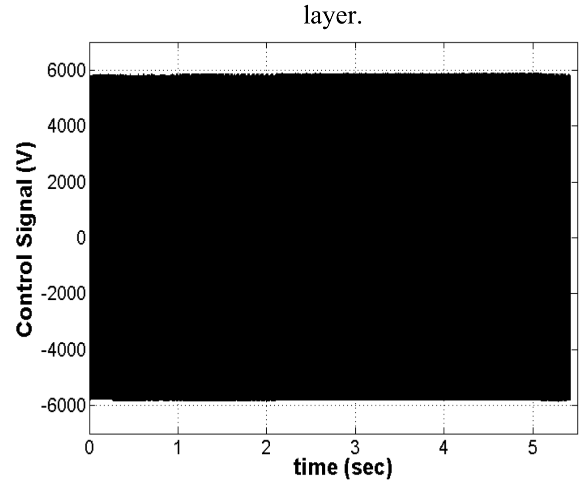

Fig. 5. Control signal, $u(t)$ with sliding mode controller without boundary layer.

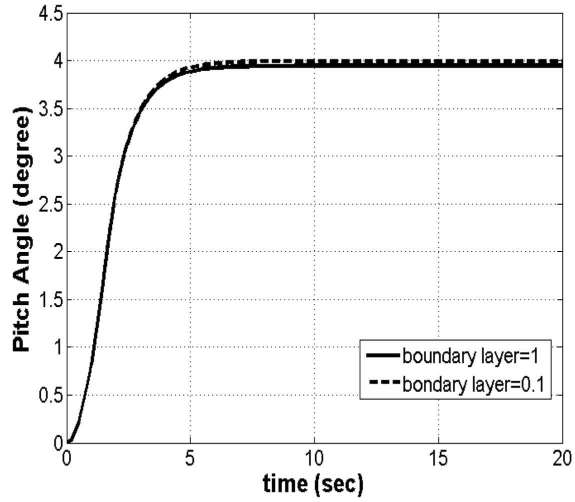

Fig. 6. Step response of 4° at t=0s using sliding mode controller with boundary layer

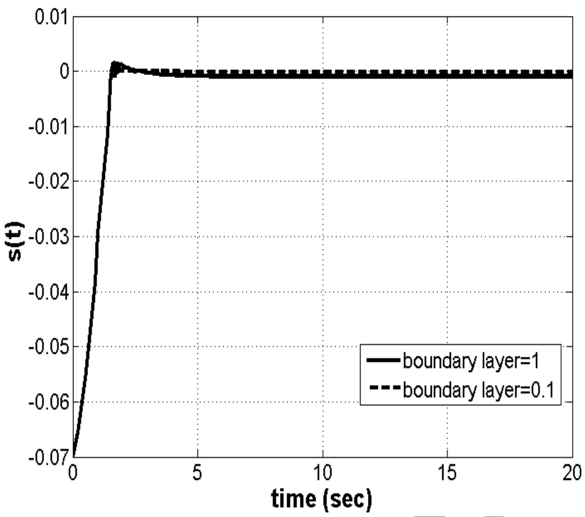

Fig. 7. $s(t)$ in sliding mode controller with boundary layer

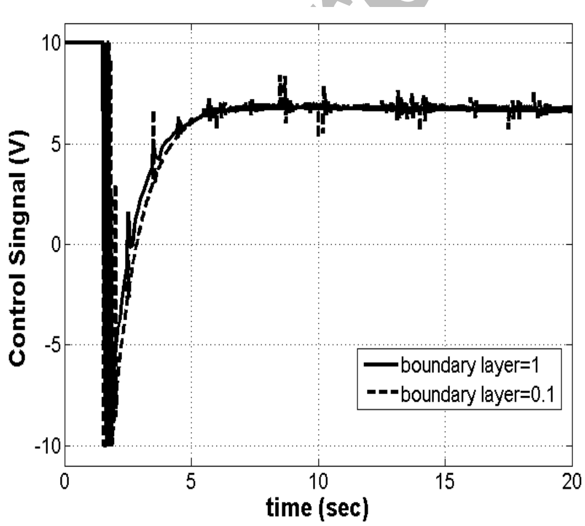

Fig. 8. Control signal, u (limited to ±10 V) with sliding mode controller with boundary layer

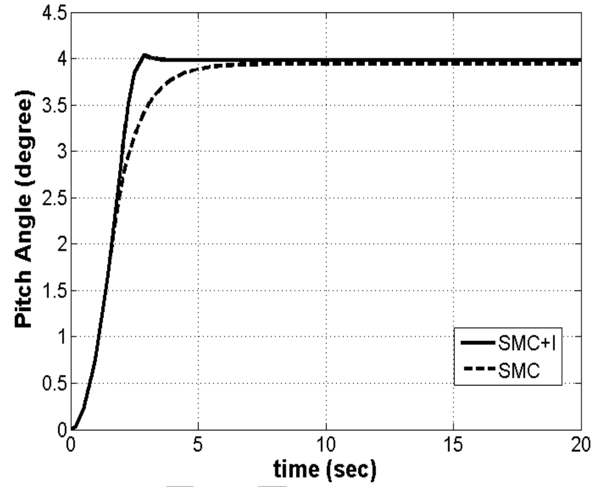

Fig. 9. Step response of 4° at t=0s using SMC and SMC+I controller with boundary layer equal 1

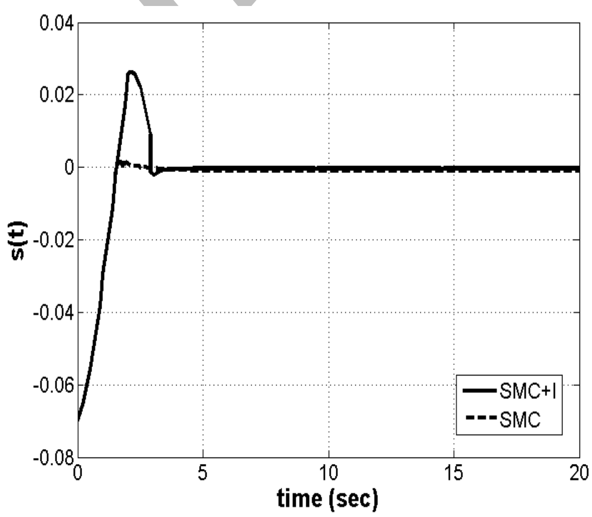

Fig. 10. $s(t)$ in SMC and SMC+I with boundary layer equal 1

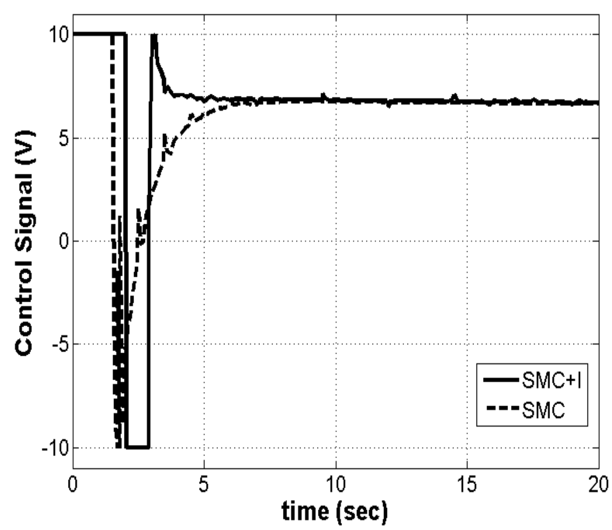

Fig. 11. Control signal, $u(t)$ (limited to ±10 V) with SMC and SMC+I with boundary layer equal 1

6. CONCLUSION

The helicopter pitch angle control problem has been simulated using a sliding mode controller. From the simulation results carried out in Matlab and Simulink environment, this controller is capable to control the plant effectively in spite of the plant uncertainty. In sliding control, chattering effect on the control signal can be reduced by adjusting the boundary layer, φ. But it is important to take in mind that by increasing φ, the steady state error may increase. So adjusting the boundary layer is very serious in sliding mode controller in order to make a suitable response.

In this paper, a sliding mode control with an integral augmented sliding surface (SMC+I) has been proposed to improve the control performance of systems and the current theoretical study showed that by including integral action the conventional sliding surface is improved where smaller rise time, settling time and steady-state error in magnitude were obtained from the SMC+I model.

In sliding mode controller, the controller is guaranteed to be able to control the plant within the specified range of uncertainty. Therefore, it is called robust controller and is proven to have robust performance as it did for this laboratory scale bench-top helicopter case study.

REFERENCES

[1] Kaynak O., Kaynak, O., Erbatur, K., & Ertugnrl, M.; **"The Fusion of Computationally Intelligent Methodologies and Sliding-Mode Control-A Survey",** *IEEE Transactions on Industrial Electronics*, Vol. 48, pp. 4-17, (2001)

[2] Utkin V., Guldner, J., Shi, J., & Shijun, M.; **Sliding Mode Control in Electromechanical Systems:** CRC, (1999)

[3] Boiko I. and Fridman L.; **"Analysis of Chattering in Continuous Sliding-Mode Controllers"**, *IEEE Transactions on Automatic Control*, Vol. 50, pp. 1442-1446, (2005)

[4] Perruquetti W. and Barbot J.; **Sliding Mode Control in Engineering:** CRC, (2002)

[5] Slotine J. and Li W.; **Applied Nonlinear Control:** Prentice-Hall Englewood Cliffs, NJ, (1991)

[6] Wai R. and Su K.; **"Adaptive Enhanced Fuzzy Sliding-Mode Control for Electrical Servo Drive"**, *IEEE Transactions on Industrial Electronics*, Vol. 53, pp. 569-580, (2006)

[7] Eker I. and Ak nal A.; **"Sliding Mode Control with Integral Augmented Sliding Surface: Design and Experimental Application to an Electromechanical System"**, *Electrical Engineering*, Vol. 90, pp. 189-197, (2008)

[8] Chen H., Renn, J., & Su, J., **"Sliding Mode Control with Varying Boundary Layers for an Electro-Hydraulic Position Servo System"**, *The International Journal of Advanced Manufacturing Technology*, Vol. 26, pp. 117-123, (2005)

[9] Eker I. and Akinal S., **"Sliding Mode Control with Integral Action and Experimental Application to an Electromechanical System"**, ICSC Congress on Computational Intelligence Methods and Applications, p. 6, (2005)

[10] Seshagiri S. and Khalil H.; **"On Introducing Integral Action in Sliding Mode Control"**, in 41st IEEE Conference on Decision and Control, Vol. 2, pp. 1473-1478, (2002)

[11] Eker I.; **"Sliding Mode Control with PID Sliding Surface TND Experimental Application to An Electromechanical Plant"**, *ISA Transactions*, Vol. 45, pp. 109-118, (2006)

[12] Wai R., Lin, C., & Hsu, C., **"Adaptive Fuzzy Sliding-Mode Control for Electrical Servo Drive"**, *Fuzzy Sets and Systems*, Vol. 143, pp. 295-310, (2004)

[13] Houpis C. and Su, K.; **Quantitative Feedback Theory: Fundamentals and Applications:** CRC Press, (2006)

[14] Mansor H., Zaeri, A. H., Mohd Noor, S. B., Kamil, R. M., & Saleena, F., **"Design of QFT Controller for a Bench-top Helicopter"**, *presented at the 2nd International Conference on Control, Instrumentation and Mechatronic Engineering (CIM09) Malacca*, Malaysia, (2009)

[15] Zaeri A.H. and Mohd Noor S.B., **"Design of Sliding Mode Controller for a Bench-top Helicopter"**, *presented at the 2nd International Conference on Control, Instrumentation and Mechatronic Engineering (CIM09) Malacca*, Malaysia, (2009)

[16] Esfroghy H. and Ebrahimi-Rad H.; **"New Robust Nonlinear Controller Design Based on Predictive Control for Industrial Processes"**, *Majlesi Journal of Electrical Engineering*, Vol. 1, No. 3, (2007)